A note on a new prime gap constant

Waldemar Puszkarz
Los Angeles, California
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Abstract

We propose a new mathematical constant related to the gaps between consecutive primes obtained by concatenating the digits in the prime gap numbers.

In this brief note we introduce a new mathematical constant.

To the best of our knowledge this constant has not yet been proposed. We could not find it, for instance, in the Online Encyclopedia of Integer Sequences, a most natural place to search for any number.

It is an artificially constructed constant in the spirit of other constants that result from concatenating fundamental integer sequences.

One of such constants, the original one, is the Champernowne (Mahler) constant (OEIS A033307) [1]. One arrives at it by concatenating consecutive natural numbers; the constant emerges in the limit of this process (OEIS A007908) [2].

More precisely, the fractional part of the decimal expansion of this constant is 1234567891011121314151920 ..., with the leading zero in its integer part added to form a finite real number from what otherwise would be an infinite string of natural numbers.

This leads to 0.1234567891011121314151920 ..., the number known as the Champernowne or the Mahler constant.

Another such a constant is the Copeland-Erdos constant (OEIS A033308) [3] that one forms by concatenating the sequence of all prime numbers (OEIS A019518) [4] and using this infinite string of integers as if it were the fractional part of the decimal expansion of some constant, with the integer part of it being zero by definition.

In other words, the Copeland-Erdos constant is 0.23571113171923293137414347 ...

While the sequence of natural numbers and the sequence of prime numbers are fundamental integer sequences, so is, in our view, the sequence of gaps between consecutive primes for it informs us about the structure of the spectrum of prime numbers.

Using this particular sequence, we can form another constant, similar, at least in its spirit, to the Copeland-Erdos constant.

Its fractional part is given as the limit of concatenating the digits in the gaps (OEIS A001223) [5] between consecutive primes, while its integer part is zero by definition.

This results in the prime gap constant as follows

0.1224242462642466264264684242414462102664662102421212424621066626421014
4241461024686646848102102646842412848461221861066261066266421210246621246
81081086648648414101221024210144241442420481084661446686 1 ...,

where the first 200 digits, obtained thanks to Mathematica, were included in its fractional part.

Compared to the other two constants, it is the smallest of the three.

Yet another way it is different from the other constants mentioned is that there are no odd digits, save for 1, amongst its first 200 digits, but such digits do appear when more gaps are taken into account, [6]. This strongly suggest that this is not a normal number, but proving that a given number is normal is quite difficult even for the numbers that look perfectly normal at first glance, a good example here being the Euler constant, e [7].

It would be rather interesting, if not counterintuitive, if this number proved to be normal just as the other two constants turned out to be.

More research is needed to find out more about the properties of this new constant. We invite all interested parties to contribute to it.

References

[1] https://oeis.org/A033307

[2] https://oeis.org/A007908

[3] https://oeis.org/A033308

[4] https://oeis.org/A019518

[5] https://oeis.org/A001223

[6] http://mathworld.wolfram.com/PrimeGaps.html

[7] http://mathworld.wolfram.com/NormalNumber.html