New formula of the mobius function

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Theorem1

$$\sum_{1\le n\le x}\mu(n)[\frac{x}{n}]=1$$

Proof)

$$x=1\Rightarrow 1=1$$

$$x=2\Rightarrow 2-1=1$$

$$x=3\Rightarrow 3-1-1=1$$

$x=29$ and $x=30$ cases
$1-1-1-1+1+1+1-1=0$ increase.

$$\sum_{1\le n\le 30}\mu(n)[\frac{30}{n}]=1$$

General cases are similally.

Theorem1

$$\sum_{1\le n\le x}\mu(n)[\frac{x}{n}]=1$$

Onother proof)

$$\sum_{n|m}\mu(n)=1(m=1)$$

$$\sum_{n|m}\mu(n)=0(m\neq 1)$$

Clearly,theorem1 is got.

More genelally,

Theorem2
For real number x.(For example $x = 1.5$)

$$\sum_{1 \le n \le x} \mu(n)[\frac{x}{n}] = 1$$

Remark:

$$\sum_{1 \le n \le x} \mu(n) = O(\sum_{1 \le n \le x} \mu(n)[\frac{x}{n}]) = O(1)$$

is not true.