Conjecture that states that any Carmichael number is a cm-composite

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Abstract. In two of my previous papers I defined the notions of c-prime respectively m-prime. In this paper I will define the notion of cm-prime and the notions of c-composite, m-composite and cm-composite and I will conjecture that any Carmichael number is a cm-composite.

Introduction:

Though, as I mentioned in Abstract, I already defined the notions of c-prime and m-prime in previous papers, in order to be, this paper, self-contained, I shall define them here too.

Definition 1:

We name a c-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), p(1) < q(1), with the property that the number q(1) – p(1) + 1 is either prime either semiprime p(2)*q(2) with the property that the number q(2) – p(2) + 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 4979 is a c-prime because 4979 = 13*383, where 383 – 13 + 1 = 371 = 7*53, where 53 – 7 + 1 = 47, a prime.

Definition 2:

We name a m-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), with the property that the number p(1) + q(1) - 1 is either prime either semiprime p(2)*q(2) with the property that the number p(2) + q(2) - 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 5411 is a m-prime because 5411 = 7*773, where 7 + 773 - 1 = 779 = 19*41, where 19 + 41 - 1 = 59, a prime.

Definition 3:

We name a cm-prime a number which is both c-prime and m-prime.

Definition 4:

We name a c-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) - p(h) + 1 is a c-prime.

Definition 5:

We name a m-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) - 1 is a m-prime.

Definition 6:

We name a cm-composite a number which is both c-composite and m-composite.

Note: We will consider the number 1 to be a prime in the six definitions from above; we will not discuss the controversed nature of number 1, just not to repeat in definitions "a prime or number 1".

Conjecture: Any Carmichael number is a cm-composite.

Verifying the conjecture(for the first 11 Carmichael numbers):

For 561 = 3*11*17 we have:
: the number 3*17 – 11 + 1 = 41, a prime;
: the number 3*17 + 11 – 1 = 61, a prime.
For 1105 = 5*13*17 we have:
: the number 5*17 – 13 + 1 = 73, a prime;
: the number 5*17 + 13 – 1 = 97, a prime.
For 1729 = 7*13*19 we have:
: the number 7*13 – 19 + 1 = 73, a prime;
: the number 7*13 + 19 – 1 = 109, a prime.
For 2465 = 5*17*29 we have:
: the number 5*17 – 29 + 1 = 57 = 3*19, a c-prime because 19 – 3 + 1 = 17, a prime;

: the number 5*17 + 29 - 1 = 113, a prime.

For 2821 = 7*13*31 we have:

: the number 7*31 – 13 + 1 = 205 = 5*41, a c-prime because 41 – 5 + 1 = 37, a prime;

: the number 7*31 + 13 - 1 = 229, a prime.

For 6601 = 7*23*41 we have:

: the number 23*41 - 7 + 1 = 937, a prime;

: the number 23*41 + 7 – 1 = 949 = 13*73, a m-prime because 13 + 73 – 1 = 85 = 5*17 and 5 + 17 – 1 = 21 = 3*7 and 3 + 7 – 1 = 9 = 3*3 and 3 + 3 – 1 = 5, a prime.

For 8911 = 7*19*67 we have:

: the number 7*19 – 67 + 1 = 67, a prime;

: the number 7*19 + 67 – 1 = 199, a prime.

For 10585 = 5*29*73 we have:

: the number 5*29 – 73 + 1 = 73, a prime;

: the number 5*29 + 73 – 1 = 217 = 7*31, a m-prime because 7 + 31 – 1 = 37, a prime.

For 15841 = 7*31*73 we have:

: the number 7*31 – 73 + 1 = 145 = 5*29, a c-prime because 29 – 5 + 1 = 25 and 5 – 5 + 1 = 1;

: the number 7*31 + 73 – 1 = 289, a m-prime because 17 + 17 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, a prime.

For 29341 = 13*37*61 we have:

: the number 13*37 – 61 + 1 = 421, a prime;

: the number 13*37 + 61 – 1 = 541, a prime.

For 41041 = 7*11*13*41 we have:

: the number 11*41 – 7*13 + 1 = 361, a c-prime because 19 – 19 + 1 = 1;

: the number 11*41 + 7*13 – 1 = 541, a prime.