

An infinite sequence based on mar function that abounds in primes

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Abstract. I introduced, in one of my previous paper, namely "The mar reduced form of a natural number", the notion of mar function, which is, essentially, nothing else than the digital root of a number, but defined as an arithmetical function, in such way that it could be used in various applications (Diophantine analysis of different types of numbers etc). In this paper I present a sequence based on a relation between a number and the value of its mar reduced form (of course not the intrinsic one), sequence that seem to be interesting because many of its terms are primes or are equal to 1 and very few composites.

Let's consider the sequence of numbers $m(n)$, where n is odd, and m is the smallest integer such that $m + n$ is divisible both with $x = 2 \cdot \text{mar } n + 2$ and $y = 2 \cdot \text{mar } n - 2$, if $y \neq 0$, respectively just with x if $y = 0$:

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:   for n = 1, we have mar n = 1, x = 4 and y = 0 so m = 3;
:   for n = 3, we have mar n = 3, x = 8 and y = 4; the
      smallest integer such that n + m is divisible both with 4
      and 8 is 5 so m = 5;
:   for n = 5, we have mar n = 5, x = 12 and y = 8 so m = 19;
:   for n = 7, we have mar n = 7, x = 16 and y = 12 so m =
      41;
:   for n = 9, we have mar n = 9, x = 20 and y = 16 so m =
      71;
:   for n = 11, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 13, we have mar n = 4, x = 10 and y = 6 so m =
      17;
:   for n = 15, we have mar n = 6, x = 14 and y = 10 so m =
      55;
:   for n = 17, we have mar n = 8, x = 18 and y = 14 so m =
      109;
:   for n = 19, we have mar n = 1, x = 4 and y = 0 so m = 1;
:   for n = 21, we have mar n = 3, x = 8 and y = 4 so m = 3;
:   for n = 23, we have mar n = 5, x = 12 and y = 8 so m = 1;
:   for n = 25, we have mar n = 7, x = 16 and y = 12 so m =
      23;
:   for n = 27, we have mar n = 9, x = 20 and y = 16 so m =
      43;
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:   for n = 29, we have mar n = 2, x = 6 and y = 2 so m = 23;
:   for n = 31, we have mar n = 4, x = 10 and y = 6 so m =
29;
:   for n = 33, we have mar n = 6, x = 14 and y = 10 so m =
37;
:   for n = 35, we have mar n = 8, x = 18 and y = 14 so m =
91;
:   for n = 37, we have mar n = 1, x = 4 and y = 0 so m = 3;
:   for n = 39, we have mar n = 3, x = 8 and y = 4 so m = 1;
:   for n = 41, we have mar n = 5, x = 12 and y = 8 so m = 7;
:   for n = 43, we have mar n = 7, x = 16 and y = 12 so m =
5;
:   for n = 45, we have mar n = 9, x = 20 and y = 16 so m =
35;
:   for n = 47, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 49, we have mar n = 4, x = 10 and y = 6 so m =
11;
:   for n = 51, we have mar n = 6, x = 14 and y = 10 so m =
19;
:   for n = 53, we have mar n = 8, x = 18 and y = 14 so m =
73;
:   for n = 55, we have mar n = 1, x = 4 and y = 0 so m = 1;
:   for n = 57, we have mar n = 3, x = 8 and y = 4 so m = 7;
:   for n = 59, we have mar n = 5, x = 12 and y = 8 so m =
13;
:   for n = 61, we have mar n = 7, x = 16 and y = 12 so m =
35;
:   for n = 63, we have mar n = 9, x = 20 and y = 16 so m =
17;
:   for n = 65, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 67, we have mar n = 4, x = 10 and y = 6 so m =
23;
:   for n = 69, we have mar n = 6, x = 14 and y = 10 so m =
1;
:   for n = 71, we have mar n = 8, x = 18 and y = 14 so m =
55;
:   for n = 73, we have mar n = 1, x = 4 and y = 0 so m = 3;
:   for n = 75, we have mar n = 3, x = 8 and y = 4 so m = 5;
:   for n = 77, we have mar n = 5, x = 12 and y = 8 so m =
19;
:   for n = 79, we have mar n = 7, x = 16 and y = 12 so m =
17;
:   for n = 81, we have mar n = 9, x = 20 and y = 16 so m =
79;
:   for n = 83, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 85, we have mar n = 4, x = 10 and y = 6 so m = 5;
:   for n = 87, we have mar n = 6, x = 14 and y = 10 so m =
53;
:   for n = 89, we have mar n = 8, x = 18 and y = 14 so m =
37;
:   for n = 91, we have mar n = 1, x = 4 and y = 0 so m = 1;

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:   for n = 93, we have mar n = 3, x = 8 and y = 4 so m = 3;
:   for n = 95, we have mar n = 5, x = 12 and y = 8 so m = 1;
:   for n = 97, we have mar n = 7, x = 16 and y = 12 so m =
47;
:   for n = 99, we have mar n = 9, x = 20 and y = 16 so m =
61;
:   for n = 101, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 103, we have mar n = 4, x = 10 and y = 6 so m =
17;
:   for n = 105, we have mar n = 6, x = 14 and y = 10 so m =
35;
:   for n = 107, we have mar n = 8, x = 18 and y = 14 so m =
19;
:   for n = 109, we have mar n = 1, x = 4 and y = 0 so m = 3;
:   for n = 111, we have mar n = 3, x = 8 and y = 4 so m = 1;
:   for n = 113, we have mar n = 5, x = 12 and y = 8 so m =
7;
:   for n = 115, we have mar n = 7, x = 16 and y = 12 so m =
29;
:   for n = 117, we have mar n = 9, x = 20 and y = 16 so m =
43;
:   for n = 119, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 121, we have mar n = 4, x = 10 and y = 6 so m =
29;
:   for n = 123, we have mar n = 6, x = 14 and y = 10 so m =
17;
:   for n = 125, we have mar n = 8, x = 18 and y = 14 so m =
1;
:   for n = 127, we have mar n = 1, x = 4 and y = 0 so m = 1;
:   for n = 129, we have mar n = 3, x = 8 and y = 4 so m = 7;
:   for n = 131, we have mar n = 5, x = 12 and y = 8 so m =
13;
:   for n = 133, we have mar n = 7, x = 16 and y = 12 so m =
11;
:   for n = 135, we have mar n = 9, x = 20 and y = 16 so m =
25;
:   for n = 137, we have mar n = 2, x = 6 and y = 2 so m = 1;
:   for n = 139, we have mar n = 4, x = 10 and y = 6 so m =
11;
:   for n = 141, we have mar n = 6, x = 14 and y = 10 so m =
69;
:   for n = 143, we have mar n = 8, x = 18 and y = 14 so m =
109;
:   for n = 145, we have mar n = 9, x = 20 and y = 16 so m =
35;
:   for n = 147, we have mar n = 3, x = 8 and y = 4 so m = 5;
:   for n = 149, we have mar n = 5, x = 12 and y = 8 so m =
19;
:   for n = 151, we have mar n = 7, x = 16 and y = 12 so m =
41;

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- : for $n = 153$, we have $\text{mar } n = 9$, $x = 20$ and $y = 16$ so $m = 7$;
- : for $n = 155$, we have $\text{mar } n = 2$, $x = 10$ and $y = 6$ so $m = 25$;
- : for $n = 157$, we have $\text{mar } n = 4$, $x = 10$ and $y = 6$ so $m = 23$;
- : for $n = 159$, we have $\text{mar } n = 6$, $x = 14$ and $y = 10$ so $m = 41$.

So the sequence $m(n)$ is:

3, 5, 19, 41, 71, 1, 17, 55, 109, 1, 3, 1, 23, 43, 23,
 29, 37, 91, 3, 1, 7, 5, 35, 1, 11, 19, 73, 1, 7, 13, 35,
 17, 1, 23, 1, 55, 3, 5, 19, 17, 79, 1, 5, 53, 37, 1, 3,
 1, 47, 61, 17, 35, 19, 3, 1, 7, 29, 43, 1, 29, 17, 1, 1,
 7, 13, 11, 25, 1, 11, 69, 109, 5, 19, 41, 7, 25, 23, 41
 (...)

Comments:

- (1) It is notable that, from the first 78 terms of this sequence, just 11 terms are composites, 16 terms are equal to one and the rest of 51 terms are primes!
- (2) From obvious reasons (because, corresponding to $\text{mar } n = 8$, respectively to $[x, y] = [18, 14]$, the largest number which should divide $m + n$ is 126), the sequence above can't have any term larger than 125.