The notions of mar constants and Smarandache mar constants

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Abstract. In one of my previous paper, "The mar reduced form of a natural number", I introduced the notion of mar function, which is, essentially, nothing else than the digital root of a number, but defined as an aritmethical function, on the operations of addition, multiplication etc. in such way that it could be used in various applications (Diophantine equations, divizibility problems and others). In this paper I present two notions, useful in Diophantine analysis of Smarandache concatenated sequences or different classes of numbers (sequences of squares, cubes, triangular numbers, polygonal numbers, Devlali numbers, Demlo numbers etc).

(I) THE NOTION OF MAR CONSTANTS

Definition:

We understand by "mar constants" the numbers with n digits obtained by concatenation from the mar values of the first n terms of an infinite sequence of non-null positive integers, if the mar values of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n. We consider that it is interesting to see, from some well known sequences of positive integers, which one is characterized by a mar constant and which one it isn't.

Example:

The mar values of the terms of the cubic numbers sequence (1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, ...) are 1, 8, 9, 1, 8, 9 (...) so these values form a sequence with a periodicity equal to three, the terms 1, 8, 9 repeating infinitely. Concatenating these three mar values is obtained a mar constant, i.e. the number 189.

Let's take the following sequences:

(1) The cubic numbers sequence

S_n is the sequence of the cubes of positive integers and, as it can be seen in the example above, is characterized by a mar constant with three digits, the number 189.

(2) The square numbers sequence

S_n is the sequence of the square of positive integers (A000290 in OEIS): 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441 (...) and is characterised by a mar constant with nine digits, the number 149779419.

(3) The triangular numbers sequence

S_n is the sequence of the numbers of the form (n*(n + 1))/2 = 1 + 2 + 3 + ... + n (A000217 in OEIS): 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300 (...) and is characterised by a mar constant with nine digits, the number 136163199.

(4) The centered square numbers sequence

S_n is the sequence of the numbers of the form m = 2*n*(n + 1) + 1 (A001844 in OEIS): 1, 5, 13, 25, 41, 61, 85, 113, 145, 181, 221, 265, 313, 365, 421, 481, 545, 613 (...) and is characterised by a mar constant with nine digits, the number 154757451.

(5) The centered triangular numbers sequence

S_n is the sequence of the numbers of the form m = 3*n*(n + 1)/ 2 + 1 (A005448 in OEIS): 1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274, 316, 361, 409, 460 (...) and is characterised by a mar constant with three digits, the number 141.

(6) The Devlali numbers sequence

S_n is the sequence of the Devlali numbers (defined by the Indian mathematician D.R. Kaprekar, born in Devlali), which are the numbers that can not be expressed like n + S(n), where n is integer and S(n) is the sum of the digits of n. The sequence of these numbers is (A003052 in OEIS): 1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86, 97, 108, 110, 121, 132, 143, 154, 165, 176, 187, 198 (...).

This sequence is characterized by a mar constant with 9 digits, the number 135792468.

(7) The Demlo numbers sequence

S_n is the sequence of the Demlo numbers (defined by the Indian mathematician D.R. Kaprekar and named by him after a train station near Bombay), which are the numbers of the form (10^n – 1)/9)^2. The sequence of these numbers is (A002477 in OEIS): 1, 121, 12321, 1234321, 123454321, 12345654321, 1234567654321, 123456787654321, 12345678987654321, 1234567900987654321 (...).

This sequence is characterized by a mar constant with 9 digits, the number 149779419.

Comment:

I conjecture that any sequence of polygonal numbers, *i.e.* numbers with generic formula ((k^2*(n – 2) – k*(n – 4))/2, is characterized by a Smarandache mar constant:

: The sequence of pentagonal numbers, numbers of the form n*(3*n – 1)/2, *i.e.* 1, 5, 12, 22, 35, 51, 70, 92, 117, 145, 176, 210, 247, 287, 330, 376, 425, 477, 532, 590, ...(A000326) is characterized by the mar constant 153486729;
: The sequence of hexagonal numbers, numbers of the form n*(2*n – 1), *i.e.* 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435, 496, 561, 630, 703, 780, ...(A000326) is characterized by the mar constant 166193139 etc.

Conclusion:

We found so far eight mar constants, six with nine digits, *i.e.* the numbers 149779419, 136163199, 154757451, 135792468, 153486729, 166193139 and two with three digits, *i.e.* the numbers 189 and 141.

(II) THE VALUE OF MAR FUNCTION FOR FEW KNOWN CLASSES OF NUMBERS

There are some known sequences of integers whose terms can only have some certain values for mar function, though these few values don’t seem to have a periodicity, in other words the sequences don’t seem to be characterized by a mar constant. Such sequences are:

(1) The EPRN numbers sequence

S_n is the sequence of the EPRN numbers (defined by the Indian mathematician Shyam Sunder Gupta), which are the numbers that can be expressed in at least two different ways as the product of a number and its reversal (for instance, such a number is 2520 = 120*021 = 210*012). The sequence of these numbers is (A066531 in OEIS): 2520, 4030, 5740, 7360, 7650, 9760, 10080, 12070, 13000, 14580, 14620, 16120, 17290, 18550, 19440 (...). Though the value of mar function for the terms of this sequence can only be 1, 4, 7 or 9, the sequence of the values of mar function (9, 7, 7, 7, 9, 4, 9, 1, 4, 9, 4, 1, 1, 1, 9, ...) don't seem to have a periodicity.

(2) The congrua numbers sequence

S_n is the sequence of the congrua numbers n, numbers which are the possible solutions to the *congruum problem* (n = x^2 – y^2 = z^2 – x^2). The sequence of these numbers is (A057102 in OEIS): 24, 96, 120, 240, 336, 384, 480, 720, 840, 960, 1320, 1344, 1536, 1920, 1944, 2016, 2184, 2520, 2880, 3360 (...). Though the value of mar function for the terms of this sequence can only be 3, 6 or 9, the sequence of the values of mar function (6, 6, 3, 6, 3, 6, 3, 9, 3, 6, 6, 3, 6, 3, 9, 9, 6, 9, ...) don't seem to have a periodicity.

(III) THE SMARANDACHE CONCATENATED SEQUENCES AND THE DEFINITION OF SMARANDACHE MAR CONSTANTS

Definition:

We understand by "Smarandache mar constants" the numbers with n digits obtained by concatenation from the mar values of the first n terms of a Smarandache concatenated sequence, if the mar values of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n. Note that not every Smarandache concatenated sequence is characterized by a Smarandache mar constant, just some of them; it is interesting to study what are the properties these sequences have in common; it is also interesting that sometimes more such sequences have the same value of Smarandache mar constant and also to study what these have in common.

Example:

The mar values of the terms of the Smarandache consecutive sequence (12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910, 1234567891011,

...) are: 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9 (...) so these values form a sequence with a periodicity equal to nine, the terms 1, 3, 6, 1, 6, 3, 1, 9, 9 repeating infinitely. Concatenating these nine mar values is obtained a Smarandache mar constant, i.e. the number 136163199.

Let's take the following Smarandache concatenated sequences:

(1) The Smarandache consecutive numbers sequence

S_n is defined as the sequence obtained through the concatenation of the first n positive integers. The first ten terms of the sequence (A007908 in OEIS) are 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

This sequence is characterized by a Smarandache mar constant with 9 digits, the number 136163199. Note that, obviously, the same constant will be obtained from the Smarandache reverse sequence (A000422), defined as the sequence obtained through the concatenation of the first n positive integers, in reverse order.

(2) The Smarandache concatenated odd sequence

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n - 1). The first ten terms of the sequence (A019519 in OEIS) are 1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 149779419.

(3) The Smarandache concatenated even sequence

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n-th term of the sequence is formed through the concatenation of the even numbers from 1 to 2*n). The first ten terms of the sequence (A019520 in OEIS) are 2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 263236299.

(4) The concatenated cubic sequence

S_n is defined as the sequence obtained through the concatenation of the first n cubes: 1(2^3)(3^3)...(n^3). The first ten terms of the sequence (A019522 in OEIS) are 1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

This sequence is characterized by a Smarandache mar constant with three digits, the number 199.

(5) The antysimmetric numbers sequence

S_n is defined as the sequence obtained through the concatenation in the following way: 12...(n)12...(n). The first ten terms of the sequence (A019524 in OEIS) are 11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 26323629. Note that the same Smarandache mar constant characterizes the Smarandache concatenated even sequence.

(6) The "n concatenated n times" sequence

S_n is defined as the sequence of the numbers obtained concatenating n times the number n. The first ten terms of the sequence (A000461 in OEIS) are 1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 149779419. Note that the same Smarandache mar constant characterizes the Smarandache concatenated odd sequence.

(7) The permutation sequence

S_n is defined as the sequence of numbers obtained through concatenation and permutation in the following way: 13...(2*n – 3)(2*n – 1)(2*n)(2*n – 2)(2*n – 4)...42. The first seven terms of the sequence (A007943 in OEIS) are 12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 313916619.

(8) The Smarandache n2*n sequence

S_n is defined as the sequence for which the n-th term a(n) is obtained concatenating the numbers n and 2*n. The first twelve terms of the sequence (A019550 in OEIS) are 12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224.

This sequence is characterized by a Smarandache mar constant with three digits, the number 369.

(9) The Smarandache nn^2 sequence

S_n is defined as the sequence for which the n-th term a(n) is obtained concatenating the numbers n and n^2. The first fifteen terms of the sequence (A053061 in OEIS) are 11, 24, 39, 416, 525, 636, 749, 864, 981, 10100, 11121, 12144, 13169, 14196, 15225.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 26323629. Note that the same Smarandache mar constant characterizes the Smarandache concatenated even sequence and the Smarandache antysimmetric numbers sequence.

(10) The Smarandache power stack sequence for k = 2

$S_n(k)$ is the sequence for which the n-th term is defined as the positive integer obtained by concatenating all the powers of k from k^0 to k^n. The first ten terms of the sequence are 1, 12, 124, 1248, 12416, 1241632, 124163264, 124163264128, 124163264128256, 124163264128256512.

This sequence is characterized by a Smarandache mar constant with six digits, the number 137649.

Comments:

(1) I conjecture that any sequence of the type nk*n is characterized by a Smarandache mar constant:

: for k = 3 the sequence 13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236 is characterized by the Smarandache mar constant 483726159;
: for k = 4 the sequence 14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248 is characterized by the Smarandache mar constant 516273849 etc.

(2) I conjecture that any sequence of the type nn^k is characterized by a Smarandache mar constant:

: for k = 3 the sequence 11, 28, 327, 464, 5125, 6216, 7343, 8512, 9729, 101000, 111331 is characterized by the Smarandache mar constant 213546879 etc.

(3) Not any power stack sequence is characterized by a Smarandache mar constant:

: for k = 3 the Smarandache sequence is 1, 13, 139, 13927, 1392781, 1392781243, 1392781243729, 13927812437292187 and the values of mar function for the terms of the sequence are 1, 4, 4, 4 (...), the digit 4 repeating infinitely so is not a sequence characterized by a Smarandache mar constant.

(4) I conjecture that not any sequence with the general term of the form 1(2^k)(3^k)...(n^k) is characterized by a Smarandache mar constant:

: the values of mar function for the terms of the concatenated square sequence 1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100, ... (A019521 in OEIS) are 1, 5, 5, 3, 1, 1, 5, 6, 6, 7, 2, 2 (...) and so far has not been shown any periodicity.

Conclusion:

We found so far 10 Smarandache mar constants, 7 with nine digits, *i.e.* the numbers 136163199, 149779419, 26323629, 313916619, 483726159, 516273849, 213546879, two with three digits, *i.e.* the numbers 199 and 369, and one with six digits, the number 137649.