Determining Power Product Relationship between Physical Quantities by Generalized Relational Expression

Yong Bao（包 勇）

100 Renmin South Road, Luoding 527200, Guangdong, China

E-mail: baoyong9803@163.com

The power product relationship between physical quantities (PQs) is determined by the generalized relational expression (GRE). Selecting two, three and four PQs in the GRE separately, we find the corresponding general formulas. Ordering the exponent of physical constants in formulas is zero or fit numbers, we gain many famous equations without factors, such as Einstein mass-energy relation, formula of event horizon temperature of Schwarzschild black hole (SBH), equation of holographic dark energy (HDE) model, Casimir effect equation, Planck black body radiation formula, Stefan-Boltzmann law, Einstein field equation, Newtonian attraction law, Schrodinger equation, Coulomb law, Newtonian second law, Clapeyron equation, power law of superconducting films, two formulas of critical temperature of LSCO, centrifugal force formula, and so on. Some new relations are given, including the square of total energy with energy density of HDE, square of energy with its density of SBH, energy density with sextic radii of SBH, pressure in SBH centre and entropy density of SBH center etc. We show that the GRE can determine the power product relationship between two, three and four PQs, it is useful and significant.

1. Introduction

It is the grail that a general theory can be found and deduced other physical equations in theoretical physics. The standard model [1] succeeds in uniting the weak interaction, electromagnetic force and strong interaction. Predictive 62 elementary particles are detected, including the Higgs boson recently [2], but it doesn't embrace the gravity. Supersymmetry [3], superstring/M theory [4], loop quantum gravity theory [5], N. Wu's quantum gauge general relativity [6], A. Garrett Lisi's exceptionally simple theory of everything [7] T. Ma and S.H. Wang produced unified field equations [8], Zh-Y. Shen's SQS theory [9], and Y-L. Wu's Quantum Field Theory of Gravity and Hyperunified Field Theory [10] described the four forces respectively, but they all aren't detected [11]. Yin Ye, Hu Suhui considered the symmetric approximation [12]; Y. Bao produced the generalized relational expression (GRE) [13]. These are making progress.

This paper is organized as follows. In Sec. 2, we determine the power product relationship between two physical quantities (PQs); find the corresponding general formulas (GFs); obtain many famous equations, such as Einstein mass-energy relation [14], temperature of event horizon of Schwarzschild black hole (SBH) [15], equation of holographic dark energy (HDE) model [16], Casimir effect equation [17], Planck black body radiation formula [18], Stefan-Boltzmann law [19], Einstein field equation [20], etc. In Sec. 3, we have the power product relationship between three PQs; find the corresponding GFs also; get Newtonian attraction law [21], Schrodinger equation [22], Coulomb law [23], Newtonian second law [21], Clapeyron equation [24], power law of superconducting films [25] and so on. In Sec. 4, we give the power product relationship between four PQs; gain the centrifugal force formula [21]. We conclude in Sec. 5.

2. Power Product Relationship between Two PQs

In this section, we obtain the relationship of power products when n = 2 in the GRE; and find the corresponding GFs; obtain the Einstein mass-energy relation, temperature of event horizon of SBH, equation of HDE model, Casimir effect equation, Planck black body radiation formula, Stefan-Boltzmann law, Einstein field equation, etc.

2.0 The basic relationship [13] is

$$A \sim \mathrm{A_P} = [\hbar^{(\delta+\varepsilon+\zeta+\eta)} \mathrm{G}^{(\delta-\varepsilon+\zeta-\eta)} \mathrm{c}^{-(3\delta-\varepsilon+5\zeta-5\eta)} \kappa^{-2\eta} \mathrm{e}^{2\lambda}]^{1/2} \quad (1)$$

Where A is any physical quantity, $[A] = [\mathrm{L}]^{\delta}[\mathrm{M}]^{\varepsilon}[\mathrm{T}]^{\zeta}[\Theta]^{\eta}[\mathrm{Q}]^{\lambda}$ its dimensions, L, M, T, Θ and Q are the dimensions of length, mass, time, temperature and electric charge separately (here we use the LMTΘQ units [13]), $\mathrm{A_P}$ the corresponding Planck scale of A, δ, ε, ζ, η and λ the real number, ħ, G, c, κ and e the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge separately.

The GRE [13] is

$$\prod_{\mathrm{i}=1}^{\mathrm{n}} A_{\mathrm{i}}^{\alpha_{\mathrm{i}}} \sim \prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{A}_{\mathrm{iP}}^{\alpha_{\mathrm{i}}};\ \mathrm{i} = 1, 2, 3\ldots n \quad (2)$$

where A_{i} is the physical quantity, α_{i} the real number, and A_{iP} the corresponding Planck scale. When n = 2, we obtain

$$A_1^{\alpha_1}A_2^{\alpha_2} \sim A_{1P}^{\alpha_1}A_{2P}^{\alpha_2} \quad (3)$$

Instructing $\alpha_1 = 1$, $\alpha_2 = \alpha$, $A_1 = A$ and $A_2 = B$, we gain

$$AB^{\alpha} \sim A_P B_P^{\alpha} \quad (4)$$

Especially when $\alpha = 1$, we obtain

$$AB \sim A_P B_P \quad (5)$$

When $\alpha = -1$, we gain

$$A \sim A_P B / B_P \quad (6)$$

Therefore we can determine the power product relationship between two PQs. For example

2.1 Assuming that energy E has relations with mass M only, we find

$$EM^{\alpha} \sim E_P M_P^{\alpha} = (\hbar c^5/G)^{1/2}(\hbar c/G)^{\alpha/2} = \hbar^{(1+\alpha)/2}G^{-(1+\alpha)/2}c^{(5+\alpha)/2} \quad (7)$$

where $E_P = \sqrt{\hbar c^5/G}$ is the Planck energy and $M_P = \sqrt{\hbar c/G}$ the Planck mass (From basic relationship (1)). It is the GF for energy and mass.

2.1.1 Ordering $1+\alpha = 0, \rightarrow \alpha = -1$, we obtain

$$E \sim Mc^2$$

This is the Einstein mass-energy relation.

2.1.2 Instructing $5+\alpha = 0, \rightarrow \alpha = -5$, we have

$$E \sim G^2M^5 / \hbar^5 \ ?$$

2.1.3 Ordering $\alpha = 1$, we gain

$$EM \sim \hbar c^3 / G$$

Substituting $E \sim \kappa T$ into above formula, we obtain

$$T \sim \hbar c^3 / \kappa GM$$

where T is the temperature. It is the temperature of event horizon of SBH, but it hasn't $1 / 8\pi$.

2.2 Supposing that energy E has relations with frequency ω merely, we find

$$E\omega^{\alpha} \sim E_P\omega_P^{\alpha} = \hbar^{(1-\alpha)/2}G^{-(1+\alpha)/2}c^{5(1+\alpha)/2} \quad (8)$$

where $\omega_P = \sqrt{c^5/\hbar G}$ is the Planck frequency. This is the GF for energy and frequency.

2.2.1 Instructing $1+\alpha = 0, \rightarrow \alpha = -1$, we gain

$$E \sim \hbar\omega$$

It is the light quantum relation [26].

2.2.2 Ordering $1-\alpha = 0, \rightarrow \alpha = 1$, we obtain

$$E\omega \sim c^5 / G$$

Substituting $E \sim Mc^2$ into above formula, we gain

$$\omega \sim c^3 / GM$$

where $\omega \sim v_G$. This is the inverse correlation between high-frequency quasi-periodic oscillation and black hole mass [27].

2.2.3 Instructing $\alpha = -3$, we have

$$E \sim \hbar^2G\omega^3 / c^5 \ ?$$

2.3 Assuming that energy E has relations with energy density ρ only, we find

$$E\rho^{\alpha} \sim E_P\rho_P^{\alpha} = \hbar^{(1-2\alpha)/2}G^{-(1+4\alpha)/2}c^{(5+14\alpha)/2} \quad (9)$$

where $\rho_P = c^7 / \hbar G^2$ is the Planck energy density. It is the GF for energy and energy density.

2.3.1 Ordering $1-2\alpha = 0, \rightarrow \alpha = 1 / 2$, we obtain

$$E^2 \sim c^{12} / G^3\rho$$

From M. Li *et al.* HDE model [16], $\rho_{de} = 3c_L^2c^3M_{pl}^2L^{-2}$, $E = \rho V$ and $V \sim L^3$, we gain

$$E_{de}^2 \sim 27c_L^6c^{12} / 512\pi^3G^3\rho_{de}$$

where E_{de} is total energy of HDE, ρ_{de} the HDE density, $c_L \geq 0$ a dimensionless model parameter, $M_{Pl} = \sqrt{\hbar c/8\pi G}$ the reduced Planck mass, L the cosmic cutoff and V its volume. So the above formula is the square of total energy with its density of HDE, hasn't $27c_L^6 / 512\pi^3$.

2.3.2 Instructing $1+4\alpha = 0, \rightarrow \alpha = -1 / 4$, we obtain

$$E^4 \sim \hbar^3c^3\rho$$

This is the relationship between biquadratic quanta energy and its density [19].

2.3.3 Ordering $\alpha = -1 / 2$, we gain

$$E^2 \sim \hbar^2G\rho / c^2$$

From $M_H V_H \sim \hbar^2G / c^4$ [13], $E = \rho V$ and $E = Mc^2$, where M_H is the mass of SBH, and V_H its volume, we obtain the above formula with square of energy and its density of SBH.

2.4 Supposing that distance R has relations with mass M merely, we find

$$RM^{\alpha} \sim L_P M_P^{\alpha} = \hbar^{(1+\alpha)/2}G^{(1-\alpha)/2}c^{-(3-\alpha)/2} \quad (10)$$

where $L_P = \sqrt{\hbar G/c^3}$ is the Planck length. This is the GF for distance and mass.

2.4.1 Instructing $1+\alpha = 0, \rightarrow \alpha = -1$, we obtain

$$R \sim GM / c^2$$

It is the radius of event horizon of stationary black holes [28].

2.4.2 Ordering $1-\alpha = 0, \rightarrow \alpha = 1$, we gain

$$R \sim \hbar / Mc$$

This is A.H. Compton wavelength formula [29].

2.4.3 Instructing $3-\alpha = 0, \rightarrow \alpha = 3$, we have

$$R \sim \hbar^2 / GM^3 \ ?$$

2.4.4 Ordering $\alpha = -3$, we obtain

$$R \sim G^2M^3 / \hbar c^3$$

Substituting $R = ct$ into above formula, we gain

$$t \sim G^2M^3 / \hbar c^4 \propto M^3$$

It is the age of SBH [15].

From $R \sim GM / c^2$, we obtain $V \sim R^3 \sim G^3M^3 / c^6$, substituting $t \sim G^2M^3 / \hbar c^4$, we gain

$$V \sim \hbar G\, t / c^2$$

That is the relation between the volume of event horizon of stationary black holes and its age. For the SBH, $R = 2GM / c^2$, $V = 32\pi G^3M^3 / 3c^6$ and $t \approx 15360\pi G^2M^3 / \hbar c^4$, we have $V \approx \hbar G\, t / 1440c^2$.

2.5 Assuming that energy density ρ has relations with distance R

only, we find

$$\rho R^{\alpha} \sim \rho_{\rm P}{\rm L}_{\rm P}^{\alpha} = \hbar^{-(2-\alpha)/2}{\rm G}^{-(4-\alpha)/2}{\rm c}^{(14-3\alpha)/2} \quad (11)$$

This is the GF for energy density and distance.

2.5.1 Instructing $2-\alpha = 0, \rightarrow \alpha = 2$, we obtain

$$\rho \sim {\rm c}^4 / {\rm G}R^2$$

where $R \sim L$. This is the equation of HDE model $\rho_{de} = 3c_L^2{\rm c}^3{\rm M}_{\rm pl}^2L^{-2}$ [16], hasn't $3c_L^2$ / 8π.

2.5.2 Ordering $4-\alpha = 0, \rightarrow \alpha = 4$, we gain

$$\rho \sim \hbar{\rm c} / R^4$$

where $\rho \sim f / \omega$, f is the force of H. Casimir effect (confer to 2.6.1).

2.5.3 Instructing $14-3\alpha = 0, \rightarrow \alpha = 14 / 3$, we have

$$\rho^3 \sim \hbar^4{\rm G} / R^{14} \ ?$$

2.5.4 Ordering $\alpha = 6$, we obtain

$$\rho \sim \hbar^2{\rm G} / {\rm c}^2R^6$$

From $M_{\rm H}V_{\rm H} \sim \hbar^2{\rm G} / {\rm c}^4$, $E = \rho V$, $E = M{\rm c}^2$ and $V \sim R^3$, we gain the above formula. This is the energy density with sextic radii of SBH.

2.6 Supposing that per area force f has relations with distance R merely, we find

$$fR^{\alpha} \sim {\rm f}_{\rm P}{\rm L}_{\rm P}^{\alpha} = \hbar^{-(2-\alpha)/2}{\rm G}^{-(4-\alpha)/2}{\rm c}^{(14-3\alpha)/2} \quad (12)$$

where ${\rm f}_{\rm P} = {\rm c}^7 / \hbar{\rm G}^2$ is the Planck per area force. It is the GF for per area force and distance.

2.6.1 Instructing $4-\alpha = 0, \rightarrow \alpha = 4$, we gain

$$f \sim \hbar{\rm c} / R^4$$

This is H. Casimir effect formula, hasn't $-\pi^2$ / 240.

2.6.2 Ordering $2-\alpha = 0, \rightarrow \alpha = 2$, we obtain

$$f \sim {\rm c}^4 / {\rm G}R^2 = {\rm F}_{\rm P} / R^2$$

where ${\rm F}_{\rm P} = {\rm c}^4 / {\rm G}$ is the Planck force, and ${\rm M}_{\rm P}^2 = \hbar{\rm F}_{\rm P} / {\rm c}^3$. From $\rho_{de} = 3c_L^2{\rm c}^3{\rm M}_{\rm pl}^2L^{-2}$ and $p = \omega\rho$, we gain

$$p_{de} = 3w_{de}c_L^2{\rm c}^3{\rm M}_{\rm pl}^2L^{-2} = 3w_{de}c_L^2{\rm F}_{\rm Pl}L^{-2}$$

where $w_{de} < 0$ is the coefficient of state, ${\rm F}_{\rm Pl} = {\rm F}_{\rm P}$ / 8π the reduced Planck force. So it is the negative pressure $p_{de} \sim f$ of HDE [16], hasn't $3w_{de}$.

2.6.3 Instructing $14-3\alpha = 0, \rightarrow \alpha = 14 / 3$, we have

$$f^3 \sim \hbar^4{\rm G} / R^{14} \ ?$$

2.6.4 Ordering $\alpha = 6$, we obtain

$$f \sim \hbar^2{\rm G} / {\rm c}^2R^6$$

From 2.5.4 and $p = \omega\rho$, we gain

$$p \sim \omega\hbar^2{\rm G} / {\rm c}^2R^6$$

This is the pressure $p \sim f$ in SBH centre.

2.7 Assuming that radiation density ρ_r has relations with frequency γ only, we find

$$\rho_r\gamma^{\alpha} \sim \rho_{r{\rm P}}\gamma_{\rm P}^{\alpha} = \hbar^{-(1+\alpha)/2}{\rm G}^{-(3+\alpha)/2}{\rm c}^{(9+5\alpha)/2} \quad (13)$$

where $\rho_{r{\rm P}} = \sqrt{{\rm c}^9/\hbar{\rm G}^3}$ is the Planck radiation density, and $\gamma_{\rm P}$ the Planck frequency. It is the GF for radiation density and frequency.

2.7.1 Instructing $3+\alpha = 0, \rightarrow \alpha = -3$, we obtain

$$\rho_r \sim \hbar\gamma^3 / {\rm c}^3$$

Comparing M. Planck black body radiation formula, it hasn't 8π / $({\rm e}^{\hbar\gamma/\kappa T}-1)$.

2.7.2 Ordering $1+\alpha = 0, \rightarrow \alpha = -1$, we gain

$$\rho_r \sim {\rm c}^2\gamma / {\rm G} \ ?$$

2.7.3 Instructing $9+5\alpha = 0, \rightarrow \alpha = -9 / 5$, we have

$$\rho_r^5 \sim \hbar^2\gamma^9 / {\rm G}^3 \ ?$$

2.7.4 Ordering $\alpha = -5$, we get

$$\rho_r \sim \hbar^2{\rm G}\gamma^5 / {\rm c}^8 \ ?$$

2.8 Supposing that energy density ρ has relations with temperature T merely, we find

$$\rho T^{\alpha} \sim \rho_{\rm P}{\rm T}_{\rm P}^{\alpha} = \hbar^{-(2-\alpha)/2}{\rm G}^{-(4+\alpha)/2}{\rm c}^{(14+5\alpha)/2}\kappa^{-\alpha} \quad (14)$$

2.8.1 Instructing $4+\alpha = 0, \rightarrow \alpha = -4$, we obtain

$$\rho \sim \kappa^4T^4 / \hbar^3{\rm c}^3$$

That is Stefan-Boltzmann law, hasn't π^2 / 15.

2.8.2 Ordering $2-\alpha = 0, \rightarrow \alpha = 2$, we gain

$$\rho \sim {\rm c}^{12} / {\rm G}^3\kappa^2T^2 \ ?$$

2.8.3 Instructing $14+5\alpha = 0, \rightarrow \alpha = -14 / 5$, we obtain

$$\rho^5 \sim \kappa^{14}T^{14} / \hbar^6{\rm G}^3 \ ?$$

2.8.4 Ordering $\alpha = -2$, we get

$$\rho \sim {\rm c}^2\kappa^2T^2 / \hbar^2{\rm G}$$

It is the gravitational energy density far from the horizon inside SBH [30].

2.9 Assuming that acceleration a has relations with temperature T only, we find

$$aT^{\alpha} \sim a_{\rm P}{\rm T}_{\rm P}^{\alpha} = \hbar^{-(1-\alpha)/2}{\rm G}^{-(1+\alpha)/2}{\rm c}^{(7+5\alpha)/2}\kappa^{-\alpha} \quad (15)$$

where $a_{\rm P} = \sqrt{{\rm c}^7/\hbar{\rm G}}$ is the Planck acceleration. It is the GF for acceleration and temperature.

2.9.1 Instructing $1+\alpha = 0, \rightarrow \alpha = -1$, we gain

$$a \sim {\rm c}\kappa T / \hbar$$

This is Unruh formula [32], hasn't 1 / 2π.

2.9.2 Ordering $1-\alpha = 0, \rightarrow \alpha = 1$, we obtain

$$aT \sim {\rm c}^6 / \kappa{\rm G} \ ?$$

2.9.3 Instructing $7+5\alpha = 0, \rightarrow \alpha = -7 / 5$, we have

$$a^5 \sim {\rm G}\kappa^7T^7 / \hbar^6 \ ?$$

2.9.4 Ordering $\alpha = -3$, we obtain

$$a \sim {\rm G}\kappa^3T^3 / \hbar^2{\rm c}^4 \ ?$$

2.10 Supposing that entropy density s has relations with temperature T merely, we find

$$sT^{\alpha} \sim s_{\rm P}{\rm T}_{\rm P}^{\alpha} = \hbar^{-(3-\alpha)/2}{\rm G}^{-(3+\alpha)/2}{\rm c}^{(9+5\alpha)/2}\kappa^{(1-\alpha)} \quad (16)$$

where $s_{\rm P} = \sqrt{\kappa^2{\rm c}^9/\hbar^3{\rm G}^3}$ is the Planck entropy density. It is the GF for entropy density and temperature.

2.10.1 Instructing $3+\alpha = 0, \rightarrow \alpha = -3$, we obtain

$$s \sim \kappa^4T^3 / \hbar^3{\rm c}^3$$

This is entropy density with cube of temperature [31] [33].

2.10.2 Ordering $3-\alpha = 0, \rightarrow \alpha = 3$, we have

$$sT^3 \sim c^{12} / G^3\kappa^2 \ ?$$

2.10.3 Instructing 9＋5α = 0, → α =－9 / 5, we get

$$s^5 \sim \kappa^{14}T^9 / \hbar^{12}G^3 \ ?$$

2.10.4 Ordering 1－α = 0, → α = 1, we gain

$$sT \sim c^7 / \hbar G^2 \ ?$$

2.10.5 Instructing α = －1, we obtain

$$s \sim \kappa^2c^2T / \hbar^2G$$

It is the entropy density of SBH center [30].

2.11 Assuming that curvature tensor $R_{\mu\nu}$ has relations with energy- momentum tensor $T_{\mu\nu}$ only, we find

$$R_{\mu\nu}T_{\mu\nu}{}^{\alpha} \sim R_{\mu\nu P}T_{\mu\nu P}{}^{\alpha} = \hbar^{-(1+\alpha)}G^{-(1+2\alpha)}c^{(3+7\alpha)} \quad (17)$$

where $R_{\mu\nu P}$= $c^3 / \hbar G$ is the Planck curvature tensor, and $T_{\mu\nu P}$= $c^7 / \hbar G^2$ the Planck energy-momentum tensor. This is the GF for curvature tensor and energy-momentum tensor.

2.11.1 Ordering 1＋α = 0, → α = －1, we gain

$$R_{\mu\nu} \sim GT_{\mu\nu} / c^4$$

It is Einstein field equation, hasn't $-Rg_{\mu\nu} / 2$ and －8π.

2.11.2 Instructing 1＋2α = 0, → α = －1 / 2, we obtain

$$R_{\mu\nu}{}^2 \sim T_{\mu\nu} / \hbar c \ ? \text{ or } R_{\mu\nu}R^{\mu\nu} \sim T_{\mu\nu} / \hbar c \ ?$$

2.11.3 Ordering 3＋7α = 0, → α = －3 / 7, we have

$$R_{\mu\nu}{}^7 \sim T_{\mu\nu}{}^3 / \hbar^4G \ ?$$

2.12 Supposing that Lagrange density function φ has relations with electromagnetic field tensor $F_{\mu\nu}$ merely, we find

$$\varphi F_{\mu\nu}{}^{\alpha} \sim \varphi_P F_{\mu\nu P}{}^{\alpha} = \hbar^{-(1+\alpha)}G^{-(2+\alpha)}c^{(7+3\alpha)}e^{\alpha}$$
$$\sim \hbar^{-(2+\alpha)/2}G^{-(2+\alpha)}c^{7(2+\alpha)/2} \quad (18)$$

where $\varphi_P = c^7 / \hbar G^2$ is the Planck Lagrange density function, $F_{\mu\nu}$ = e $c^3 / \hbar G$ the Planck electromagnetic field tensor, and e $\sim \sqrt{\hbar c}$. This is the GF for Lagrange density function and electro-magnetic field tensor.

2.12.1 Instructing 2＋α = 0, → α = －2, we obtain only

$$\varphi \sim F_{\mu\nu}{}^2 \sim F_{\mu\nu}F^{\mu\nu}$$

It is electromagnetic Lagrange density function under Lorentz gauge [34], hasn't －1 / 4 and $-(\partial_\mu A^\mu)^2 / 2$.

3. Power Product Relationship between Three PQs

In this section, we obtain the relationship of power products when n = 3 in the GRE; find the corresponding GFs also; and obtain the Newtonian attraction law, Schrodinger equation, Coulomb law, Newtonian second law, Clapeyron equation, power law of superconducting films, two formulas of critical temperature of LSCO, etc.

3.0 Similarly when n = 3, we obtain

$$A_1^{\alpha_1}A_2^{\alpha_2}A_3^{\alpha_3} \sim A_{1P}^{\alpha_1}A_{2P}^{\alpha_2}A_{3P}^{\alpha_3} \quad (19)$$

Ordering $\alpha_1 = 1$, $\alpha_2 = \alpha$, $\alpha_3 = \beta$, $A_1 = A$, $A_2 = B$, and $A_3 = C$, we give

$$AB^{\alpha}C^{\beta} \sim A_PB_P^{\alpha}C_P^{\beta} \quad (20)$$

when β = 0, (4) is recovered. Thus we can determine the power product relationship between three PQs. For example

3.1 Assuming that energy E has relations with mass M and distance r, we find

$$EM^{\alpha}r^{\beta} \sim \hbar^{(1+\alpha+\beta)/2}G^{-(1+\alpha-\beta)/2}c^{(5+\alpha-3\beta)/2} \quad (21)$$

This is the GF for energy, mass and distance.

3.1.1 Instructing 1＋α＋β = 0, and 5＋α－3β = 0 → α = －2 and β = 1, we obtain

$$E \sim GM^2 / r \sim GMm / r$$

It is Newtonian attraction law, hasn't －1.

3.1.2 Ordering 1＋α－β = 0, and 5＋α－3β = 0 → α = 1 and β = 2, we gain

$$E \sim \hbar^2 / Mr^2$$

Substituting E→ iħ ∂ / ∂t and 1 / $r^2 \to \nabla^2$ into above formula, we obtain

$$i\hbar \, \partial \psi / \partial t \sim \hbar^2\nabla^2\psi / M$$

where ψ is wave function. This is Schrodinger equation, hasn't －1 / 2.

3.1.3 Instructing 1＋α＋β = 0, and 1＋α－β = 0 → α = －1 and β = 0, we obtain

$$E \sim Mc^2$$

It is Einstein mass-energy relation again.

3.1.4 Ordering α = －1 and β = 2, we gain

$$E \sim \hbar GM / cr^2$$

From Unruh formula $T = 2\pi\hbar a / c\kappa$, $a \sim g$ and $g = GM / r^2$, we have

$$T = 2\pi\hbar G M / c\kappa r^2$$

so it is the temperature $T \sim E / \kappa$ in Newtonian attraction, hasn't 2π.

3.2 Supposing that energy E has relations with electric charge Q and distance r, we find

$$EQ^{\alpha}r^{\beta} \sim \hbar^{(1+\beta)/2}G^{-(1-\beta)/2}c^{(5-3\beta)/2}e^{\alpha}$$
$$\sim \hbar^{(1+\alpha+\beta)/2}G^{-(1-\beta)/2}c^{(5+\alpha-3\beta)/2} \quad (22)$$

It is the GF for energy, electric charge and distance.

3.2.1 Ordering 1＋α＋β = 0, and 1－β = 0 → α = －2 and β = 1, also 5＋α－3β = 0, we gain only

$$E \sim Q^2 / r \sim Q_1Q_2 / r$$

This is Coulomb law.

3.3 Assuming that acceleration a has relations with force F and mass M, we find

$$aF^{\alpha}M^{\beta} \sim \hbar^{(1-\beta)/2}G^{-(1+2\alpha+\beta)/2}c^{(7+8\alpha+\beta)/2} \quad (23)$$

It is the GF for acceleration, force and mass.

3.3.1 Instructing 1－β = 0, and 1＋2α＋β = 0 → α = －1 and β = 1, also 7＋8α＋β = 0, we obtain merely

$$a \sim F / M$$

This is Newtonian second law [21].

Only ordering $1-\beta = 0$, $\rightarrow \beta = 1$, we gain

$$a \sim \mathrm{G}^{-(1+\alpha)}\mathrm{c}^{4(1+\alpha)}F^{-\alpha} / M$$

when $\alpha = -2$, we have

$$a \sim \mathrm{G}\, F^2 / M\mathrm{c}^4 \ ?$$

3.4 Supposing that acceleration a has relations with mass M and distance r, we find

$$aM^{\alpha}r^{\beta} \sim \hbar^{-(1-\alpha-\beta)/2}\mathrm{G}^{-(1+\alpha-\beta)/2}\mathrm{c}^{(7+\alpha-3\beta)/2} \quad (24)$$

It is the GF for acceleration, mass and distance.

3.4.1 Ordering $1-\alpha-\beta = 0$, and $7+\alpha-3\beta = 0 \rightarrow \alpha = -1$ and $\beta = 2$, we gain

$$a \sim \mathrm{G}M / r^2$$

This is Newtonian gravitational acceleration [21].

3.4.2 Instructing $1+\alpha-\beta = 0$, and $7+\alpha-3\beta = 0 \rightarrow \alpha = 2$ and $\beta = 3$, we have

$$a \sim \hbar^2 / M^2 r^3 \ \rightarrow\ r \sim \sqrt[3]{\hbar^2 / aM^2}$$

It is $h_n = \sqrt[3]{9[(n-\frac{1}{4})\pi\hbar/m]^2 / 8g}$ [35], where $h_n \sim r$ is the height of the nth energy level, $m \sim M$ the neutron mass and $g \sim a$ the Earth's gravitational acceleration.

3.4.3 Ordering $1-\alpha-\beta = 0$, and $1+\alpha-\beta = 0 \rightarrow \alpha = 0$ and $\beta = 1$, we obtain

$$a \sim \mathrm{c}^2 / r$$

From $\rho_{de} = 3c_L^2\mathrm{c}^3\mathrm{M}_{\mathrm{pl}}^2 L^{-2}$, $p = \omega\rho$, $F \sim pL^2$, $a \sim F / M$, $M\mathrm{c}^2 = \rho V$ and $V \sim L^3$, we gain

$$a \sim 3w_{de}\mathrm{c}^2 / 8\pi L$$

where $r \sim L$. It is the acceleration of HDE, hasn't $3w_{de} / 8\pi$.

3.5 Assuming that pressure p has relations with volume V and temperature T, we find

$$pV^{\alpha}T^{\beta} \sim \mathrm{p_P V_P^{\alpha} T_P^{\beta}} = \hbar^{-(2-3\alpha-\beta)/2}\mathrm{G}^{-(4-3\alpha+\beta)/2}\mathrm{c}^{(14-9\alpha+5\beta)/2}\kappa^{-\beta} \quad (25)$$

where $\mathrm{p_P} = \mathrm{c}^7 / \hbar\mathrm{G}^2$ is the Planck pressure. This is the GF for pressure, volume and temperature.

3.5.1 Instructing $2-3\alpha-\beta = 0$, and $4-3\alpha+\beta = 0 \rightarrow \alpha = 1$ and $\beta = -1$, also $14-9\alpha+5\beta = 0$, we obtain only

$$pV \sim \kappa T$$

This is Clapeyron equation, hasn't WN_{A} / M, where W is the gaseous mass, N_{A} the Avogadro constant and M the mass of gaseous mole molecule.

3.6 Assuming that thickness d has relations with temperature T and resistance R, we find

$$dT^{\alpha}R^{\beta} \sim \mathrm{L_P T_P^{\alpha} R_P^{\beta}} = \hbar^{(1+\alpha+2\beta)/2}\mathrm{G}^{(1-\alpha)/2}\mathrm{c}^{-(3-5\alpha)/2}\mathrm{e}^{-2\beta}\kappa^{-\alpha} \quad (26)$$

where $\mathrm{R_P} = \hbar / \mathrm{e}^2$ is the Planck resistance. This is the GF for thickness, temperature and resistance.

3.6.1 Ordering $1-\alpha = 0 \rightarrow \alpha = 1$, we obtain

$$dT \sim \hbar^{(1+\beta)}\mathrm{c}\,\kappa^{-1}\mathrm{e}^{-2\beta}R^{-\beta}$$

It is the power law of superconducting films $d\,T_{\mathrm{c}} = AR_{\mathrm{S}}^{-B}$ [25], where T_{c} is critical temperature, R_{S} sheet resistance, A and B are fitting parameters. When $\beta = 1$, we get $dT \sim \hbar^2\mathrm{c}\,\kappa^{-1}\mathrm{e}^{-2}R^{-1}$.

3.6.2 Instructing $1+\alpha+2\beta = 0$, and $1-\alpha = 0 \rightarrow \alpha = 1$ and $\beta = -1$, we gain

$$dT \sim \mathrm{c}\,\kappa^{-1}\mathrm{e}^2 R \ ?$$

3.6.3 Ordering $1+\alpha+2\beta = 0$, and $3-5\alpha = 0 \rightarrow \alpha = 3 / 5$ and $\beta = -4 / 5$, we have

$$d^5T^3 \sim \mathrm{G}\,\kappa^{-3}\mathrm{e}^8R^4 \ ?$$

3.7 Supposing that temperature T has relations with superfluid density ρ_s and mass m, we find

$$T\rho_s{}^{\alpha}m^{\beta} \sim \mathrm{T_P}\rho_{\mathrm{sP}}^{\alpha}\mathrm{M_P^{\beta}} = \hbar^{(1-2\alpha+\beta)/2}\mathrm{G}^{-(1+2\alpha+\beta)/2}\mathrm{c}^{(5+6\alpha+\beta)/2}\kappa^{-1} \quad (27)$$

where $\rho_{\mathrm{sP}} = \mathrm{c}^3 / \hbar\mathrm{G}$ is the Planck superfluid density. It is the GF for temperature and superfluid density.

3.7.1 Ordering $1+2\alpha+\beta = 0$, and $5+6\alpha+\beta = 0 \rightarrow \alpha = -1$, $\beta = 1$, we obtain

$$T \sim \hbar^2\rho_s / \kappa\, m$$

That is the Uemura's law $T_{\mathrm{c}} \propto n_{sO} / m^*$ [36] or one of the two formulas of critical temperature of LSCO and its superfluid density $T_{\mathrm{c}} = T_0 + \alpha\rho_{s0}$, where n_{sO} is the density of superconducting electrons, m^* the electron effective mass, $T_0 = (7.0 \pm 0.1)$K and $\alpha = 0.37 \pm 0.02$ [36].

Instructing $1+2\alpha+\beta = 0$ and $\beta = 0$, $\rightarrow \alpha = -1 / 2$, we gain

$$T \sim \hbar\mathrm{c}\sqrt{\rho_s} / \kappa$$

It is the other one of the two formulas of LSCO $T_{\mathrm{c}} = \gamma\sqrt{\rho_{s0}}$, where $\gamma = (4.2 \pm 0.5)\ \mathrm{K}^{1/2}$ [36].

Ordering $T = \mathrm{D}\hbar\,\mathrm{c}\sqrt{\rho_s} / \kappa = \gamma\sqrt{\rho_{s0}}$, we get $\mathrm{D} = \kappa\,\gamma / \hbar\mathrm{c} = 1834.2$. Similarly $T = T_0 + \mathrm{C}\,\hbar^2\rho_s / \kappa\, m = T_0 + \alpha\rho_{s0}$, we have $\mathrm{C} = \alpha\,\kappa\, m / \hbar^2$.

3.7.2 Ordering $1-2\alpha+\beta = 0$ and $5+6\alpha+\beta = 0$, $\rightarrow \alpha = -1 / 2$, $\beta = -2$, we gain

$$T \sim \mathrm{G}\, m^2\sqrt{\rho_s} / \kappa\ ?$$

3.8 Supposing that force F has relations with Hamiltonian function H and curvature k, we find

$$FH^{\alpha}k^{\beta} \sim \mathrm{F_P H_P}^{\alpha}\mathrm{k_P}^{\beta} = \hbar^{-(\alpha-\beta)/2}\mathrm{G}^{-(2+\alpha+\beta)/2}\mathrm{c}^{(8+5\alpha+3\beta)/2} \quad (28)$$

where $\mathrm{H_P} = \sqrt{\hbar\mathrm{c}^5/\mathrm{G}}$ is the Planck Hamiltonian function and $\mathrm{k_P} = \sqrt{\mathrm{c}^3/\hbar\mathrm{G}}$ the Planck curvature. It is the GF for force, Hamiltonian function and curvature.

3.8.1 Instructing $\alpha-\beta = 0$, and $2+\alpha+\beta = 0 \rightarrow \alpha = -1$ and $\beta = -1$, also $8+5\alpha+3\beta = 0$, we obtain merely

$$F \sim H\,k$$

That is the generalized CFL $dP / dt = -2H\,k\mathrm{n}$ [37], where P is the momentum, n the local unit normal vector, and $F \sim dP / dt$, hasn't $-2\mathrm{n}$.

4. Power Product Relationship between Four PQs

In this section, we obtain the relationship of power products when n = 4 in the GRE; obtain the centrifugal force formula.

4.0 Similarly when n = 4, we obtain

$$A_1^{\alpha_1} A_2^{\alpha_2} A_3^{\alpha_3} A_4^{\alpha_4} \sim \mathrm{A}_{1\mathrm{P}}^{\alpha_1} \mathrm{A}_{2\mathrm{P}}^{\alpha_2} \mathrm{A}_{3\mathrm{P}}^{\alpha_3} \mathrm{A}_{4\mathrm{P}}^{\alpha_4} \quad (29)$$

Instructing α_1 = 1, $\alpha_2 = \alpha$, $\alpha_3 = \beta$, $\alpha_3 = \gamma$, $A_1 = A$, $A_2 = B$, $A_3 = C$ and $A_4 = D$, we gain

$$AB^{\alpha} C^{\beta} D^{\gamma} \sim \mathrm{A_P} \mathrm{B_P^{\alpha}} \mathrm{C_P^{\beta}} \mathrm{D_P^{\gamma}} \quad (30)$$

when γ = 0, (20) is recovered. Therefore we can determine the power product relationship between four PQs. For example

4.1 Supposing that force F has relations with mass M, speed v and distance r, we find

$$FM^{\alpha} v^{\beta} r^{\gamma} \sim \hbar^{(\alpha+\gamma)/2} \mathrm{G}^{-(2+\alpha-\gamma)/2} \mathrm{c}^{(8+\alpha+2\beta-3\gamma)/2} \quad (31)$$

This is the GF for force, mass, speed and distance.

4.1.1 Ordering $\alpha + \gamma = 0$, $2 + \alpha - \gamma = 0$ and $8 + \alpha + 2\beta - 3\gamma = 0$ → $\alpha = -1$, $\beta = -2$ and $\gamma = 1$, we obtain

$$F \sim M v^2 / r$$

It is the centrifugal force formula. And so on.

5. Conclusion

In this paper we determine the power product relationship between PQs by the GRE. We find the following results:

1) The power product relationship between two PQs is determined when n = 2 in the GRE. Specially two PQs have direct proportion or inverse relation when their exponents are equal to −1 or 1.

2) The corresponding GFs are found by Assuming the relations between energy and mass, energy and frequency, energy and energy density, distance and mass, energy density and distance, per area force and distance, radiation density and frequency, radiation density and temperature, acceleration and temperature, entropy density and temperature, curvature tensor and energy-momentum tensor, Lagrange density function and electromagnetic field tensor respectively etc.

3) Many famous equations without corresponding factors are obtained, including Einstein mass-energy relation [14], temperature of event horizon of SBH [15], light quantum relation [26], inverse correlation between high-frequency quasi-periodic oscillation and black hole mass [27], biquadratic quanta energy with its density [19], radius of event horizon of stationary black holes [28], A.H. Compton wavelength formula [29], age of SBH [15], equation of HDE model [16], radiation formula for early universe [31], Casimir effect equation [17], negative pressure of HDE [16], Planck black body radiation formula [18], Stefan-Boltzmann law [19], Unruh formula [32], entropy density with cube of temperature [31] [33], Einstein field equation [20], electromagnetic Lagrange density function under Lorentz gauge [34], and so on.

4) Some new relations are found, including square of total energy with energy density of HDE, square of energy with its density of SBH, energy density with sextic radii of SBH, pressure in SBH centre, gravitational energy density far from the horizon inside SBH, entropy density of SBH center, etc.

5) The relations of power products of three and four PQs are determined when n = 3 and 4 in the GRE.

6) The corresponding GFs are found also by Assuming the relations between energy, mass and distance; energy, electric charge and distance; acceleration, force and mass; acceleration, mass and distance; pressure, volume and temperature; thickness, temperature and resistance; force, mass, speed and distance respectively and so on.

7) Also some famous equations without factors are gained, including Newtonian attraction law [21], Schrodinger equation [22], Coulomb law [23], Newtonian second law [21], Newtonian gravitational acceleration [21], height of the nth energy level of neutrons in the Earth's gravitational field [35], acceleration of HDE, Clapeyron equation [24], power law of superconducting films [25], Uemura's law, two formulas of critical temperature of LSCO [36], generalized CFL [37], and centrifugal force formula [21] etc.

8) Some relations which are given can't be understood.

9) The GRE can determine the power product relationship between two, three and four PQs, but can't give the corresponding factors. It is useful and significant.

Refences

[1] M. Kaku, "Quantum Field Theory, A Modern Introduction", Oxford University Press, 1993.

[2] F. Englert, R. Brout, *Phys. Rev. Lett.*, **13** (1964) 9: 321–23; P. W. Higgs, *Phys. Rev. Lett.*, **13** (1964), 508-509; G.S. Guralnik, C. R. Hagen, T. W. B. Kibble, *Phys. Rev. Lett.* **13** (1964) 20: 585–587; P. Higgs, *Phys. Lett.*, **12** (1964) 2: 132–133; O'Luanaigh, C, "New results indicate that new particle is a Higgs boson", *CERN*. 2013-10-09; Bryner, J, "Particle confirmed as Higgs boson", *NBC News*. 2013-03- 14; Heilprin, J, "Higgs Boson Discovery Confirmed After Physicists Review Large Hadron Collider Data at CERN", *The Huffington Post*. 2013-03-14.

[3] D. Clowe *et al.*, Astrophys. J. 648, L109 (2006), eprint: astro-ph/0608407; S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1.

[4] M.B. Green, J.H. Schwarz and E. Witten, "Superstring Theory", Cambridge University Press, 1987.

[5] C. Revelli, L. Smolin, *Nuclear physics* B **331**, 80 (1990).

[6] Wu Ning, *Commun. Theor. Phys.* **42**, 542 (2004); arXiv: gr-qc/0309041.

[7] A. G. Lisi, arXiv: hep-th/0711.0770; arXiv: gr-qc/1006.4908; arXiv: gr-qc/1506.08073.

[8] T. Ma and SH.H. Wang, arXiv: gen-ph/1210.0448; T. Ma, "See Physical World from Mathematical Point of View: Elementary Particle and Unified Field Theory", Science Press, 2014.

[9] Z-Y. Shen, *Journal of Modern Physics*, **4**, (2013), 1213-1380.

[10] Yue-Liang Wu, *Phys. Rev.* D **93**, 024012 (2016); *Eur. Phys.* J. C (2018) 78:28, https://doi.org/10.1140/epjc/s10052-017-5504-3.

[11] J.J. Hudson, D.M. Kara, I.J. Smallman, B.E. Sauer, M.R. Tarbutt & +*et al.*, *Nature* **473** (2011) 493-496.

[12] Yin Ye, Hu Suhui, *Frontier Science*, **1**, (2014), 8, 29.

[13] Y. Bao, vixra: 1502.0101.

[14] A. Einstein, *Ann. Physik*, **18** (1905), 639.

[15] S.W. Hawking, *J. Commun. Math. Phys.*, **43** (1975) 3: 199-220.

[16] M. Li, *Phys. Lett.* B **603**, 1 (2004); arXiv: hep-th/0403127; LI Miao, LI Xiao-Dong, WANG Shuang, and WANG Yi, *Commun. Theor. Phys.*, **56** (2011), 525-640.

[17] H.B.G. Casimir, *Proc. K. Ned. Akad. Wet.*, **51**. 793 (1948); E.M. Lifshitz, *Sov. Phys. JETP* **2**, 73 (1956).

[18] M. Planck, *Verhl. d. D. Phys. Ges.*, **2** (1900). 2002, 237; *Ann. Physik*, **1** (1900), 69, 719; 4 (1901) 553.

[19] J. Stefan, *Wien. Ber.*, **79** (1879), 391; L.E. Boltzmann, *Wie. Ann. Phys.*, **22** (1884), 291.

[20] A. Einstein, *Ann. Physik*, **49** (1916), 769.

[21] I. Newton, "Philosophiae naturalis Principia mathematica", 1687.

[22] E. Schrodinger, *Ann. Physik*, **79** (1926), 361, 489; **80** (1926), 437; **81** (1926), 109.

[23] C.A.de Coulomb, *J. de Phys.*, **27** (1785), 116; *Mém. Acad. Sci.*, **1785** (1788), 569, 578, 612; **1786** (1788), 67; **1787** (1791), 617; **1789** (1799), 455.

[24] Peng Huanwu, Xu Xiseng, "The Fundamentals of Theoretical Physics", Peking University Press, 1998, p.144.

[25] Y. Ivry *et al.*, arXiv: 1407.5945; Yong Tao, arXiv: 1504.07097, *Sci. Rep* **6**. 23863, doi: 10.1038/srep23863 (2016).

[26] A. Einstein, *Ann. Physik*, **17** (1905), 132.

[27] M.A. Abramowicz, T. Bulik, M. Bursa, & W. Kluźniak, *A&A*, **404**, L21, 2003; M.A. Abramowicz, & W. Kluźniak, *A&A*, **374**, L19, 2001; M.A. Abramowicz, W. Kluźniak, J.E. McClintock, & R.A. Remillard, *ApJL*, **609**, L63, 2004; M.A. Abramowicz,, & F.K. Liu, *A&A*, **548**, 3, 2012,; R.A. Remillard, M.P. Muno, J.E. McClintock, & J.A. Orosz, *ApJ*, **580**, 1030, 2002; R.A. Remillard, *AIPC*, **714**, 13, 2004; R.A. Remillard, & J.E. McClintock, *ARA&A*, **44**, 49, 2006; ; X-L. Zhou, W.M. Yuan, H-W. Pan, and Z. Hiu, *ApJ. Letter*, **798** (2015) L5; arXiv: cond-mat /1411.7731.

[28] S.W. Hawking, F.R. Ellis, "The large scale structure of space-time", Cambridge University Press, 1973.

[29] A.H. Compton, *Phys. Rev*., **21** (1923), 483; **22** (1923), 409.

[30] Y. Bao, vixra: 1409.0159.

[31] S. Weinberg, "GRAVITATION and COSMOLOGY, Principles and Applications of the General Theory of Relativity", Wiley, 1972.

[32] W.G. Unruh and R.M. Wald, *Phys. Rev.* D **25**, 942; *Gen. Rel. Grav.*, **15** 195 (1983); *Phys. Rev.* D **27**, 2271 (1983).

[33] MENG Qing-miao, *Journal of Heze Teachers College*, **26**, 2 (2004).

[34] C-F. Qiao, "Introduction to Quantum Field Theory", College of Physical Sciences, Graducate University, Chinese Academy of Science, 2008.

[35] V. V. Nesvizhevsky *et al.*, *Phys. Rev.* D **67**, 102002 (2003), DOI: 10.1103/PhysRevD.67.102002; arXiv: hep-th/0306198.

[36] Y.J. Uemura *et al.*, *Phys. Rev. Lett.* **62**, 2317 (1989); Y.J. Uemura *et al.*, *Phys. Rev. Lett.* **66**, 2317 (1991); I. Bozovic, X. He, J. Wu ＆ A.T. Bollinger, *Nature* **536**, 309-311 (2016), DOI: 10.1038/nature19061.

[37] L. D. Hu, D. K. Lian, ＆ Q. H. Liu, *Eur. Phys. J.* C (2016) **76**: 655. doi: 10.1140/epjc/s10052-016-4473-2.