Evidence combination from an evolutionary game theory perspective

Xinyang Deng, Deqiang Han, Jean Dezert, Yong Deng*, and Yu Shyr

Abstract

Dempster-Shafer evidence theory is a primary methodology for multi-source information fusion since it allows to deal with uncertain information. This theory is based on Dempster's rule of combination to synthesize multiple evidences from various information sources. However, in some cases, counter-intuitive results may be obtained based on Dempster's rule of combination. Lots of improved or new methods have been proposed to suppress the counter-intuitive results based on a physical perspective that minimizes the lost or deviation of original information. In this paper, inspired by evolutionary game theory, a biological and evolutionary perspective is considered to study the combination of evidences. An evolutionary combination rule (ECR) is proposed to mimick the evolution of propositions in a given population and finally find the biologically most supported proposition which is called as evolutionarily stable proposition (ESP) in this paper. Our proposed ECR provides new insight for the combination of multi-source information. Experimental results show that the proposed method is rational and effective.

Index Terms

Dempster-Shafer evidence theory, Belief function, Evolutionary game theory, Evolutionarily stable strategy, Replicator equation, Multi-source information fusion.

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I. Introduction

Multi-source information fusion aims to integrate multiple data and knowledge from multiple information sources into a consistent, comprehensive, and useful estimation for objects that beyond the performance of single information source. Many issues in disciplines can be translated to canonical multi-source information fusion problems. Because of that, multi-source information fusion is extensively used in many fields [1]–[3]. A crucial issue in the multi-source information fusion is that how to represent and dispose the imprecise, fuzzy, ambiguous, even inconsistent and incomplete information. Several branches of theories have been proposed for the processing of uncertain information. Typically, there are fuzzy set theory [4], rough set theory [5], [6], possibility theory [7], and so forth. Among these theories, Dempster-Shafer evidence theory [8], [9] has attracted increasing interest from scientific communities because of its inherent advantages in representing and handling uncertain information [10]–[17].

As a tool of reasoning in uncertain environment, Dempter-Shafer evidence theory has conceived a rounded system for the management of uncertainty. In this theory, the data from each information source is represented as a mass function, or called as evidence, Dempster's rule of combination is provided to combine multiple evidences for the fusion of multi-source information. However, the combination rule is controversial. Even though it is of many desirable characteristics, such as commutativity, associativity, and fast and clear convergence towards a solution, but in some cases, especially when the evidences to be combined are highly conflicting, Dempster's rule of combination may bring out counter-intuitive results. Typical examples contains Zadeh's paradox [18], evidence shifting [19], [20], dictatorial power of Dempster's rule [21], [22], and so on.

For the reasons of producing counter-intuitive results, there exists many debates, which lead to various alternatives of Dempster's rule or new methods for the combination of evidences, for example conjunctive rule [23], disjunctive rule [24], cautious conjunctive rule and bold disjunctive rule [25], and so on [20], [26]–[36]. Regarding the debate and improvement about Dempster's rule of combination [37]–[43], ones hold that Dempster's rule is inadequate so that they modify the original rule or propose new rules to synthesize various information, ones defend Dempster's rule and advocate the modification or preprocess of original evidences [21]. Corporately, both of them are expected to bring out an exactly synthetic evidence which can best represent the system consisting of all original evidences. At present, this is an traditional and mainstream view. During the process of information fusion, a primary hidden criterion is to minimize the lost or deviation between the information included in the synthetic evidence and that contained in original evidences. Essentially, this is a typical perspective of physics or engineering.

Instead, inspired by evolutionary game theory, in this paper we use a biological and evolutionary perspective to study the information fusion. The process of combining evidences to obtain the most acceptable conclusion is compared to the evolution of species to find the individuals with the highest fitness who can survive in a population. During this process, evolutionary game theory [44], [45] provides a theoretical framework. Specifically, the supported propositions in evidences are treated as strategies adopted by individuals in a given population. First, by interacting with others, each individual receives payoff which determines its fitness in the population. Then, individuals with high fitness have more chances to reproduce so as to increase their rate in the population. Finally, through the evolution of population, individuals with the highest fitness survive, and the strategy adopted by them, i.e. proposition, wins out to become the most supported or acceptable conclusion. The proposed method is called as evolutionary combination rule (ECR). Here, even it is named as a rule, the ECR actually is not a traditional combination rule which is expected to obtain a synthetic evidence, but just want to find the most supported propositions in the given multi-evidence system. Because it can directly find the object which has the highest fitness, potentially, the proposed ECR provides a fast decision-making support for evidence-based multi-source information fusion without relying on a transformation function between evidence and probability distribution.

In the rest of this paper, it is organized as follows. Section II gives a brief introduction to Dempster-Shafer evidence theory and evolutionary game theory. In Section III, the proposed ECR is presented, which mainly contains five steps including evidence weighted averaging, construction of Jaccard matrix game, evolutionary dynamics, evolution of averaging evidence, and two-dimensional measure output. After that, some illustrative examples are given to show the effectiveness of the proposed method in Section IV. Finally, Section V concludes this paper.

II. Preliminaries

A. Dempster-Shafer evidence theory

Dempster-Shafer evidence theory [8], [9], also called Dempster-Shafer theory (DST) or evidence theory, is used to handle uncertain information. This theory needs weaker conditions than the Bayesian theory of probability, so it is often regarded as an extension of the Bayesian theory – see discussion in [22]. As a theory of reasoning under the uncertain environment, DST has an advantage of directly expressing the "uncertainty" by assigning the probability to the subsets of the set composed of multiple objects, rather than to each of the individual objects. The probability assigned to each subset is limited by a lower bound and an upper bound, which respectively measure the total belief and the total plausibility for the objects in the subset. DST offers a method to combine distinct and cognitively independent bodies of evidence. Recently, DST has attracted many concern from various fields, such as parameter estimation [46]–[48], classification and clustering [49]–[51], decision-making [52]–[56], etc [57]–[61]. As an extension of this theory, Dezert-Smarandache Theory (DSmT) of plausible and paradoxical reasoning [62]–[64] is also given much attention [36], [65]–[67].

For completeness of the explanation, a few basic concepts on DST are introduced as follows.

Let Ω be a set of mutually exclusive and collectively exhaustive events, indicated by

$$\Omega = \{E_1, E_2, \cdots, E_i, \cdots, E_N\} \tag{1}$$

where set Ω is called a frame of discernment (FOD). The power set of Ω is indicated by 2^Ω, namely

$$2^\Omega = \{\emptyset, \{E_1\}, \cdots, \{E_N\}, \{E_1, E_2\}, \cdots, \{E_1, E_2, \cdots, E_i\}, \cdots, \Omega\} \tag{2}$$

The elements of 2^Ω or subset of Ω are called propositions. For example if $A \in 2^\Omega$, A is called a proposition.

For a FOD $\Omega = \{E_1, E_2, \cdots, E_N\}$, a mass function is a mapping m from 2^Ω to $[0, 1]$, formally defined by:

$$m: \quad 2^\Omega \to [0, 1] \tag{3}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{\mathrm{A} \in 2^\Omega} \mathrm{m(A)} = 1 \tag{4}$$

In DST, a mass function is also called a basic probability assignment (BPA), or a belief function, or a piece of evidences. The assigned probability $m(A)$ measures the belief exactly assigned to A and represents how strongly the evidence supports A. If $m(A) > 0$, A is called a focal element, and the union of all focal elements is called the core of the mass function.

Associated with each mass function is the belief measure and plausibility measure, Bel function and Pl function respectively. For a proposition $A \subseteq \Omega$, the belief function $Bel\ :\ 2^\Omega \to [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

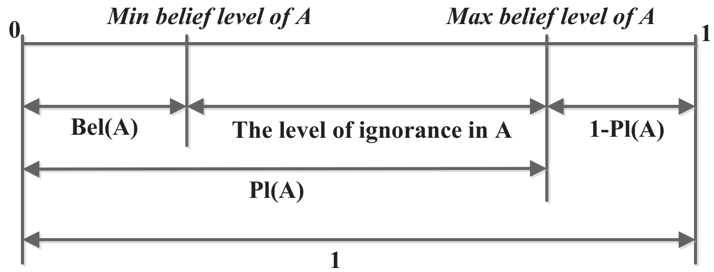

Fig. 1. The relationship of belief and plausibility.

The plausibility function $Pl : 2^{\Omega} \to [0,1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \tag{6}$$

where $\bar{A} = \Omega \setminus A$.

Obviously, $Pl(A) \geq Bel(A)$, these functions Bel and Pl are the lower limit function and upper limit function of the probability to which proposition A is supported, respectively. According to Shafer's explanation [9], the difference between the belief and the plausibility of a proposition A expresses the ignorance of the assessment for the proposition A. The relationship of belief and plausibility is shown in Fig. 1.

Consider two independent pieces of evidence characterized by their BPAs m_1 and m_2 on the FOD Ω, in DST, m_1 and m_2 are combined with Dempster's rule of combination, denoted by $m = m_1 \oplus m_2$, defined as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum\limits_{B \cap C = A} m_1(B) m_2(C) \ , & A \neq \emptyset; \\ 0 \ , & A = \emptyset. \end{cases} \tag{7}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{8}$$

where K is a normalization constant, called conflict coefficient between the two BPAs. Note that Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$. Dempster's rule of combination is the core of DST, it satisfies commutative and associative properties, i.e., (i) $m_1 \oplus m_2 = m_2 \oplus m_1$ and (ii) $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$. Thus if there exist multiple belief structures, the combination of them can be carried out in a pairwise way with any order. The vacuous BPA $m(\Omega) = 1$ is neutral element of Dempster's rule.

B. Evolutionary game theory

Evolutionary game theory (EGT) [44], [45] has been developed by John Maynard Smith to study the interaction among different players or populations. In recent years, EGT has become a paradigmatic framework to understand the emergence and evolution of cooperation among unrelated individuals [68]–[81]. The main idea of the EGT is to track the change of each strategy's frequency in the population during the evolutionary process. In EGT, evolutionarily stable strategy (ESS) [44] and replicator equation (RE) [82], [83] are the two central and key concepts.

1) Evolutionarily stable strategy (ESS): In a given environment, an ESS is such a strategy, adopted by a population, that can not be invaded by any other alternative strategy which is initially rare. The condition required by an ESS can be formulated as [44], [45]:

$$E(S,S) > E(T,S), \tag{9}$$

or

$$E(S,S) = E(T,S), \qquad and \qquad E(S,T) > E(T,T), \tag{10}$$

for all $T \neq S$, where strategy S is an ESS, T is an alternative strategy, and $E(T,S)$ is the payoff of strategy T playing against strategy S.

An evolutionarily stable strategy biologically means that it can not be successfully invaded by any mutant strategies. In game theory, there are two kinds of strategies, pure strategies and mixed strategies. A pure strategy defines an absolutely certain action or move that a player will play in every possible attainable situation in a game. In contrast, a mixed strategy is an assignment of a probability to pure strategies so as to allow a player to randomly select a pure strategy. In other words, in a mixed strategy, there are two or more pure strategies that can be selected by chance. If the evolutionarily stable strategy (ESS) S is a pure strategy, S is called a pure ESS. On the contrary, once S is a mixed strategy, S becomes the so-called mixed ESS. In [45], it has been proven that a game with two pure strategies always has an ESS (pure ESS or mixed ESS).

Hawk-Dove game is a classical and paradigmatic example of pure and mixed ESS. Assume there is a population of animals, in which each individual aggressiveness is different during the interaction with others. Accordingly, their behaviors can be divided into two types: the aggressive type and the cooperative type. The aggressive type corresponds to strategy "Hawk" (H), the cooperative type is associated with strategy "Dove" (D). Within each interaction, two animals meet and compete for a resource V ($V > 0$). When two Hawks meet, they will fight so that both of them have the opportunity to get $(V-C)/2$, where C is the cost of injury in the fight. When two Doves meet, they will share the resource, which means each individual obtains $V/2$. If, however, a Hawk meets a Dove, the former will fight and the latter can only escape. As a result, the Hawk obtains the entire resource without any cost of injury, the Dove is left with nothing. In this sense, the payoff matrix of Hawk-Dove game is shown in Eq. (11).

$$A = \begin{array}{c} \\ H \\ D \end{array} \begin{array}{c} \begin{array}{cc} H & D \end{array} \\ \begin{pmatrix} (V-C)/2 & V \\ 0 & V/2 \end{pmatrix} \end{array} \tag{11}$$

Here, $E(H,H) = (V-C)/2$, $E(H,D) = V$, $E(D,H) = 0$, $E(D,D) = V/2$, where $E(s_1, s_2)$ is the payoff of s_1 playing against s_2. In the Hawk-Dove game, there are two pure strategies, namely H and D, and innumerable mixed strategies $qH + (1-q)D$ where $q \in (0,1)$. According to the condition of ESS, in this game, strategy H is a pure ESS if $V > C$ because $E(H,H) > E(D,H)$. Conversely, if $V < C$, the mixed strategy $p^* = V/C$ which means that H and D are respectively selected with probability p^* and $1-p^*$, is a mixed ESS because, $\forall p \in (0,1)$ and $p \neq p^*$, $E(p^*,p^*) = E(p,p^*)$ and $E(p^*,p) > E(p,p)$, where $E(p^*,p^*) = \frac{V}{2}(1-\frac{V}{C})$, $E(p,p^*) = \frac{V}{2}(1-\frac{V}{C})$, $E(p^*,p) = \frac{V}{2}(1+\frac{V}{C}-2p)$, $E(p,p) = \frac{V}{2}(1-\frac{C}{V}p^2)$. It is noted that the payoff of mixed strategy $\vec{p} = (p, 1-p)$ playing against mixed payoff $\vec{q} = (q, 1-q)$ is calculated by $\vec{p}A\vec{q}$.

2) Replicator equation (RE): In EGT, the replicator equation (RE) [82], [83] plays a key role to determine the evolutionary process of population, which has provided a frequency-dependent evolutionary dynamics to a well-mixed population.

Assume there exists n strategies in a well-mixed population. A game payoff matrix $A = [a_{ij}]$ determines the payoff of a player with strategy i if he meets another player who carries out strategy j. The fitness of strategy i is defined by:

$$f_i = \sum_{j=1}^{n} x_j a_{ij}, \quad i = 1, \cdots, n \tag{12}$$

where x_j is the relative frequency of strategy j in the population. The average fitness of all strategies is denoted as ϕ, which is defined by:

$$\phi = \sum_{i=1}^{n} x_i f_i \tag{13}$$

The relative frequency of strategy i, namely x_i, is changed with time by this following differential equation:

$$\frac{dx_i}{dt} = x_i(f_i - \phi), \quad i = 1, \cdots, n \tag{14}$$

Eq.(14) is the so called replicator equation, which implies that the change of x_i depends on the fitness of strategy i and x_i itself. By solving $\frac{dx_i}{dt} = 0$, $i = 1, \cdots, n$, the fixed points of this evolutionary system, denoted as $(x_1^*, \cdots, x_n^*)$, can be found. The stability of each fixed point is the most focus of concern. Regarding the stability of each fixed point $(x_1^*, \cdots, x_n^*)$, a theorem is usually used to verify whether the fixed point is stable or not, which is given as below.

Theorem 1: [83] Given a set of replicator equations $\frac{dx_i}{dt} = x_i(f_i - \phi), \quad i = 1, \cdots, n$, the fixed point $p^* = (x_1^*, \cdots, x_n^*)$ is asymptotically stable if all eigenvalues associated with p^* are negative numbers or have negative real parts.

For more details on Theorem 1, please refer to [83]. Since the replicator equation is nonlinear, an exact solution is difficult to find. Hofbauer and Sigmund have proved that each ESS of A is definitely an asymptotically stable point of the replicator equation [83]. So the ESSs are often taken as the asymptotically stable solutions of the replicator equation.

III. Evolutionary combination rule (ECR)

The evolution of species abides by natural selection which provides an excellent mechanism to find individuals with higher fitness. Analogously, information fusion aims to find the most supported proposition by roundly synthesizing information coming from multiple sources. If the information is expressed by the means of probability distributions, the state with the biggest probability is always concerned intensely. In DST, since the introduction of subjective or epistemic uncertainty, the mass function, also called evidence or basic probability assignment (BPA), is employed to represent the uncertain information.

Traditional solution of combining evidences aims to obtain a mass function that best synthesizes all information. Then, based on the obtained synthetic evidence, a decision can be made. In this paper, instead of seeking the best synthetic evidence, our purpose is to find the best supported proposition (of course, it can also be seen as an evidence who only has a focal element), analogous to find the most probable state in probability theory. Obviously, it is not rational that simply let the proposition with the biggest belief be the candidate, because this approach ignores the interaction between propositions. Natural selection and the concept of fitness have inspired us consider this problem from a biological and evolutionary standpoint.

In this paper, we propose a new method to combine evidences coming from different information sources based on evolutionary game theory. The proposed method is called as evolutionary combination rule (ECR). In the ECR, propositions in the FOD are naturally

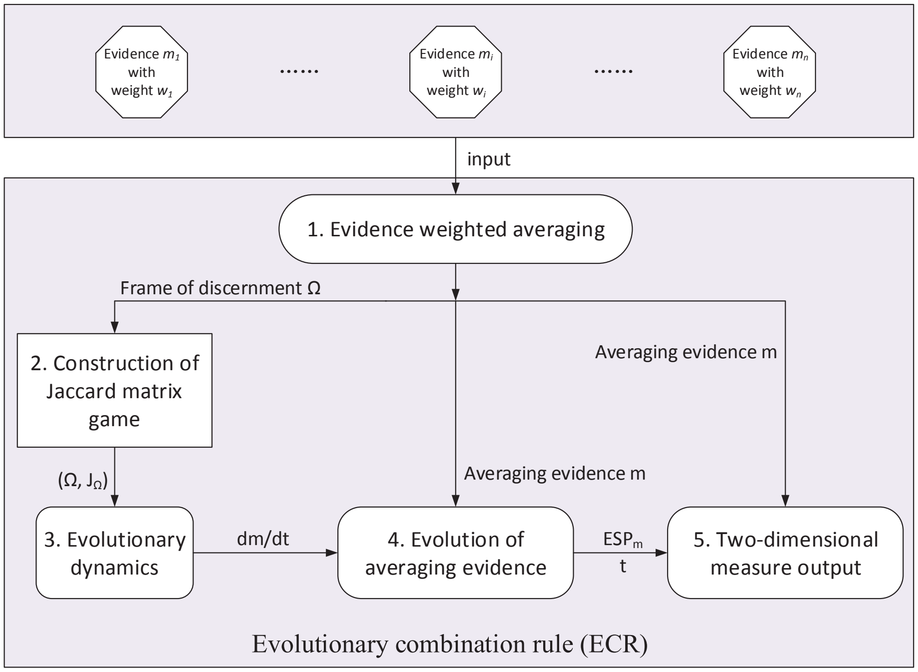

Fig. 2. Framework of the proposed evolutionary combination rule

seen as strategies that can be adopted by a population given an environment. Through an evolutionary process of strategies, propositions with the highest fitness can be found so as to facilitate the subsequent analysis and decision. Fig. 2 shows a framework of the proposed ECR. Mainly, there are five steps including evidence weighted averaging, construction of Jaccard matrix game, evolutionary dynamics, evolution of averaging evidence, two-dimensional measure output, which will be detailed in the following content of this section, respectively. Most notably, in the proposed ECR, there are two basic problems: (i) what are the interactive relationships between propositions (strategies) ? (ii) how do propositions evolve in a population? These questions will also be answered in the following content.

A. Evidence weighted averaging

Given multiple evidences from different information sources, in accord with the traditional assumption, these evidences are mutually independent. For these independent evidences, by considering the difference of importance among information sources, the weighted averaging approach is used to integrate multiple evidences.

Given a FOD Ω, assume there are n evidences or BPAs indicated by $m_1, \cdots, m_n$, each evidence has a weighting factor indicated by w_i, $\sum\limits_{i=1}^{n} w_i = 1$. the averaging evidence is denoted as m, which is obtained by

$$m(A) = \sum_{i=1}^{n} w_i m_i(A), \quad A \subseteq \Omega. \tag{15}$$

B. Jaccard matrix game (JMG)

In the ECR, propositions, subsets of the FOD, are treated as strategies. A key problem is to define the interaction rule between these propositions. Given a FOD Ω, suppose proposition A meets proposition B, how many payoff will be obtained for A, and for B as well? To solve this issue, in this paper the similarity degree between sets is used to express the payoffs of propositions since propositions are represented as sets. This idea is based on that an individual can obtain more benefit if he interacts with individuals being similar with him. In biology, there is a so-called greenbeard effect which shows that cooperative behaviors more likely appear between individuals with similar phenotypes [84]. Therefore, each proposition obtains more if similar propositions meet, but less if not similar. Specially, in a pair interaction, each can obtain 1 if these two propositions are identical, conversely 0 if they are totally different. Mathematically, Jaccard similarity coefficient gives the similarity degree between two sets [85]. In this paper, a Jaccard matrix game (JMG) is proposed to formalize the interaction relationship between propositions. The definition of the JMG is given as follows.

Definition 1: Given a FOD Ω, a Jaccard matrix game (JMG) on Ω is defined as

$$\Gamma = (\Omega, J_\Omega) \tag{16}$$

where the set of strategies is composed by propositions of the FOD (i.e., the nonempty subsets of Ω), the payoff matrix is $J_\Omega = [J_\Omega(A,B)]_{A,B \subseteq \Omega}$ in which $J_\Omega(A,B)$ represents the payoff of proposition A playing against proposition B, and it is defined by the Jaccard similarity coefficient between sets A and B, i.e., $J_\Omega(A,B) = \frac{|A \cap B|}{|A \cup B|}$.

Next, an example is given to show the JMG.

Example 1: Given a FOD with two elements $\Omega_1 = \{a, b\}$, a JMG on Ω_1, denoted as (Ω_1, J_{Ω_1}), can be constructed. In this game, the set of all potential strategies is $\{a, b, ab\}$ (for the sake of presentation, set $\{x, y\}$ is abbreviated as xy), and the payoff matrix is calculated easily

$$J_{\Omega_1} = \begin{array}{c} \\ a \\ b \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} a & b & ab \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{array}\right] \end{array} \tag{17}$$

Similarly, given a FOD with three elements $\Omega_2 = \{a, b, c\}$, a new JMG (Ω_2, J_{Ω_2}) is obtained, where the set of strategies is $\{a, b, c, ab, ac, bc, abc\}$, and the payoff matrix is shown as follows.

$$J_{\Omega_2} = \begin{array}{c} \\ a \\ b \\ c \\ ab \\ ac \\ bc \\ abc \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & ab & ac & bc & abc \end{array} \\ \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 & 1/3 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 & 1/3 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 & 1/3 \\ 1/2 & 1/2 & 0 & 1 & 1/3 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 & 1/3 & 1 & 1/3 & 2/3 \\ 0 & 1/2 & 1/2 & 1/3 & 1/3 & 1 & 2/3 \\ 1/3 & 1/3 & 1/3 & 2/3 & 2/3 & 2/3 & 1 \end{array}\right] \end{array} \tag{18}$$

Again, for each proposition $A \subseteq \Omega$ in DST, it is equivalent to a pure strategy in a JMG. For example, given a FOD $\Omega = \{a, b\}$, there are three propositions a, b ab, each is a pure strategies in the JMG (Ω, J_Ω). All propositions consist of the set of pure strategies adopted by a population. Moreover, in a JMG, a pure strategy corresponds to a proposition which also can be seen as a mass function who only has a focal element, and a mixed strategy is associated with a set of propositions which is a mass function with multiple focal elements. As shown in the above example, a JMG can be constructed if a FOD is given. Formally, the complication of a JMG is determined by the size of the given FOD Ω. However, regardless of the size of Ω, some self-evident corollaries about the JMG are obtained.

Corollary 1: A JMG (Ω, J_Ω) is symmetric, namely $J_\Omega(A, B) = J_\Omega(B, A)$.

Corollary 2: Let Δ_Ω^{ESS} be the set of all ESSs (evolutionarily stable strategies) in a JMG (Ω, J_Ω), then

$$\Delta_\Omega^{ESS} = \{A | A \subseteq \Omega, A \neq \emptyset\}$$

Corollary 2 shows a one-to-one correspondence between pure strategies and ESSs in JMGs. Due to the equivalence between propositions and pure strategies in JMGs, in this paper we also call the evolutionarily stable strategy (ESS) as evolutionarily stable proposition (ESP). Assume the set of ESPs in a JMG is indicated by Δ_Ω^{ESP}, then we have $\Delta_\Omega^{ESP} \Leftrightarrow \Delta_\Omega^{ESS}$.

C. Replicator dynamics on the JMG

Once the interaction relationship between propositions has been determined, the next problem is that how the propositions evolve. In other words, what kind of evolutionary dynamics is adopted in this population? To address this issue, the replicator equation is used to mimick the evolutionary process of population based on two reasons. At first, it commendably simulates the evolution of population in a well-mixed environment where individuals could randomly interact with other members of the populations. More important, the replicator equation is mathematically equivalent to the Lotka-Volterra equation of ecology [83] which describes the dynamics of species in an interacting biological system. The Lotka-Volterra equation is very close to real facts since it does not rely on the rationality assumption which is fundamental but challenged in the classical game theory. So, the equivalent replicator equation also gets rid of the puzzle caused by the controversial rationality assumption. The replicator dynamics on a JMG is defined as follows.

Definition 2: Given an evidence m on a FOD Ω, the belief or basic probability for proposition A, i.e. $m(A)$, evolves in time according to the equation

$$\frac{dm(A)}{dt} = m(A)\left(f_A - \phi\right), \quad A \subseteq \Omega, A \neq \emptyset \tag{19}$$

where f_A is the fitness of proposition A, and it is given in terms of a JMG (Ω, J_Ω) by

$$f_A = \sum_{B \subseteq \Omega, B \neq \emptyset} m(B) J_\Omega(A, B) \tag{20}$$

whereas the average fitness of propositions in the population is

$$\phi = \sum_A \sum_B m(A) J_\Omega(A, B) m(B), \quad A, B \subseteq \Omega, and\ A, B \neq \emptyset \tag{21}$$

Regarding the differential dynamic system shown in Eqs.(19)-(21), people are mainly concerned with the asymptotically stable points (ASPs). Hofbauer and Sigmund [83] provided a theorem to show the relationship between the ASPs of replicator dynamics and ESSs of a symmetric two-player game.

Theorem 2: [83] Given a symmetric two-player game G, strategy x^* is an ESS of G if and only if x^* is an ASP of the replicator equation evolving on G.

Theorem 2 shows that there is an one-to-one correspondence between ASPs and ESSs in a symmetric two-player game. Let us denote the set of ASPs of the replicator equation in a JMG by Δ_Ω^{ASP}, evolutionarily stable propositions (ESPs) by Δ_Ω^{ESP}. Since JMGs are symmetric, the following corollary is naturally satisfied.

Corollary 3: In JMGs, $m^* \in \Delta_\Omega^{ESP}$ if and only if $m^* \in \Delta_\Omega^{ASP}$.

Here, an example is given to help understand this relationship.

Example 2: Given a FOD $\Omega = \{a, b\}$, a JMG $\Gamma = (\Omega, J_\Omega)$ is constructed, its payoff matrix J_Ω is shown as follows.

$$J_\Omega = \begin{array}{c} \\ a \\ b \\ ab \end{array} \begin{array}{c} \begin{array}{ccc} a & b & ab \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{array} \right] \end{array} \tag{22}$$

Now let us study the replicator dynamics on the game Γ. In this JMG, there are three pure strategies, a, b, and ab. For the sake of presentation, let the basic probabilities of these propositions be $m(a) = x$, $m(b) = y$, $m(ab) = z$, where $x + y + z = 1$. According to the replicator equation, we can write (replacing, as usual, the time derivatives of x, y, z by $\dot{x}$, $\dot{y}$, $\dot{z}$),

$$\begin{cases} \dot{x} = x(f_x - \phi) \\ \dot{y} = y(f_y - \phi) \\ \dot{z} = z(f_z - \phi) \end{cases} \tag{23}$$

where

$$\begin{cases} f_x = x + z/2 \\ f_y = y + z/2 \\ f_z = x/2 + y/2 + z \end{cases} \tag{24}$$

and

$$\phi = (x, y, z)J_\Omega(x, y, z)^T = x(x + z/2) + y(y + z/2) + z(x/2 + y/2 + z) \tag{25}$$

According to the stability theory of differential equations, the fixed point of the replicator dynamic system represented by Eqs. (23)-(25) should satisfy $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$, where $x + y + z = 1$. So all fixed points can be obtained, they are $(x_1^*, y_1^*, z_1^*) = (1, 0, 0)$, $(x_2^*, y_2^*, z_2^*) = (0, 1, 0)$, $(x_3^*, y_3^*, z_3^*) = (0, 0, 1)$, $(x_4^*, y_4^*, z_4^*) = (0, 0.5, 0.5)$, $(x_5^*, y_5^*, z_5^*) = (0.5, 0.5, 0)$, $(x_6^*, y_6^*, z_6^*) = (0.5, 0, 0.5)$. The stability of each fixed point can be checked by a Jacobian matrix, JM, as follows.

$$JM = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{x}}{\partial z} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial z} \\ \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial y} & \frac{\partial \dot{z}}{\partial z} \end{pmatrix} \tag{26}$$

Let us take (x_1^*, y_1^*, z_1^*) as an example. For this fixed point,

$$A = JM|_{(x=x_1^*,\, y=y_1^*,\, z=z_1^*)} = \begin{pmatrix} -1 & 0 & -0.5 \\ 0 & -1 & 0 \\ 0 & 0 & -0.5 \end{pmatrix}. \tag{27}$$

TABLE I

FIXED POINTS AND THEIR STABILITY IN EXAMPLE 2

Fixed points (Beliefs of a, b and ab)	**Associated eigenvalues**	**Stablity**
(1, 0, 0)	-1, -1, -0.5	stable (ASP)
(0, 1, 0)	-1, -1, -0.5	stable (ASP)
(0, 0, 1)	-1, -0.5, -0.5	stable (ASP)
(0, 0.5, 0.5)	-0.75, -0.5, 0.25	unstable
(0.5, 0.5, 0)	-0.5, 0, 0.5	unstable
(0.5, 0, 0.5)	-0.75, -0.5, 0.25	unstable

The stability of fixed point (x_1^*, y_1^*, z_1^*) is determined by the eigenvalues of the following characteristic equation

$$\det(A - \lambda I) = 0 \tag{28}$$

where det is the determinant of a matrix, and I is an identity matrix. So, the eigenvalue λ can be readily calculated, and they are $\lambda_1 = -1$, $\lambda_2 = -1$, $\lambda_3 = -0.5$. According to Theorem 1 [83], the fixed points (x_1^*, y_1^*, z_1^*) is an asymptotically stable point (ASP) since all eigenvalues are negative numbers. Meanwhile, ASP (x_1^*, y_1^*, z_1^*) corresponds to $m(a) = 1$, so we say proposition a is an evolutionarily stable proposition (ESP).

By means of this approach, the stability of other fixed points can be found. The results are shown in Tab. I. As shown in Tab. I, there are three ASPs, namely $m(a) = 1$, $m(b) = 1$ and $m(ab) = 1$, which are in one-one corresponding with ESPs, i.e., $\Delta_\Omega^{ESP} = \Delta_\Omega^{ASP}$. Graphically, a two-dimensional space, called simplex, can clearly represent the evolutionary dynamics of propositions a, b and ab, as shown in Fig. 3. In the simplex, every vertex of means that there only exists a sole proposition (i.e., strategy) in the population, edges represent that at least a proposition is missing in the population. The interior of the simplex corresponds to the case of all propositions coexistence. At each point of the simplex, the sum of the belief of all propositions is 1. In addition, in Fig. 3, arrows represent the directions of evolution, black circles indicate ASPs, white circles are unstable fixed points. From Fig. 3, it clearly shows that there are three ASPs that are precisely ESPs of Γ, and three unstable fixed points that are located on the midpoints of there edges.

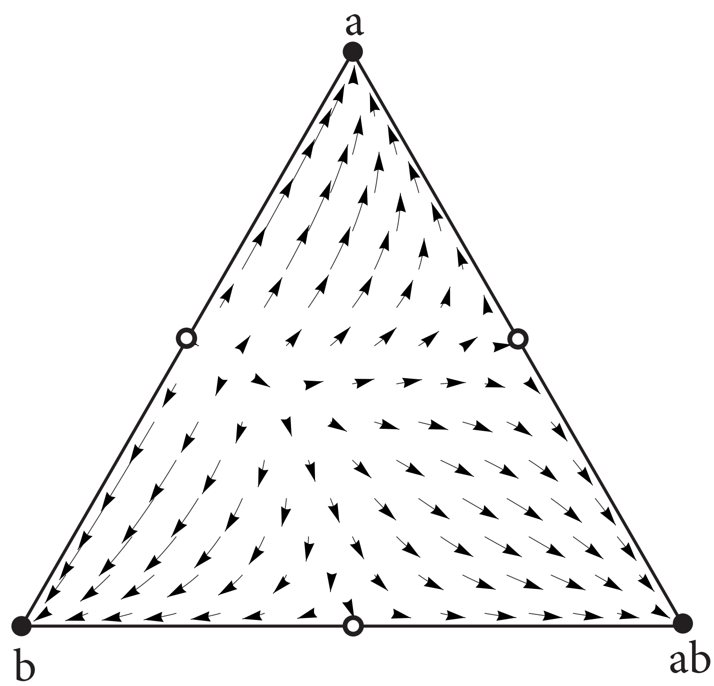

Fig. 3. Evolutionary dynamics of propositions a, b and ab in Example 2

D. Evolution of averaging evidence

As aforementioned, given an evidence which is usually the average of multiple evidences in multi-source information fusion, the replicator dynamics determines which proposition this evidence will evolve to. Here is an example to show the evolutionary process.

Example 3: Given an evidence m on a FOD $\Omega = \{a, b, c\}$,

$m(a) = 0.25$, $m(b) = 0.25$, $m(c) = 0.25$,

$m(ab) = 0.05$, $m(ac) = 0.1$, $m(bc) = 0.05$, $m(abc) = 0.05$.

We want to determine which is the most possible object among objects a, b, c, in terms of m. As we see, the support degrees for objects a, b and c are very similar in m. Facing this situation, we use the proposed ECR for analysis. Let m be the initial configuration of this population at $t = 0$, and $m_t(A)$ be the mass value of $m(A)$ at time t, where $A \subseteq \Omega$. The evolutionary process is illustrated by

$$m_{t+\Delta t}(A) = m_t(A) + \frac{dm_t(A)}{dt}\Delta t \tag{29}$$

In this paper, we simulate the evolutionary process of each proposition by using the fourth-order Runge-Kutta method, as shown in Fig. 4, where the horizontal axis “Time” indicates time points in the Runge-Kutta method. According to Fig. 4, given an initial configuration

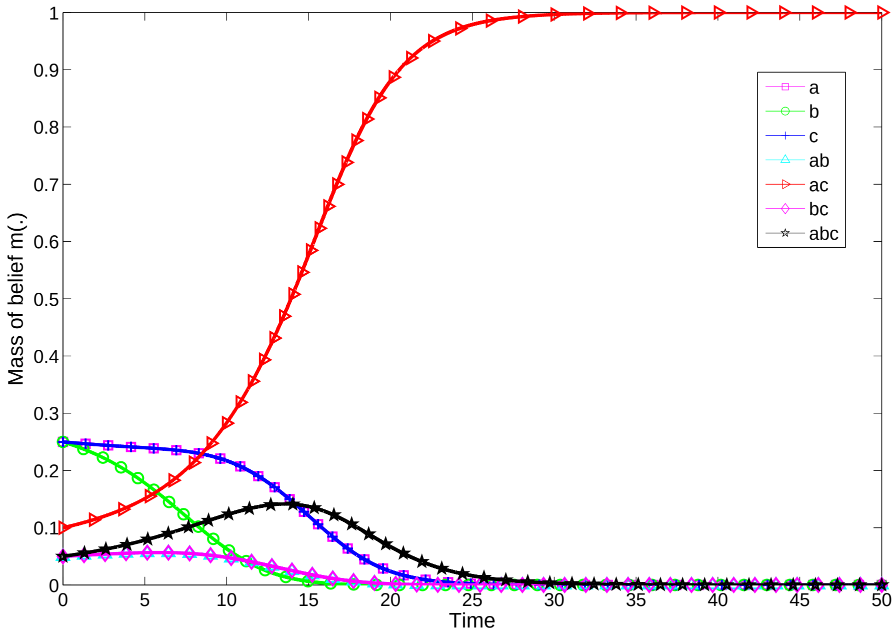

Fig. 4. The evolutionary process of propositions in Example 3

determined by evidence m, in the end of evolution, proposition ac survives and finally occupies the population, while other propositions become extinct. So, ac is the only ESP for evidence m, denoted as $ESP_m = ac$. It implies that proposition ac has the highest fitness. As a result, it suggests that the most supported object is either a or c. Moreover, as shown in Fig. 4, by using the Runge-Kutta method, we can track the evolutionary curve of each proposition, and obtain the time required for reaching equilibrium. In this paper, we assume that the replicator dynamics equation reaches equilibrium if the maximum increment or decrement of propositions' mass values is less than 10^{-3} between two adjacent time points.

E. Two-dimensional measure

By using our proposed ECR method, an equilibrium state can be evolved for any given evidences. It could be an ESP if the equilibrium state is stable, as mentioned above; otherwise, it is an unstable equilibrium point, as while circles shown in Fig. 3. However, the problem is not totally solved. Considering this case that two different evidences evolve to a same equilibrium state, how can we distinguish them? An effective measure is necessary. Fortunately, since the evolutionary process is dynamic, we find that, the time evolving to the stable or unstable equilibrium state provides a reasonable measure to reflect such difference.

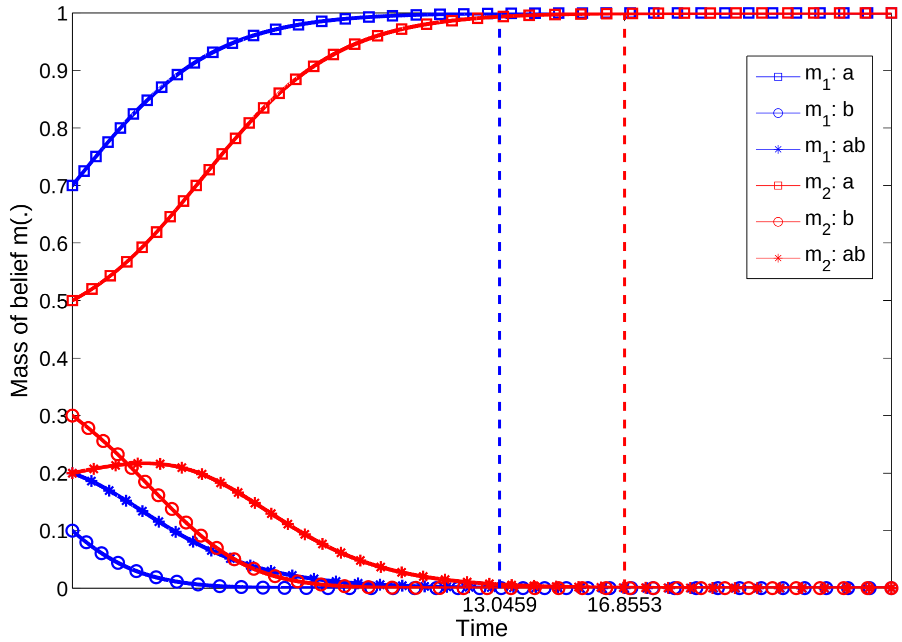

Fig. 5. The evolutionary process of propositions in m_1 and m_2 of Example 4

1) Case of evolving to an ESP: Let us first consider the case that the population evolves to a stable equilibrium state, namely an ESP.

Example 4: Given two evidences, indicated by m_1 and m_2, on a FOD $\Omega = \{a, b\}$,

$m_1(a) = 0.7$, $m_1(b) = 0.1$, $m_1(ab) = 0.2$;

$m_2(a) = 0.5$, $m_2(b) = 0.3$, $m_2(ab) = 0.2$.

The evolutionary process of propositions in m_1 and m_2 are shown in Fig. 5. For evidences m_1 and m_2, the ESPs are both a, namely $ESP_{m_1} = a$, $ESP_{m_2} = a$. But the time evolving to the equilibrium state are different. For m_1 and m_2, the time of reaching proposition a are $t_1 = 13.0459$ and $t_2 = 16.8553$, respectively. From the view of evolution, these results indicate that m_1 is more close to ESP a since proposition a is more supported in m_1.

Example 5: Given a FOD $\Omega = \{a, b, c\}$, there is an evidence m shown as follows,

$m(a) = x$, $m(b) = 0.9 - x$, $m(bc) = 0.05$, $m(abc) = 0.05$.

where $x \in [0, 0.9]$. Now let us investigate the ESP of m and the time evolving to that ESP denoted as t_{ESP} with the change of x from 0 to 0.9 where every increment is 0.01.

Fig. 6 illustrates the results. From the figure, we can see that the ESP of m is proposition b if $x \leq 0.45$, and proposition a if $x \geq 0.46$. When $x \leq 0.45$, t_{ESP} of evolving to proposition b is increasing with the decline of the belief of b due to the ascent of x. When $x \geq 0.46$, $ESP_m = a$, and the belief of a increases with the rise of x, as a result t_{ESP} of evolving

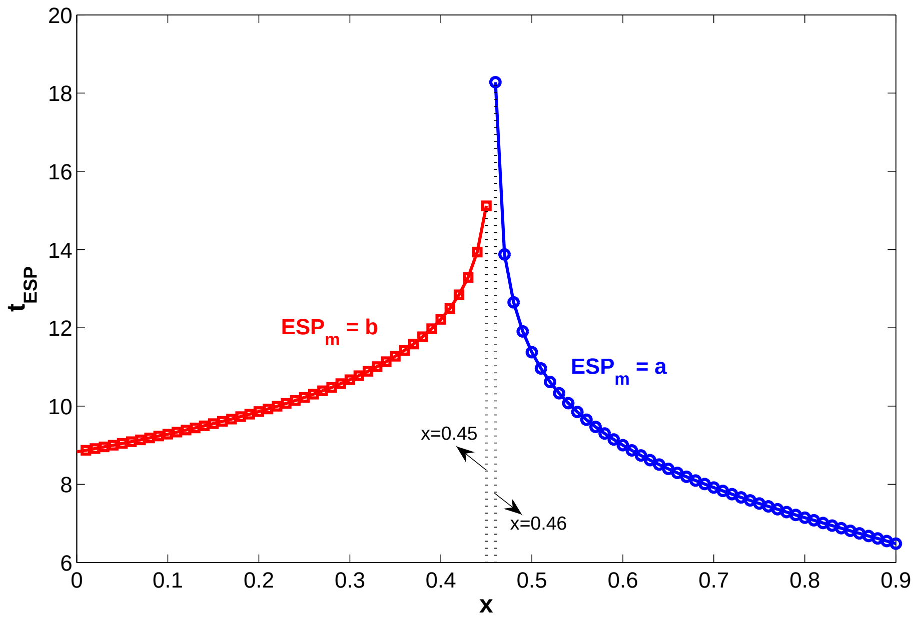

Fig. 6. Evolutionary results in Example 5

to ESP a decreases. This example shows that, regardless the ESP is either a or b, t_{ESP} is changed with the initial configuration of evidence m. Also, even if two evidences evolve to a same ESP, t_{ESP} is lower for this evidence which supports the associated ESP more. In other words, the higher fitness of ESP in an evidence leads to a lower t_{ESP} of real evolving to that ESP.

Therefore, we can say that t_{ESP} measures the temporal cost of an evidence evolving to its associated ESP. From an biology perspective, t_{ESP} is an evolutionary distance from the given evidence to its associated ESP. Based on this consideration, we use t_{ESP} to further depict the relationship between an evidence and its associated ESP. A two-dimensional measure is defined as follows.

Definition 3: Assume there are n evidences indicated by $m_1, \cdots, m_n$, let m be the average of these n evidences, if m evolves to an ESP, the evolutionary output of using the proposed ECR is represented as

$$< ESP_m, t_{ESP} >= f_{ECR}(m_1, \cdots, m_n) \tag{30}$$

where ESP_m is the ESP of m, and t_{ESP} is the time of m evolving to that ESP as a measure of evolutionary distance.

Again, in the ECR, we just want to find the most supported proposition, but not an exactly synthetical evidence, the time of evolving to an ESP provides a reference for the evolutionary distance between the averaging evidence and its associated ESP.

2) Case of evolving to an unstable equilibrium state: It is mentioned that, by using the ECR, the averaging evidence m may do not evolve to a ESP, but to a state where two or more propositions coexist. Mathematically, those states correspond to unstable fixed points, graphically illustrated as the white circles in Fig. 3. A very slight disturbance will cause that m deviates such unstable equilibrium states and goes to an ESP. Therefore, the probability of evolving to such states is very small in general. Even so, considering the completeness of methodology, we also give a definition to formalize the output of ECR in this situation.

Definition 4: Given n evidences m_1, $\cdots$, m_n, for their averaging evidence m which will evolves to an unstable equilibrium state, the evolutionary output of using the proposed ECR can be expressed as

$$< P_m, t_{ES} >= f_{ECR}(m_1, \cdots, m_n) \tag{31}$$

where P_m is set of existent propositions in this unstable equilibrium state, and t_{ES} is the time of m evolving to that equilibrium state.

Examples will be given in the following section. It is worthy noticed that, sometimes, if the population evolves to an unstable equilibrium state, it implies that there are highly conflicting information among the original evidences m_1, $\cdots$, m_n. In this sense, the unstable equilibrium state sometimes can act as an alarm to report the existence of high conflict.

IV. Illustrative examples and analysis

A. Combination of highly conflicting evidences

Conflicting evidence combination [86], [87] is a main concern to verify the effectiveness of combination rules in multi-sources information fusion. Here we will give several classical cases in DST for the verification of the proposed ECR.

Example 6: Zadeh's paradox [18]. Two doctors diagnose a patient, and they agree that the patient suffers from one of these three diseases including meningitis (M), brain tumor (T), and concussion (C). Hence, a FOD is determined as $\Omega = \{M, T, C\}$. Both of these doctors think that a tumor is unlikely, but they hold different opinions for the likely cause. Two diagnosis are given as follows.

$m_1(M) = 0.9$, $m_1(T) = 0.1$, $m_1(C) = 0.0$.

$m_2(M) = 0.0$, $m_2(T) = 0.1$, $m_2(C) = 0.9$.

We can find that these two evidences are highly conflicting. If using the classical Dempter's rule of combination to combine them, as shown in Eqs.(7) and (8), the combination result is

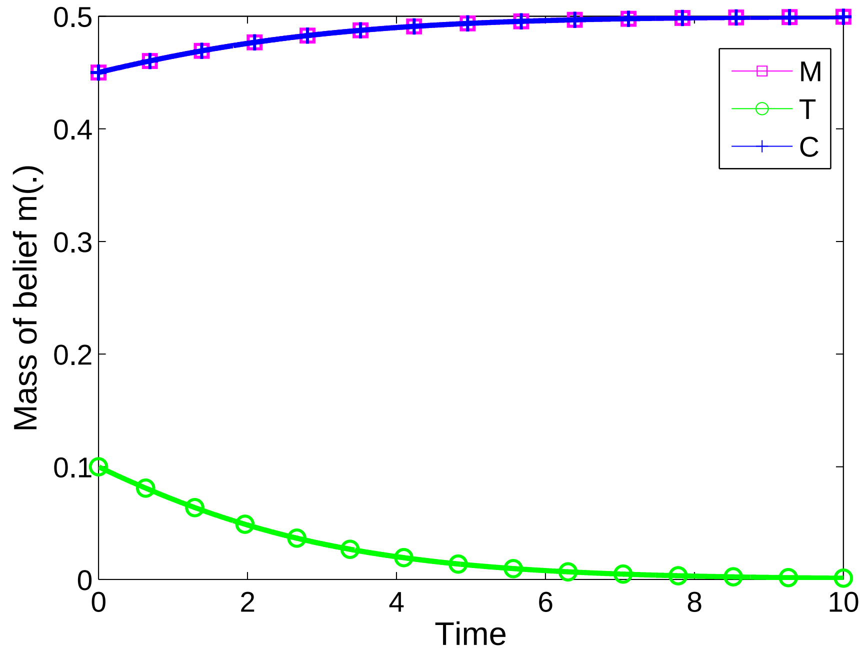

Fig. 7. Evolutionary result of Zadeh's paradox

$m_{\oplus}(M) = 0$, $m_{\oplus}(T) = 1$, $m_{\oplus}(T) = 0$.

and the conflict coefficient $K = 0.99$. This is an apparently counter-intuitive result. The patient most likely does not suffer from tumor in each doctor's diagnosis, but the synthesizing result shows that the patient suffers from tumor with 100%. So it is counter-intuitive. This classical example is first given by Zadeh to show the doubts on the validity of Dempster's rule when information are highly conflicting [18].

Now, let's use the proposed ECR to integrate these two evidences. Assume the weight factor of each doctor is identical, the averaging evidence is calculated as $m(M) = 0.45$, $m(T) = 0.1$, $m(C) = 0.45$. The evolutionary process of each proposition in m is shown in Fig. 7. From this figure, we can find that with the increase of evolutionary time the belief of T goes to 0, and finally propositions M and C averagely share the total belief 1. At the end of evolution, the belief of each proposition becomes $m(M) = 0.5$, $m(T) = 0.0$, $m(C) = 0.5$, which means that M and C coexist finally. The time of evolving to this unstable equilibrium stable is $t_{ES} = 8.7759$. Besides, the instability of this equilibrium stable implies that there are highly conflicting information in m caused by the original evidences m_1 and m_2. The ECR effectively identifies this situation.

Example 7: Modified Zadeh's paradox. Regarding Zadeh's paradox, now let us assume that the two doctors give two outright conflicting diagnosis as follows.

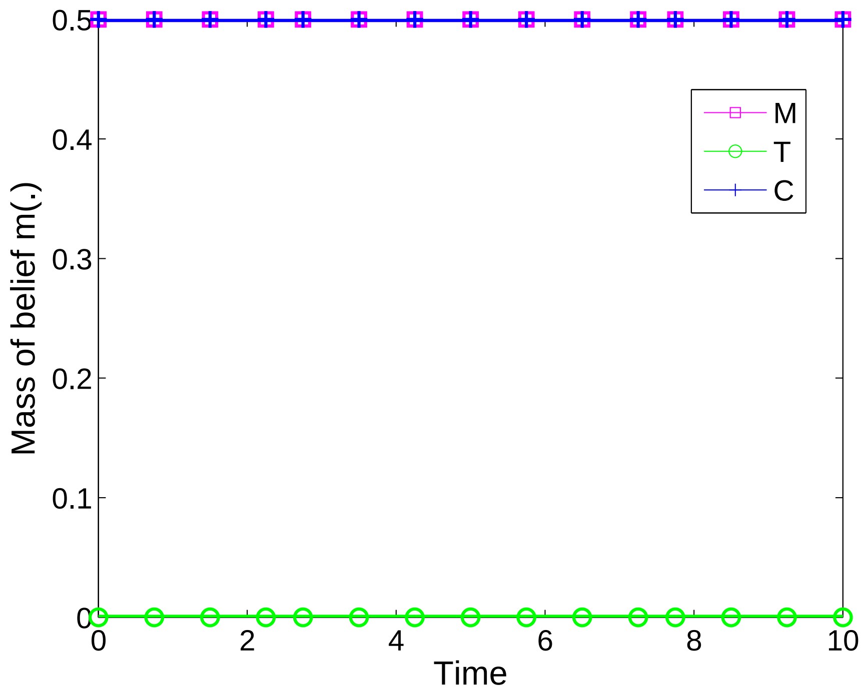

Fig. 8. Evolutionary result of modified Zadeh's paradox

$m_1(M) = 1$, $m_1(T) = 0$, $m_1(C) = 0$.

$m_2(M) = 0$, $m_2(T) = 0$, $m_2(C) = 1$.

Now there is not any agreement between m_1 and m_2. According to Dempster's rule of combination, the conflict coefficient $K = 1$. In this situation, Dempster's rule becomes invalid and can not be used to synthesize them. Instead, let us consider them by using the proposed ECR. The averaging evidence is $m(M) = 0.5$, $m(C) = 0.5$. The evolutionary results are shown in Fig. 8. From that figure, we can find that the belief of each proposition in m does not show any change, as the evolutionary time increases. The average evidence m is exact located at an unstable equilibrium point. Therefore, as a measure of evolutionary distance, the time reaching the equilibrium state is $t_{ES} = 0$ in this case.

These above two examples clearly show that the proposed ECR is effective for these cases where Dempster's rule is questionable. Moreover, by using the ECR, Zadeh's paradox is transformed to a simple mathematical problem, and then explained by the instability of equilibrium point. Even Zadeh's paradox is objectively existent, but it is extremely unstable, and a slight numerical change of the inputs could break away from this dilemma.

B. More illustrative examples

In the following content, we will examine the proposed ECR by using more examples.

Example 8: In [21], [22], the authors have presented an emblematic example to show the inadequate behavior of Dempster's rule of combination. The authors called that behavior as the *dictatorial power* (DP) of Dempster's rule. Specifically, in that example the level of conflict can be chosen at any low or high value, which means that the example is not related to the level of conflict between evidences. A simple version [20] of that example is shown as follows.

Given a FOD $\Theta = \{\theta_1, \theta_2, \theta_3\}$, there are four evidences as below:

$m_1(\theta_1) = a$, $m_1(\theta_1\theta_2) = 1 - a$.

$m_i(\theta_3) = b$, $m_i(\Theta) = 1 - b$, where $i = 2, 3, 4$.

When using Dempster's rule of combination, one gets:

$m_\oplus(\theta_1) = a = m_1(\theta_1)$, $m_\oplus(\theta_1\theta_2) = 1 - a = m_1(\theta_1\theta_2)$.

It clearly shows that Dempster's rule is not responding to the combination of different evidences. It seems that evidence m_1 dominates other evidences since the combination result is always m_1, which does not accord with people's expectation or intuition.

Now let us reconsider this example by using the proposed ECR. Moreover, for the sake of comparison, several improved methods for the combination of evidences, including Murphy's simple average [28], Deng's weighted average [29], Han's sequential weighted combination [20], proportional conflict redistribution (PCR6) rule [63], have also been used to test the results. Because PCR6 is not associative, to get optimal results, the PCR6 rule is implemented in this paper by combining all evidences altogether at the same time. Here, assume $a = 0.7$, $b = 0.6$. The results are listed in Tab. II. As illustrated in that table, the combination results are always the same as m_1 if using Dempster's rule of combination, which is counter-intuitive. However, the counter-intuitive results have been eliminated in the results of Murphy's simple average, Deng's weighted average, Han's sequential weighted combination, and PCR6 rule. Similarly, in the proposed ECR, the counter-intuitive behaviors are also suppressed, which shows the advantage of the proposed method. In every case of combination, the most supported proposition obtained by the ECR totally accords with the results obtained by other reasonable methods. Moreover, the decrease of t_{ESP} indicates that the evolutionary distance to the ESP is reducing with the accumulation of evidences, which further coincides with people's expectation.

Example 9: In [20], the authors studied the problem of target recognition in a multisensor system. In a multisensor-based automatic target recognition system, assume the FOD

TABLE II

RESULTS FOR EXAMPLE 8

Evidences	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4
Dempster's rule of combination	$m(\theta_1) = 0.7000$, $m(\theta_1\theta_2) = 0.3000$.	$m(\theta_1) = 0.7000$, $m(\theta_1\theta_2) = 0.3000$.	$m(\theta_1) = 0.7000$, $m(\theta_1\theta_2) = 0.3000$.
Murphy's simple average	$m(\theta_1) = 0.5250$, $m(\theta_1\theta_2) = 0.1179$, $m(\theta_3) = 0.3000$, $m(\Theta) = 0.0571$.	$m(\theta_1) = 0.3379$, $m(\theta_1\theta_2) = 0.0615$, $m(\theta_3) = 0.5622$, $m(\Theta) = 0.0384$.	$m(\theta_1) = 0.1794$, $m(\theta_1\theta_2) = 0.0292$, $m(\theta_3) = 0.7711$, $m(\Theta) = 0.0203$.
Deng's weighted average	$m(\theta_1) = 0.5250$, $m(\theta_1\theta_2) = 0.1179$, $m(\theta_3) = 0.3000$, $m(\Theta) = 0.0571$.	$m(\theta_1) = 0.1032$, $m(\theta_1\theta_2) = 0.0290$, $m(\theta_3) = 0.8122$, $m(\Theta) = 0.0555$.	$m(\theta_1) = 0.1032$, $m(\theta_1\theta_2) = 0.0093$, $m(\theta_3) = 0.9344$, $m(\Theta) = 0.0246$.
Han's sequential weighted combination	$m(\theta_1) = 0.5250$, $m(\theta_1\theta_2) = 0.1179$, $m(\theta_3) = 0.3000$, $m(\Theta) = 0.0571$.	$m(\theta_1) = 0.2362$, $m(\theta_1\theta_2) = 0.0363$, $m(\theta_3) = 0.6369$, $m(\Theta) = 0.0906$.	$m(\theta_1) = 0.0676$, $m(\theta_1\theta_2) = 0.0089$, $m(\theta_3) = 0.8298$, $m(\Theta) = 0.0937$.
PCR6 rule	$m(\theta_1) = 0.5062$, $m(\theta_1\theta_2) = 0.1800$, $m(\theta_3) = 0.3138$.	$m(\theta_1) = 0.3432$, $m(\theta_1\theta_2) = 0.1028$, $m(\theta_3) = 0.4306$, $m(\Theta) = 0.1234$.	$m(\theta_1) = 0.2464$, $m(\theta_1\theta_2) = 0.0642$, $m(\theta_3) = 0.4921$, $m(\Theta) = 0.1973$.
The proposed ECR	$< ESP = \theta_1$, $t_{ESP} = 21.4913 >$	$< ESP = \theta_3$, $t_{ESP} = 21.6188 >$	$< ESP = \theta_3$, $t_{ESP} = 16.0809 >$

is $\Theta = \{\theta_1, \theta_2, \theta_3\}$. For an unknown target, the system has collected five evidences shown as follows.

$m_1(\theta_1) = 0.60$, $m_1(\theta_2) = 0.10$, $m_1(\theta_2, \theta_3) = 0.30$.

$m_2(\theta_1) = 0.65$, $m_2(\theta_2) = 0.10$, $m_2(\theta_3) = 0.25$.

$m_3(\theta_1) = 0.00$, $m_3(\theta_2) = 0.90$, $m_3(\theta_2, \theta_3) = 0.10$.

$m_4(\theta_1) = 0.55$, $m_4(\theta_2) = 0.10$, $m_4(\theta_2, \theta_3) = 0.35$.

$m_5(\theta_1) = 0.55$, $m_5(\theta_2) = 0.10$, $m_5(\theta_2, \theta_3) = 0.35$.

By using different methods, the combination results are derived, as shown in Tab. III. From Tab. III, in the combination results based on Dempter's rule, $m(\theta_1)$ always equals to

TABLE III

RESULTS FOR EXAMPLE 9

Evidences	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
Dempster's rule of combination	$m(\theta_1) = 0.7723$, $m(\theta_2) = 0.0792$, $m(\theta_3) = 0.1485$.	$m(\theta_1) = 0.0000$, $m(\theta_2) = 0.8421$, $m(\theta_3) = 0.1579$.	$m(\theta_1) = 0.0000$, $m(\theta_2) = 0.8727$, $m(\theta_3) = 0.1273$.	$m(\theta_1) = 0.0000$, $m(\theta_2) = 0.8981$, $m(\theta_3) = 0.1019$.
Murphy's simple average	$m(\theta_1) = 0.7716$, $m(\theta_2) = 0.0790$, $m(\theta_3) = 0.1049$, $m(\theta_2\theta_3) = 0.0444$.	$m(\theta_1) = 0.3526$, $m(\theta_2) = 0.5978$, $m(\theta_3) = 0.0380$, $m(\theta_2\theta_3) = 0.0116$.	$m(\theta_1) = 0.5167$, $m(\theta_2) = 0.4573$, $m(\theta_3) = 0.0189$, $m(\theta_2\theta_3) = 0.0071$.	$m(\theta_1) = 0.6706$, $m(\theta_2) = 0.3169$, $m(\theta_3) = 0.0088$, $m(\theta_2\theta_3) = 0.0037$.
Deng's weighted average	$m(\theta_1) = 0.7716$, $m(\theta_2) = 0.0790$, $m(\theta_3) = 0.1049$, $m(\theta_2\theta_3) = 0.0444$.	$m(\theta_1) = 0.6013$, $m(\theta_2) = 0.3302$, $m(\theta_3) = 0.0532$, $m(\theta_2\theta_3) = 0.0153$.	$m(\theta_1) = 0.7987$, $m(\theta_2) = 0.1698$, $m(\theta_3) = 0.0227$, $m(\theta_2\theta_3) = 0.0088$.	$m(\theta_1) = 0.8975$, $m(\theta_2) = 0.0897$, $m(\theta_3) = 0.0088$, $m(\theta_2\theta_3) = 0.0040$.
Han's sequential weighted combination	$m(\theta_1) = 0.7716$, $m(\theta_2) = 0.0790$, $m(\theta_3) = 0.1049$, $m(\theta_2\theta_3) = 0.0444$.	$m(\theta_1) = 0.5591$, $m(\theta_2) = 0.4021$, $m(\theta_3) = 0.0297$, $m(\theta_2\theta_3) = 0.0091$.	$m(\theta_1) = 0.6781$, $m(\theta_2) = 0.3054$, $m(\theta_3) = 0.0071$, $m(\theta_2\theta_3) = 0.0093$.	$m(\theta_1) = 0.8103$, $m(\theta_2) = 0.1797$, $m(\theta_3) = 0.0014$, $m(\theta_2\theta_3) = 0.0086$.
PCR6 rule	$m(\theta_1) = 0.7371$, $m(\theta_2) = 0.0644$, $m(\theta_3) = 0.1370$, $m(\theta_2\theta_3) = 0.0615$.	$m(\theta_1) = 0.4224$, $m(\theta_2) = 0.4729$, $m(\theta_3) = 0.0483$, $m(\theta_2\theta_3) = 0.0564$.	$m(\theta_1) = 0.4755$, $m(\theta_2) = 0.3849$, $m(\theta_3) = 0.0351$, $m(\theta_2\theta_3) = 0.1045$.	$m(\theta_1) = 0.5111$, $m(\theta_2) = 0.3244$, $m(\theta_3) = 0.0276$, $m(\theta_2\theta_3) = 0.1369$.
The proposed ECR	$< ESP = \theta_1$, $t_{ESP} = 7.5708 >$	$< ESP = \theta_2$, $t_{ESP} = 19.3900 >$	$< ESP = \theta_1$, $t_{ESP} = 11.5065 >$	$< ESP = \theta_1$, $t_{ESP} = 10.6493 >$

0 after combining m_3, while regardless of the support for θ_1 in m_4 and m_5. Evidently, this is counter-intuitive. In contrast, as we can see in Tab. III, based on the other four methods, the counter-intuitive results are suppressed. In Deng's weighted average and Han's sequential weighted combination, the most supported is always θ_1 for any case, and the support for θ_1 is increasing after the arrival of m_4 and m_5. For Murphy's simple average, PCR6 rule, and the proposed ECR, even though the most supported is θ_2 in the case of combining m_1, m_2 and m_3, but it promptly changes to θ_1 as soon as the arrival of m_4, which also overcomes the counter-intuitive behavior. Moreover, in the proposed ECR, the increase of the support

TABLE IV

RESULTS FOR EXAMPLE 10

The number of evidences l	$l = 10$	$l = 25$	$l = 50$
Dempster's rule of combination	$m(\theta_1) = 0.4614$, $m(\Theta) = 0.5386$.	$m(\theta_1) = 0.7871$, $m(\Theta) = 0.2129$.	$m(\theta_1) = 0.9547$, $m(\Theta) = 0.0453$.
Murphy's simple average	$m(\theta_1) = 0.4614$, $m(\Theta) = 0.5386$.	$m(\theta_1) = 0.7871$, $m(\Theta) = 0.2129$.	$m(\theta_1) = 0.9547$, $m(\Theta) = 0.0453$.
Deng's weighted average	$m(\theta_1) = 0.4614$, $m(\Theta) = 0.5386$.	$m(\theta_1) = 0.7871$, $m(\Theta) = 0.2129$.	$m(\theta_1) = 0.9547$, $m(\Theta) = 0.0453$.
Han's sequential weighted combination	$m(\theta_1) = 0.3195$, $m(\Theta) = 0.6805$.	$m(\theta_1) = 0.3684$, $m(\Theta) = 0.6316$.	$m(\theta_1) = 0.3924$, $m(\Theta) = 0.6076$.
PCR6 rule	$m(\theta_1) = 0.4614$, $m(\Theta) = 0.5386$.	$m(\theta_1) = 0.7871$, $m(\Theta) = 0.2129$.	$m(\theta_1) = 0.9547$, $m(\Theta) = 0.0453$.
The proposed ECR	$< ESP = \Theta$, $t_{ESP} = 6.4296 >$	$< ESP = \Theta$, $t_{ESP} = 6.4296 >$	$< ESP = \Theta$, $t_{ESP} = 6.4296 >$

for θ_1 is expressed by means of the decline of t_{ESP} which implies the evolutionary distance to θ_1 is reduced. Therefore, our proposed ECR is still effective in this example.

Example 10: Evidence shifting paradox [19], [20] describes another counter-intuitive behavior in Demspter-Shafer evidence theory. Let us consider this scene that a target is evaluated by l different experts with the same importance. The FOD is $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Each expert gives an identical assessment as below.

$m_i(\theta_1) = 0.06$, $m_i(\Theta) = 0.94$, where $i = 1, \cdots, l$.

By using Dempster's rule, the combination result is shown as follows.

$m_{\oplus}(\theta_1) = 1 - 0.94^l$, $m_{\oplus}(\Theta) = 0.94^l$.

If l is a big number, for example 100, $m_{\oplus}(\theta_1) = 1 - 0.94^{100} = 0.9979$ which is very large although for each evidence to be combined $m_i(\theta_1) = 0.06$ which is very small. The result shows that the aggregation of the wisdom of crowds may generate counter-intuitive results by using Dempster's rule of combination.

Now, let's study this paradox by using the proposed ECR as well as other improved methods. The results derived based on different methods are listed in Tab. IV. As illustrated in Tab. IV, with the rise of the number of evidences to be combined, Dempster's rule of combination, Murphy's simple average, Deng's weighted average, and PCR6 rule all generate counter-intuitive results. Although Han's sequential weighted combination brings out reasonable results, but in that method $m(\theta_1)$ has a rising trend as l increases. A counter-intuitive result that $m(\theta_1) > 0.5$ would also produce when l becomes large enough. Only the proposed ECR brings out the most reasonable result that the most supported proposition is always Θ, and the evolutionary time also does not change with the increase of l. Here, the ECR presents a similarly idempotent character which helps to settle the evidence shifting paradox.

V. Conclusion

In this paper, we have proposed an evolutionary combination rule (ECR) for the evidence-based multi-source information fusion from an evolutionary game theory perspective. Within the proposed framework of ECR, original evidences are averaged by their weights, and a Jaccard matrix game is presented to formalize the interaction relationship between propositions, then we use the replicator dynamics equation to mimick the evolution of population, finally a proposition with the highest fitness is identified as the biologically most supported or possible conclusion. Experimental results show that the proposed ECR has suppressed the counter-intuitive results caused by the classical Dempster's rule of combination, which demonstrates the rationality and effectiveness of the proposed method.

In this work, we import a biological and evolutionary idea into DST, which is not presented in previous studies and contributes a new insight for multi-source information fusion. Based on our proposed ECR, the most supported proposition, in the biological sense, can be found for decision-making. Of course, the ECR is still of some problems to be solved in the future research. We summarized several noticeable issues as follows.

Firstly, the ECR is not associative, all evidences must be combined together at the same time. Two main reasons, average mechanism and replicator equation, lead to the non-associativity. In the initial stage of ECR, multi-source information is synthesized by average mechanism which is also used in many existing evidence combination approaches, such as Murphy's simple average [28], Deng's weighted average [29], Han's sequential weighted combination [20]. As a result, all these approaches are not associative. In addition, the replicator equation ruling the evolution of mass value of each proposition does not meet associativity as well.

Secondly, the ECR does not preserve the neutrality for a vacuous evidence $m_v(\Theta) = 1$ where Θ is the FOD, namely $m \oplus_{ECR} m_v \neq m$. In the framework of ECR, as same as other proposition $A \subseteq \Theta$, the FOD Θ is regarded as a strategy which can be adopted by individuals in an assumed population. The vacuous evidence $m_v(\Theta) = 1$ does not means totally unknown, but means that every individual adopts strategy Θ in the population. So, the ECR does not meet the neutrality.

Thirdly, some counter-intuitive results are still hard to interpret in the framework of ECR at present. For example, in [37] the author presented an example that combines $m_1(A) = m_1(B, C) = 0.5$ and $m_2(C) = m_2(A, B) = 0.5$. Dempster's rule gives $m(A) = m(B) = m(C) = 1/3$. As reported by Voorbraak [37], this result is counter-intuitive, since intuitively B seems to share twice a probability mass of 0.5, while both A and C only have to share once 0.5 with B and are once assigned 0.5 individually. So intuitively, B is less confirmed than A and C, but they are equally confirmed by Dempster's rule. By using the proposed ECR, we obtain $m(AB) = 0.5$, $m(BC) = 0.5$ which corresponds to an unstable equilibrium state. The results seem to also be counter-intuitive, but the ECR is unable to interpret currently.

In summary, although there are some drawbacks in the proposed ECR at present, this work is still meaningful and innovative because of the exploiting of biological and evolutionary standpoint for evidence combination. In many cases, it is effective and useful. In the future research, we will continue to improve and perfect the framework of ECR.

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