The restoration of locality: the axiomatic formulation of the modified Quantum Mechanics

Jiří Souček

Charles University in Prague
Faculty of Philosophy
Ke kříži 8, Prague 5, 158 00
jiri.soucek@ff.cuni.cz

Abstract

From the dichotomy "nonlocality vs non-realism" which is the consequence of Bell Inequalities (BI) we shall choose the non-realism. We shall present here the modified Quantum Mechanics (modQM) in the axiomatic form. ModQM was introduced in [5] and we shall show its non-realism in the description of an internal measurement process. ModQM allows the restoration of locality, since BI cannot be derived in it. In modQM it is possible to solve: the measurement problem, the collapse problem, the problem of a local model for EPR correlations (see[5]). ModQM is a unique explicit realization of non-realism in QM. ModQM should be preferred as an alternative to the standard QM mainly since it restores the locality.

1 Introduction

The derivation of Bell inequalities (BI) has created the inconsistency in the quantum theory (QT). BI disagree with quantum mechanics (QM) and also with the results of experiments. BI do not assert anything on the quantum reality (they are false), but they assert that something in the theory is not in order. The theory is wrong since it was possible to derive BI (the derivation uses also some hidden assumptions: locality and realism).

Thus the basic problem in QT is to ensure the non-derivability of BI. But this needs deep changes in QM and in physics in general. There are two possibilities

(i) The nonlocality of QM. Some authors even claim that the nonlocality is a direct consequence of QM (see [3], [7], [9], [11]).

(ii) The non-realism of QM. This possibility was discussed only in general terms ([8], [10], [12], [13], [15], [16], [17]), but the explicit formulation of the non-realistic variant of QM was never presented (at least as we are informed).

The only concrete proposal of the non-realism was the modified QM (modQM) presented in [5]. The presentation of modQM in the paper [5] is based on the ideas coming from the non-standard probability theory called the extended probability theory (see [14]) and from the study of real QM.

The purpose of this paper is to give the clear formulation of modQM using the axiomatic formulation (sect. 2).

The other purpose is to show the functioning of the measurement process in modQM, since this process does not make a part of axioms. The measurement process is in modQM replaced by the concept of an observation (using axioms **Ax6 - Ax8**) and this is done in sect. 3.

The next goal is to describe the relation between modQM and the standard QM (stQM). These two theories are fundamentally different, but they have the same experimental consequences. The difference between these theories can be simply seen from the following implications

$$\text{locality} + \text{stQM } \textit{do imply} \text{ BI},$$

$$\text{locality} + \text{modQM } \textit{do not imply} \text{ BI}.$$

Moreover we describe exactly what means the non-reality of modQM (sect. 4).

There are two main differences between modQM and stQM:

(i) There are differences in the concept of individual states (the individual state is the state of the individual system). In stQM there is the von Neumann's axiom (see [4]) stating that each pure state is an individual state (or equivalently, the ensemble in the pure state is homogeneous). On the contrary, in modQM we require that each two individual states must be orthogonal (as a consequence we obtain the anti-superposition principle used in [5]).

(ii) Axioms of modQM do not contain the concept of the measurement (they also do not contain the concept of an observable). Instead of this the axioms contain the concept of an observation of the individual state of the measuring system (as a consequence we obtain the absence of the measurement problem in modQM).

In the actual presentation of modQM there is a certain generalization with respect to [5]. The system where it is possible to observe its individual state is called the observable system. Here we require that in each dimension there exists at least one observable system. In [5] it was required that each system is observable and this is a rather non-realistic assumption.

Then we show (in sect. 5) that there are important advantages in favor of modQM: no BI, locality, no measurement problem, no collapse problem, the local model for EPR correlations (see [5]). The price to be paid for these advantages is the only one: the non-realism as it is expressed in **Ax5**. This is a quite comfortable price, since we know that experimental consequences of modQM and stQM are the same and differences exist only in the theory.

2 Axioms of the modified QM

At first we shall list axioms which are shared by stQM and modQM. These are axioms **Ax1**,..., **Ax4**. Then we shall list axioms **Ax5**,..., **Ax8**, which are specific for modQM and are not part of stQM.

Ax1. For every system S there is defined its Hilbert space $\mathcal{H}_S$. It is assumed that $\mathcal{H}_S$ is the complex, finite dimensional (for simplicity) Hilbert space. The Hilbert space $\mathcal{H}_S$ is called the base of S.

The ensemble of systems is the set of systems $S_1, \dots, S_N$ with the same base $\mathcal{H}_S$ which are generated by some preparation procedure

$$\mathbb{E} = \{S_1, \dots, S_N\},\ N \to \infty.$$

Ax2. The set of possible states of an ensemble $\mathbb{E}$ with the base $\mathcal{H}_S$ is the set $St(\mathcal{H}_S)$ of all density operators in $\mathcal{H}_S$.

An operator in $\mathcal{H}_S$ is a density operator if it is hermitean and positive with the trace equal to one.

Definition 2.1. The state $\varrho \in St(\mathcal{H}_S)$ is a pure state , if there exists a vector $\psi \in \mathcal{H}_S$ satisfying $\varrho = \psi \otimes \psi^*,\ ||\psi|| = 1$.

The pure state is uniquely characterized by the ray

$$\vec{\psi} = \{\alpha\psi \mid \alpha \in \mathbb{C},\ |\alpha| = 1\},\ ||\psi|| = 1$$

The set of all rays is the complex projective space (see [1])

$$\mathcal{P}\mathcal{H}_S = \{\vec{\psi} \mid \psi \in \mathcal{H}_S,\ ||\psi|| = 1\}$$

Ax3. The time evolution $\varrho(t)$ of the state of an ensemble is given by

$$\varrho(t) = \mathcal{U}_t \varrho(0) \mathcal{U}_t^*,\ t \in \mathbb{R},$$

where $\mathcal{U}_t = \exp(-iHt),\ t \in \mathbb{R}$ is one-parametric unitary group in $\mathcal{H}_S$.

Ax4. For the composite system

$$S = S_1 \oplus S_2$$

we have

$$\mathcal{H}_S = \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_1}.$$

If $\mathbb{E}_i$ is an ensemble in the state ϱ_i (based on $\mathcal{H}_{S_i}$) for $i = 1, 2$ and systems $S_i^1 \in \mathbb{E}_1$ are independent from systems $S_i^2 \in \mathbb{E}_2$, then the state of the composite ensemble will be

$$\varrho = \varrho_1 \otimes \varrho_2$$

The differences between modQM and stQM are concentrated on two topics:

(i) The set of possible states of an individual system - i.e. the concept of an individual state (sometimes it is called the "ontic" state - see [1]). In modQM the set of individual states is substantially restricted with respect to stQM.

(ii) The axioms describing the measurement process. In modQM there is a concept an *observation* of the measuring apparatus instead of the concept of a measurement of a system. This fundamental change gives the possibility to solve the measurement problem in QM. In modQM the measuring process is the intrinsic process and in axioms of modQM the concept of a measurement is absent (this was the requirement of John Bell - "against measurement").

Now we shall consider the problem of the concept of an individual state (=state of individual systems). The first person considering this question seriously was von Neumann in his classical book on QM (see [3]). He started with the following definition. The state ϱ of an ensemble $\mathbb{E}$ is homogeneous if all systems $S \in \mathbb{E}$ are in the same individual state. It is clear that this is equivalent to the definition of an individual state: ϱ is an individual state iff the ensemble in the state ϱ is homogeneous. This is of course the circular definition. We shall define the concept of an individual state axiomatically.

Let us assume that for each system S there is defined a set

$$\tilde{D_S} \subset St(\mathcal{H}_S)$$

which denote the set of all possible individual states of the system S.

In stQM the set $\tilde{D_S}$ was defined by von Neumann in the axiom which we shall call the von Neumann axiom $\mathbf{Ax}_{vN}$ stating that

$$\tilde{D_S} = \mathcal{P}\mathcal{H}_S$$

i.e. that individual states are pure states. The experimental verification of $\mathbf{Ax}_{vN}$ is impossible since QM predicts only probabilities. But this does not mean that there could not exist theoretical arguments against $\mathbf{Ax}_{vN}$. Let us note that $\mathbf{Ax}_{vN}$ implies the superposition principle in QM.

The main difference between modQM and stQM consists in fact that $\mathbf{Ax}_{vN}$ is replaced by the opposite anti-von Neumann axiom $\mathbf{Ax}_{avN}$ introduced in [4]

($\mathbf{Ax}_{avN}$) each two different individual states must be orthogonal.

$\mathbf{Ax}_{avN}$ clearly implies the principle of anti-superposition which was the basic assumption of [5]: no non-trivial superposition of two different individual states can be the individual state.

We shall use also another technical assumption stating that individual states generate all $\mathcal{H}_S$. Together with $\mathbf{Ax}_{avN}$ this gives the following axiom.

Ax5. For each system S the set of its individual states $\tilde{D_S}$ consists of an orthogonal base in $\mathcal{H}_S$. Each system S is (in given a instant of time) in a certain state $s \in \tilde{D_S}$.

Let $n = dim\mathcal{H}_S$. In modQM we have

$$\tilde{D_S} = \{\vec{s_1}, \dots, \vec{s_n}\}.$$

Let us note that we can define: $\vec{\psi}_1, \dots, \vec{\psi}_n$ are orthogonal iff $\psi_1, \dots, \psi_n$ are orthogonal and we can define that $\vec{\psi}$ is a linear combination of $\vec{\psi}_1, \dots, \vec{\psi}_k$ iff ψ is a linear combination of $\psi_1, \dots, \psi_k$, since in both cases the results do not depend on the choice of representants.

Let $s_1, \dots, s_n$ be the set of representants of $\vec{s_1}, \dots, \vec{s_n}$ resp. Then we define

$$D_S = \{s_1, \dots, s_n\}.$$

The set D_S is not uniquelly determinated since the set

$$\{\alpha_1 s_1, \dots, \alpha_n s_n\}, \ |\alpha_1| = \dots = |\alpha_n| = 1 \tag{2.1}$$

is another representation of $\tilde{D_S}$. Later (in App.2) we shall show that the possible variability, given by $\alpha_1, \dots, \alpha_n$ does not change the final results in modQM.

Using the representation $D_S = \{s_1, \dots, s_n\} \subset \mathcal{H}_S$ we can represent each $\varrho \in St(\mathcal{H}_S)$ by a function $q : D_S \times D_S \to \mathbb{C}$ given by

$$q(s, s') = \langle s', \varrho(s) \rangle, \ s, s' \in D_S \tag{2.2}$$

This representation has following properties

(i) $q(s', s) = q(s, s')^*$

(ii) $\sum_{s,s' \in D_S} q(s, s') f(s) f(s')^* \geq 0, \ \forall f : D_S \to \mathbb{C}$

(iii) $\sum_s q(s, s) = 1.$

It can be simply shown that for each q satisfying (i)-(iii) there exists unique $\varrho \in St(\mathcal{H}_S)$ which satisfies (2.2).

So we have (for a given D_S) the 1-1 map $q \leftrightarrow \varrho$ given by (2.2).

Now we shall consider the concept of an observation. In [5] we have postulated the following principle:

For each system S it is possible to observe in which individual state the system occurs.

Here we shall consider this requirement as non-realistic. We shall assume that only for some systems it is possible to observe its actual individual state.

Definition 2.2.. A system S is called an observable system if it is possible to observe in which individual state this system is.

There is an axiom postulating that there exist sufficient number of observable systems.

Ax6.

(i) For each $n = 2, 3, \ldots$ there exists at least one observable system S such that

$$dim\mathcal{H}_S = |D_S| = n.$$

(ii) If S_1 and S_2 are observable systems then the composite system $S = S_1 \oplus S_2$ is the observable system, too.

It is clear that if $S = S_1 \oplus S_2$ is the composite system, then

$$D_S = D_{S_1} \times D_{S_2}.$$

Definition 2.3.. The set $A \subset D_S$ is called the observable domain if

(i) the system S can be written as a composition $S = S_1 \oplus S_2$

(ii) the subsystem S_1 (resp. S_2) is an observable system

(iii) there exists a set $A_1 \subset D_{S_1}$ (resp. $A_2 \subset D_{S_2}$) such that

$$A = A_1 \times D_{S_2} \ (A = D_{S_1} \times A_2 \ resp.)$$

The observable event is an event of the type $[s \in A]$, where A is an observable domain. We are then interested in the probability of this event when the system is in a given state.

Let the ensemble $\mathbb{E}$ be in the state $\varrho \in St(\mathcal{H}_S)$ and let $S \in \mathbb{E}$. Let $q : D_S \times D_S \to \mathbb{C}$ is the function representing ϱ through the formula (2.2)

Ax7. Let $A \subset D_S$ be an observable domain and let q be the state of the system S. Then the probability of an event $[s \in A]$ is given by

$$prob([s \in A]|q) = \sum_{s \in A} q(s, s).$$

Let $A \subset D_S$ be an observable domain and let

$$\mathbb{E} = \{S_1, \ldots, S_N\}$$

is an ensemble in the state $\varrho \in St(\mathcal{H}_S)$ and let $q : D_S \times D_S \to \mathbb{C}$ represents ϱ.

Definition 2.4.. The ensemble $\mathbb{E}\lfloor A$ (the restriction of $\mathbb{E}$ onto A) is defined by $\mathbb{E}\lfloor A = \{S \in \mathbb{E} \mid \text{individual state of } S \in A\}$.

Ax8. Let $\mathbb{E}$ be an ensemble in the state $\varrho \in St(\mathcal{H}_S)$ represented by $q : D_S \times D_S \to \mathbb{C}$, and let $A \subset D_S$ be the observable domain. Then the ensemble $\mathbb{E}\lfloor A$ will be in the state $q\lfloor A$ defined by

$$(q\lfloor A)(s, s') = q(s, s')\mathcal{X}(A; s)\psi(A; s') \cdot N_A^{-1}, \;\; s, s' \subset D_S$$

where $\mathcal{X}(A; s)$ is the characteristic function of A (i.e. $\mathcal{X} = 1$ iff $s \in A$ and $\mathcal{X} = 0$ iff $s \notin A$) and

$$N_A = \sum_{s \in A} q(s, s),$$

$q\lfloor A$ is defined only if $N_A > 0$.

The process $\mathbb{S}_A : \mathbb{E} \mapsto \mathbb{E}_A$ was called the selection process in [5]. The selection process does not mean primarily the change of the state, but the change of the ensemble. As a consequence of the change of an ensemble also the state changes: the new ensemble will be in a new state. Instead of the collapse of the wave function we have in modQM the change of the ensemble as a result of the selection precess.

I.e. the change $q \mapsto q\lfloor A$ is a consequence of the change $\mathbb{E} \mapsto \mathbb{E}\lfloor A$. The selection process must be considered as a consequence of equiring a new information that the individual state of S lies in A.

The typical situation is the following. We consider the composite system $S = S_1 \oplus S_2$, where S_1 is the observable system. We observe S_1 and find that the individual state of S_1 is equal to $s_1 \in D_{S_1}$. From this we can deduce that the individual state of S will be in

$$A_{s_1} = \{s_1\} \times D_{S_2}.$$

From **Ax7** we obtain that

$$prob\,([\text{ indiv. state of } S \in A_{s_1}]|q) = \sum_{s \in A_{s_1}} q(s,s).$$

The state of an ensemble $\mathbb{E}\lfloor A$ will be $q\lfloor A$ defined in **Ax8**.

3 The intrinsic realization of the measurement. The non-realism of modQM.

Definition 3.1. Two systems S_1, S_2 are similar if $dim\mathcal{H}_{S_1} = dim\mathcal{H}_{S_2} = n \geq 2$.

Definition 3.2. Let M and S be two similar systems and let $m_0 \in D_M$. Then the orthogonal map

$$\mathcal{E} : \mathcal{H}_{M\oplus S} \to \mathcal{H}_{M\oplus S}$$

is called an (M, S, m_0)-entangling map if there exist representations of D_M and D_S.

$$D_M = \{m_0, \dots, m_{n-1}\}$$

$$D_S = \{s_0, \dots, s_{n-1}\}$$

such that

$$\mathcal{E}(m_i \otimes s_j) = m_{i\oplus j} \otimes s_j$$

where

$$i \oplus j = i + j, \text{ if } i + j \leq n - 1$$

$$i \oplus j = i + j - n, \text{ if } i + j \geq n.$$

In particular we have

$$\mathcal{E}(m_0 \otimes s_j) = m_j \otimes s_j.$$

Definition 3.3. Let M and S are similar systems. Let $\mathcal{E}$ be an (M, S, m_0)-entangling map and let $\mathcal{U} : \mathcal{H}_S \to \mathcal{H}_S$ be a unitary map. Then the unitary map

$$\mathfrak{M} : \mathcal{H}_{M\oplus S} \to \mathcal{H}_{M\oplus S}$$

given by

$$\mathfrak{M} = (I_M \otimes \mathcal{U}^{-1}) \circ \mathcal{E} \circ (I_M \otimes \mathcal{U})$$

is called the $(M, S, m_0, \mathcal{E}, \mathcal{U})$-measuring map.

The unitary *map* $\mathcal{U}$ can be equivalently given by the orthogonal base $\{\psi_i\}$, where

$$\mathcal{U}(\psi_i) = s_i, \ i = 0, \dots, n - 1.$$

Then we have

$$\mathfrak{M}(m_i \otimes \phi_j) = m_{i\oplus j} \otimes \phi_j, \ i, j = 0, \dots, n - 1$$

and in particular

$$\mathfrak{M}(m_0 \otimes \phi_i) = m_i \otimes \phi_i, \ i = 0, \dots, n - 1.$$

The measurement process is parametrized by the following objects:

(i) M and S are similar systems, M is an observable system.

(ii) $m_0 \in D_M$

(iii) $\mathcal{E}$ is an (M, S, m_0)-entangling map

(iv) $\mathcal{U}$ - a unitary map in $\mathcal{H}_S$ and consequently $\{\phi_j\}$, $\phi_j = \mathcal{U}^{-1}(s_j)$ is an orthogonal base of $\mathcal{H}_S$.

Definition 3.4. Then the corresponding measurement process has the following steps

(i) We consider the ensemble $\mathbb{E}_0$ of $(M \oplus S)$-systems in the initial state $m_0 \otimes \Psi$, where

$$\Psi = \sum_{j=0}^{n-1} a_j \phi_j$$

(ii) The measuring transformation is applied and we obtain

$$\mathfrak{M}(m_0 \otimes \Psi) = \sum a_j \mathfrak{M}(m_0 \otimes \phi_j) = \sum a_j m_j \otimes \phi_j$$

and this is the new state of the ensemble $\mathbb{E}_0$

(iii) The measuring system M is observed and it is found that M is in the individual state m_{i_0}, $0 \le i_0 \le n-1$.

(iv) The observable domain $A_{i_0} \subset D_M \times D_S$ is defined by

$$A_{i_0} = \{m_{i_0}\} \times D_S.$$

and the new ensemble $\mathbb{E}_0 \lfloor A_{i_0}$ is created by the selection process following Definition 2.4.

(v) From (ii) we see that $\sum a_j m_j \otimes \phi_j$ is the new state of the ensemble $\mathbb{E}_0$. Using the decomposition $\phi_j = \sum b_{ik} s_k$, where (b_{ik}) is a unitary matrix we obtain that

$$\sum a_i m_i \otimes \phi_i = \sum_{i,j} a_i b_{ik} m_i \otimes s_k$$

we have $A_{i_0} = \{m_{i_0} \otimes s_k \mid k = 0, \ldots, n-1\}$. For

$$((m_i, s_k), (m_j, s_k)) \in D_{M \oplus S} \times D_{M \oplus S}$$

we have

$$q\left((m_i, s_k), (m_j, s_k)\right) = a_i b_{ik} \cdot a_j^* b_{jl}^*$$

then we obtain (**Ax7**) that

$$\sum_{A_{i_0}} q\left((m_i, s_k), (m_i, s_k)\right) = \sum_k a_{i_0} b_{i_0 k} a_{i_0}^* b_{i_0 k}^* = |a_{i_0}|^2 \cdot \sum_k b_{i_0 k} b_{i_0 k}^* = |a_{i_0}|^2,$$

since $\sum_k b_{i_0 k} b^*_{i_0 k} = 1$ Then by **Ax7**.

$$prob\left([(m_i, s_k) \in A_{i_0}] \mid q\right) = |a_{i_0}|^2.$$

i.e. the probability of finding the measuring system M in the individual state m_{i_0} is equal to $|a_{i_0}|^2$.

(vi) Let us assume that $|a_{i_0}|^2 > 0$. Then by **Ax8**. we obtain that the ensemble $\mathbb{E}_0 \lfloor A_{i_0}$ will be in the state

$$m_{i_0} \otimes \phi_{i_0}$$

This is the separated state (see [5]) where the measuring system M is in the individual state m_{i_0} and the measured system is in the collective state ϕ_{i_0}.

In this way the measurement process is described as an internal process inside of modQM. As an external process we need only the observational process described in **Definition 2.2** and in **Ax6**.

We note that the observational process is completely classical. In fact

(i) possible outcomes are $m_0, \ldots, m_{n-1}$

(ii) probabilities of outcomes are given by **Ax7**

(iii) the state of the ensemble after having made the observation is given by **Ax8**.

There is no problem in the observation process and we can state that in modQM the so-called "measurement problem" was completely solved.

The non-realism features of modQM. At first we shall define the concepts of an individual state and a collective state:

$$\text{indiv.state}(S) = s \in D_S.$$

The preparation procedure P prepares the ensemble of systems $\mathbb{E}$ in the state $\varrho \in St(\mathcal{H}_S)$. Each element of this ensemble $S \in \mathbb{E}$ is in the collective state ϱ.

The extremely important fact is that the evolution operator transforms collective states onto collective states (the collective state ϱ is transformed onto the collective state $\mathcal{U}\varrho\mathcal{U}^*$). There is no operator of the evolution of individual states. Ensemble $\mathbb{E}$ is in an individual state iff it is homogeneous (i.e. there exists $s \in D_S$ such that each $S \in \mathbb{E}$ is in the individual state s).

Thus we arrive at the fact that the collective state of $S \in \mathbb{E}$ (this is the state of $\mathbb{E}$) is equivalent to the preparation procedure by which $\mathbb{E}$ was created. Thus collective states are the basic instruments for QM. The concept of an individual state is really relevant only for observable systems, when we are able (**Ax6**) to observe in which individual state the individual system is.

In the measurement process we have the following situation

(i) the measuring system is in the individual state m_{i_0}

(ii) the measured system is in the collective stat ϕ_{i_0} (in the general case)

Thus we obtain

(i) the individual state of $M \not\Rightarrow$ the individual state of S

(ii) the individual state m_{i_0} of $M \Rightarrow$ the collective state ϕ_{i_0} of S.

This means that the individual property of $M \not\Rightarrow$ the individual property of S in the measurement process.

In stQM we have the individual state of $M \Rightarrow$ the individual state of S assuming that the system $M \oplus S$ is in a pure state.

We shall now show that Bell inequalities (BI) cannot be derived in modQM. Let us assume that the system $S_1 \oplus S_2$ is in the standard EPR state. This is the collective state and the individual states of S_1 and S_2 are not interconnected with collective EPR state. Thus we have no correlation between individual states of S_1 and S_2 and the key argument of the derivation of BI cannot go trough.

The anti-correlation of S_1 and S_2 is (in a general case) given by the collective state of $S_1 \oplus S_2$.

The resulting situation is following

(i) the measuring systems are anti-correlated in the individual state of $M_1 \oplus M_2$

(ii) the measured systems are anti-correlated collectively in the collective state of $S_1 \oplus S_2$.

Details to this discussion can found in [5].

If modQM $\not\Rightarrow$ BI, then locality+modQM$\not\Rightarrow$ contradiction thus modQM $\not\Rightarrow$ non-locality.

We can expect that modQM will restore the locality in QM.

4 The comparison between stQM and modQM

The differences between stQM and modQM are lying mainly in the concept of an individual state. In modQM the set of individual states is restricted to the orthogonal base of $\mathcal{H}_S$. All other states of S are mixed states which cannot be attributed to the individual system.

Let us consider stQM formulated as an operational theory. This formulation is based on concepts: the preparation procedure, the measurement procedure and the probability [1]

$$p(k \mid M, P)$$

of obtaining the outcome k of the measurement M given the preparation P. In our case we shall consider the standard measurement given by the orthogonal base $\{\phi_i\}_0^{n-1}$ in $\mathcal{H}_S$ and the corresponding set of projectors $P_k = |\phi_k\rangle\langle\phi_k| = \phi_k \otimes \phi_k^*$, $k = 0, \dots, n-1$. Every preparation procedure is associated with a density operator ϱ in $\mathcal{H}_S$. The probability of obtaining the outcome k is in stQM given by the Born rule

$$p(k \mid M, P) = tr(\varrho P_k).$$

In modQM we have another situation. The measurement process is the internal process based on the measuring transformation $\mathfrak{M}$ and on the process of an observation of the individual state of the measuring system M.

Let us consider the initial state (see notation above and in [5])

$$|m_0\rangle|s_{j_0}\rangle \sim \delta_{m_0} \otimes \delta_{s_{j_0}}.$$

In modQM we obtain

$$prob = |\langle\phi_{k_0}, s_{j_0}\rangle|^2$$

while in stQM we obtain

$$tr(P_{k_0}\varrho) = |\langle\phi_{k_0}, s_{j_0}\rangle|^2.$$

Thus we have obtained the same value.

The proofs of above statements can be found in the Appendix 1 bellow.

It follows that in the domain of the operational approach the results of stQM and modQM are the same.

The differences exist in the theoretical parts of the theories. Especially the considerations concerning individual states are in these theories completely different.

5 Conclusions

The modQM is the alternative to stQM and it has the following features:

(i) modQM restores the locality of QM (and, as a consequence, the locality of quantum theory and of all physics)

(ii) modQM preserves the consistency of the quantum theory (see [4],[6]).

(iii) modQM solves so-called measurement problem of QM - in fact, the measurement process is substituted by the observation process, which does not create problems

(iv) the problem of BI - i.e. the impossibility to prove BI is solved in modQM

(v) modQM and stQM have the same experimental consequences (at least at the extend of the operional formulation of QM)

(vi) the modQM is non-realistic since

$$\text{individual state of } M \nRightarrow \text{ individual state of } S$$

in the situation where M measures S.

(vii) there is no concept of the observable in modQM, it is replaced by the concept of an observation of the individual state of the observable system

The main difference between modQM and stQM lies in the treatment of individual states:

1. stQM asserts the von Neumann axiom ($\mathbf{Ax}_{vN}$) that each pure state is an individual state
2. modQM asserts the anti-von Neumann axiom ($\mathbf{Ax}_{avN}$) which says that different individual states must be orthogonal

The modQM offers the locality, consistency and no measurement problem at the price of non-realism.

This means that the creation of an individual system S in the state ϱ is only the short expression of the fact that the ensemble $\mathbb{E}$ was created by a certain preparation procedure and that $S \in \mathbb{E}$.

So we have the consequence that modQM and stQM are very different on the theoretical level but they coincide at the experimental level.

There is a question why modQM is considered as a new theory and not as a new interpretation of the standard theory. It may happen that two different theories have the same content, but the equality needs that each axiom of one theory is a theorem in another theory. This is not true for modQM and stQM. $\mathbf{Ax}_{vN}$ is the axiom of stQM and it is not true in modQM, where $\mathbf{Ax}_{avN}$ holds and viceversa for $\mathbf{Ax}_{avN}$. Moreover, the concept of the measurement in stQM is completely different from the concept of the observation in modQM. Both these arguments imply that modQM must be considered as the new theory.

Another argument was already mentioned:

$$\text{locality+stQM} \Rightarrow \text{BI}$$

while

$$\text{locality+modQM} \nRightarrow \text{BI}.$$

ModQM is the unique concrete proposal (as I know) which is local and non-realistic. ModQM solves some foundational problems of QM: locality, BI, inconsistency of quantum theory ([6]), the measurement problem, the collapse problem.

We think that modQM is a good candidate to replace stQM. We also think that the root of many problems in QM is the von Neumann's axiom $\mathbf{Ax}_{vN}$ (and its consequence, the principle of superposition).

We can say that the choice of the nonlocality (instead of the non-realism) was not a good choice: it brought many troubles and almost no advantages.

The choice of non-realism (i.e. modQM) brings many advantages and almost no problems. The experimental consequences are the same and there is the main benefit: the restoration of locality.

Appendix 1. Here we shall give the technical proof of the agreement of modQM and stQM.

The calculation of the probability in modQM.

We shall consider the state $|m_0\rangle|s_{j_0}\rangle$ which is denoted by $\delta_{m_0} \otimes \delta_{s_{j_0}}$. The state vector is

$$v = \sum v_{ij} \delta_{m_i} \otimes \delta_{s_j}, \ \ v_{ij} = \delta_{i0}\delta_{jj_0}.$$

After the measuring transformation (see [5]) we obtain the vector $v' = \sum v'_{kl} \delta_{m_k} \otimes \delta_{s_l}$, where

$$v'_{kl} = \sum \mathfrak{M}_{kl,ij} v_{ij} = \sum g_{k\ominus i,l} g^*_{k\ominus i,j} \delta_{i0}\delta_{jj_0} = g_{kl} g^*_{kj_0}$$

where $\phi_k = \sum g_{kl}\delta_{s_l}$, $k = 0, \ldots, n-1$. We have

$$v' = \sum g_{kl} g^*_{kj_0} \delta_{m_k} \otimes \delta_{s_l}$$

and then

$$v' \otimes v'^* = \sum g_{kl} g^*_{kj_0} g^*_{\overline{k}\overline{l}} g_{\overline{k}j_0} (\delta_{m_k} \otimes \delta_{s_l}) \otimes (\delta_{m_{\overline{k}}} \otimes \delta_{s_{\overline{l}}}).$$

Let us assume that the measuring system was observed in the state m_{k_0}. The corresponding selection domain is

$$A_{k_0} = \{(k_0, l) \mid l = 0, \ldots, n-1\}.$$

The diagonal elements of A_{k_0} are specified by the term $\delta_{kk_0}\delta_{\overline{k}k_0}\delta_{l\overline{l}}$.

We obtain

$$\sum g_{kl} g^*_{kj_0} g^*_{\overline{k}\overline{l}} g_{\overline{k}j_0} \cdot \delta_{kk_0}\delta_{\overline{k}k_0}\delta_{l\overline{l}} = \sum g_{k_0 l} g^*_{k_0 j_0} g^*_{k_0 l} g_{k_0 j_0} = g^*_{k_0 j_0} g_{k_0 j_0} = |g_{k_0 j_0}|^2$$

since

$$\sum g_{k_0 l} g^*_{k_0 l} = \delta_{k_0 k_0} = 1$$

Thus the probability of the outcome k_0 providing that the initial state was s_{j_0} is equal to $|g_{k_0 j_0}|$.

The calculation of this probability in stQM.

Let us consider the state vector $|s_{j_0}\rangle \approx \delta_{s_{j_0}}$.

The corresponding density operator is $\varrho = \delta_{s_{j_0}} \otimes \delta^*_{s_{j_0}}$

For the projector $P_{k_0} = |\phi_{k_0}\rangle\langle\phi_{k_0}|$ onto the vector $\phi_{k_0} = \sum g_{k_0 r}\delta_{s_r}$ we have

$$P_{k_0} = \sum g_{k_0 r}\delta_{s_r} \otimes g^*_{k_0\overline{r}}\delta_{s_{\overline{r}}} = \sum g_{k_0 r} g^*_{k_0\overline{r}} \delta_{s_r} \otimes \delta_{s_{\overline{r}}}.$$

Then ($\circ$ denotes the composition of operators)

$$P_{k_0}\varrho = \sum g_{k_0 r} g^*_{k_0\overline{r}} (\delta_{s_r} \otimes \delta_{s_{\overline{r}}}) \circ (\delta_{s_{j_0}} \otimes \delta^*_{s_{j_0}}) = \sum g_{k_0 r} g^*_{k_0\overline{r}} \cdot \delta_{\overline{r}j_0} \delta_{s_r} \otimes \delta_{s_{j_0}}$$

and we obtain

$$tr(P_{k_0}\varrho) = |g_{k_0 j_0}|^2 = tr_{A_{k_0}}\ (v' \otimes v'^*)$$

and this proves that the probability of an outcome k_0 is the same in modQM as in stQM.

Moreover we shall prove the collapse rule in modQM. We have to prove that the selection rule (**Ax8**) gives the same result as the von Neumann's collapse rule. We have written above the formula for $g' = v' \otimes v'^*$

In **Ax8** there are two restrictions onto A_{k_0}. They will be represented by δ_{kk_0} and $\delta_{\overline{k}k_0}$. Together we obtain (after recordering the tensor product) the new $\tilde{g}'$ obtained by the selection rule **Ax8** is

$$\begin{aligned}
\tilde{g}' &= \sum g_{k_0 l} g^*_{k_0 j_0} g^*_{k_0 \overline{l}} g_{k_0 j_0} \delta_{m_{k_0}} \otimes \delta_{m_{k_0}} \otimes \delta_{s_l} \otimes \delta_{s_{\overline{l}}} \cdot N^{-1}_{A_{k_0}} = \\
&= |g_{k_0 j_0}|^2 \delta_{m_{k_0}} \otimes \delta_{m_{k_0}} \otimes (\sum g_{k_0 l} \delta_{s_l}) \otimes (\sum g^*_{k_0 \overline{l}} \delta_{s_{\overline{l}}}) \cdot N^{-1}_{A_{k_0}} = \\
&= |g_{k_0 j_0}|^2 \delta_{m_{k_0}} \otimes \delta_{m_{k_0}} \otimes P_{k_0} \cdot N^{-1}_{A_{k_0}}
\end{aligned}$$

since

$$P_{k_0} = \phi_{k_0} \otimes \phi^*_{k_0},\ \ \phi_{k_0} = \sum g_{k_0 l} \delta_{s_l}.$$

Here clearly

$$N_{A_{k_0}} = |g_{k_0 j_0}|^2.$$

Thus we see that the selected subensemble will be in the state P_{k_0}.

Appendix 2. The another choice of the representants $s_1, \ldots, s_n$ in (2.1).

Above we have proved that the probability formula in **Ax7** and the projection formula in **Ax8** coincide with the corresponding rules in stQM (App.1). This implies that the another choice of representants (say $\alpha_1 s_1, \ldots, \alpha_n s_n$) will have no influence on result from **Ax7** and **Ax8**. And this says that experimental consequences do not depend on the choice of representants $(s_1, \ldots, s_n)$ of $(\vec{s_1}, \ldots, \vec{s_n})$.

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