Fermi Energy of Metals: a New Derivation

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Abstract-Two different ways of computing the time between collisions related to the electrical conductivity of metals are presented. The combination of them leads to the formula for the Fermi energy of metals.

1 - Introduction

The Fermi energy of metals is usually determined by considering the conduction electrons as free particles living in a box, where the occupancy of the energy levels is done by taking in account the Pauli exclusion principle, reflecting the fermionic character of the charge carriers [1,2,3]. The process also takes in account the energy levels of a particle confined in a cubic box, where the number of occupied states is fixed by the value of the Fermi energy. As a result, it is obtained the next relation for the Femi energy, E_F, in three dimensions

$$E_F = [\, h^2 / (8\, m)]\, (3/\pi)^{2/3}\, n^{2/3},$$

where h is the Planck's constant, m the electron mass and n the density of conduction electrons.

In this paper we are going to deduce the relation for the Fermi energy of metals in an alternative way. We consider two different ways of determine the time between collisions related to the physics of the electrical conductivity in metals. The first one treats this collision time as a particle lifetime. The second one makes use of the Drude relation for the electrical conductivity of metals, jointly with the ideas advanced by Landauer [4], which states that: "conduction is transmition" (please see also [5 and 6]).

2 – Average collision time as a particle lifetime

There are two characteristics linear momenta that we can associate to the free electrons responsible for the electrical conductivity of metals. They are the Fermi momentum mv_F and the Compton momentum mc. By taking into account the fermionic character of the electron, we will write a non-linear Dirac-like equation describing the "motion" of this particle. We have [7]

$$\partial\Psi / \partial x - (1/c)\, \partial\Psi / \partial t = [(m\, v_F)/\hbar]\, \Psi - [(mc)/\hbar]\, |\Psi^*\Psi|\Psi. \qquad (1)$$

We see that eq. (1) contains only first order derivatives of the field Ψ. Besides this, the field Ψ exhibits not a spinorial character. Taking the zero of (1) and solving for $|\Psi^*\Psi|$, we get

$$|\Psi^*\Psi| = v_F / c. \qquad (2)$$

On the other hand in the collision process, the conduction's electron loss its memory. We may think that this feature looks similar to the annihilation of a particle-anti particle pair, each of mass-energy equal to E_F. Putting this in a form of the uncertainty principle yields

$$2\, E_F\, \Delta t = h / 2 \qquad \text{or} \qquad h\, \upsilon / 2 = 2\, E_F. \qquad (3)$$

Solving equation (3) for υ, we get

$$\upsilon = 1 / \Delta t = 4\, E_F / h. \qquad (4)$$

By combining the results of (2) and (4) we obtain the line width Γ tied to the "particle" decay

$$\Gamma = (1/3)\, \upsilon \,|\, \Psi^{*}\Psi| = 4\, E_F\, v_F/(\, 3h\, c\,). \qquad (5)$$

We have introduced the factor one third in relation (5), thinking that equation (1) refers to a case of spherical symmetry, whereas in the presence of an electrical field we have an explicit brake of symmetry , conferring a linear character to the problem.

The averaged time between collisions τ is then given by

$$\tau = 1/\Gamma = (3\, h\, c\,)/(\, 4\, E_F\, v_F\,). \qquad (6)$$

2- Maximum time between collisions

Drude formula for the electrical conductivity of metals can be written as

$$\sigma = (e^2\, n\, \tau)/m, \qquad (7)$$

where e is the quantum of electric charge, n is the number of charge carriers per unit of volume, τ is the average time between collisions and m is the mass of the charge carriers.

Besides this in reference [5], starting from Landauer's paradigm: conduction is transmission [4,6], the relation for the electrical conductivity can be put in the form

$$\sigma = e^2 / (\pi \hbar \ell_0), \tag{8}$$

where ℓ_0 is the size of the channel of conduction.

In the case of the charge carrier being the electron, the maximum conductivity is reached when the length, ℓ_0, becomes equal to the reduced Compton wavelength of it, namely

$$\ell_0 = \hbar / (mc). \tag{9}$$

Inserting (9) into (8) we get

$$\sigma_{max} = (e^2 m c) / (\pi \hbar^2). \tag{10}$$

Making the identification between the two relations for the electrical conductivity, namely equaling (7) and (10), and solving for τ, we obtain for the maximum time between collisions the expression

$$\tau_{max} \equiv \tau = (m^2 c) / (n \pi \hbar^2). \tag{11}$$

3 – Fermi energy formula and concluding remarks

Making the equality between (6) and (11), we obtain that the Fermi energy of metals could be expressed as

$$E_F = [h^2 / (8 m)] (3 / \pi)^{2/3} n^{2/3}, \tag{12}$$

which is identical to the known formula exhibited in literature (please see [1,2,3,7].

Relation (3) can be interpreted as: the vacuum fluctuations, represented by the ground state energy of a harmonic oscillator, leads to the creation (annihilation) of a particle-antiparticle pair having mass-energy equals to $2E_F$. Meanwhile, this result combined with the Dirac-like equation (1), can be considered as an effective field theory account of the electrical conduction in metals. Finally formulas (11) and (12) imply that the basic parameters tied to the electrical conduction in metals are fixed by the number density of charge carriers.

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