

The Mass of the Electron – Part II

A formula for the mass of the electron is derived from a lepton mass factor, a Koidean ratio for quarks and a dimensionless factor based on the fine-structure constant. If this formula were found to be correct it would prove a profound relationship between the masses of leptons and quarks.

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1. Nomenclature

The following are the symbols for the constants I shall use in this paper

m_e = electron rest mass

m_μ = muon rest mass

m_τ = tau particle rest mass

m_{up} = up quark nude mass

m_{down} = down quark nude mass

$m_{strange}$ = strange quark nude mass

m_{charm} = charm quark nude mass

m_{bottom} = bottom quark nude mass

m_{top} = top quark nude mass

R_{quarks} = Koidean ratio for quarks

α = fine-structure constant (atomic structure constant)

f = dimensionless factor

$F_{JMeV} = 1.602\,176\,564 \times 10^{-13} \frac{J}{MeV}$ (conversion factor between Jules and MeV)

2. The Formula for the Mass of the Electron in Terms of the Masses of Leptons and Quarks and the Fine Structure Constant

Lets us define the Koide formula for quarks which is similar to the Koide formula for leptons

$$R_{quarks} \equiv \frac{m_{up} + m_{down} + m_{strange} + m_{charm} + m_{bottom} + m_{top}}{\left(\sqrt{m_{up}} + \sqrt{m_{down}} + \sqrt{m_{strange}} + \sqrt{m_{charm}} + \sqrt{m_{bottom}} + \sqrt{m_{top}}\right)^2} \quad (2.1)$$

We shall also define a dimensionless factor, f , as

$$f \equiv 1 + \frac{\alpha}{\pi} - \frac{\alpha}{\pi^2} + \frac{\alpha}{\pi^3} - \frac{\alpha}{\pi^4} + \frac{\alpha}{\pi^5} - \frac{\alpha}{\pi^6} + \frac{\alpha}{\pi^7} - \frac{\alpha}{\pi^8} + \frac{\alpha}{\pi^9} - \frac{\alpha}{\pi^{10}} + \frac{\alpha}{\pi^{11}} - \frac{\alpha}{\pi^{12}} + \frac{\alpha}{\pi^{13}} - \frac{\alpha}{\pi^{14}} \quad (2.2)$$

The following is the formula for the mass of the electron I discovered in 2012:

$$m_e \approx \left(\frac{m_\mu^2}{\pi^3 m_\tau} \right) \left(\frac{f}{R_{quarks}^2} \right) \quad (2.3)$$

The formula relates the masses of leptons with the masses of quarks. Using equation (2.2) we can expand equation (2.3). This expansion gives

$$(2.4)$$

$$m_e \approx \left(1 + \frac{\alpha}{\pi} - \frac{\alpha}{\pi^2} + \frac{\alpha}{\pi^3} - \frac{\alpha}{\pi^4} + \frac{\alpha}{\pi^5} - \frac{\alpha}{\pi^6} + \frac{\alpha}{\pi^7} - \frac{\alpha}{\pi^8} + \frac{\alpha}{\pi^9} - \frac{\alpha}{\pi^{10}} + \frac{\alpha}{\pi^{11}} - \frac{\alpha}{\pi^{12}} + \frac{\alpha}{\pi^{13}} - \frac{\alpha}{\pi^{14}} \right) \left(\frac{m_\mu^2}{\pi^3 m_\tau} \right) \left(\frac{1}{R_{quarks}^2} \right)$$

We can also expressed the above formula in a more compact form as follows

$$m_e \approx \left(1 + \sum_{i=1}^{14} \frac{(-1)^{i+1} \alpha}{\pi^i} \right) \left(\frac{m_\mu^2}{\pi^3 m_\tau} \right) \left(\frac{1}{R_{quarks}^2} \right) \quad (2.5)$$

The masses of the quarks: up, down and strange were taken from a web page published by *phys.org* [1] and based on the research of professor G. P. Lepage et al. [2]

“According to their results, the up quark weighs approximately 2 mega electron volts (MeV), which is a unit of energy, the down quark weighs approximately 4.8 MeV, and the strange quark weighs in at about 92 MeV.”

I have calibrated the values of the masses of the charm, bottom and top quarks to produce the correct value of the mass of the electron. These three calibrated values are marked with an asterisk on table 1 and they fall within the mass range published by Wikipedia. Table 1 shows the quarks masses we adopted in this paper. The reader will surely realize that different calibration values could have been adopted instead of the values proposed by the author. However, only more accurate calculations of all quark masses will indicate the accuracy of the above formula.

Quark Name	Symbol (Particle)	Symbol (Antiparticle)	Mass Range (MeV/c^2) (source Wikipedia)	Mass (used in this paper) (MeV/c^2) Set 1	Mass (used in this paper) (MeV/c^2) Set 2	References
up	u	\bar{u}	1.5 - 3.3	2	2	[1][2]
down	d	\bar{d}	3.5 - 6.0	4.8	4.8	[1][2]
strange	s	\bar{s}	70 - 130	92	92	[1][2]
charm	c	\bar{c}	1,160 - 1,340	1237(*)	1270(*)	[3]
bottom	b	\bar{b}	4,130 - 4,670	4670 (*)	4599 (*)	[4]
top	t	\bar{t}	169,100 - 173,300	171987.5 (*)	171992.05 (*)	[5][6][7]

Table 1: The masses of quarks. Note that there are two sets of quark values.

3. The Predicted Mass of the Electron

Values from Table 1-Set 1

Let us calculate the value of the mass of the electron predicted by the formula (2.5). Let us start with the dimensionless factor, f

$$f = 1.00176196752951$$

The quark Koidean ratio is

$$R_{quarks} = \frac{(2 + 4.8 + 92 + 1237 + 4670 + 171987.5) MeV/c^2}{(\sqrt{2} + \sqrt{4.8} + \sqrt{92} + \sqrt{1237} + \sqrt{4670} + \sqrt{171987.5})^2 MeV/c^2}$$

$$R_{quarks} = 0.630\ 274\ 316$$

The inverse of the square of the quark Koidean ratio is

$$\frac{1}{R_{quarks}^2} = 2.517\ 333\ 646$$

According to NIST 2010, the values of the masses of the muon (second generation of matter) and tau particle (third generation of matter) are

$$m_{\tau} = 3.16747(29) \times 10^{-27} Kg \quad (\text{NIST 2010})$$

$$m_{\mu} = 1.883531475(96) \times 10^{-28} Kg \quad (\text{NIST 2010})$$

We shall define the value for the muon square mass over the tauon mass times π^3 as

$$m_{leptons} \equiv \left(\frac{m_{\mu}^2}{\pi^3 m_{\tau}} \right) = \frac{(1.883531475(96) \times 10^{-28} \text{ Kg})^2}{\pi^3 \times 3.16747 \times 10^{-27} \text{ Kg}} = 3.61229854 \times 10^{-31} \text{ Kg}$$

Thus, the value of the predicted mass of the electron turns out to be

$$m_e = 1.00176196753 \times 3.61229854 \times 10^{-31} \times 2.517333646 \text{ Kg}$$

$$m_e = 9.109382859 \times 10^{-31} \text{ Kg}$$

Values from Table 1-Set 2

Similarly if we use the second set of quark values given in Table 1 we get

$$m_e = 9.109382872 \times 10^{-31} \text{ Kg}$$

4. The Measured Mass of the Electron

According to NIST [8], the measured value for the rest mass of the electron is

$$m_{e \text{ NIST } 2010} = 9.10938291(40) \times 10^{-31} \text{ Kg} \quad (\text{NIST } 2010)$$

It is customary to express the mass of a particle in MeV/c^2 . Thus we use the conversion factor, F_{JMeV} , between Joules and MeV

$$F_{\text{JMeV}} = 1.602176564 \times 10^{-13} \frac{\text{J}}{\text{MeV}}$$

This yields

$$\frac{m_{e_exp} c^2}{F_{\text{JMeV}}} = \frac{9.10938291 \times 10^{-31} \times (299792458)^2 \text{ J}}{1.602176564 \times 10^{-13} \text{ J/MeV}} = 0.5109989279 \text{ MeV}$$

$$m_{e_exp} = 0.5109989279 \text{ MeV}/c^2 \approx 0.511 \text{ MeV}/c^2$$

5. Conclusions

When we calibrate the masses of the charm, the bottom and the top quark, the formula presented in this paper yields the value of the electron mass very accurately. Because all the masses used in this calculation fall within the mass range given in Table 1, it is reasonable to think that the formula is capable of yielding a very accurate value for the electron mass. The reason of performing a calibration is due to the fact that we do not know the exact values of the naked

quark masses. This hole in our knowledge tell us that a more refined calculation of the quark masses is needed to determine the degree of accuracy and validity of the above formula.

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