

Explanation of the Allais Effect by Gravitational Waves Emitted from the Center of the Sun

F. Winterberg
University of Nevada, Reno

Abstract

An attempt is made to explain the Allais effect, the anomalous behavior of pendulums during a total solar eclipse, by Poisson diffraction of gravitational waves into the lunar shadow, with the waves emitted through large mass motion in the center of the sun by a thermonuclear fusion reaction driven magnetohydrodynamic dynamo. Thermomagnetic currents in the tachocline shield the strong magnetic field in the solar core.

1. Introduction

The Allais effect [1], (see Fig. 1 and Fig. 2) first reported by Maurice Allais, a French winner of the Nobel prize and amateur scientist, is the influence a total solar eclipse seems to have on a paraconical (Foucault) pendulum. The same or similar events were later recorded by Saxl and Allen [2] and other investigators. The effect led Allais to the conjecture that Einstein's law of gravity might have to be changed. A critical examination of all these observations together with possible conventional explanations was made by C. P. Duif [3], with the conclusion: "That all the proposed conventional explanations either qualitatively or quantitatively fail to explain the observations."

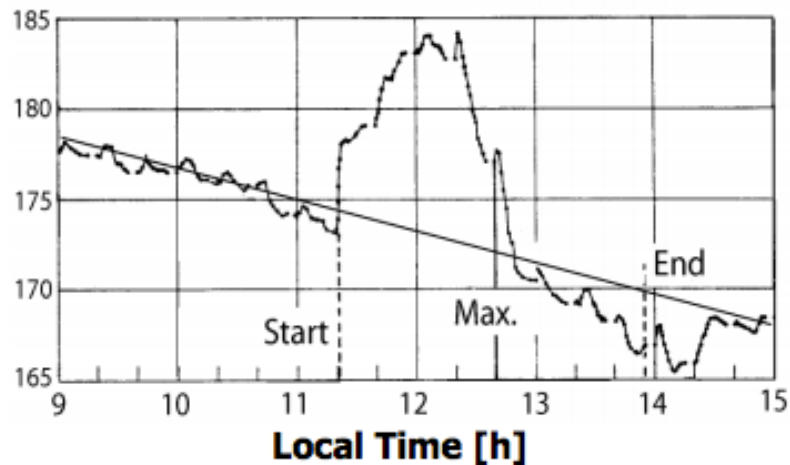


Fig. 1. Measurements during the June 30, 1954 solar eclipse with a paraconical pendulum by Allais [All59a, All97]. Vertical axis is the azimuth angle of the pendulum trajectory (in grades, 100 grades=90°). Also a linear trend line is shown. Each point is the accumulated effect of 14 min of measurement, pendulum release is every 20 min.

Figure 1 [3]

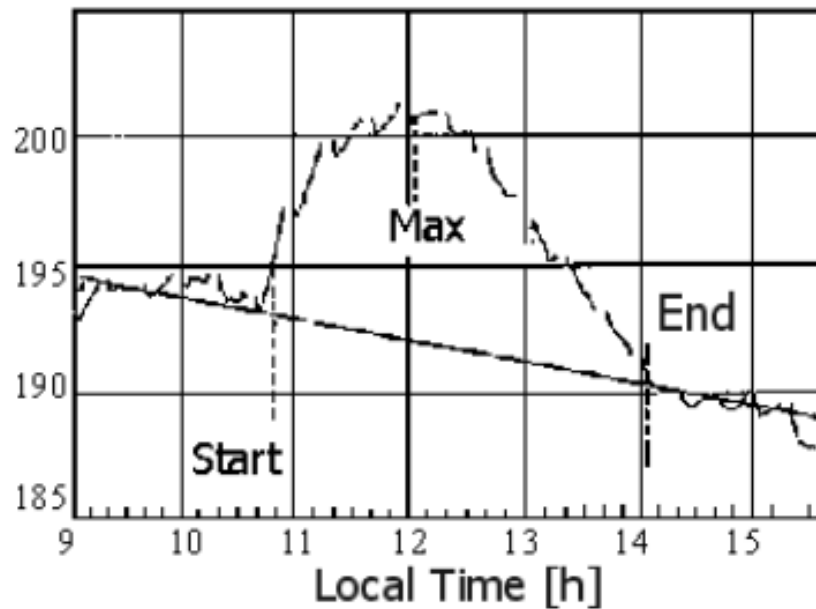


Fig. 2. Measurements during the 2 October 1959 solar eclipse with a paraconical pendulum by Allais [All97]. Vertical axis is the azimuth angle of the pendulum trajectory (grades).

Figure 2 [3]

In this communication an explanation is given in full agreement with Einstein's theory. It makes the assumption that large motions in the center of the sun by a magnetohydrodynamic dynamo driven by a thermonuclear fusion reaction is the source of gravitational waves, which in reaching the moon are focused by Poisson diffraction into the axis of the lunar shadow.

2. Magnetohydrodynamic Dynamo in the Center of the Sun

A magnetohydrodynamic dynamo is ruled by the equation

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B} + \text{curl} \mathbf{v} \times \mathbf{B} \quad (1)$$

in addition to the fluid dynamic Euler equation of motion, the energy equation, the equation of mass conservation, the equation of state and Ohm's law [4]. In (1) \mathbf{B} is the magnetic field strength in Gauss, \mathbf{v} is the fluid velocity driving the dynamo, σ is the electric conductivity, and c is the speed of light. At the high plasma temperature in the center of the sun one can set $\sigma = \infty$ and one obtains

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl} \mathbf{v} \times \mathbf{B} \quad (2)$$

Neglecting viscous forces the Euler equation with the magnetic body force $(1/c) \mathbf{j} \times \mathbf{B}$ where \mathbf{j} is the electric current density, is given by

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho} \nabla p - \frac{1}{2} \nabla v^2 + \mathbf{v} \times \text{curl} \mathbf{v} + \frac{1}{\rho c} \mathbf{j} \times \mathbf{B} \quad (3)$$

Putting

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \quad (4)$$

where $\boldsymbol{\omega} = (1/2) \text{curl} \mathbf{v}$, and with $(4\pi/c) \mathbf{j} = \text{curl} \mathbf{B}$, one thus has for (3)

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho} \nabla p - \boldsymbol{\omega}^2 \mathbf{r} - 2\boldsymbol{\omega} \times \mathbf{v} - \frac{1}{4\pi\rho} \mathbf{B} \times \text{curl} \mathbf{B} \quad (5)$$

with the magnetohydrodynamic instabilities arising from the last term on the r.h.s. of (5).

For $(1/2)\rho v^2 \gg B^2/8\pi$ the fluid stagnation pressure $(1/2)\rho v^2$ overwhelms the magnetic pressure forces $B^2/8\pi$, and the magnetic lines of force align themselves with the streamlines of the fluid flow. Then, if likewise the electric current flow lines \mathbf{j} align themselves with $\boldsymbol{\omega}$, one has $\mathbf{j} \perp \mathbf{B}$, since $\boldsymbol{\omega} \perp \mathbf{v}$. With $\boldsymbol{\omega} = (1/2) \text{curl} \mathbf{v}$ and $\mathbf{j} = (c/4\pi) \text{curl} \mathbf{B}$ one can write for $(1/2)\rho v^2 \gg B^2/8\pi$

$$|\mathbf{v} \times \text{curl} \mathbf{v}| \gg \left| \frac{\mathbf{B} \times \text{curl} \mathbf{B}}{4\pi\rho} \right| \quad (6)$$

or that

$$v_A < v \quad (7)$$

where

$$v_A = \frac{B}{\sqrt{4\pi\rho}} \quad (8)$$

is the Alfvén velocity. Initially, one may set in equation (4) for $|\boldsymbol{\omega}|$ the value for the slowly rotating sun, but with the build-up of the magnetic by (2), \mathbf{B} will eventually reach the value $B = \sqrt{4\pi\rho}v$, where $v_A = v$. In approaching this magnetic field strength, the magnetic pressure

forces begin to distort the fluid flow which becomes unstable. In a plasma this leads to the

formation of unstable pinch current discharges, where $p = \frac{B^2}{8\pi}$. For a hydrogen plasma of

temperature T and particle number density n one has there the Bennett equation

$$B = \sqrt{16\pi nkT} \quad (9)$$

The pinch instability which can be seen as the breakdown of the plasma into electric current filaments, and determined by the $\mathbf{B} \times \text{curl} \mathbf{B}$ term, the $\mathbf{v} \times \text{curl} \mathbf{v}$ term is responsible for the breakdown of the plasma in vortex filaments. But while the breakdown into current filaments is unstable, the opposite is true for the breakdown into vortex filaments. This can be seen as follows: Outside a linear pinch discharge one has $\text{curl} \mathbf{B} = 0$, and outside a linear vortex filament

$\text{curl}\mathbf{v}=0$. Because of $\text{curl}\mathbf{B}=0$, the magnetic field strength gets larger with a decreasing distance from the center of curvature of magnetic field lines of force. For $\text{curl}\mathbf{v}=0$, the same is true for the velocity of a vortex line. But whereas in the pinch discharge a larger magnetic field means a larger magnetic pressure, a larger fluid velocity means a smaller pressure by virtue of Bernoulli's theorem. Therefore, whereas a pinch column is unstable with regard to its bending, the opposite is true for a line vortex. This suggests that one should place a pinch column into a line vortex.

What is true for the $m=0$ kink pinch instability is also true for the $m=0$ sausage instability by the conservation of circulation

$$Z = \oint \mathbf{v} \cdot d\mathbf{r} = \text{const.} \quad (10)$$

And because of the centrifugal force, the vortex also stabilizes the plasma against the Rayleigh-Taylor instability.

The magnetohydrodynamic dynamo in the sun is driven by thermonuclear reactions. In the presence of a magnetic field this leads to large plasma velocities magnifying the magnetic field up to the value given by (9), where the Alfvén velocity and plasma velocity are about equal to the thermal proton velocity $v_o \approx \sqrt{kT/M}$, where $T=1.6 \times 10^7 \text{K}$ is the temperature in the solar core and M the mass of a proton. At this temperature one has $v_o = 5.3 \times 10^7 \text{cm/s}$, and for the density $\rho = nM = 1.5 \times 10^2 \text{g/cm}^3$ ($n = 9 \times 10^{25} / \text{cm}^3$), and one finds that $B = 3 \times 10^9 \text{G}$, much larger than the magnetic dipole field of the sun which is on the order of 1 Gauss. It is conjectured that the mass motion of the dynamo in the center of the sun is a source of gravitational radiation, responsible for the Allais effect. But this strong magnetic field must be shielded from leaking out

which is possible by thermomagnetic currents in the tachocline, separating the region where the fusion reactions take place from the rest of the sun.

A toroidal mass flow configuration in the center of the sun was proposed by Alfvén [5], with the axis of the torus aligned with the axis of the rotating sun. In the context of the very large mass motion proposed here, the dynamo must have at least two such tori, to conserve angular momentum. However, since no sizable oblateness of the sun is observed, which would be caused by a large centrifugal force, more than one torus is needed to make the centrifugal force equal in all directions. The most simple topological configuration having this property is the 3-vortex Borromean configuration, which also minimizes the volume it occupies [6].

3. Magnetar Analogue

A better understanding of the proposed dynamo is by thinking of it as a white dwarf magnetar placed in the center of the sun. The mass of the solar core is $m=6.8 \times 10^{32} \text{g}$. Setting the core radius $r=1.4 \times 10^{10} \text{cm}$, the gravitational acceleration at the surface of the core is (G Newton's constant)

$$g = -\frac{Gm}{r^2} = -1.3 \times 10^5 \text{ cm/s}^2 \quad (11)$$

Equating this attractive acceleration with the repulsive centrifugal acceleration

$$g = \omega^2 r \quad (12)$$

one obtains

$$\omega = \sqrt{g/r} = 9.6 \times 10^{-3} \text{ s}^{-1} \quad (13)$$

For the repulsive centrifugal force given by

$$\mathbf{F} = \omega^2 \mathbf{r} \quad (14)$$

one has

$$\text{div}\mathbf{F} = 2\omega^2 \quad (15)$$

Following Hund [7], one can assign a mass of density ρ for the repulsive centrifugal force by putting

$$\text{div}\mathbf{F} = -4\pi G\rho \quad (16)$$

which in combination with (15) gives a negative mass with the mass density

$$\rho = \frac{\omega^2}{2\pi G} = -2.2 \times 10^2 \text{ g/cm}^3 \quad (17)$$

comparable in absolute values to the positive mass density of the solar core $\rho = 1.5 \times 10^2 \text{ g/cm}^3$. The mass density of the gravitational field is given by

$$\frac{g^2}{8\pi Gc^2} = -1.1 \times 10^{-5} \text{ g/cm}^2 \quad (18)$$

very much as the positive mass density of the electric field is given by $E^2/8\pi c^2$. But besides the centrifugal force there is the Coriolis force

$$\mathbf{C} = 2c\boldsymbol{\omega} = 5.8 \times 10^8 \text{ cm/s}^2 \quad (19)$$

Its mass density is given by

$$\rho_c = -\frac{\mathbf{C}^2}{8\pi Gc^2} = -\frac{\omega^2}{2\pi G} = -2.2 \times 10^2 \text{ g/cm}^3 \quad (20)$$

equal to the negative mass density (17). It thus follows that the negative mass of the Coriolis field is the cause for the repulsive centrifugal force. The equation of motion in a rotating reference system is given by

$$\ddot{\mathbf{r}} = \mathbf{F} + \frac{\mathbf{v}}{c} \times \mathbf{C} \quad (21)$$

very much as the equation of motion of a charged particle in the presence of an electric and magnetic field

$$\underline{\mathbf{r}} = \frac{e}{m} \left[\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right] \quad (22)$$

This makes it plausible that for the functioning of the dynamo in the solar core the Coriolis force is of critical importance.

4. Emission of Gravitational Waves from the Core of the Sun

According to a theorem by Birkhoff, a completely spherical symmetric mass flow pattern cannot lead to the emission of gravitational radiation, but according to a theorem by Cowling [8], a hydromagnetic dynamo does not have a spherical flow pattern. However, to make an estimate about the magnitude of the energy loss due to gravitational radiation, we may take the expression derived by Eddington [9] for a spinning rod:

$$\mathbf{P} = \frac{32GI_m^2\omega^6}{5c^5} \quad (23)$$

or as given by Weber [10] in cgs units

$$\mathbf{P} = 1.73 \times 10^{-59} I_m^2 \omega^6 \text{ [erg / s]} \quad (24)$$

where I_m is the inertial momentum of the rod and ω its angular velocity. For I_m we set $I_m = mr^2$, and for the angular velocity $\omega = v_o/r$. With $m=6.8 \times 10^{32} \text{g}$, $r=1.4 \times 10^{10} \text{cm}$, $v_o = 5.3 \times 10^7 \text{cm/s}$, one obtains

$$\mathbf{P} \approx 10^{33} \text{ erg/s} \quad (25)$$

At the distance of $R=150$ million km from the earth-moon system the intensity of gravitational radiation per cm^2 is reduced by $4\pi R^2 = 3 \times 10^{27} \text{cm}^2$ to the respectable value of

$$\mathbf{P}_E \approx 3 \times 10^5 \text{ erg/cm}^2 \text{ s} \quad (26)$$

This large amount of gravitational radiation emitted by the sun, only $\sim 1/5$ smaller than the solar constant ($1.4 \times 10^6 \text{erg/cm}^2 \text{s}$) comes as a surprise. The reason for it is the ω^6 dependence of equation (23). However, it is still enough to be in agreement with the age of main sequence stars and the universe. But because Eddington's formula gives us only an estimate, it could still be smaller. It is through the high velocity large mass motion of the conjectured dynamo that ω becomes large. In theories of stellar structure [11], ignoring the possible existence of a powerful dynamo, the convection velocity is estimated to be of the order 10^4cm/s , not the 10^7cm/s . At these much smaller velocities there would be no appreciable emission of gravitational radiation.

5. Screening the Large Magnetic Field of the Dynamo Inside the Sun

The consensus regarding the observed stellar and planetary magnetic fields is that they are caused by a rotationally driven magnetohydrodynamic dynamo with the Coriolis force

making the dynamo to work. However, the assumed large dynamo as an explanation for the Allais effect leads to very large magnetic fields, not observed on the surface of the sun. This requires an explanation of how these fields are screened.

The temperature dependence of the thermonuclear reaction rate in the region of 10^7K goes in proportion to $T^{4.5}$ [11]. This means there is a sharp boundary between a much hotter region where most of the thermonuclear reactions occur and a cooler region where they are largely absent. This boundary is the tachocline. It is the thermomagnetic Nernst effect which by the large temperature gradient between the hotter and cooler region leads to large currents in the tachocline shielding the large magnetic field of the dynamo.

For a fully ionized hydrogen plasma the Nernst effect leads to a current density given by

$$\mathbf{j}_N = \frac{3knc}{2B^2} \mathbf{B} \times \text{grad}T \quad (27)$$

with the force density

$$\mathbf{f} = \frac{1}{c} \mathbf{j} \times \mathbf{B} = \frac{3}{2} \frac{nk}{B^2} (\mathbf{B} \times \text{grad}T) \times \mathbf{B} \quad (28)$$

or with $\text{grad}T$ perpendicular to \mathbf{B}

$$\mathbf{f} = \frac{3}{2} nk \times \text{grad}T \quad (29)$$

leading to the magnetic equilibrium condition

$$\mathbf{f} = \text{grad}p \quad (30)$$

with $p=2nkT$, one has $\text{grad}p=2nk \text{ grad}T+ 2kT \text{ grad}n$, hence

$$2nk \times \text{grad}T + 2kT \times \text{grad}n = \frac{3}{2}nk \times \text{grad}T \quad (31)$$

which upon integration yields

$$Tn^4 = \text{const.} \quad (32)$$

Taking a cartesian coordinate system with z directed along grad T, the magnetic field into the x-direction and the Nernst current into the y-direction, one has

$$j = j_y = -\frac{3knc}{2B} \frac{dT}{dz} \quad (33)$$

From Maxwell's equation $4\pi\mathbf{j}/c = \text{curl}\mathbf{B}$, one has

$$j_y = \frac{c}{4\pi} \frac{dB}{dz} \quad (34)$$

and thus

$$2B \frac{dB}{dz} = -12\pi kn \frac{dT}{dz} \quad (35)$$

From (32) one has

$$n = \frac{n_o T_o^{\frac{1}{4}}}{T^{\frac{1}{4}}} \quad (36)$$

where for large values of z (towards the center of the sun) $n = n_o$, $T = T_o$. Inserting (36) into (35)

one finds

$$dB^2 = -\frac{12\pi kn_o T_o^{\frac{1}{4}}}{T^{\frac{1}{4}}} dT \quad (37)$$

and hence

$$\frac{B_o^2}{8\pi} = 2n_o kT_o \quad (38)$$

expressing the fact that the magnetic field of the thermomagnetic current in the tachocline neutralizes the magnetic field of the dynamo reaching the tachocline.

One can see this result in still a different way. Very much as in the Meissner effect, a fully ionized high temperature plasma has a very high conductivity acting like a superconductor where induced surface currents prevent a magnetic field from penetrating the plasma. The thermonuclear energy releasing-dynamo generating the magnetic field is not subject to such a shielding effect. Since the plasma is not infinitely conducting, some magnetic field can still leak out, but by turbulently enhanced resistivity even this residual magnetic field is weakened by resistive heating of the plasma.

5. The Moon as a Gravitational Wave Antenna

It was the idea by Weber [12], to use meter-size metallic cylinders with a large Q-value as an antennas to detect gravitational waves coming from space. He needed at least two such cylinders, separated from each other by a large distance, to look for coincident signals to overcome the large background noise. His experiment failed because the sensitivity of his antennas was not great enough. And they are not even large enough even for the Ligo-type antennas.

The situation is changed if one uses the moon as an antenna in place of a Weber bar. Not only is the mass of the moon much larger than the mass of a Weber bar, but unlike the earth it

also has a large Q-value, which for the moon has been determined to be about $Q=5000$ [13].

Using the moon with its large Q value as an antenna for gravitational waves from the sun, these waves are focused by Poisson diffraction into the center line of the lunar shadow during a total eclipse.

The scattering cross section S of the moon for gravitational waves is given by [10]

$$S = \frac{15\pi GmQ(\beta r)^2}{8\omega c} \quad (39)$$

where $m=7.9 \times 10^{27}$ g is the mass and $r=1.74 \times 10^8$ cm is the radius of the moon. $\beta \equiv k_r$ is the wave propagation vector of the gravitational wave, with $\beta=2\pi/\lambda$, where λ is the wavelength. Setting $\lambda=2\pi r$, one has $\beta r=1$. Making this assumption one finds that $S=1.3 \times 10^{17}$ cm², and $\sqrt{S} \approx 2r$. This result means that during a total eclipse the moon forms a large disk with regard to the incoming gravitational radiation with a radius equal to $\lambda=2\pi r \approx 10^9$ cm= 10^4 km, about one order of magnitude smaller than the radius of the solar core, the hypothesized source of the gravitational radiation. The thusly formed disc leads to Poisson diffraction of the incoming radiation, focusing the radiation onto the axis of the lunar shadow. The moon therefore acts like a large lens magnifying the incoming radiation, which may explain the small observed distortion in the motion of high Q value pendulums during the totality of an eclipse.

6. Conclusion

It was pointed out by Duif [3] that all the different proposals trying to give a conventional explanation of the Allais effect taken together may explain it , but they cannot make a convincing case, but that this effect has been observed can hardly be disputed.

The explanation of the Allais effect I have here proposed is in full agreement with Einstein's general theory of relativity. It makes the assumption that the core of the sun is a large magnetohydrodynamic generator driven by the thermonuclear reactions in the solar core. The magnetic body forces of the dynamo acting on the high temperature plasma can lead to velocities much larger than those by gravity-driven buoyant forces as in the conventional theory by Schwarzschild [11]. While the gravity-driven buoyant forces lead to convection velocities of the order 3×10^3 cm/s, the much stronger magnetic forces can lead to velocities about equal to the thermal motion velocities of the plasma, for the temperatures in the center of the sun are of the order of several 10^7 cm/s or about 10^4 times larger. The diameter of buoyant driven convection cells are on the order of (1/10) of the solar radius, and are within an order of magnitude equal to the radius r of the solar core, the ω -value entering equation (23) is by the relation $\omega = v_o/r$, about 10^4 times larger, and the ω^6 factor in equation (23) is about 10^{24} times larger. Without this factor the generation of gravitational waves by solar convection would be insignificant. And with the moon acting as a kind of a lens, increasing the intensity of the gravitational waves coming from the sun in the center of the lunar shadow by Poisson diffraction, it becomes plausible that the wave intensity might become large enough to influence the motion of a high Q-value pendulum.

Appendix

A. Connection to the Dynamo Theory of Earth and Stellar Magnetism

According to Cowling [8], a two-dimensional hydromagnetic dynamo is not possible. The symmetry is broken by rotational motion, leading to the Coriolis force inside the rotating sun. In addition to rotation, a heat source is needed to drive the dynamo. For the earth it is provided by the radioactive decay, in particular by the β -decay of K40. For the sun it is provided by thermonuclear reactions in its center. In the presence of gravity it leads to convective motion amplifying the magnetic field through the second term $\text{curl}\mathbf{v}\times\mathbf{B}$ on the r.h.s of (1). For a non-vanishing resistivity $\rho=1/\sigma$, the current establishing the magnetic field is dissipated by the first term on the r.h.s. of (1). And for a self-exciting dynamo the second term must exceed the first term one has a rising magnetic field. This happens if the magnetic Reynolds number

$$\text{Rem} = \frac{4\pi\sigma vL}{c^2} > 1 \quad (\text{A.1})$$

or if

$$v > \frac{c^2}{4\pi\sigma L} \quad (\text{A.2})$$

At the temperature in the center of the sun $T=1.6\times 10^7\text{K}$, one finds that $\sigma\approx 2\times 10^{18}/\text{s}$. Taking the convection velocity given by Schwarzschild [10], $v\approx 3\times 10^3\text{cm/s}$, and for the diameter L of the thermonuclear producing core of the sun, $L=10^{10}\text{cm}$, one finds that $\text{Rem}\approx 10^{12}\gg 1$, or that $v\geq 10^{-8}\text{cm/s}$. It is for this reason that the assumption $\sigma=\infty$ is well justified. However, as indicated,

the field cannot raise indefinitely and is limited by $B^2/8\pi \leq 2nkT$, which for the sun means that $B \leq 10^9$ Gauss.

The exhaustive treatment of the dynamo theory was pioneered by Walter Elsasser [14]. The complete solution of the dynamo problem involves more than just equation (1) because this equation must be solved in conjunction with the equation of motion (Euler equation with magnetic force term) and the equations of continuity and energy. Only recently has it become possible to obtain solutions with the advent of supercomputers. Because of these difficulties it was already proposed by the author back in 1963, to obtain solutions experimentally by simulating such dynamos in liquid metals (such as liquid sodium), brought into rapid rotation with a propeller to simulate convection [15]. This idea has been more recently adapted by a number of research groups all over the world [16], with the group in Maryland acknowledging the origin of this idea [17].

With the dynamo action depending on the magnetic Reynolds number, a similarity law for the solutions exists, but because of the largeness of the product σLv in the magnetic Reynolds number, and with v limited by $v \leq c$, this similarity law cannot be used for a laboratory experiments, unlike what is possible with the 'ordinary' Reynolds number in laboratory simulation experiments, as for the simulation of a river flow.

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