New Version of General Relativity that Unifies Mass and Gravity in a Common 4D Higgs Compatible Theory

Jacky Jerome, Frederic Jerome

AC42 - Dpt. Physics, 42120, Le Coteau, France.
diaverre@sfr.fr

Recent enigmas of astrophysics such as dark energy or accelerating universe need to update General Relativity. A thorough examination of the original Einstein Field Equations (EFE) highlights three inconsistencies concerning the nature of spacetime. Here we solve these inconsistencies. As a consequence, this article proposes a Higgs compatible 4D expression of mass, $m = f_{(x,y,z,t)}$, and a new explanation of gravity based on Le Sage Push Gravity. This paper is interesting and important because it touches the weakest nerve of General Relativity by asking "how mass curves spacetime"? Moreover, this article is supported by several mathematics demonstrations such as a new version of the Newton Law, a new version of the Schwarzschild Metric, and a 4D rewriting of the Energy-Momentum Tensor and Einstein Constant.

---

1. Introduction

When Einstein developed General Relativity (GR) [1-11, 16-28, 50-79], knowledge in physics was very poor. In the 1900s, the atom was regarded as a pudding in which electrons were represented by raisins [12,13]. The proton and neutron were discovered later, in, respectively, 1919 and 1932. So, in the 1910s, GR was grounded on an erroneous view of reality.

Based upon the fluid mechanics, Einstein identified spacetime to a fluid. He developed GR upon this assumption and his theory was fully verified by Sir Arthur Eddington in 1919. The mathematics of GR, i.e. the Einstein Field Equations (EFE), Eq. (1), are well known, but the mechanism of curvature of spacetime by mass remains somewhat obscure [14].

Since the 1910s, physics has evolved considerably. So, to date, we must reconsider GR in the current context, excluding the pudding model that does not exist anymore, but including the Higgs Theory. This leads to a new version of GR which explains several enigmas of physics, such as:

– How can mass curve spacetime?
– What is the mechanism of the creation of mass? (in accordance with the Higgs Boson);
– What is gravity?
– What is the mechanism of the increase of the mass of relativistic particles?......

Note - The full paper is split in two parts: 1/ The main article (6 pages), which explains the curvature of spacetime and mass, and 2/ Mathematical demonstrations (Appendices - 34 pages), which cover an explanation of a new Le Sage-type gravity, a new version of the Newton's Law, a new calculus of the Schwarzschild Metric, the 4D mass calculus, and a rewriting of the EFE.

2. Towards a new GR

GR can be studied in different ways. This article is grounded on the 1910s original version [11] from the fluid mechanics. The examination of the EFE highlights three inconsistencies which are described in this section. The EFE, without the cosmological constant Λ, may be written as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_0}{c^4} T_{\mu\nu} \qquad (1)$$

;

2.1 Inconsistency #1: Direction of the Curvature of Spacetime

In fluid mechanics, the curvature of the fluid made by an object is *convex* (Fig. 1a). In the scientific literature, the curvature of spacetime produced by a mass such as a black hole is *concave* (Fig. 1b). Since GR is grounded on the Fluid Mechanics, the curvature of spacetime in GR must be *convex*, not *concave*.

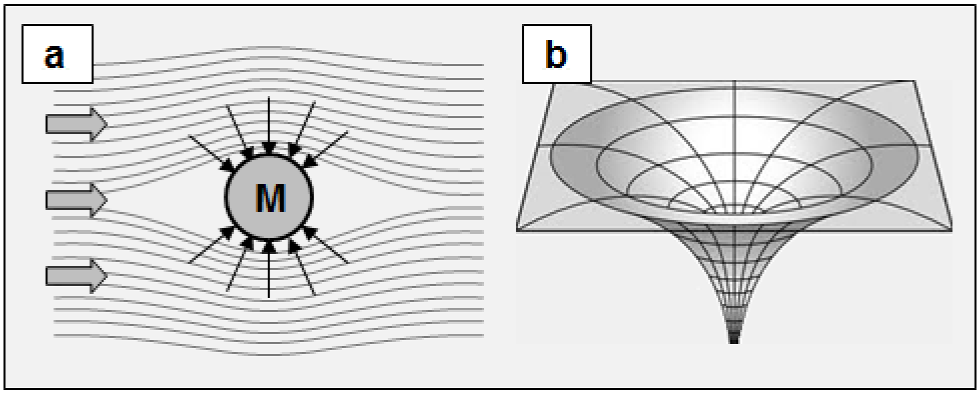

Figure 1. The curvature of the fluid is convex in Fluid Mechanics (a). Since GR is grounded on the Fluid Mechanics, the curvature of spacetime in GR must be convex too, not concave (b).

2.2 Inconsistency #2: Pressure or attractive force?

The fluid exerts a *pressure* on objects (black arrows in Fig. 1a). This is expressed by the Cauchy Stress Tensor where each element has a pressure-like dimension (Fig. 2a).

Since the energy–momentum tensor is based on the Cauchy stress tensor, each term of the two tensors must have the same meaning (Fig. 2a and 2b). Especially, the trace of the two tensors must represent an **isostatic pressure in both cases**. Therefore, it is not logical to have a pressure force in Fluid Mechanics, and an attractive force in GR.

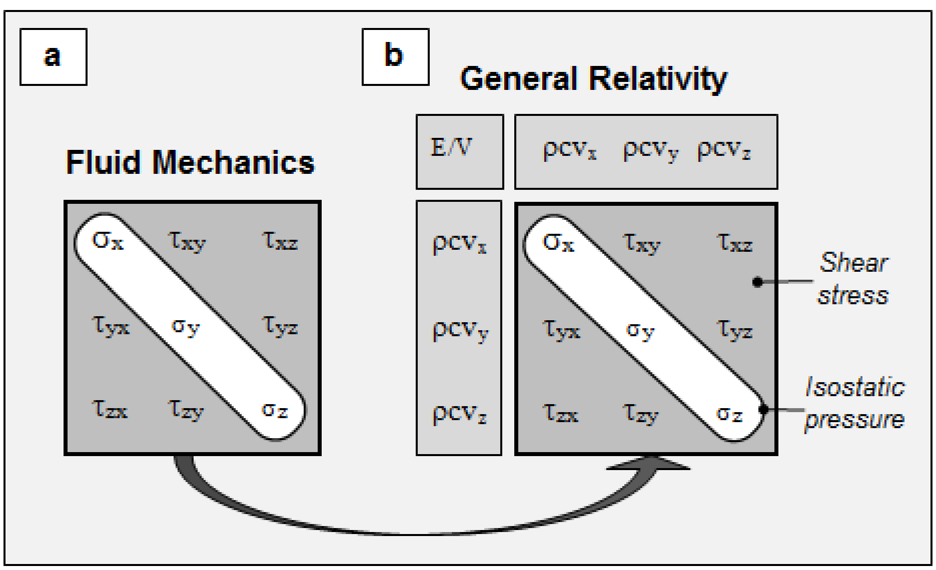

Figure 2. The origin of the energy–momentum tensor is the Cauchy stress tensor (a). Since this tensor is included in the energy–momentum tensor, the two tensors must have the same meaning, at least in the spatial part.

2.3 Inconsistency #3: Mass or Volume?

In fluid mechanics, the displacement of the fluid is made by the *volume* of the object, not by its *mass*, as shown in Fig. 1a. Here too, we must keep the original meaning of the fluid mechanics and replace *"Mass"* by *"Volume"*. However, this poses a problem that will be solved later.

2.4 Discussion

Inconsistencies #1 and #2 are easily solved because they cancel each other. For example, a pressure on one side of a sheet of paper produces the same effects than an attractive force on the other side (Appendix A). The curvature of the sheet is also inverted: concave on one side, convex on the other.

Convex curvature + Pressure force = Concave curvature + Attractive force

If we assign by convention a positive sign to a convex curvature of spacetime and to a pressure force, the above assertion gives $(++) = (--)$. In both cases, the results are identical. As we see, this does not change the mathematics of GR.

Inconsistency #3 is less obvious. Indeed, experimentations prove that spacetime is curved by mass, but Fig. 1a shows that spacetime is curved by volume. In reality, spacetime is not curved by mass as is thought, nor by volume, but by a **special kind of volume**.

The second part of this article, sections 3 to 6, explains why spacetime is curved by this special kind of volume instead of mass.

3. Relation Between Mass and Volume

One tends to assume that there exists only one kind of volume. This view must be dropped because some volumes curve spacetime, others do not. Since spacetime is at the heart of GR, then to understand the mechanism of the curvature of spacetime, the first thing to do is to separate the volumes that curve spacetime from those that do not.

In reality, we have not only one type of volume, as we think, but five. The three major classes of volumes are described below. Appendix K covers the two remaining classes

3.1 Matter Volumes

These volumes are those of nuclear matter and are well known since the 1930s. Experiments show that the radius of a nucleus is $R = A^{1/3} r_0$, where r_0 is about 1.25 fm. Taking the mass number $A = 1$ and the nucleon mass as 1.67×10^{-27} kg, we get:

$$M = kV = f_{(x,y,z)} \qquad (k = 2.3 \times 10^{17} \text{kg/m}^3) \qquad (1)$$

A more accurate formula, which gives better results, is proposed in Appendix D:

$$M \;=\; \frac{c^2}{G_0} \epsilon_v \frac{V}{S}\, \rho_s = f_{(x,y,z)} \qquad (2)$$

... where ϵ_v is elasticity of spacetime, and ρ_s is the density of spacetime relative to a flat spacetime.

It is important to see that, whatever Eq. (1) or (2), mass is directly connected to volume:

$$M = f_{(Matter Volume)} \qquad (3)$$

3.2 Empty Volumes

Empty Volumes are simply vacuum, but can also be found in various forms, such as the volumes of the orbitals of atoms. They exist but are transparent regarding spacetime. Empty Volumes have no mass, even therein virtual particles would be present. Therefore, they do not curve spacetime.

3.3 Apparent Volumes

Objects we use daily are mainly made of molecules, which are combinations of Matter Volumes (nuclei and electrons) that have mass, and Empty Volumes (orbitals, gaps between atoms or molecules...), without mass:

$$\boxed{\text{Apparent Volumes} = \sum \text{Matter Volumes} + \sum \text{Empty Volumes}}$$

4. What We Call "Volume"

4.1 Volumes in Daily Life and General Physics

In daily life and in general physics, we use exclusively Apparent Volumes. We have the feeling that Mass and Volume are two different quantities because in Apparent Volumes, the proportion of *Matter Volumes/Empty Volumes* varies from one atom to another, from one molecule to another, from one object to another. Out of GR, we can continue using Apparent Volumes.

4.2 Volumes in GR

In some domains of physics such as GR and astrophysics, we must exclude Empty Volumes by a thought experiment since these volumes are inert regarding spacetime. There remains Matter Volumes, i.e. the only volumes that curve spacetime. This shows that mass and matter volume are two different views of the same quantity, since mass can be converted into matter volume and conversely, Eq. (1-3). In other words, we must disregard Empty Volumes which are massless, and we must think *"Matter Volume"* instead of *"Mass"*.

4.3 Substituting Mass by Matter Volume

Since $M = f_{(MatterVolume)}$, it becomes possible to replace mass by Matter Volume. This means that, instead of saying:

"Spacetime is curved by Mass"

we should say:

$$\boxed{\textit{“Spacetime is curved by Matter Volume”}}$$

The substitution of M by V is very important because it explains the curvature of spacetime, as shown in Fig. 1a, and, consequently, many enigmas of modern physics.

4.4 Example

If we plunge an object into a glass filled with water, as expected, <u>it is the volume</u> of the object, not its mass, that displaces the water. This example can be transposed to spacetime as follows.

Consider an electron of volume V crossing a cube of volume 1,000,000 V. Here, the particle is a Matter Volume, whereas the cube contains a vacuum, i.e. Empty Volumes. Both kinds of volumes have their own spacetime. The internal spacetime of the particle will "push" the surrounding spacetime to make room. Since spacetime has elasticity properties (Einstein), the cube will contain 1,000,001 elements inside a volume of 1,000,000 V. In other words, spacetime will be curved in a convex manner around the particle. Here too, <u>it is the volume</u> of the particle, not its mass, that curves spacetime, as shown in Fig. 1a.

To summarize, since $Mass = f_{(MatterVolume)}$, we can use indifferently "Mass" or "Matter volumes" in our calculus. In both cases, the result is identical.

5. Expression of Mass

The above conclusions fully explain the mass creation. The mechanism is as follows:

– A Matter Volume (not a mass) curves spacetime;
– This curvature is convex, not concave, as shown in Fig. 1a;
– It also has elasticity properties (Einstein);
– The elasticity of spacetime exerts a pressure on the surface of the Matter Volume;
– This pressure increases the resistance to movement of the Matter Volume;
– ...which leads to a “Mass Effect”: $M = f_{(MatterVolume)}$ (Appendix D).

In other words, any matter volume curves spacetime and, therefore, creates its own mass effect.

These observations are partially included in the Higgs Mechanism. However, the Higgs Field has a problem: it does not have an associated potential, as any field does. Saying that the Higgs Field is a 5th dimension, created at the Big Bang, which permeates all space is pure speculation. This contradicts the Poisson’s law and Einstein RG. In reality, the Higgs Field is nothing but a Newtonian Field in spacetime created by a Matter Volume, which acts as a potential.

6. What is Gravity?

Gravity is an application of the curvature of spacetime and is covered by Appendix A.

In section 2.1, we have demonstrated that the curvature of spacetime is not concave but convex. Therefore, gravity must be “inverted” too. In consequence, gravity should not be an attractive force but a pressure force. In all cases, the vector of gravity remains the curvature of spacetime. The mechanism of gravity is explained and mathematically demonstrated in the appendices.

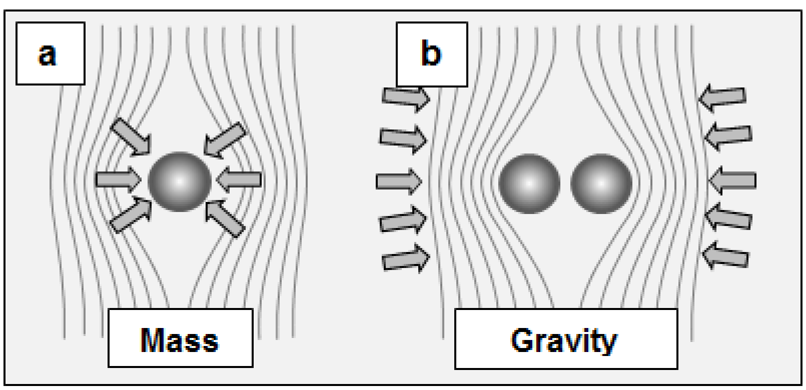

Figure 1. Mass and gravitation are two similar phenomena. The vector of gravity is the elasticity of spacetime (Einstein) which exerts a pressure on one (mass) or more (gravity) “Matter Volumes”. See Appendices A, B, C and E.

7. Conclusions

To summarize, the originality of this study lies in some simple observations, supported by several mathematical demonstrations (see the appendices, section 8).

• **Direction of the curvature of spacetime:** The Fluid Mechanics shows that the curvature of spacetime is convex, not concave;
• **Origin of the curvature of spacetime:** Spacetime is not curved by mass as we think, but by a special kind of volumes, “Matter Volumes”;

• **The Mass Effect:** The origin of the mass effect is the curvature of spacetime, which exerts a pressure force on “Matter Volumes”;
• **Mass or Volume?** Since $Mass = f_{(matter\ volume)}$, mass can be replaced by “Matter Volumes”. The 4D expression of mass is given in Eq. (2);
• **Gravity:** The trace of the Cauchy Tensor represents an isostatic pressure. Therefore, gravity is not an attractive force but a pressure force (Le Sage “Push Gravity”).

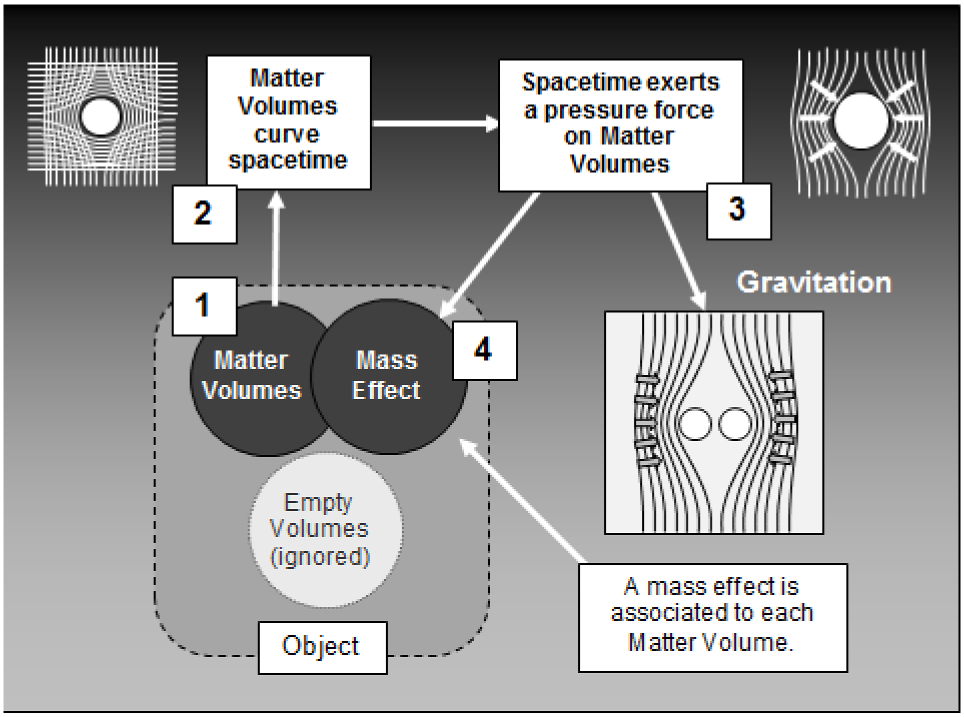

Figure 2. Each object has a matter volume [1] which curves spacetime [2]. Then, elasticity of spacetime exerts a pressure on the surface of the matter volume [3], which produces a “Mass Effect” [4]. The same principle also applies to gravity.

8. Appendices

The appendices are available as a 34 page PDF document (*):

A – What is Gravity?
B – New version of Newton’s Law;
C – New version of the Schwarzschild Metric;
D – Expression of the mass effect in 4D;
E – Partial Rewriting of the EFE: $R_{\mu\nu} - 1/2 g_{\mu\nu} R = (8\pi\delta_v)/S\ .\ J_{\mu\nu}$;
F – Connexion with the Von Laue Geodesics;
G – Explanation of the Increase of the Mass of Relativistic Particles;
H – Equivalence Principle using the Curvature of Spacetime;
J – Bethe-Weizsacker SEMF;
K – Other Classes of Volumes;
Bibliography.

(*) http://www.spacetime-model.com/files/appendices.pdf

Summary

What is Gravity?

A-1 Introduction

In the current life, we see that two objects are attracted one toward the other. So, our immediate deduction is that gravity is an attractive force.

The fact that 99,99% of people think that gravity is an attractive force is not a proof per se. In reality, since no one knows the origin of the gravity force, we are faced with two possibilities:

1/ Gravity is an attractive force (probability = 0.5);
2/ Gravity is a force of pressure (probability = 0.5).

For example, a sheet of paper (Fig. A-1) can be curved either by attracting it (A), or by pushing it (B). In both cases, the result is the same. Gravity follows the same principle.

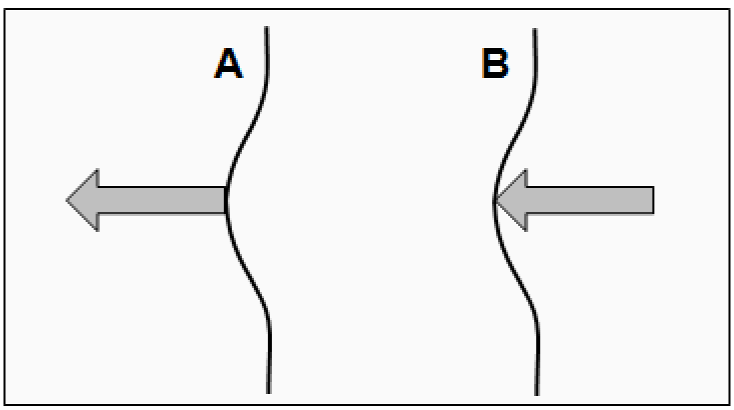

Figure A-1: An attractive force (A) produces the same effect than a pressure force (B).

A-2 The Vector of Gravity

In the 1910s, Einstein thought that the vector of gravity could be spacetime. So, he identified Spacetime to a fluid, and developed General Relativity (GR) grounded on this assumption. The result is probably the most genius theory of all times: GR.

Since 1919, hundreds experimentations has validated GR. To date, the physicist community considers that gravity is a consequence of the curvature of spacetime. This is 100% right.

However, the knowledge of the curvature of spacetime is not sufficient to explain the nature of the force of gravity. Is it an attractive or a pressure force? What is the mechanism of gravity?

To explain the curvature of spacetime, i.e. gravity, physicists use metaphors such as those of Fig. A-2 (next page) or Fig. 1b of the main article. These two figures are identical to thousands similar scientific figures that try to represent the curvature of spacetime. However, these metaphors never explain the mechanism of gravity.

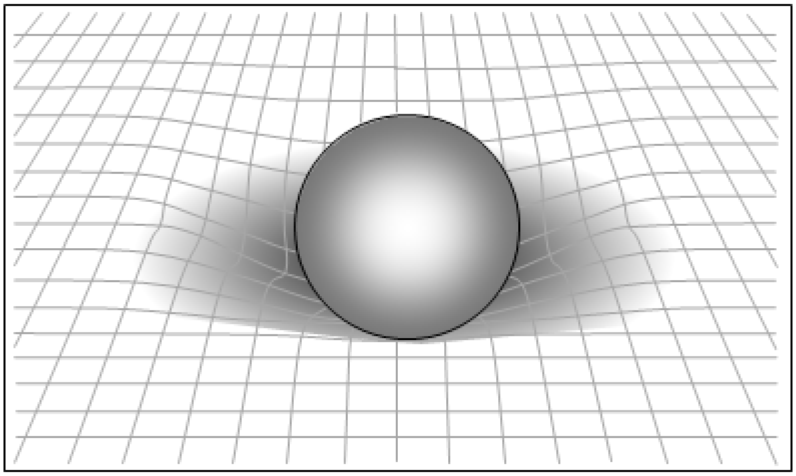

Figure A-2: This metaphor, as thousands similar scientific figures,
tries to represent the curvature of spacetime by a mass.

A-3 Curvature of Spacetime vs. Gravity

Fig. A-3 tries to explain gravity, i.e. the attraction of a mass M_1 by another mass M_2. On this figure we see that mass M_1 falls in the hole produced by the larger mass M_2 (black arrow).

This figure raises a problem: theoretically, M_1 must move toward the center of M_2, not toward its bottom. Therefore, despite its popularity, this kind of figures does not represent the reality of the curvature of spacetime, and therefore gravity.

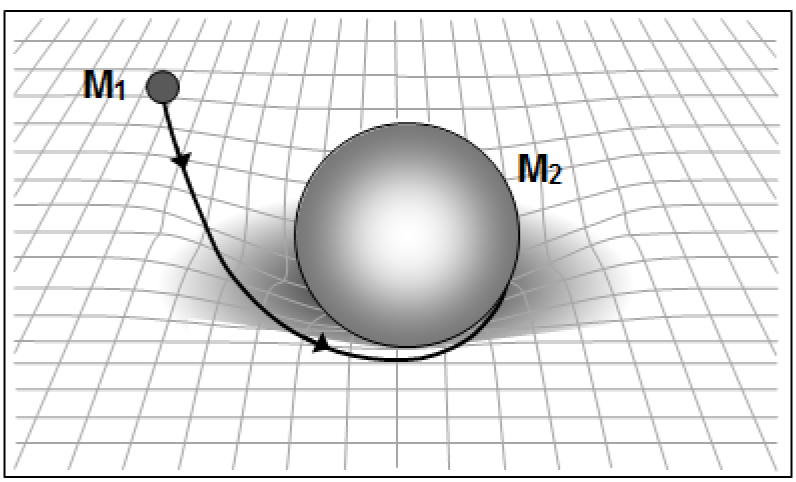

Figure A-3: This figure is wrong because theoretically, M_1
should be attracted toward the center of M_2, not toward its bottom.

Another more pertinent inconsistency is shown in Fig. A-4, next page.

Each mass makes its own curvature of spacetime. Even if the two masses are close to each other, we do not see by which mystery these two masses could be attracted one toward the other. This means that representing the curvature of spacetime by a deformation of a 2D plane is wrong.

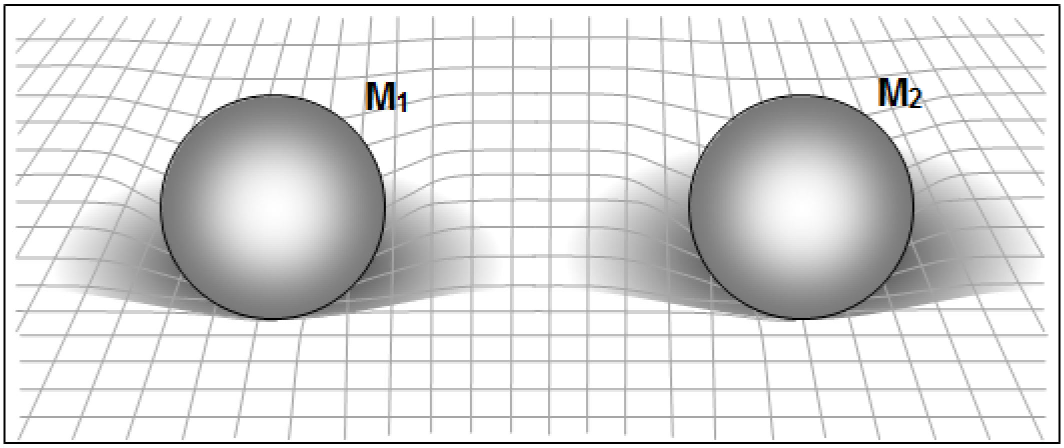

Figure A-4: Each mass, M_1 and M_2, produces its own curvature of spacetime. We do not see by which mechanism these two masses could be attracted one toward the other.

A-4 The Einstein Field Equations

A thorough examination of the Einstein Field Equations, or "EFE" (see the main article), gives the solution to the gravity enigma. Gravity would be a pressure force, not an attractive force.

Fig. 1a of the main article shows that the fluid exerts a pressure on a "Matter Volume". As explained below, extending this pressure to several volumes gives the mechanism of gravity.

A second argument is given by the trace of the energy-momentum tensor T_{uv} in the main article. This mathematical object represents the isostatic pressure force on the surface of a "Matter Volume". Here too, if we extend this external pressure force to several objects, we get gravity.

A-5 What is "Push Gravity"

It seems that the first suggestion of gravity as a pressure force comes from a friend of Newton, Nicolas Fatio de Duillier [29-31]. In 1690, he wrote a letter to Huygen in which he suggested that gravity could be a pressure force. His theory is dated 1690-1743.

In 1748, Georges-Louis Le Sage completed the De Duillier's theory [32-34]. His "Push Gravity", also known as "Shadow Gravity", describes gravity as a pressure force. In the Le Sage's theory, space could be filled with tiny particles called "Ultramundane Corpuscles".

During the last century, several physicists have tried unsuccessfully to explain gravity. Even if the vector of gravity is spacetime, the mechanism of gravity remains an enigma.

For example, in 1904, J. J. Thomson considered a Le Sage-type model in which the ultramundane flux consisted of radiations such as x-rays. In the 1930s, De Broglie thought that Le Sage Corpuscles were neutrinos. However, whatever the nature of the flux, "Push Gravity" must be reconsidered in a spacetime context.

Here we show that gravity is a Le Sage "Push Gravity", not a Newton Attractive Force. However, two major differences exist between the Le Sage "Push Gravity" and our explanation:

1/ The external pressure force comes from the curvature of spacetime;
2/ Mass must be replaced by Matter Volume.

To summarize, Push Gravity seems much more logical than the well known Newton-type gravity. The pressure force does not come from Le Sage hypothetical Ultramundane Corpuscles but from the curvature of spacetime produced by Matter Volumes.

A-6 Proposed Theory

The mechanism of gravity becomes clear and easy to understand if we think “Matter Volume” instead of “Mass” because a mass can not curve spacetime. A volume does.

We have demonstrated in the main article (sections 2 and 3) that spacetime exerts a pressure on any matter volume. Gravity is nothing but an extension of this ascertainment (Fig. A-5):

Gravity would not be an attractive force between masses,
but a pressure force exerted by the curvature of spacetime on
“Matter Volumes”, which tends to bring them closer to each other.

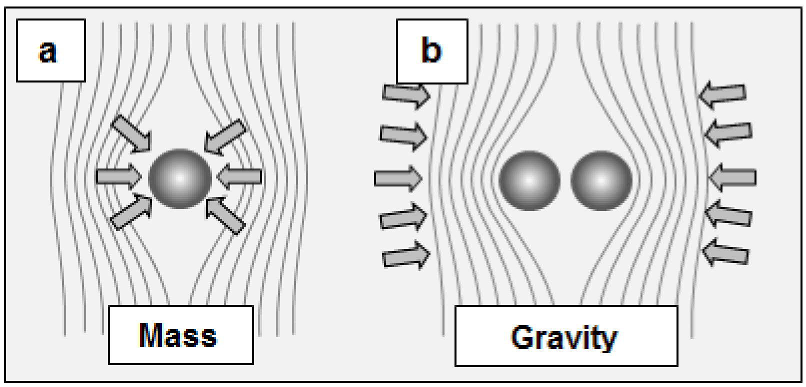

Figure A-5: Mass and gravity are two similar phenomena

Convex vs. Concave:
Section 4-2 and Fig. 1 of the main article show that the curvature of spacetime is *convex*, not *concave* (Fig. A-5).

Pressure Force vs. Attractive Force:
Section 4-4 of the main article and Fig. A-5 also show that the convex curvature of spacetime exerts a *pressure force* on objects, not an *attractive force*.

Results:
As shown in the introduction and Fig. A-1, the results are identical in both cases. Therefore, we can combine these assertions into the following expression concerning spacetime:

$$\boxed{\text{Concave curvature + Attractive force} \equiv \text{Convex curvature + Pressure force}}$$

If we assign by convention a positive sign to a convex curvature of spacetime and to a pressure force, the above assertion gives $(--) = (++)$. As we see, this does not change the EFE.

This means that gravity is a combination of three elements:

- The Le Sage’s Push Gravity
- The Einstein’s Curvature of Spacetime
- Our explanation of “Matter Volumes” and EFE inconsistencies (main article).

This combination gives a rational and consistent explanation of gravity.

Moreover, this new explanation of gravity is fully demonstrated with mathematics in Appendices B, C and E in which we replace “Mass” by “Matter Volume”.

A-7 Curvature of Spacetime vs. Pressure

Fig. A-6a shows the curvature of spacetime produced by the "Matter Volumes" of two spheres.

On the left sphere, spacetime has two different magnitudes of curvature:

– **Point L** (Left side): The curvatures of spacetime of the two spheres are added.
– **Point R** (Right side): The curvature of spacetime of the right sphere is subtracted from that of the left one because the two curvatures are in opposition.

The difference of the curvature of spacetime on each side of the two spheres leads to a difference of pressure (black and grey arrows on Fig. A-6b). In reality, Fig. A-6a and Fig. A-6b are two different views of the same phenomenon: *curvature of spacetime vs. pressure*.

The same principle also applies if the two spheres are the Earth and the Moon.

At the Lagrangian point (oval on Fig. A-6a), the curvature of spacetime produced by the Earth is canceled by that produced by the Moon. The curvature of spacetime is null at this point and, as a consequence, gravity disappears.

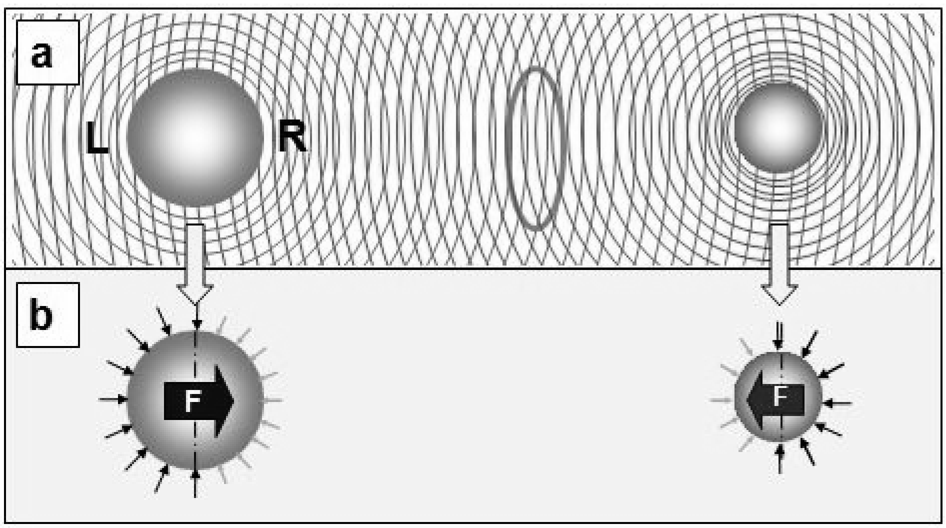

Figure A-6: Curvature of spacetime vs. pressure on the surface of each sphere (gravity).

A-8 Conclusions

This appendix shows that the combination "*Le Sage Push Gravity + Einstein Curvature of Spacetime + Our theory about volumes and EFE inconsistencies*" solves the enigma of gravity. The original "Push Gravity" must be modified as follows:

1/ "Ultramundane particles" must be replaced by the curvature of spacetime.
2/ Spacetime is curved by "Matter volumes", not by mass;
3/ The curvature of spacetime is convex, not concave;
4/ Spacetime exerts a pressure force on objects which is "Gravity".

New Version of the Newton's Law

B-1 Introduction

The Newton's Law is obtained from the Einstein Field Equations (EFE). It is a particular solution for a spherical static symmetry object using the weak field approximation. Here we show that the Newton's Law can be easily explained and obtained replacing mass by "Matter Volume". This method is simpler than calculating the Newton's Law from the EFE.

This new version of the Newton's Law is very important because it demonstrates that gravity is a pressure force exerted by the curvature of spacetime (see Appendix A).

B-2 Bulk Modulus

The bulk modulus K_B of a substance measures the substance's resistance to uniform compression (Fig. B-1). It is defined as the pressure increase needed to cause a given relative decrease in volume, Eq. (1).

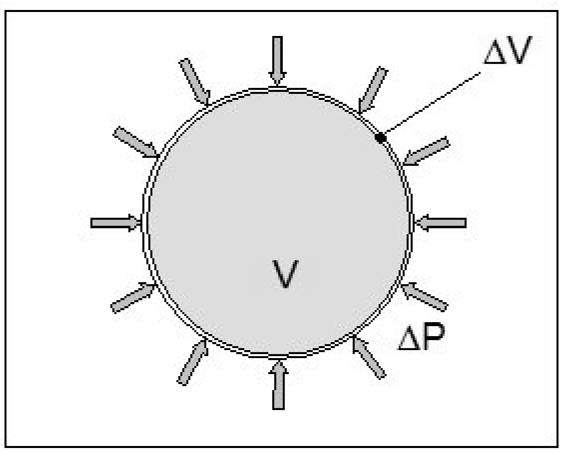

Figure B-1: Bulk modulus

$$K_B = -V\frac{\Delta P}{\Delta V} \qquad (1)$$

Starting with the Navier-Stokes Equations of the Fluid Mechanics, Einstein, helped by Grossmann, demonstrated in the 1910s that spacetime:

- Can be identified to a Newtonian fluid,
- Returns to its rest shape after having removal of the applied stress (elasticity).

Therefore, the Bulk Modulus, Eq. (1), which is a version of the Cauchy Tensor of the Fluid Mechanics, can also be applied to spacetime. It means that the curvature of spacetime made by a "Matter Volume" exerts a pressure on the surface of the latter (fig. B-1).

B-3 Elasticity Law

Elasticity phenomena follow the well-known logarithmic law:

$$\epsilon = ln\left(\frac{R-\Delta R}{R}\right) \qquad (2)$$

where ϵ = coefficient of elasticity (in fact "true strain").

The Schwarzschild Metric gives an order of magnitude of the curvature (compression) of spacetime, which is infinitesimal. For example, the ratio curvature of spacetime/radius of the Schwarzschild Metric, or $\Delta R/R = GM/Rc^2$, is 1.4166×10^{-39} for the proton, with M = 1.672×10^{-27} kg, R = 8.768×10^{-16} m, G = 6.674×10^{-11}, $c^2 = 8.987 \times 10^{16}$ m/s.

Under these conditions, whatever the formula used, logarithmic or not, the curvature of spacetime can be considered as a linear function since we are working on an infinitesimal segment near to the point zero. So, Eq. (2) becomes in first order approximation:

$$\epsilon_R \approx \frac{\Delta R}{R} \qquad (3)$$

or, with volumes:

$$\epsilon_V \approx \frac{\Delta V}{V} \qquad (4)$$

For the moment, the linear ϵ_R and volumetric ϵ_v coefficients of elasticity (in fact "strain") of spacetime are unknown.

B-4 Curvature of Spacetime

A matter volume V inserted into spacetime pushes it to make room (Fig. B-2). So the following volumes are identical:

$$V = V_1 = V_2 ... = V_n \qquad (5)$$

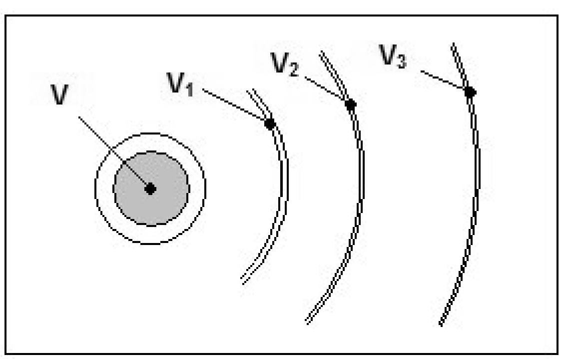

Figure B-2: Volumes V, V1, V2, V3... are identical

Since the curvature of spacetime is a linear function, the coefficient of elasticity of spacetime ϵ_v can be considered constant. So, combining Eqs. (4) and (5) gives:

$$\Delta V = \Delta V_1 = \Delta V_2 ... = \Delta V_n \qquad (6)$$

However, there should not be any confusion between a simple displacement of spacetime, V_x, produced by the insertion of a matter volume into a flat spacetime, and the curvature $\Delta V_x = \epsilon_v V_x$ due to the elasticity of spacetime (fig. B-3).

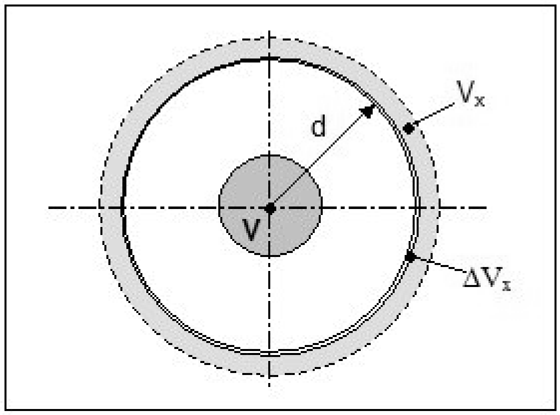

Figure B-3: Simple displacement (V_x) vs. curvature of spacetime (ΔV_x)

B-5 Solving $\Delta \mathbf{x} = \mathbf{f}_{(\Delta \mathbf{R})}$

Since the ΔV's are infinitesimal, the volume ΔV_x is simply the product of Δx by the surface S_x (fig. B-4):

$$\Delta V_x = \ S_x \, \Delta x \ = \ 4\pi d^2 \Delta x \qquad (7)$$

The volume ΔV_R is also the product of ΔR by the surface S_R:

$$\Delta V_R = \ S_R \, \Delta R \ = \ 4\pi R^2 \Delta R \qquad (8)$$

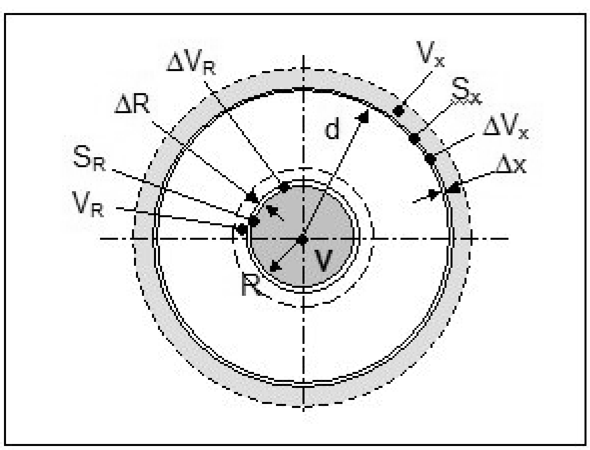

Figure B-4: Displacement and curvature at distances R and d

From Eq. (6) we have:

$$\Delta V_R = \Delta V_x \qquad (9)$$

Combining Eqs. (7), (8) and (9) gives:

$$4\pi R^2 \Delta R = 4\pi d^2 \Delta x \qquad (10)$$

Finally, we get:

$$\Delta x = \frac{R^2}{d^2} \Delta R \qquad (11)$$

Where:

- R is the radius of the matter volume V_R,
- ΔR is the curvature of spacetime on the surface of the matter volume V_R,
- d is the distance of the point of measurement,
- Δx is the curvature of spacetime at distance d.

B-6 Curvature Δx vs Mass M

As explained in the main text, the “Mass Effect” is a combination of the volume and the surface of a “Matter Volume”.

On one hand, the “Mass Effect” is proportional to the curvature of spacetime, i.e. to the“Matter Volume” since it is the latter that curves spacetime. On the other hand, it is inversely proportional to the surface S since the origin of the “Mass Effect” is a pressure. If we combine these two statements, we get the following dimensional quantity:

$$[M] \equiv [L^3][1/L^2] \equiv [L] \qquad (12)$$

At this point, we do not know the relation between ΔR and M but, referring to Einstein’s works, we have good reasons to believe that this relation is a simple linear function like:

$$\Delta R = KM \qquad (13)$$

. . . where K is a constant having the dimensional quantity of [L/M]. This value is in line with the dimensional quantity of some terms of the Schwarzschild Metric: $2\epsilon = 2\Delta r/r = 2GM/rc^2$ (sometimes, we use the Schwarzschild Radius, $\epsilon = r_s/r$).

The challenge, now, is to calculate K to find back the Newton Law.

B-7 The Newton Law

Porting Eq. (13) in Eq. (11) gives:

$$\Delta x = \frac{R^2}{d^2}\, KM \qquad (14)$$

or

$$\frac{\Delta x}{R^2} = K\frac{M}{d^2} \qquad (15)$$

Since x = ct, replacing R^2 by c^2t^2 gives:

$$\frac{\Delta x}{c^2t^2} = K\frac{M}{d^2} \qquad (16)$$

or :

$$\frac{\Delta x}{t^2} = c^2K\frac{M}{d^2} \qquad (17)$$

The value $\Delta x/t^2$ has the dimensional quantity of an acceleration $[L/T^2]$. So, replacing this fraction by the acceleration symbol "a" (see the note below), we get:

$$a = c^2K\frac{M}{d^2} \qquad (18)$$

Notes: Here, Δx is an infinitesimal quantity, not a differential quantity such as dx. Moreover, we are working in a linear segment of the elasticity of spacetime. In such a situation, $\Delta x/\Delta t \approx x/t$.

The multiplication of a constant c^2 by a second constant K gives a constant. So, we can replace the product c^2K by a new and unknown constant for the moment, G for example:

$$c^2K \; = \; G \qquad (19)$$

or *(the following equation is not necessary here but will be used further)*:

$$K \; = \; \frac{G}{c^2} \qquad (20)$$

Porting Eq. (19) in Eq. (18) gives:

$$a = G\frac{M}{d^2} \qquad (21)$$

To be consistent, this unknown constant G must have the same dimensional quantity of the product c^2K, Eq. (19):

- c^2 : Dimensional quantity $\Rightarrow [L^2/T^2]$
- K : Dimensional quantity $\Rightarrow [L/M]$ (see section B-6)

So,

The dimensional quantity of this new constant G is
$[c^2K] = [L^2/T^2][L/M] = [L^3/MT^2]$.

On the other hand, we know that a force F is the product of an acceleration a by a mass m. Therefore, Eq. (21) can be written as follows:

$$\boxed{F = G\,\frac{Mm}{d^2}} \qquad (22)$$

For the moment, G is unknown but we must note that:

• G is a constant,
• Its dimensional quantity is $[L^3/MT^2]$.

So, we can identify G to the constant of gravitation issued from experimentation:

$$G = 6,67428 \times 10^{-11} \qquad (23)$$

In other words,

Eq. (22) can be identified to the
Newton Law of Universal Gravitation.

This result is very important because the mathematic development in this appendix is based exclusively on “Matter Volumes” instead of “Mass”.

New Version of the Schwarzschild Metric

C-1 Introduction

Here we show that the Schwarzschild Metric can be easily obtained using matter volumes instead of masses. The following demonstration does not require the knowledge of tensor calculus.

C-2 The Minkowski Metric

The expression of the Minkowski Metric, in spherical coordinates, is:

$$ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (1)$$

The Schwarzschild Metric refers to a static object with a spherical symmetry. It is built from a Minkowski Metric, in spherical coordinates, with two unknown functions: A(r) and B(r):

$$ds^2 = -B_{(r)}c^2dt^2 + A_{(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (2)$$

The Minkowski Equation must follow the Lorentz Invariance in Special Relativity (SR) or General Relativity (GR). To get this invariance, we must set $A_{(r)} = 1/B_{(r)}$. Details of calculus are described in books concerning SR and GR [11, 50-79]. So:

$$B_{(r)}A_{(r)} = 1 \qquad (3)$$

C-3 The Schwarzschild Metric

To calculate the Schwarzschild Metric, we can start with fig. C-1 (next page), which is issued from the main article, where:

- d_{rout} is an elementary differential radial variation outside of any "Matter Volume", i.e. outside of any mass;
- d_{rin} is an elementary differential radial variation inside a Schwarzschild spacetime;
- r is the point of measurement.

Section B-3 of Appendix B shows that the order of magnitude of the coefficient of elasticity ϵ_R (here noted ϵ) is infinitesimal. So, we can use the first order approximation Eq. (3) of Appendix B. Since ϵ is a simple coefficient, we can calculate the relation between two differential elementary radius dr(out) and dr(in), out and in a gravitational field, closed one to the other:

$$dr_{in} = (1+\epsilon)\, dr_{out} \qquad (4)$$

Since $\epsilon \ll 1$, Eq. (4) can be written as:

$$dr_{in} = \frac{1}{(1-\epsilon)}\, dr_{out} \qquad (5)$$

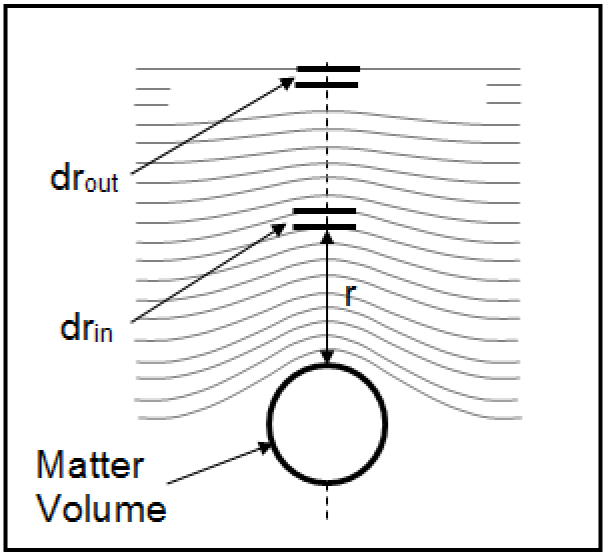

Figure C-1: Spacetime has been reduced to 1D

or, elevating in square:

$$dr_{in}^2 = \frac{1}{(1-\epsilon)^2}\, dr_{out}^2 \qquad (6)$$

Developing the denominator $(1-\epsilon)^2 = 1 - 2\epsilon + \epsilon^2$ and ignoring the last term ϵ^2, we obtain:

$$dr_{in}^2 = \frac{1}{(1-2\epsilon)}\, dr_{out}^2 \qquad (7)$$

This result is nothing but the radial component of the Schwarzschild Metric, i.e. the function A(r) of dr^2 in Eq. (2). Then, the calculation of B(r) is immediate, taking into account that $A(r)B(r) = 1$ from Eq. (3). So:

$$A_{(r)} = \frac{1}{(1-2\epsilon)} \qquad (8)$$

$$B_{(r)} = (1-2\epsilon) \qquad (9)$$

Thus, Eq. (2) becomes:

$$ds^2 = -(1-2\epsilon)c^2dt^2 + \frac{1}{(1-2\epsilon)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (10)$$

On the other hand, Eqs. (3), (13) and (20) of Appendix B can be rewritten as:

$$\epsilon = \frac{\Delta R}{r} = \frac{KM}{r} = \frac{GM}{rc^2} \qquad (11)$$

Finally, porting this expression in Eq. (10) gives:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)}\, dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (12)$$

C-4 Conclusions

This new calculus of the Schwarzschild Metric, which is exclusively based on matter volumes instead of mass (Fig. C-1), gives identical results than developing the EFE in the case of a static spherical symmetry. This is due to the fact that the origin of EFE is the Fluid Mechanics, which is itself based on volumes, not on masses.

Another point: it is remarkable to see the simplicity of this 2 page demonstration v.s. reducing the EFE to the case of a static sphere with a spherical symmetry which requires about 15 pages of complex equations (tensors).

Expression of the Mass Effect in 4D

D-1 Expression of "M"

In the main paper and appendix B, we have seen that the displacement of spacetime V_R is equal to that of the matter volume, V, which produces this displacement (fig. D-1):

$$V_R = V \qquad (1)$$

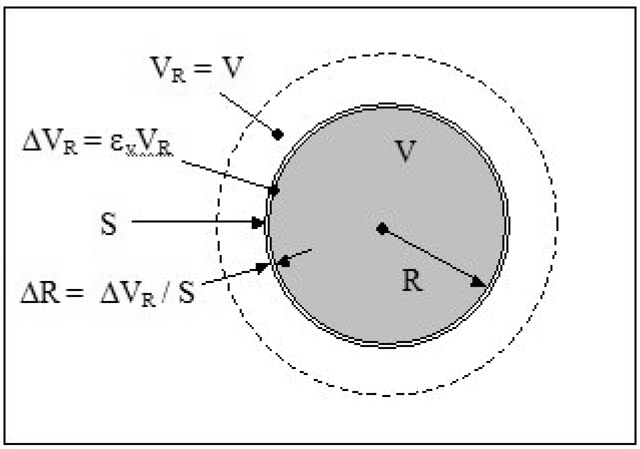

Figure D-1: The curvature of spacetime

On the other hand, the curvature of spacetime is (Eq. (4), section B-3, Appendix B):

$$\Delta V_R = \epsilon_v \, V_R \qquad (2)$$

Porting Eq. (1) in Eq. (2) gives:

$$\Delta V_R = \epsilon_v \, V \qquad (3)$$

Since the volume ΔV_R is infinitesimal, we can consider that the radial curvature of spacetime, ΔR, at the surface of M, is calculated dividing the volume by the surface:

$$\Delta R = \frac{\Delta V_R}{S} \qquad (4)$$

Replacing ΔV_R by Eq. (3) gives:

$$\Delta R = \epsilon_v \frac{V}{S} \qquad (5)$$

Porting Eq. (13), Section B-6, Appendix B, in Eq. (5) gives:

$$KM = \epsilon_v \frac{V}{S} \qquad (6)$$

or

$$M = \frac{\epsilon_v}{K}\frac{V}{S} \qquad (7)$$

Porting in Eq. (7) the expression of K given in Eq. (20) of Appendix B gives the expression of the "Mass Effect". Here, we have added a new coefficient, ρ_s (see explanation below):

$$\boxed{M = \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\, \rho_s} \qquad (8)$$

with:
M = Mass effect (kg)
V = Volume of the "Matter Volume" (m^3)
S = Surface of the "Matter Volume" (m^2)
ϵ_v = Coefficient of volumetric elasticity of spacetime. This parameter is unknown but can be calculated from the mass/diameter of spherical particles such as some leptons or "magic" nuclei. See section D-3.
c = Speed of the light (m/s)
G_0 = Universal constant of gravitation
ρ_s = Density of surrounding spacetime relative to a flat spacetime. This parameter is equal to 1 in a Minkowsky Spacetime.

It seems useful to differentiate ϵ_v, the coefficient of elasticity of spacetime, and ρ_s, the density of surrounding spacetime. We could merge these two parameters in one common parameter since we are faced with two dimensionless coefficients. In both cases, result is identical.

The coefficient of elasticity of spacetime ϵ_v is an intrinsic coefficient of spacetime. This coefficient does not change. On the contrary, the density of surrounding spacetime, ρ_s, can change from one point of the universe to another. This is why the two parameters have been separated.

For example, the particle may be located in a Riemannian spacetime, such as near a black hole. In this case, the "Mass Effect" of the particle will increase because the curvature of spacetime (ρ_s) near a black hole is huge.

To summarize, Eq. (8) shows that the "Mass Effect" is not constant and may vary. For example, the mass of electron is 510.998928(11) KeV/c^2 on Earth, but it can be for example 498 KeV/c^2 outside our solar system in a flat spacetime, or more than 100 GeV/c^2 near a black hole.

D-2 Case of a sphere

In the particular case of a sphere, we have $V = 4/3\pi R^3$ and $S = 4\pi R^2$. Thus, Eq. (8) becomes:

$$\boxed{M = \frac{\epsilon_v R c^2}{3\, G_0}\, \rho_s} \qquad (9)$$

D-3 Nuclei

Nuclei aren't spherical generally. Since we don't know exactly their shape, it is not possible to apply Eq. (9) to calculate their mass effect.

The radius R of a nucleus is often defined as $R = 1.2 \times A^{1/3}$, A being the mass number. In reality, the right member does not mean that mass follows a $A^{1/3}$ law. This rule concerns the arrangement of nucleons inside the nucleus. The result is equivalent but the significance is different.

We must also note that a surface component also exists in the Bethe-Weizsacker expression. It means that, early in 1937, Bethe and Weizsacker predicted the Eq. (8) here demonstrated, by connecting the "Mass Effect" to the volume and surface. This topic is developed in Appendix J.

We must keep in mind that nuclei are made of empty volumes and matter volumes (see Appendix J). However, the nucleus has a behaviour of "matter volume" because the empty volumes are enclosed inside the nucleus. The space between nucleons may vary from one nucleus to another. Thus, it is necessary to know the arrangement of nucleons with accuracy before any calculations. A particular case are magic nuclei (nuclei having a null quadripolar moment) because they are supposed to be spherical. It will be interesting to make accurate experiments on the relation volume/surface/mass effect of magic nuclei.

On the other hand, some nuclei have a halo made of empty volumes that are not relevant in mass calculation (see section J-2, Appendix J). This is the case for example of the ^{11}Li (3p8n), which has empty volumes between the ^{9}Li and the 2n orbitals. These exceptions highlight the difficulty to make accurate calculations of the mass effect. In all cases, before any calculation, we must know exactly the geometry of matter volumes and empty volumes inside the nucleus. It means that the calculus of the mass from the geometry of the nucleus, Eq. (8), is not as simple as it sounds.

As a direct consequence of the proposed theory, it could be possible that a relationship exists between the sphericity of particles or nuclei and the accuracy of measurements. This deduction suggests that leptons could be spherical since their mass effect is known with an excellent accuracy. This is also the case of some particles such as the proton, neutron, or π meson. Inversely, this is not the case of quarks. This means that quarks could have a non-spherical and complex shape.

Partial Rewriting of the Einstein Field Equations (EFE)

E-1 Original EFE

The original EFE are fully described in [11] and in several books [50-79].

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G_0}{c^4}T_{\mu\nu} \qquad (1)$$

with:
$R_{\mu\nu}$ = Ricci Curvature Tensor;
$g_{\mu\nu}$ = Metric Tensor;
R = Scalar Curvature;
$8\pi G_0/c^4$ = Einstein Coefficient;
$T_{\mu\nu}$ = Energy-Momentum Tensor. See below and Fig. 2 of the main text.

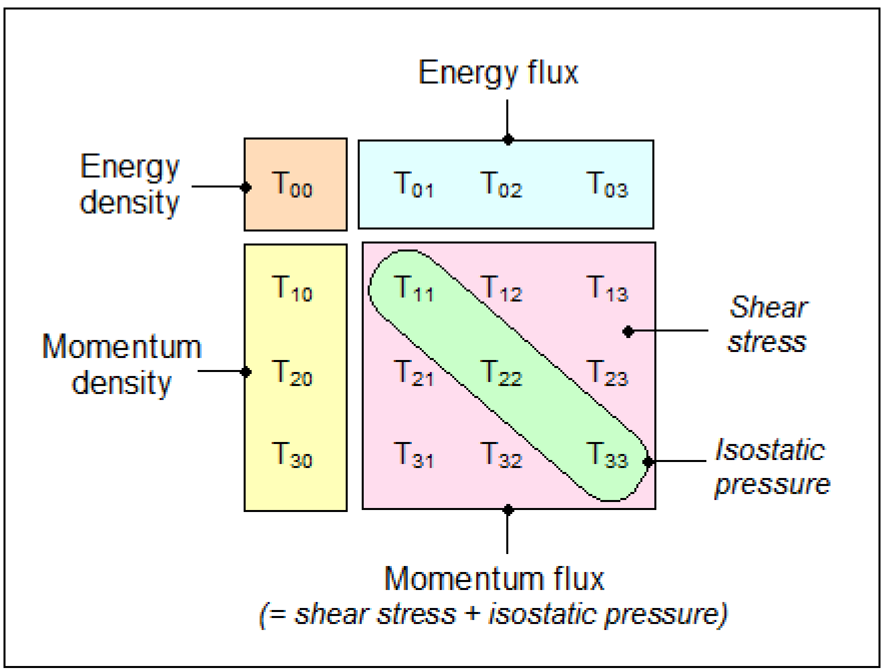

Figure E-1: Energy-Momentum Tensor

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix} = \begin{bmatrix} E/V & \rho_m c v_x & \rho_m c v_y & \rho_m c v_z \\ \rho_m c v_x & \sigma_x & \tau_{xy} & \tau_{xz} \\ \rho_m c v_y & \tau_{yx} & \sigma_y & \tau_{yz} \\ \rho_m c v_z & \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad (2)$$

E-2 Expression of Mass

Eq. (8) of Appendix D shows that $Mass = f_{(MatterVolume)}$:

$$M = \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\ \rho_s \qquad (3)$$

with:
M = Mass effect (kg)
c = Speed of the light (m/s)
G_0 = Universal constant of gravitation
ϵ_v = Coefficient of volumetric elasticity of spacetime.
V = Volume of the matter volume (m^3)
S = Surface of the matter volume (m^2)
ρ_s = Density of surrounding spacetime relative to a flat spacetime.

E-3 Density of Matter

In the case of non-relativistic objects, the density of matter, ρ_m, is:

$$\rho_m = \frac{M}{V} \qquad (4)$$

Porting Eq. (3) in Eq. (4) gives, after simplification by V:

$$\rho_m = \frac{c^2}{S\ G_0}\epsilon_v\ \rho_s \qquad (5)$$

E-4 Components of $T_{\mu\nu}$

Components of the $T_{\mu\nu}$ tensor are given in Eq. (2).

Element $\mathbf{T_{00}}$

$$T_{00} = \frac{E}{V} = \frac{mc^2}{V} \qquad (6)$$

Since $m = \rho_m V$ (ρ_m = density of matter):

$$T_{00} = \frac{\rho_m V c^2}{V} \qquad (7)$$

Porting Eq. (5) in Eq. (7), after simplification by V:

$$T_{00} = \frac{c^4}{S\ G_0}\epsilon_v\ \rho_s \qquad (8)$$

Elements $\mathbf{T_{01}, T_{02}, T_{03}, T_{10}, T_{20}, T_{30}}$

$$T_{\mu\nu} = \rho_m\ c\ v_{\mu\nu} = \frac{c^2}{S\ G_0}\epsilon_v\ \rho_s\ c\ v_{\mu\nu} \qquad (9)$$

or:

$$T_{\mu\nu} = \frac{c^4}{S\ G_0}\epsilon_v\ \rho_s \cdot \frac{v_{\mu\nu}}{c} \qquad (10)$$

Elements $\mathbf{T_{11}, T_{22}, T_{33}}$
$\sigma_{\mu\nu}$ is a normal pressure and $a_{\mu\nu}$ is a normal acceleration:

$$\sigma_{\mu\nu} = \frac{F_{\mu\nu}}{S} = \frac{ma_{\mu\nu}}{S} = \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\rho_s \cdot \frac{a_{\mu\nu}}{S} \qquad (11)$$

or:

$$\sigma_{\mu\nu} = \frac{c^4}{S\ G_0}\ \epsilon_v\ \rho_s \cdot \frac{V}{S\ c^2} a_{\mu\nu} \qquad (12)$$

Elements $\mathbf{T_{12}, T_{13}, T_{23}, T_{21}, T_{31}, T_{32}}$
$\tau_{\mu\nu}$ is a tangential pressure and $a_{\mu\nu}$ is a tangential acceleration.

$$\tau_{\mu\nu} = \frac{F_{\mu\nu}}{S} = \frac{ma_{\mu\nu}}{S} = \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\rho_s \cdot \frac{a_{\mu\nu}}{S} \qquad (13)$$

or:

$$\tau_{\mu\nu} = \frac{c^4}{S\ G_0}\ \epsilon_v\ \rho_s \cdot \frac{V}{S\ c^2} a_{\mu\nu} \qquad (14)$$

E-5 New Energy-Momentum Tensor

Eqs. (8-10-12-14) have the following common expression:

$$\frac{c^4}{S\ G_0}\ \epsilon_v\ \rho_s \qquad (15)$$

Extracting this expression from each component Eqs. (8-10-12-14) leads to a new tensor. To avoid confusion with the traditional Energy-Momentum tensor $T_{\mu\nu}$, this new tensor is called $J_{\mu\nu}$ (this symbol is different to the Bessel Symbol J_ν used in some countries). We get:

$$J_{\mu\nu} = \begin{bmatrix} 1 & v_{01}/c & v_{02}/c & v_{03}/c \\ v_{10}/c & (V/Sc^2)a_{11} & (V/Sc^2)a_{12} & (V/Sc^2)a_{13} \\ v_{20}/c & (V/Sc^2)a_{21} & (V/Sc^2)a_{22} & (V/Sc^2)a_{23} \\ v_{30}/c & (V/Sc^2)a_{31} & (V/Sc^2)a_{32} & (V/Sc^2)a_{33} \end{bmatrix} \qquad (16)$$

This gives the following equivalence:

$$T_{\mu\nu} = \frac{c^4}{S\ G_0}\ \epsilon_v\ \rho_s\ J_{\mu\nu} \qquad (17)$$

Appending the Einstein Constant to Eq. (17), the full right hand side of the EFE becomes:

$$\frac{8\pi G_0}{c^4} \times T_{\mu\nu} = \frac{8\pi G_0}{c^4} \times \frac{c^4}{S\ G_0}\ \epsilon_v\ \rho_s\ J_{\mu\nu} \qquad (18)$$

After simplifying by G_0 and c^4 the right hand side we get the new EFE:

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi\epsilon_v\rho_s}{S}J_{\mu\nu}} \qquad (19)$$

where:

ϵ_v = Coefficient of volumetric elasticity of spacetime.
S = Surface of the matter volume (m^2)
ρ_s = Density of surrounding spacetime relative to a flat spacetime.

E-6 Conclusions

1 - Spacetime Density

This new version of EFE includes a constant, ρ_s, which is the density of the surrounding spacetime. Here, it is a constant but, in reality, it is a variable (a partial derivative) because it varies from one point of the universe to another. It means that this new version of EFE may give different results depending of the location of the object(s) in the universe.

For example, we could suppose that the density of spacetime ρ_s is greater in the center of the universe than on its borders since the whole spacetime of the universe exerts a pressure toward the center. This density of spacetime, ρ_s, can be calculated on each point of the universe using computers. Result is a new vision of the universe that could explain dark matter and dark energy.

In some sense, ρ_s, can be assimilated to the Cosmological Constant $\Lambda g_{\mu\nu}$ that Einstein removed from his EFE. The only difference is that, contrary to $\Lambda g_{\mu\nu}$, ρ_s is fully explained (Appendix D).

2 - Using traditional EFE

This new version of EFE needs to know the volume and the surface of the matter volume which produces the curvature of spacetime. Moreover, it also needs to know the coefficient of curvature of spacetime, ϵ_v, and the density of surrounding spacetime relative to a flat spacetime, ρ_s. In that sense, it would be more convenient to continue using the traditional Energy-Momentum Tensor on Earth. In astrophysics, we must bear in mind that ρ_s varies at each point of the universe. In that particular case, using the traditional Energy-Momentum Tensor could lead to erroneous results.

E-7 Case of a static sphere

In this case we have:

$$V = \frac{4}{3}\pi r^3 \qquad (20)$$

and

$$S = 4\pi r^2 \qquad (21)$$

hence

$$\frac{V}{S} = \frac{4\pi r^3}{3}\frac{1}{4\pi r^2} = \frac{r}{3} \qquad (22)$$

Porting this expression in the tensor $J_{\mu\nu}$, Eq. (16), gives, after simplification, a new tensor $K_{\mu\nu}$:

$$K_{\mu\nu} = \begin{bmatrix} 1 & v_{01}/c & v_{02}/c & v_{03}/c \\ v_{10}/c & (r/3c^2)a_{11} & (r/3c^2)a_{12} & (r/3c^2)a_{13} \\ v_{20}/c & (r/3c^2)a_{21} & (r/3c^2)a_{22} & (r/3c^2)a_{23} \\ v_{30}/c & (r/3c^2)a_{31} & (r/3c^2)a_{32} & (r/3c^2)a_{33} \end{bmatrix} \qquad (23)$$

Finally, replacing "S" of the right hand side of Eq. (19) by its expression in Eq. (21) gives, after simplification, a reduced version of the EFE in the case of a static sphere:

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{2\epsilon_v\rho_s}{r^2}K_{\mu\nu}} \qquad (24)$$

Von Laue Geodesics

F-1 Introduction

A set of concentric circles is drawn in (fig. F-1a). These lines represent the geodesics of spacetime far from any mass, in a Minkowski spacetime. If a static spherical symmetry matter volume is inserted in the centre (fig. F-1b), spacetime will be curved, as explained in the main article. This figure F-1b has been duplicated in fig. F-2.

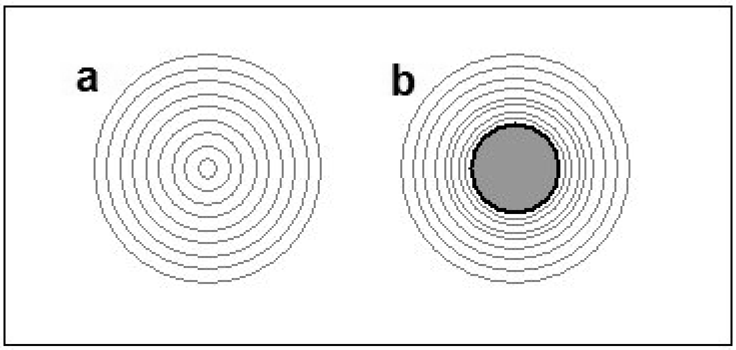

Figure F-1: Curvature of spacetime

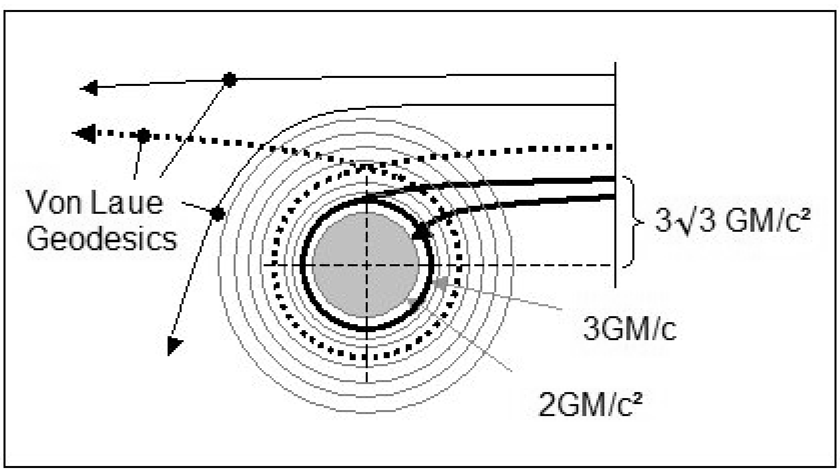

Figure F-2: Von Laue Geodesics

The Von Laue Geodesics [32] has been drawn over these concentric circles. Result is that the Von Laue Geodesics match EXACTLY the concentric circles.

In other words, it seems that Von Laue, early as the 1920s, predicted the theory presented here. It is obvious that **his diagram shows volumes, not masses,** even if the Von Laue Formulas are related to the mass of the body.

Explanation of the Increase of the Mass of Relativistic Particles

G-1 Introduction

The increase of the mass of relativistic particles is covered by special relativity. However, this phenomenon remains somewhat obscure. To date, we are still unable to explain with simple words, i.e. without using mathematics, why the mass of relativistic particles increases. This appendix proposes a simple and rational explanation of this strange phenomenon using the curvature of spacetime and “Matter Volume” instead of mass.

G-2 Length contraction

Special relativity states that, at relativistic speed, times *expand,* lengths *contract* and angles are *modified* [1-11]. The length contraction is defined by the formula

$$l_m = l_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad (1)$$

where
- l_m = Measured length
- l_0 = Proper length
- v = Speed of object
- c = Speed of light

G-3 Mass increase

Consider a particle at rest (Fig. G-1a, next page). Its matter volume produces a curvature of the spacetime. Geodesics of spacetime are spaced of l_0.

If this particle moves at a relativistic speed “v” (Fig. G-1b), spacetime geodesics seems to shrink (length contraction). As a result, the curvature of spacetime, ΔR_0 and ΔR_m below, increases. In other words, the density of the curvature of spacetime is inversely proportional to the space between two geodesics (also see note 1). Therefore, Eq. (1) becomes:

$$\Delta R_m = \frac{\Delta R_0}{\sqrt{1 - v^2/c^2}} \qquad (2)$$

where:
- ΔR_m = Infinitesimal element of the curvature of spacetime measured (external observer)
- ΔR_0 = Infinitesimal element of the proper curvature of spacetime
- v = Speed of the particle
- c = Speed of light

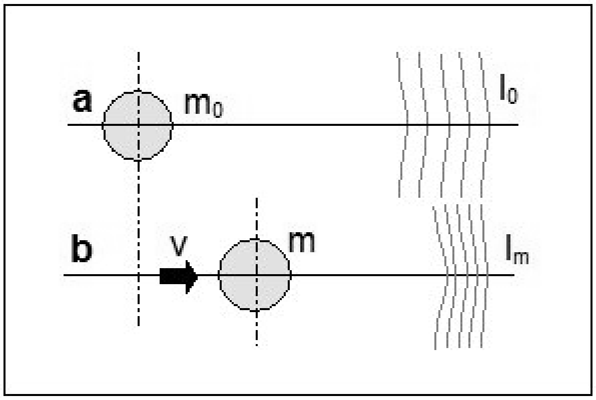

Figure G-1: Since the spacetime is more dense in (b), the mass effect increases.

Using Eq. (13) of Appendix B, $\Delta R = KM$, we can replace the curvatures of spacetime ΔR_m and ΔR_0, by the measured mass effect "m" and the proper mass effect "m_0". See notes 2 and 3. After simplification by K, we get:

$$\boxed{m = \frac{m_0}{\sqrt{1 - v^2/c^2}}} \qquad (3)$$

In other words, as shown in Fig. G-1, the mass or volume of a relativistic particle does not increase. In reality, the curvature of spacetime becomes more condensed at relativistic speed and produces an opposition force which has the same effect as an increase of mass.

Note 1: The spacetime curvature is the difference of displacement ΔR of a geodesic vs. to the same geodesic in a Minkowski space. As shown in Appendix C, the curvature of spacetime is infinitesimal. Thus, regardless of the function used, the portion of the curve on which we work is linear. Taking this linearity into account, there is no objection to consider that the curvature of spacetime is inversely proportional to the space between two geodesics.

Note 2: The nature of expression $\Delta R = KM$ is not relevant because this section covers exclusively the calculation of the coefficient to be applied to a proper value to get the measured value. This coefficient is noted γ (or $1/\gamma$) in scientific literature. This means that the relationship between the spacetime curvature and the mass effect is not affected by this study. For example, if we make 5 measurements of the curvature of spacetime at different speeds, we will not have 5 different relationships between ΔR and M, but only one applicable in all cases ...but we will have 5 different coefficients γ.

Note 3: The principles of special relativity state that the measurement of the mass of a relativistic particle increases. However, the converse is also true if we swap the reference systems. If we could pick up a measuring device on a particle in movement, this device would indicate that our spacetime, that in which we live, is much more dense as we see it. Thus, a section of the LHC for example, with a mass of 3 tons, measured from a device located on the particle in motion, would have a mass of 3000 tons if $\gamma = 1000$. From our point of view, the mass of a relativistic particle increases, but from the particle's point of view, it is our world that increases. In all cases, the proper mass of the particle or that of our world remains unchanged. This "relative view" is often misunderstood.

Note 4: It is wrong to think that the mass or the "matter volume" of relativistic particles really increases. In reality, it is the mass effect due to the apparent compression of spacetime that increases. The volume remains unchanged. On Earth, we consider that "mass" is an intrinsic value of a particle, such as the volume. It is not true. Since the mass effect comes from the pressure of spacetime on the particle, it is a virtual effect, such as pressure, speed, force, energy... On the other hand, the mass effect depends of the surrounding density of spacetime. Thus, for example, if the spacetime density was two times higher, the mass effect would be twice as important as well, but the intrinsic characteristics of the particle, more specifically the volume, would remain unchanged.

Equivalence Principle using the Curvature of Spacetime

H-1 Demonstration

Let's consider a static object on Earth (fig. H-1). "Matter volume" cause a curvature of spacetime that exerts a gravitational force on the surface of the object. Gravity is $g = 9.81 m.s^{-2}$ on the surface of Earth.

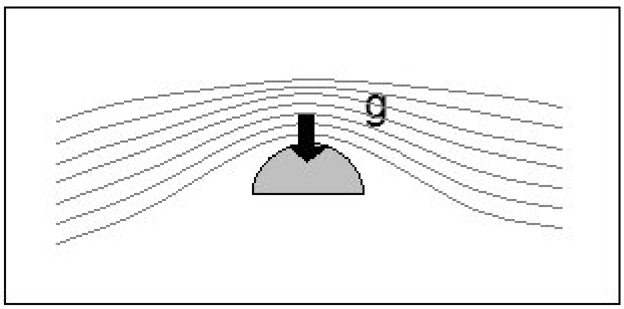

Figure H-1: Gravitationnal force using the spacetime curvature

Let's now consider the same object accelerated out of any gravitational field. We can represent this object in two different views (fig. H-2a and H-2b).

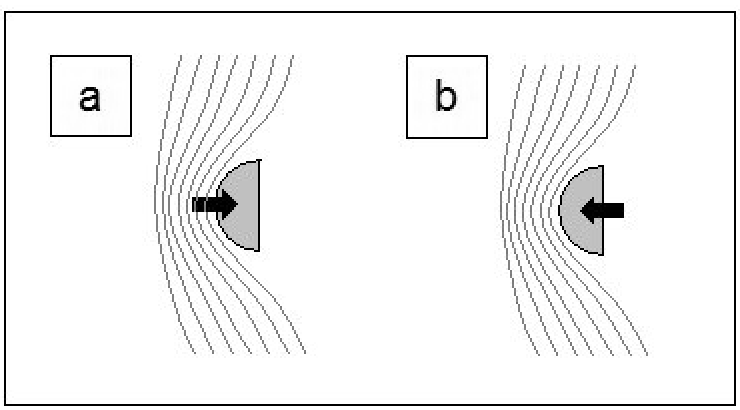

Figure H-2: Inertial acceleration using the spacetime curvature

In both cases, acceleration "a" is supposed to be identical to g:

$$a = g = 9.81 m.s^{-2}$$

Without any reference, a local observer cannot know if the curvature of spacetime is due to a pressure on the object (fig. H-2a) or to its acceleration (fig. H-2b). In fact, these two figures are identical and depend on where the observer stands, as described in Special Relativity.

Since:

• By definition, g = 9.81 $m.s^{-2}$ (fig. H-1) is identical to a = 9.81 $m.s^{-2}$ (fig. H-2a and H-2b).
• These examples use the same object. Therefore, the curvature of spacetime produced by the matter volume of this object is identical.
• The two precedent points show that the "mass effect" produced by these curvatures are identical.

We deduce that the "gravitational mass effect" (fig. H-1) is identical to the "inertial mass effect" (fig. H-2):

Gravitational mass effect
=
Inertial mass effect
=
Effect from spacetime curvature

As shown, this demonstration of the Equivalence Principle is fully explained using "Matter Volume" instead of "Mass".

The Bethe-Weizsacker SEMF

J-1 History

The Bethe-Weizsacker's formula, sometimes also called semi-empirical mass formula (SEMF), is used to approximate the binding energy of an atomic nucleus from its number of protons and neutrons. It is based partly on theory and partly on empirical measurements.

The theory is based on the liquid drop model proposed by George Gamow, which can account for most of the terms in the formula and gives rough estimates for the values of the coefficients. It was first formulated in 1935 by Carl Friedrich von Weizsacker, and although refinements have been made to the coefficients over the years, the structure of the formula remains the same today.

The SEMF gives a good approximation but, as explained, this formula is grounded on the "liquid drop model", which is not a "pure theory".

This appendix shows that the SEMF and Equ. (8) of Appendix D have strong similarities. In particular, the two main terms of each expressions, the Volume V, and the Surface S, are present in both equations. It means that the theory here proposed also explains the SEMF.

J-2 The formula

In the following formula, let "A" be the total number of nucleons, "Z" the number of protons, and "N" the number of neutrons, so that "A=Z+N".

The mass of an atomic nucleus is given by:

$$m = Zm_p + Nm_n - \frac{E_B}{c^2} \qquad (1)$$

where m_p and m_n are the rest mass of a proton and a neutron, respectively, and E_B is the binding energy of the nucleus. The SEMF states that the binding energy will take the following form:

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} - \delta(A, Z) \qquad (2)$$

The coefficients are determined empirically. Each term is summarized below.

Volume term

The term "$a_V A$" is known as the "volume term". The volume of the nucleus is proportional to "A", so this term is proportional to the volume.

Surface term

The term "$a_S A^{2/3}$" is known as the "surface term". This term, based on the strong force, is a correction to the volume term. If the volume of the nucleus is proportional to "A", then the radius should be proportional to "$A^{1/3}$" and the surface area to "$A^{2/3}$". This explains why the surface term is proportional to "$A^{2/3}$". It can also be deduced that "a_S" should have a similar order of magnitude as "a_V".

Coulomb term

The term "$a_C(Z^2/A^{1/3})$" is known as the "Coulomb" or "electrostatic term". The basis for this term is the electromagnetic force electrostatic repulsion between protons.

Asymmetry term

The term "$a_A((A-2Z)^2/A)$" is known as the "asymmetry term".

Pairing term

The term "$\delta(A,Z)$" is known as the "pairing term", possibly also known as the pairwise interaction.

J-3 Explanation of the SEMF

The proposed theory gives the mass formula in Appendix D, Equ. (8):

$$\boxed{M \ = \ \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\ \rho_s} \qquad (3)$$

with:
M = Mass effect (kg)
V = Volume of the "Matter Volume" (m^3)
S = Surface of the "Matter Volume" (m^2)
... etc... (see Appendix D).

In this equation, we see that the mass is directly proportional to the volume V, and inversely proportional to the surface S.

The SEMF equation is different because it represents the binding energy, not the mass of a given nucleus. However, these two expressions (fig. J-1 next page) are grounded on a common basis: the volume V, and the surface S.

$$M \ = \ \frac{c^2}{G_0}\epsilon_v \frac{V}{S}\ \rho_s$$

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A}$$

Figure J-1: The mass expression of Appendix D, Equ. (8), and SEMF equation are grounded on the same basis: the volume V, and the surface S.

J-4 Conclusions

As fig. J-1 shows, the mass expression of Appendix D, Equ. (8), and SEMF equation are grounded on the same basis: the volume V, and the surface S.

It is not a validation per se but an strong argument that supports the theory here proposed.

Other Classes of Volumes

In reality, it exists five classes of volumes, not three as indicated in the main article. The two extra classes are described below.

K-1 Hermetic Volumes

These volumes are combinations of matter and empty volumes but their global volume is hermetic regarding spacetime. Therefore, Hermetic Volumes are like Matter Volumes, they get mass. For example, a nucleus is made of nucleons (matter volumes), separated by empty space (empty volume). Whatever the content of this empty space (a vacuum, π^0 mesons...), this volume exists. Since this global volume is hermetic regarding spacetime, the whole volume of the nucleus deforms spacetime and, therefore, gets mass. This explains why the volume ($\equiv$ mass) of the nucleus is greater than those of its nucleons (mass excess). This also explains why the volume of a proton, 938 MeV/c^2, is much greater than the "matter volumes" of the three quarks, 2.3+2.3+4.8=9.4 MeV/c^2.

K-2 Special Volumes

These kinds of volumes are called "special" because we do not know their behaviour regarding spacetime. This is the case of ^{6}He, ^{8}He, ^{14}Be... For example, ^{11}Li has a core with $3p6n$ and a halo of $2n$. Since we do not know the penetration of spacetime inside these nuclei, these volumes can not be classified in any of the preceding categories.

K-3 Can we replace mass by volume?

The following questions may arise about volumes:

1/ Can we replace mass by "volume"?
No, because the word "Volume" is undefined.

2/ Can we replace mass by "apparent volumes"?
No, because apparent volumes are a combination of matter volumes, with mass, and massless empty volumes. Only matter volumes (with mass) are significant.

3/ Can we replace mass by "matter volumes" or "hermetic volumes"?
Yes, because there is a relation between matter volumes or hermetic volumes and mass (see the main article and appendix D). It is obvious that if we have $V = f_{(M)}$ and reciprocally $M = f_{(V)}$, we can use indifferently V or M in our calculus.

Bibliography

[1] A. Einstein (1905), "Zur Elektrodynamik bewegter Körper" *Annalen der Physik* **17:** 891.
[2] A. Einstein (1907), "Uber das Relativitatsprinzip" *Jahrbuch der Radioaktivitat* **4:** 411.
[3] A. Einstein (1907), "On the Inertia of Energy Required by SR" *Annalen der Physik* **23:** 371-384.
[4] A. Einstein (1915), "Die Feldgleichungen der Gravitation" *SPA der Wissenschaften*: 844-847.
[5] A. Einstein (1916), "Der allgemeinen Relativitatstheorie" *Annalen der Physik* 49.
[6] A. Einstein (1916), "Die Grundlage der allgemeinen Relativitatstheorie" *Annalen der Physik* **354 (7):** 769-822.
[7] A. Einstein (1917), "Kosmologische und Relativitatstheorie*" SPA der Wissenschaften* 142.
[8] A. Einstein (1920), "Relativity, The Special and the General Theory" *Routledge* ISBN 0-415-25384-5.
[9] A. Einstein; M. Grossmann (1913), "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation" *Zeitschrift für Mathematik und Physik* **62**: 225–261.
[10] A. Einstein; M. Grossmann (1914), "Covariance Properties of the Field Equations of the Theory of Gravitation Based on the Generalized Theory of Relativity" *Zeitschrift für Mathematik und Physik* **63**: 215–225.
[11] A. Einstein, H.A. Lorentz, H. Minkowski, H. Weyl (1952, Original: 1923), "The Principle of Relativity: a collection of original memoirs on SR and GR" *Courier Dover Publications* ISBN 0-486-60081-5.
[12] G. J. Stoney (1894), "Of the Electron or Atom of Electricity" *Philosophical Magazine* **5 38 (233):** 418–420.
[13] J. J. Thomson (1904), "On the Structure of the Atom" *Philosophical Magazine* **6 7 (39):** 237.
[14] H. Friedrich (2005), "Is general relativity understood?" *Annalen Physik* **15 (1-2):** 84-108
[15] Von Laue (1921) "Die allgemeine relativitatstheorie, vol. 2" *Friedr. Vieweg & Sohn Braunschweig*, Fig. 23, page 226
[16] R. Caldwell (2004), "Dark Energy", *Physics World 17* **(5):** 37-42
[17] A. Ashtekar (1987), "New Hamiltonian formulation of GR", *Phys. Rev. D* **36 (6):** 1587-1602
[18] C. H. Brans (1961), "Relativistic Theory of Gravitation", *Physical Review* **124 (3):** 925-935
[19] K. Schwarzschild (1916-a), "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie" *Sitzungsber. Preuss. Akad. D. Wiss.*: 189-196.
[20] K. Schwarzschild (1916-b), " Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie ", *Sitzungsber. Preuss. Akad. D. Wiss.*: 424-434.
[21] P. Havas (1964), "Newtonian mechanics their relation to GR", *Rev. Mod. Phys.* **36 (4):** 938-965
[22] A. Komar (1959), "Covariant Conservation Laws in GR", *Phys. Rev.* **113 (3):** 934-936
[23] D. Lovelock (1972), "The 4D Space and the Einstein Tensor", *J. Math. Phys.* **13 (6):** 874-876
[24] J. A. Font (2003), "Numerical Hydrodynamics in GR", *Living Rev. Relativity* 6 **doi:**10.12942
[25] R. Penrose (1969), "Gravitational collapse: the role of GR", *Riv. Nuovo Cim.* **1:** 252-276
[26] I. Shapiro (1964), "Fourth test of general relativity", *Phys. Rev. Lett.* **13 (26):** 789-791
[27] I. Shapiro (1968), "Fourth test of general relativity", *Phys. Rev. Lett.* **20 (22):** 1265-1269
226 https://archive.org/stream/dierelativitts02laueuoft#page/226/mode/2up
[28] C. M. Will (2006), "General Relativity and Experiment", *Living Review of Relativity* **9**.
[29] N. Fatio de Duillier (1690a), "Oeuvres complètes de Christiaan Huygens", *Société Hollandaise des Sciences*, Lettre N° 2570: 381–389, http://gallica.bnf.fr/ark:/12148/bpt6k778578/f388.table
[30] N. Fatio de Duillier (1701), "Die wiederaufgefundene Abhandlung von Fatio de Duillier: De la cause de la Pesanteur", Texte repris par Karl Bopp, http://fr.wikisource.org/wiki/De_la_cause_de_la_pesanteur
[31] N. Fatio de Duillier, (1743), "De la Cause de la Pesanteur", Notes and Records, *Royal Society of London* (1949) **6 (2):** 125–160
[32] G. L. Le Sage (1748), "Lucrèce Newtonien", *Memoires de l'Academie Royale des Sciences et Belles Lettres de Berlin*: 404–432, http://bibliothek.bbaw.de/bibliothek-digital/digitalequellen/schriften/anzeige/index_html?band=03-nouv/1782&seite:int=0495
[33] G. L. Le Sage (1748), "The Le Sage Theory of Gravitation", *Annual Report of the Board of Regents of the Smithsonian Institution*: 139–160, http://en.wikisource.org/wiki/The_Le_Sage_Theory_of_Gravitation
[34] P. Prevost (1818), Physique Mécanique des Georges-Louis Le Sage, in: Deux Traites de Physique Mécanique, Geneva & Paris: J.J. Paschoud, pp. 1–186, http://gdz.sub.uni-goettingen.de/dms/load/img/?PPN=PPN521099943&IDDOC=304083

Books and websites on General Relativity
[50] R. Arnowitt (1962), "The Dynamics of General Relativity", *Wiley*
[51] J. Barreau (2011), "Relativité Générale", *Dunod*
[52] F. Belabar (1999), "Einstein, Relativité Générale, Cosmologie et Theories Unitaires", *Seuil*
[53] L. Brillouin (1938) "Les Tenseurs", *Valentin*
[54] Brown, Kevin (2005), "Reflections on Relativity", *Cambridge University Press*
[55] S. Carroll (2004), "An Introduction to General Relativity", *Addison-Wesley*, ISBN 0-8053-8732-3

[56] J. E. Charon (1963), "15 Leçons sur la Relativité Générale", *Imprimerie Firmin Didot*
[57] T. Damour (2005), "Einstein Aujourd'hui", *EDP Sciences*
[58] Denis-Papin, Kauffman (1953), "Cours de Calcul Tensoriel", *Eyrolles-C4pf*
[59] G. Efstathiou (2009), "Relativité Générale", *De Boeck*
[60] J. Ehlers (1973), "Survey of General Relativity Theory", *Reidel*
[61] A. Einstein (1917-1990), "La Relativité", *Payot*
[62] A. Einstein (1916-2012), "La Théorie de la Relativité", *Dunod*
[63] A. Einstein (1915), "Die Feldgleichungen der Gravitation", *Preussische Akademie der Wissenschaften*
[64] J. Eisenstaedt (2002), "Einstein et la Relativité Générale", *CNRS*
[65] E. Elbaz (1998), "Relativité Générale et Gravitation", *Ellipses*
[66] J. Emile (1987), "La Relativité Complexe", *Albin Michel*
[67] R. Feynman, Richard (1956), "Cours de Physique", *Dunod*
[68] Gron, Oyvind, Hervik, Einstein (2007) "General Relativity", *Springer*, ISBN 978-0-387-69199-2
[69] J. B. Hartle, "An Introduction to Einstein's General Relativity", ISBN 0-8053-8662-9
[70] J. Hladik (2000), La Relativité selon Einstein (*Ellipses*, 2000), ISBN 2-7298-0124-3
[71] J. Hladik (1999), Le Calcul Tensoriel en Physique (*Ellipses*, 1999)
[72] J. Hladik (2006), "Introduction à la Relativité Générale Niveau M1", *Ellipses*
[73] Hughston, "Introduction to General Relativity", *Cambridge Univ. Press*, ISBN 0-1985-2957-0
[74] Landau, "The Classical Theory of Fields", ISBN 0-7506-2768-9
[75] M. Ludvigsen, "Relativité Générale, une Approche Géométrique", *Dunod* ISBN 2-1004-9688-3
[76] Misner, Wheeler (1973), "Gravitation", *Freeman*, ISBN 0-7167-0344-0
[77] Moor, Rafi (2006), "Understanding General Relativity", http://www.rafimoor.com/english/GRE.htm
[78] G. Nordstrom, Gunnar (1918) "On the Energy of the Gravitational Field", *Wetenschap.*
[79] B. F. Schutz (2001), "A First Course in General Relativity", ISBN 9780-521-88705-2