Two formulas based on c-chameleonic numbers which conducts to c-primes and the notion of c-chameleonic number

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Abstract. In one of my previous papers I defined chameleonic numbers as the positive composite squarefree integers C not divisible by 2, 3 or 5 having the property that the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d. In this paper I revise this definition, I introduce the notions of c-chameleonic numbers and m-chameleonic numbers and I show few interesting connections between c-primes and c-chameleonic numbers (I defined the notions of a c-prime in my paper "Conjecture that states that any Carmichael number is a cm-composite").

Definition 1:

We name a chameleonic number a number which is either c-chameleonic or m-chameleonic.

Definition 2:

We name a c-chameleonic number a positive integer, not necessary squarefree, not divisible by 2 or 3, with three or more prime factors, having the property that the absolute value of all the numbers P – d + 1, where d is one of its prime factors and P the product of all the others, is prime.

Example: 1309 = 7*11*17 is a c-chameleonic number because 7*11 – 17 + 1 = 61, prime, 7*17 – 11 + 1 = 109, prime and 11*17 – 7 + 1 = 181, prime (in fact, 1309 is the smallest c-chameleonic squarefree number with three prime factors).

Definition 3:

We name a m-chameleonic number a positive integer, not necessary squarefree, not divisible by 2 or 3, with three or more prime factors, having the property that the absolute value of all the numbers P + d - 1, where d is one of its prime factors and P the product of all the others, is prime.

Example: The Carmichael number 29341 = 13*37*61 is a m-chameleonic number because 13*37 + 61 – 1 = 541, prime, 13*61 + 37 - 1 = 829, prime and 37*61 + 13 – 1 = 2269, prime.

Observation 1:

Let p*q*r be a c-chameleonic number with three prime factors; then the number (p + 1)*(q + 1)*(r + 1) + 1 seems to be often a c-prime.

Examples:

: For p = q = 5 we have the following ordered sequence of c-chameleonic numbers:

  : 5*5*7 because 5*5 – 7 + 1 = 19, prime and 5*7 – 5 + 1 = 31, prime;
  Indeed, the number 6*6*8 + 1 = 289 = 17^2 is a c-prime (is a square of prime);

  : 5*5*13 because 5*5 – 13 + 1 = 13, prime and 5*13 – 5 + 1 = 61, prime;
  Indeed, the number 6*6*14 + 1 = 505 = 5*101 is a c-prime (101 – 5 + 1 = 97, prime);

  : 5*5*31 because 31 – 5*5 + 1 = 7, prime and 31*5 – 5 + 1 = 151, prime;
  Indeed, the number 6*6*32 + 1 = 1153 is prime, implicitly a c-prime;

  : 5*5*37 because 37 – 5*5 + 1 = 13, prime and 37*5 – 5 + 1 = 181, prime;
  Indeed, the number 6*6*38 + 1 = 1369 = 37^2 is a c-prime (is a square of prime);

  : 5*5*43 because 43 – 5*5 + 1 = 19, prime and 43*5 – 5 + 1 = 211, prime;
  Indeed, the number 6*6*44 + 1 = 1585 = 5*317 is a c-prime (317 – 5 + 1 = 313, prime);

  : 5*5*67 because 67 – 5*5 + 1 = 43, prime and 67*5 – 5 + 1 = 331, prime;
  Indeed, the number 6*6*68 + 1 = 2449 = 31*79 is a c-prime (79 – 31 + 1 = 49 = 7^2, a square of prime);

  : 5*5*127 because 127 – 5*5 + 1 = 103, prime and 127*5 – 5 + 1 = 631, prime;
  Indeed, the number 6*6*128 + 1 = 4609 = 11*419 is a c-prime (419 – 11 + 1 = 409, prime);
  (...)

Note:

A very interesting thing is that, through the formula above, is obtained from the c-chameleonic number 1309 = 7*11*17 the Hardy-Ramanujan number 1729 = 7*13*19; indeed, 8*12*18 + 1 = 1729.

Observation 2:

Let C = p*q*r be a c-chameleonic number with three prime factors; then the numbers C + 30*(p – 1), C + 30*(q – 1) and C + 30*(r – 1) seems to be often c-primes.

Examples:

: For C = 1309 = 7*11*17 we have:
: 1309 + 30*6 = 1489, prime, implicitly a c-prime;
: 1309 + 30*10 = 1609, prime, implicitly a c-prime;
: 1309 + 30*16 = 1789, prime, implicitly a c-prime.