Channel Access-Aware User Association with Interference Coordination in Two-Tier Downlink Cellular Networks

Uzma Siddique, Hina Tabassum, and Ekram Hossain

Abstract—**The diverse transmit powers of the base stations (BSs) in a multi-tier cellular network, on one hand, lead to uneven distribution of the traffic loads among different BSs when received signal power (RSP)-based user association is used. This causes under utilization of the resources at low-power BSs. On the other hand, strong interference from high-power BSs affects the downlink transmissions to the users associated with low-power BSs. In this context, this paper proposes a channel access-aware (CAA) user association scheme that can simultaneously enhance the spectral efficiency (SE) of downlink transmission and achieve traffic load balancing among different BSs. The CAA scheme is a network-assisted user association scheme that requires traffic load information from different BSs in addition to the channel quality indicators. We develop a tractable analytical framework to derive the SE of downlink transmission to a user who associates with a BS using the proposed CAA scheme. To mitigate the strong interference, the almost blank subframe (ABS)-based interference coordination is exploited first in macrocell-tier and then in smallcell-tier. The performance of the proposed CAA scheme is analyzed in presence of these two interference coordination methods. The derived expressions provide approximate solutions of reasonable accuracy compared to the results obtained from Monte-Carlo simulations. Numerical results comparatively analyze the gains of CAA scheme over conventional RSP-based association and biased RSP-based association with and without the interference coordination method. Also, the results reveal insights regarding the selection of the proportion of ABS in macrocell/smallcell-tiers for various network scenarios.**

Index Terms—**Two-tier cellular network, downlink user association, interference coordination, channel access probability, interference statistics, universal channel reuse, almost blank subframe (ABS), spectral efficiency (SE).**

I. Introduction

The deployment of low-power smallcells such as femtocells and picocells over existing macrocell networks is considered as a potential solution to boost the spectral efficiency (SE) performance (in bits/sec/Hz) of next-generation cellular networks [1]. Such kind of cellular networks are commonly referred as heterogeneous or multi-tier networks. The heterogeneity among different base stations (BSs) is due to their varying coverage areas, diverse traffic loads, transmission power limits, capital and operational expenditures [2] etc. The diversity among different BSs breeds several new challenges that may significantly impact the SE performance. For instance, due to diverse transmit powers of different BSs, most users prefer to associate with the high power BSs with the conventional received signal power (RSP)-based association scheme. This results in the uneven distribution of traffic load among different BSs and in turn underutilization of the resources at low power BSs [3].

To tackle the traffic load balancing problem, biased received signal power (BRSP)-based association (also known as cell range expansion (CRE)) is considered as a potential solution technique [4]–[6]. In such a scheme, an arbitrary bias is added to the received signal power from low power smallcell BSs (SBSs) that helps offloading more users from macro BSs (MBSs) to SBSs. Nevertheless, the SE of transmission to such users who associate to low-power BSs is affected by strong interference from the MBSs and neighboring SBSs. As such, interference coordination becomes mandatory to protect the off-loaded users from strong cross-tier as well as co-tier interferences. In this regard, almost blank subframe (ABS)-based interference coordination (also known as enhanced inter-cell interference co-ordination (eICIC)) [7] at MBSs and/or SBSs is a recommended technique. However, the feasibility and possible trade-offs of applying ABS in macrocell-tier or smallcell-tier have not been investigated yet in a comprehensive manner.

A. Related Work

A number of research works investigate the problem of joint user association, interference coordination, and/or traffic load balancing [4], [6], [8], [9]. CRE with fixed bias is considered in [4], [6] for traffic load balancing. An optimal proportion of ABS frames at MBS are derived by solving a network-wide utility maximization problem [4] and a sum-rate utility maximization problem considering full buffer and non full buffer traffic types [6]. Another interesting work to achieve traffic load balancing is [8] where centralized and distributed user association schemes are proposed. The algorithms however need to execute repetitively in order to adapt with the network variations. To cope with this issue, low-complexity signal-to-interference-plus-noise ratio (SINR) bias and rate bias-based association criteria are recommended. The SINR bias is obtained by a brute force search, and the best rate bias is the optimal BS price determined by the BS load. Both SINR bias and rate bias are same for all BSs in each tier. In [9], the framework of [8] is extended to consider interference

The authors are with the Department of Electrical and Computer Engineering at the University of Manitoba, Canada (emails: {Uzma.Siddique, Hina.Tabassum, Ekram.Hossain}.umanitoba.ca). This work was supported by a Strategic Project Grant (STPGP 430285) from the Natural Sciences and Engineering Research Council of Canada (NSERC).

coordination. The long-term network wide utility is maximized to find the optimal user association and proportion of blank resources. The scheme is centralized and maximizes the utility of all BSs and all users in the network.

Other research works focus on developing tractable mathematical frameworks to characterize the performance of CRE with fixed arbitrary bias either without interference coordination [10], [11] or with interference coordination [12]–[17]. In [10], [11], the performance of CRE is analyzed by deriving outage probability expressions. The SE performance for the offloaded users is analyzed in [12] considering ABS at MBS assuming that the distances between different users and their nearest BSs are independent. This assumption is relaxed in [13]. The outage probability and SE performances for the tagged link are analyzed in [14] considering CRE, ABS at MBS, and distributed antenna system. In [15], the mean throughput is analyzed to suggest the proportion of ABSs at the MBS and SBS given the BS placement statistics and target throughput for a typical user. In [16], SINR analysis is performed for resource partitioning in frequency domain. In [17], SE performance is analyzed by taking into consideration both CRE and eICIC at the MBS. Most of the aforementioned optimized cell-association schemes exploit significant knowledge of network information thus implying considerable signaling overhead.

The studies dealing with the theoretical performance analysis focus mainly on BRSP-based association that selects a static arbitrary bias value for all BSs of a specific tier. Although BRSP based association is simple, an optimal bias value needs to be calculated for different network scenarios. The bias value is typically not unique for each BS and, thus, cannot capture the traffic load conditions of different BSs. Note that the traffic load may significantly differ among various BSs of a specific tier. This traffic load imbalance, if not taken into account, can deteriorate the SE performance of an offloaded user (e.g., due to channel unavailability as well as strong interference from high-power or nearby BSs and poor link quality). Thus, there is a need to develop low-complexity user association schemes that can enhance the SE performance in the system by exploiting traffic load information in the different BSs (in different tiers as well as different BSs in a specific tier) in addition to the link quality information. Also, the performances of such schemes need to be characterized both in the presence of different eICIC schemes.

B. Contributions

This paper proposes a low-complexity channel access-aware (CAA) user association scheme for two-tier downlink cellular networks. In particular, a new user estimates its *channel access probability* (i.e., the probability that a channel will be available for this user) from different BSs and associates with a BS such that the product of channel access probability and received signal power from the BS is maximized. The proposed CAA association scheme exhibits the following features:

- The channel access probability from a BS reduces with increasing traffic load served by that BS. Thus, despite high received signal power, a user may not select such a congested BS.
- With the increasing number of incoming users, the CAA scheme balances the traffic load among different BSs. The channel access probability thus serves as a dynamic bias towards a given BS regardless of which tier it belongs to. This is different from the conventional BRSP in which a higher bias is always allocated to low-power BSs.
- The CAA scheme reduces to RSP-based association if channel access probabilities from all BSs turn out to be similar. On the other hand, it reduces to traffic load-based association if the received signal powers from different BSs turn out to be similar.

We develop a tractable framework to characterize the performance (in terms of SE [bits/sec/Hz]) of downlink transmission to a user who associates to a BS using the CAA scheme. Two different interference coordination schemes are exploited to overcome the strong cross-tier and co-tier interferences from MBSs and low-power SBSs, respectively, namely, (i) eICIC scheme-A in which ABS is adopted at MBSs; (ii) eICIC scheme-B in which ABS is adopted in specific number of SBSs.

Numerical results demonstrate the performance gains of CAA-based user association over conventional RSP-based and BRSP-based user association schemes with and without interference coordination. Insights are extracted related to selecting the proportion of ABS in eICIC schemes A and B and the number of smallcells that can be allowed to operate in ABS mode in eICIC scheme-B.

C. Paper Organization and Notations

The rest of the paper is organized as follows. Section II describes the system model. In Section III, we derive the statistics of the association metric for CAA scheme. In Sections IV-V, we derive the statistics of received signal and interference powers at a user who is associated to a BS using CAA scheme considering eICIC Scheme A and eICIC Scheme B, respectively. The SE is then derived in Section VI. Section VII presents the numerical and simulation results. Section VIII concludes the paper.

Notation: $\mathrm{Gamma}(\kappa_{(\cdot)}, \Theta_{(\cdot)})$ represents a Gamma distribution with shape parameter κ, scale parameter Θ and $(\cdot)$ displays the name of the random variable (RV). $\mathcal{K}_G(m_{c_{(\cdot)}}, m_{s_{(\cdot)}}, \Omega_{(\cdot)})$ represents the generalized-$\mathcal{K}$ distribution with fading parameter m_c, shadowing parameter m_s and average power Ω. $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx$ represents the Gamma function, $\Gamma_u(a;b) = \int_b^\infty x^{a-1}e^{-x}dx$ denotes the upper incomplete Gamma function, $\Gamma_l(a;b) = \int_0^b x^{a-1}e^{-x}dx$ denotes the lower incomplete Gamma function and $\Gamma(a;b_1;b_2) = \Gamma_u(a;b_1) - \Gamma_u(a;b_2) = \int_{b_1}^{b_2} x^{a-1}e^{-x}dx$ denotes the generalized Gamma function [18]. ${}_2F_1[\cdot,\cdot,\cdot,\cdot]$ denotes the Gauss's hyper geometric function. $\Pr(A)$ denotes the probability of event A. $f(\cdot)$, $F(\cdot)$, and $\mathcal{M}(\cdot)$ denote the probability density function (PDF), cumulative distribution function (CDF), and moment generating function (MGF), respectively. $\mathbb{E}[\cdot]$ denotes the expectation operator. A list of the main notations and their definitions is given in Table I.

TABLE I
SUMMARY OF THE MAIN VARIABLES AND THEIR DEFINITIONS

Variable	Definition
γ_m	Received signal power at the new user (NU) from MBS m
γ_s	Received signal power at the NU from SBS s
$\gamma_{(s)}$	Received signal power at the NU from $(s)^{\text{th}}$ ranked SBS
D_l	Distance between the reference MBS wherein the NU is located and the neighboring MBS l
S	Average number of SBSs within distance T of the NU
Q'	Number of active SBSs within distance T when S_1' SBSs are allowed to be in ABS mode
h_m	Hybrid association metric for MBS m
$h_{(s)}$	Hybrid association metric for the $(s)^{\text{th}}$ ranked SBS within distance T
$p_m^{(\cdot)}$	Association probability of the NU with MBS m; $(\cdot)$ denotes m-NABS mode or s-NABS mode or s-ABS mode
$p_{(s)}^{(\cdot)}$	Association probability of the NU with $(s)^{\text{th}}$ ranked SBS within distance T; $(\cdot)$ denotes m-NABS mode or m-ABS or s-NABS mode
$p_{(q')}^{\text{s}-\text{ABS}}$	Association probability of NU with $(q')^{\text{th}}$ ranked SBS within distance T in s-ABS mode
$r_{z,w}$	Distance between the NU located on polar coordinate (r_z, θ_w) in the reference MBS m and the neighboring MBS l, $r_z \in \{r_1, r_2, \cdots, r_{\mathcal{Z}}\}$, $\theta_w \in \{\theta_1, \theta_2, \cdots, \theta_{\mathcal{W}}\}$
$I_m^{(\cdot)}$	Cumulative interference at the NU when associated with MBS m; $(\cdot)$ denotes m-NABS mode, s-NABS mode, and s-ABS mode
$I_{(s)}^{(\cdot)}$	Cumulative interference at the NU when associated with $(s)^{\text{th}}$ ranked SBS within distance T; $(\cdot)$ denotes m-NABS mode, m-ABS, and s-NABS mode
$I_{(q')}^{\text{s}-\text{ABS}}$	Cumulative interference at the NU when associated with $(q')^{\text{th}}$ ranked SBS within distance T in s-ABS mode
ρ_m	Proportion of macrocell ABS
ρ_s	Proportion of smallcell ABS

II. SYSTEM MODEL AND ASSUMPTIONS

A. Network Model

We consider a downlink network of M circular macrocells. Each macro cell m has a coverage radius of R_m and number of users U_m. The macrocells are overlaid with S' randomly deployed smallcells. A given smallcell s has radius R_s and number of users U_s. The number of users U_m in a macrocell and U_s in a smallcell is a Poisson distributed RV with intensity λ_m and λ_s, respectively. The distribution of U_m and U_s, respectively, can therefore be given as:

$$P(U_m = u_m) = \frac{\lambda_m^{u_m}}{u_m!} e^{-\lambda_m}, \tag{1}$$

$$P(U_s = u_s) = \frac{\lambda_s^{u_s}}{u_s!} e^{-\lambda_s}. \tag{2}$$

Macro users (MUs) and smallcell users (SUs) are uniformly distributed within their corresponding macrocell and smallcell coverage area, respectively. Each MBS or SBS selects a user on a given transmission channel considering round-robin scheduling scheme. The smallcells are assumed to operate in the open access mode.

Now consider a new user (NU) entering into the two-tier network. The location of the NU is assumed to be uniformly distributed within a given macrocell region. Both MBS and SBSs possess an initial traffic load, i.e., U_m and U_s users are already associated with MBS m and SBS s, respectively. Once the NU arrives, it associates with either an SBS within a given circular region[1] of radius T around it or its nearest MBS depending on the association criterion. The number of SBSs Q which fall within distance T is random. For analytical tractability, we approximate the number of SBSs within distance T by its average, i.e., $\mathbb{E}[Q] = S$.

We consider shared spectrum in both macro-tier and smallcell-tier; however, the interference coordination at MBSs has been exercised to control macro-tier interference in eICIC scheme A. In eICIC scheme-A, all MBSs mute their transmissions in a synchronous manner for a proportion of subframes[2] ρ_m. All MBSs operate in either ABS or non-ABS mode. In the ABS mode, all MBSs mute their transmissions (referred to as m-ABS mode) and the NU will then have only option to associate with S SBSs. The associated MUs will remain in coverage hole (outage) and are not assumed to change their associations in the ABS mode. In the non-ABS mode, all MBSs operate normally (referred to as m-NABS mode) and serve their own local users. Similar to eICIC scheme-A, in eICIC scheme-B, S_1' out of S' SBSs operate in ABS mode for ρ_s proportion of time (referred to as s-ABS mode). The associated SUs will remain in outage and are not assumed to change their associations in ABS mode. On the other hand, in the non-ABS mode, all SBSs (referred to as s-NABS mode) serve their own local users.

B. Channel Model

The received signal power at NU on a given transmission channel from the MBS is defined as follows:

$$\gamma_m = P_m r_m^{-\beta} \chi_m, \qquad \forall m = 1, 2, \cdots, M, \tag{3}$$

[1]The region of radius T around the NU is considered to restrict the association distance of the NU from being very large and practically infeasible.

[2]The term proportion of subframes and duty cycle (ratio of the number of subframes in ABS mode to the total number of subframes) are used interchangeably throughout the paper.

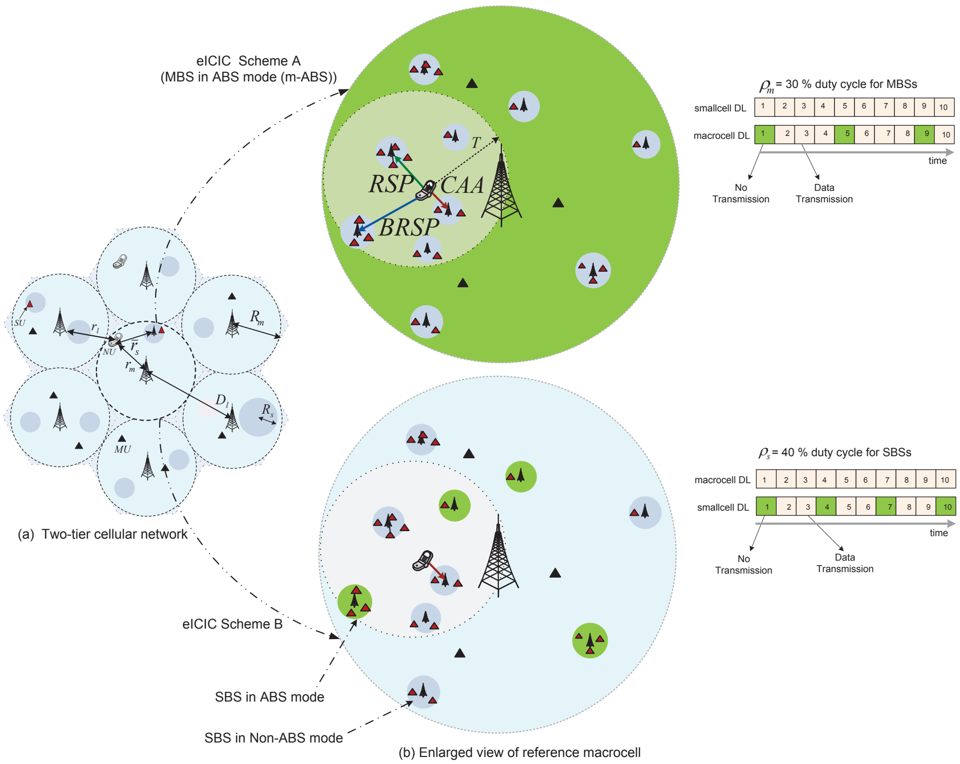

Fig. 1. (a) Graphical illustration of a two-tier smallcell network. In this part of the figure, the distances are demonstrated graphically. (b) Enlarged view of the reference macrocell illustrating eICIC scheme-A for MBS (when MBS is in ABS mode (m-ABS)) and eICIC scheme-B for SBS (when S_1' SBSs are in ABS mode (s-ABS)). Moreover, the working mechanisms of RSP, BRSP, and CAA-based downlink user associations are illustrated.

where β is the path-loss exponent, r_m denotes the distance of NU from the MBS m, P_m denotes the transmission power of m^{th} MBS per transmission channel, and χ_m represents the composite shadowing and fading channel. The received signal power at the NU from any arbitrary neighboring MBS is defined as follows:

$$X_l = P_l r_l^{-\beta} \chi_l, \qquad \forall l \neq m, \tag{4}$$

where r_l denotes the distance of NU in the reference MBS from the neighboring MBS l located at distance D_l from the reference MBS and P_l denotes the transmit power of l^{th} MBS per transmission channel.

Similarly, the received signal power at the NU on a given transmission channel from SBS s is given as follows:

$$\gamma_s = P_s \bar{r}_s^{-\beta} \zeta_s, \quad \forall s = 1, 2, \cdots, S', \tag{5}$$

where P_s represents the transmit power of SBS s, $\bar{r}_s$ denotes the distance between the NU and the SBS s, and ζ_s represents the composite shadowing and fading channel.

Generally, composite fading distributions can be used to jointly model the shadowing and fading channels. Recently, the generalized-$\mathcal{K}$ distribution has been proposed wherein Gamma distribution [19], [20] is used to model the shadowing as well as fading channels. Since the PDF, CDF, and MGF of the Generalized-$\mathcal{K}$ distribution engage computation-intensive special functions such as `Meijer-G` and `Whittaker` functions, the distribution is approximated with a more tractable Gamma distribution using the moment matching method, i.e., $\mathcal{K}_G(m_c, m_s, \Omega) \approx \text{Gamma}(\kappa, \Theta)$ [20]. By matching the first and second moments of the two distributions, the corresponding values of κ and Θ can be given as [20]:

$$\kappa = \frac{m_c m_s}{m_c + m_s + 1 - m_c m_s \epsilon}, \Theta = \frac{\Omega}{\kappa}, \tag{6}$$

where ϵ is an adjustment parameter. Thus, ζ and χ will be considered as Gamma RVs throughout the paper.

C. Channel Access-Aware (CAA) User Association

The CAA user association is a generalized scheme that considers both the channel access probability (i.e., the probability of obtaining a channel for transmission from a BS) as well as the received signal power from different BSs as the association metric. Given that the round-robin scheduling[3] is used at each BS, the channel access probability of the NU with the nearest MBS can be given as $p_m^{\text{access}} = \frac{1}{U_m+1}$ and with SBS as $p_s^{\text{access}} = \frac{1}{U_s+1}$. The association criterion can then be written as follows:

$$k^*_{\text{CAA}} = \arg\max\{h_m, h_s\}, \qquad \forall\, s = 1, 2, ..., S, \tag{7}$$

where

$$h_m = \frac{\gamma_m}{(U_m+1)} = \frac{P_m r_m^{-\beta} \chi_m}{(U_m+1)}, \tag{8}$$

[3]For ease of exposition, we focus on round-robin scheduling in this paper. However, the channel access probabilities can be derived for a variety of scheduling schemes. Interested readers may refer to [21].

$$h_s = \frac{\gamma_s}{(U_s+1)} = \frac{P_s \bar{r}_s^{-\beta} \zeta_s}{(U_s+1)}. \quad (9)$$

For comparison purposes, we consider the following user association schemes:

1) Conventional RSP-based association: in which the NU utilizes the received signal powers from all SBSs within distance T as well as its nearest MBS to select the best BS, i.e.,

$$k^*_{\mathrm{RSP}} = \arg\ \max\{\gamma_m, \gamma_s\},\ \forall s = 1,2,...,S. \quad (10)$$

2) BRSP-based association: in which an arbitrary bias value b is assigned to all SBSs. NU utilizes the biased received signal powers from all SBSs within distance T and the received power from the nearest MBS to select the best BS, i.e.,

$$k^*_{\mathrm{BRSP}} = \arg\ \max\{\gamma_m, b\gamma_s\},\ \forall s = 1,2,...,S. \quad (11)$$

D. Analytical Approach: Spectral Efficiency Evaluation

The considered eICIC scheme-A and eICIC scheme-B differ in terms of their association probabilities and in turn received signal and interference powers. Therefore, we outline their corresponding methodology of evaluating the SE of downlink transmission to the NU separately in the following.

eICIC Scheme-A:

- Derive the average number of SBSs within distance T, i.e., S.
- Derive the association probability of the NU with reference MBS ($p_m^{\mathrm{m-NABS}}$), and S SBSs ($p_{(s)}^{\mathrm{m-NABS}}$) in m-NABS mode.
- Derive the association probability of the NU with S SBSs ($p_{(s)}^{\mathrm{m-ABS}}$) in m-ABS mode.
- Derive the MGF of the received signal power and cumulative interference at NU in m-NABS and m-ABS modes.
- Derive the SE of the NU in m-ABS mode ($\mathcal{C}^{\mathrm{m-ABS}}$) and m-NABS mode ($\mathcal{C}^{\mathrm{m-NABS}}$).

eICIC Scheme-B:

- Derive the distribution of the number of active SBSs within distance T, i.e., Q' when S_1' SBSs are allowed to be in ABS mode for ρ_s proportion of time.
- Derive the association probability of the NU with reference MBS ($p_m^{\mathrm{s-NABS}}$), and S SBSs ($p_{(s)}^{\mathrm{s-NABS}}$) when all SBSs operate normally (s-NABS mode). This step is identical to m-NABS mode in eICIC scheme-A.
- Derive the association probability of the NU with MBS ($p_m^{\mathrm{s-ABS}}$), and Q' SBSs ($p_{(q')}^{\mathrm{s-ABS}}$) when $S_1^{'}$ SBSs operate in ABS mode (s-ABS mode) for ρ_s proportion of time.
- For both the s-ABS and s-NABS modes, derive the MGF of the received signal and cumulative interference at the NU.
- Derive the average SE of the NU in s-ABS mode ($\mathcal{C}^{\mathrm{s-ABS}}$) and s-NABS mode ($\mathcal{C}^{\mathrm{s-NABS}}$).

The total average SE of downlink transmission to a NU in eICIC schemes-A and B can therefore be calculated, respectively, as follows:

$$\mathcal{C}^{\mathrm{scheme-A}} = \rho_m\ \mathcal{C}^{\mathrm{m-ABS}} + (1-\rho_m)\ \mathcal{C}^{\mathrm{m-NABS}}. \quad (12)$$

$$\mathcal{C}^{\mathrm{scheme-B}} = \rho_s\ \mathcal{C}^{\mathrm{s-ABS}} + (1-\rho_s)\ \mathcal{C}^{\mathrm{s-NABS}}. \quad (13)$$

III. Statistics of h_m and h_s

In this section, first we derive the statistics of h_m. Then, we derive the the statistics of Q and h_s for eICIC scheme-A and the statistics of Q' and $h_{q'}$ for eICIC scheme-B.

A. Statistics of h_m

Since the NU is uniformly distributed in a reference macrocell, the PDF of r_m is given by:

$$f_{r_m}(r) = \frac{2r}{R_m^2}, \qquad 0 \le r \le R_m. \quad (14)$$

The distribution of h_m can then be derived by conditioning on r_m and U_m in (8), doing transformation of RV, i.e., $f_{h_m}(h_m|U_m, r_m) = \frac{r_m^\beta (U_m+1)}{P_m} f_\chi\left(h_m \frac{r_m^\beta (U_m+1)}{P_m}\right)$, and averaging over the distributions of U_m and r_m, respectively, as follows:

$$f_{h_m}(h_m) = \sum_{u_m=0}^{\infty} \frac{(u_m+1)P(U_m=u_m)}{P_m} \times \int_0^{R_m} r^\beta f_\chi\left(h_m \frac{r^\beta(u_m+1)}{P_m}\right) f_{r_m}(r) dr. \quad (15)$$

Since $\chi \sim \text{Gamma}(\kappa_\chi, \Theta_\chi)$, we can rewrite (15) by substituting the PDF of r_m given in (14), U_m given in (1), as well as χ and doing some algebraic manipulations as follows:

$$f_{h_m}(h_m) = \sum_{u_m=0}^{\infty} \frac{2(\frac{u_m+1}{P_m\Theta_\chi})^{\kappa_\chi} \int_0^{R_m} r^{\beta\kappa_\chi+1} e^{-\frac{h_m(u_m+1)r^\beta}{P_m\ \Theta_\chi}} dr}{e^{\lambda_m}\ u_m!\ R_m^2\ \lambda_m^{-u_m}\ \Gamma(\kappa_\chi) h_m^{1-\kappa_\chi}}. \quad (16)$$

After solving the integral in (16) using [18, 3.381/8], the PDF of h_m can be simplified as follows:

$$f_{h_m}(h_m) = \sum_{u_m=0}^{\infty} \frac{2\ \lambda_m^{u_m} (\frac{P_m\Theta_\chi}{1+u_m})^{\frac{2}{\beta}} \Gamma_l\left(\frac{2+\beta\kappa_\chi}{\beta}, \frac{h_m\ R_m^\beta(1+u_m)}{P_m\Theta_\chi}\right)}{e^{\lambda_m}\ h_m^{\frac{2+\beta}{\beta}}\ R_m^2\ \beta\ u_m!\ \Gamma(\kappa_\chi)}. \quad (17)$$

Consequently, using the property $\Gamma_l(\cdot) + \Gamma_u(\cdot) = \Gamma(\cdot)$, and [18, Eq.(06.06.21.0002.01)] and after some algebraic manipulations, the CDF of h_m, i.e., $F_{h_m}(h_m) = \int_0^{h_m} f_{h_m}(u) du$ can be derived as follows:

$$F_{h_m}(h_m) = \sum_{u_m=0}^{\infty} \frac{\lambda_m^{u_m} e^{-\lambda_m}\ (\frac{P_m\Theta_\chi}{1+u_m})^{\frac{2}{\beta}}}{h_m^{\frac{2}{\beta}}\ R_m^2\ u_m! \Gamma(\kappa_\chi)} \times \left(\frac{R_m^2\ \Gamma_l(\kappa_\chi, \frac{h_m R_m^\beta(1+u_m)}{P_m\Theta_\chi})}{(P_m\Theta_\chi)^{\frac{2}{\beta}}(h_m(1+u_m))^{-\frac{2}{\beta}}} - \Gamma_l\left(\kappa_\chi + \frac{2}{\beta}, \frac{h_m\ (1+u_m)}{R_m^{-\beta} P_m\Theta_\chi}\right)\right). \quad (18)$$

B. Distribution of Q and h_s

1) Distribution of Q: Since Q is a discrete random variable, the distribution of Q can be given by the Binomial distribution

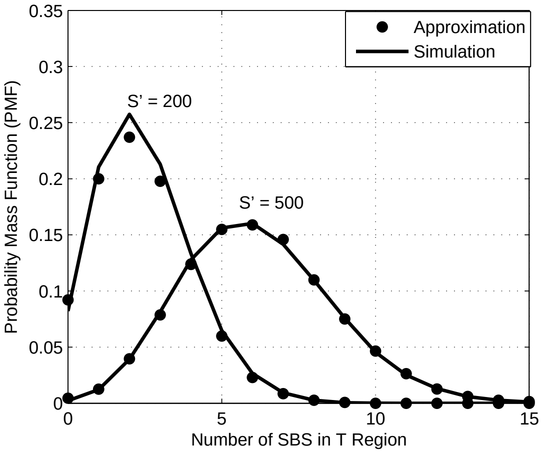

Fig. 2. Comparison between the approximated PMF of the number of SBSs within distance T given in (19) and the exact PMF obtained from Monte-Carlo simulations (for $R_m = 300$ m, $\beta = 2, T = 100$ m, $D_l = 600$ m).

as:

$$p_Q(q) = \binom{S'}{q} p^q (1-p)^{S'-q}, \; \forall \; q = 0, 1, \cdots, S', \qquad (19)$$

where p is the probability of a SBS to fall within distance T around NU. To compute p, the distribution of the distance between an arbitrary SBS and NU is required. Note that the NU can be located anywhere within R_m and the SBS can be located anywhere within $R_m + D_l$ from the reference MBS, where $D_l = 2R_m$. The exact distance distribution between NU and SBS that are distributed in different radii is unknown yet[4]. To make the framework tractable, we consider the following assumption which simplifies the distribution of the distance between an arbitrary SBS and the NU.

Assumption: Since the NU considers SBSs only within circular region of radius T around her, the impact of the location of NU within $R_m + D_l$ is not significant (specifically in the calculations that relates to the distance between NU and S SBSs within distance T). This has also been verified through Monte-Carlo simulations in Fig. 2. As such, we assume that the NU is located at origin. Thus, the exact distance distribution between NU and an arbitrary SBS ($f_{\bar{r}_s}(\bar{r}_s)$) is approximated as $f_{\bar{r}_s}(\bar{r}_s) \approx f_{r_s}(r_s) = \frac{2r_s}{(R_m+D_l)^2}$, where $0 \leq r_s \leq R_m + D_l$.

Based on this assumption, we derive the value of $p = F_{r_s}(T) = P(r_s \leq T)$, as follows:

$$F_{r_s}(T) = \int_0^T f_{r_s}(r_s) dr_s = \frac{T^2}{(R_m + D_l)^2}. \qquad (20)$$

The average number of SBSs within distance T can then be given as $S = \mathbb{E}[Q] = S'p = S' F_{r_s}(T)$. To validate the accuracy of the considered approximation, Fig. 2 compares the approximated PMF of Q obtained by using (19) with the exact PMF obtained from the Monte-Carlo simulations (in which NU can be located anywhere within R_m). The analytical approximation is found to be in a good agreement with the simulation results.

2) Statistics of h_s: To derive the statistics of h_s, we rank the distances of SBSs within distance T from NU. In particular, we derive the ranked distribution of r_s by utilizing the assumption stated above and the theory of ordered statistics such that $r_{(1)} < \cdots < r_{(S)}$, where $r_{(1)}$ and $r_{(S)}$ represent the minimum distance and maximum distance of the SBSs from the NU, respectively. The PDF of $r_{(s)}$ can be defined as:

$$f_{r_{(s)}}(r_{(s)}) = \frac{S! \Big(F_{r_s}(r_s)\Big)^{s-1} \Big(1 - F_{r_s}(r_s)\Big)^{S-s} f_{r_s}(r_s)}{(s-1)!(S-s)!}. \qquad (21)$$

Substituting $f_{r_s}(r_s)$, and $F_{r_s}(r_s)$, (21) can be rewritten as:

$$f_{r_{(s)}}(r_{(s)}) = \frac{S! \Big(\frac{r_{(s)}^2}{(R_m+D_l)^2}\Big)^{s-1} \Big(1 - \frac{r_{(s)}^2}{(R_m+D_l)^2}\Big)^{S-s} 2r_{(s)}}{(R_m + D_l)^2 (s-1)! \, (S-s)!}. \qquad (22)$$

Using Binomial expansion, (22) can be simplified as:

$$f_{r_{(s)}}(r_{(s)}) = \sum_{n=0}^{S-s} \frac{2 \; S! \; (-1)^{S-s-n} \left(\frac{r_{(s)}}{R_m+D_l}\right)^{2S-2n-1}}{(s-1)! \; (S-s-n)! \; n! \; (R_m+D_l)^2}. \qquad (23)$$

Since $0 \leq r_{(s)} \leq R_m + D_l$, we truncate this distribution to a maximum distance of T. The truncated distribution[5] of $r_{(s)}$ can be described as:

$$\tilde{f}_{r_{(s)}}(r_{(s)}) = \frac{f_{r_{(s)}}(r_{(s)})}{\int_0^T f_{r_{(s)}}(r_{(s)}) dr_{(s)}}. \qquad (24)$$

Now h_s corresponding to ranked SBS s is defined as follows:

$$h_{(s)} = \frac{P_s \; r_{(s)}^{-\beta} \zeta_s}{U_s + 1}, \quad \forall s = 1, \cdots, S. \qquad (25)$$

The PDF of $h_{(s)}$ can then be derived by conditioning on the distribution of U_s and $r_{(s)}$ in (25), doing transformation of RV, i.e., $f_{h_{(s)}}(h_{(s)}|U_s, r_{(s)}) = \frac{r_{(s)}^{\beta}(U_s+1)}{P_s} f_\zeta \left(h_{(s)} \frac{r_{(s)}^{\beta}(U_s+1)}{P_s}\right)$, and averaging over the distributions of U_s and truncated $r_{(s)}$, respectively, as follows:

$$f_{h_{(s)}}(h_{(s)}) = \sum_{u_s=0}^{\infty} \frac{\lambda_s^{u_s} e^{-\lambda_s} (\frac{u_s+1}{P_s \Theta_\zeta})^{\kappa_\zeta} h_{(s)}^{\kappa_\zeta - 1}}{u_s! \Gamma(\kappa_\zeta)} \times$$
$$\int_0^T r_{(s)}^{\beta\kappa_\zeta} f_\zeta \left(h_{(s)} \frac{r_{(s)}^{\beta}(u_s+1)}{P_s \Theta_\zeta}\right) \tilde{f}_{r_{(s)}}(r_{(s)}) dr_{(s)}, \qquad (26)$$

where $\zeta \sim \text{Gamma}(\kappa_\zeta, \Theta_\zeta)$. By substituting $\tilde{f}_{r_{(s)}}(r_{(s)})$, solving the integral and doing some algebraic manipulations,

[4]The distribution of distance between two random points uniformly distributed in the same radius can be obtainable by using random line picking theory; however, it is quite complicated [22].

[5]Truncated distribution is a conditional distribution that results from restricting the domain of $f_{r_{(s)}}(r_{(s)})$.

we can rewrite (26) as follows:

$$f_{h_{(s)}}(h_{(s)}) = \sum_{n=0}^{S-s}\sum_{u_s=0}^{\infty} \frac{K_n \lambda_s^{u_s} \Gamma_l\left(\frac{\beta\kappa_\zeta - 2n+2S}{\beta}, \frac{h_{(s)}T^\beta(1+u_s)}{p_s\Theta_\zeta}\right)}{u_s!\Gamma(\kappa_\zeta)\beta\ e^{\lambda_y} h_{(s)}(\frac{h_{(s)}(1+u_s)}{p_s\Theta_\zeta})^{\frac{2S-2n}{\beta}}}, \tag{27}$$

where

$$K_n = \frac{2\ S!\ (-1)^{S-s-n}\left(\frac{1}{R_m+D_l}\right)^{2S-2n}}{(\int_0^T f_{r_{(s)}}(r_{(s)})dr_{(s)})(s-1)!\ (S-s-n)!n!}. \tag{28}$$

Consequently, using the property $\Gamma_l(\cdot)+\Gamma_u(\cdot)=\Gamma(\cdot)$ as well as [18, 3.381/1] and after doing some algebraic manipulations, the CDF of $h_{(s)}$, i.e., $F_{h_{(s)}}(h_{(s)}) = \int_0^{h_{(s)}} f_{h_{(s)}}(u)du$, can be derived as follows:

$$F_{h_{(s)}}(h_{(s)}) = \sum_{n=0}^{S-s}\sum_{u_s=0}^{\infty} \frac{K_n\lambda_s^{u_s}}{2e^{\lambda_s}u_s!(n-S)\Gamma(\kappa_\zeta)}\times \left(\frac{\Gamma_l(\frac{\beta\kappa_\zeta-2n+2S}{\beta}, \frac{h_{(s)}\ T^\beta(1+u_s)}{p_s\Theta_\zeta})}{(\frac{h_{(s)}(1+u_s)}{p_s\Theta_\zeta})^{\frac{2S-2n}{\beta}}} - \frac{\Gamma_l(\kappa_\zeta, \frac{h_{(s)}T^\beta(1+u_s)}{P_s\Theta_\zeta})}{T^{2n-2S}}\right), \tag{29}$$

where K_n is defined in (28).

C. Distribution of Q' and $h_{(q')}$

In eICIC scheme-B, S_1' SBSs out of S' operate in ABS mode for ρ_s proportion of time. The number of active SBSs within distance T, i.e., Q' in s-ABS mode is a discrete random variable for a given ρ_s. The distribution of Q' can be given as:

$$p_{Q'}(Q') = \binom{S}{Q'} a^{Q'}(1-a)^{S-Q'},\ \forall Q' = 1,\cdots,S, \tag{30}$$

where a is the probability of an SBS within distance T to be active during s-ABS mode and it can be calculated as

$$a = (1-\rho_s)\Big(\frac{S_1'}{S'}\Big) + \Big(\frac{S'-S_1'}{S'}\Big). \tag{31}$$

Since the ABS mode is applied randomly to any SBS, the likelihood of any SBS (either within or outside T) to be active is same and can be given by considering two possibilities, i.e.,

- the probability that a SBS belongs to the group of S_1' SBSs in ABS mode $\left(\frac{S_1'}{S'}\right)$ and its probability to be active $(1-\rho_s)$.
- the probability that an SBS belongs to the group of $S'-S_1'$ SBSs in non-ABS mode $\left(\frac{S'-S_1'}{S'}\right)$ and its probability to be active is 1.

Combining the two possibilities, we obtain (31). Now, the association criterion can be calculated for each ranked active SBS within distance T as follows:

$$h_{(q')} = \frac{P_{q'}\ r_{(q')}^{-\beta}\zeta_{q'}}{U_{q'}+1},\ \forall\ q' = 1,\cdots,S. \tag{32}$$

The PDF and CDF of $h_{(q')}$ can be derived in a similar manner by replacing s with q' in (27) and (29), respectively.

IV. Association Probabilities, Statistics of Signal and Interference Powers: eICIC Scheme-A

In eICIC scheme-A, the MBS operates in ABS mode for ρ_m and non-ABS mode for $(1-\rho_m)$ proportion of subframes. First we evaluate the association probabilities of the NU with MBS ($p_m^{\mathrm{m-NABS}}$) as well as SBSs ($p_{(s)}^{\mathrm{m-NABS}}$) in m-NABS mode and with SBSs ($p_{(s)}^{\mathrm{m-ABS}}$) in m-ABS mode. Then, we derive the statistics of the received signal and interference at the NU when it gets associated with MBS or a SBS in m-NABS mode and with a SBS in m-ABS mode.

A. Association Probabilities

1) Association probabilities in m-NABS mode: In m-NABS mode, MBSs operate normally and serve their associated users. In this case, the NU has opportunity to associate with either an MBS or an SBS depending on the association probability.

Conditioned on h_m, the association probability of the NU with the nearest MBS can be derived as:

$$p_{m|h_m}^{\mathrm{m-NABS}} = \Pr(\underset{s=1,2,\ldots,S}{h_{(s)}} \le h_m) = \prod_{s=1}^{S} F_{h_{(s)}}(h_m), \tag{33}$$

where $F_{h_{(s)}}(h_{(s)})$ is given in (29). The unconditional $p_m^{\mathrm{m-NABS}}$ can then be derived by averaging over the distribution of h_m as follows:

$$p_m^{\mathrm{m-NABS}} = \int_0^\infty \prod_{s=1}^{S} F_{h_{(s)}}(h_m) f_{h_m}(h_m)dh_m. \tag{34}$$

Substituting $F_{h_{(s)}}(h_{(s)})$ and $f_{h_m}(h_m)$ from (29) and (17), respectively, (34) can be solved using standard mathematical software such as `MATHEMATICA` and `MAPLE`.

Similarly, the association probability of the NU with the s ranked SBS in m-NABS mode can be derived by conditioning on $h_{(s)}$ as follows:

$$\begin{aligned} p_{(s)|h_{(s)}}^{\mathrm{m-NABS}} &= \Pr(\underset{\substack{n=1,2,\ldots,S\\ n\neq s}}{h_{(n)}}, h_m \le h_{(s)}) \\ &= \prod_{\substack{n=1\\ n\neq s}}^{S} F_{h_{(n)}}(h_{(s)}) F_{h_m}(h_{(s)}). \end{aligned} \tag{35}$$

The unconditional $p_{(s)}^{\mathrm{m-NABS}}$ can be derived by averaging over the distribution of $h_{(s)}$ as follows:

$$p_{(s)}^{\mathrm{m-NABS}} = \int_0^\infty \prod_{\substack{n=1\\ n\neq s}}^{S} F_{h_{(n)}}(h_{(s)}) F_{h_m}(h_{(s)}) f_{h_{(s)}}(h_{(s)})dh_{(s)}. \tag{36}$$

Substituting $F_{h_{(s)}}(h_{(s)})$, $F_{h_m}(h_m)$ and $f_{h_{(s)}}(h_{(s)})$ given in (29), (18) and (27) in (36), we can solve (36) using standard mathematical softwares such as `MATHEMATICA` and `MAPLE`.

Fig. 3 compares the analytical and simulation results for the association probabilities of the NU considering no interference coordination (or 100% non-ABS mode) and two traffic load scenarios, i.e., (i) $\lambda_m = \lambda_s, S' = 500$ (ii) $\lambda_m \ge \lambda_s$ for (a) $S' = 200$ (b) $S' = 500$. Analytical results corroborate with the simulation results. It can be noted that for (i) the association

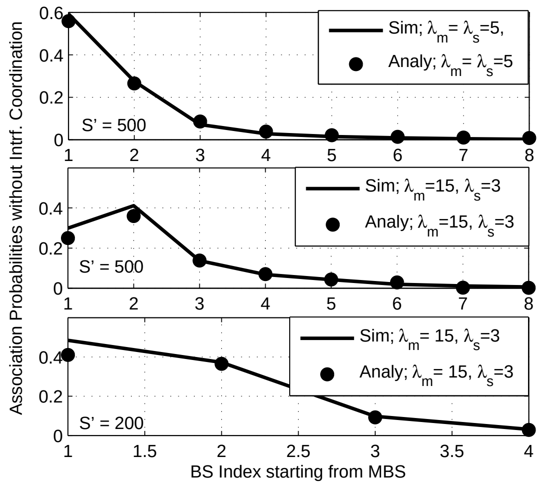

Fig. 3. Association probabilities of the NU with a reference MBS and S SBSs without interference coordination (for $R_m = 300$ m, $T = 100$ m, ρ_m = ρ_s = 0). BS indices $[2, \cdots, 4]$ represent the S SBSs within distance T.

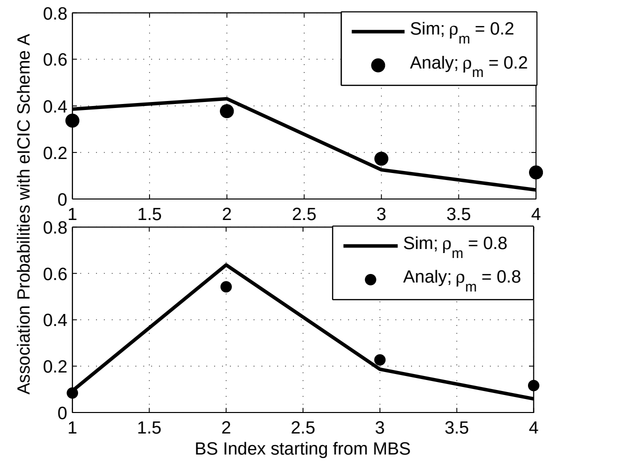

Fig. 4. Association probabilities of the NU with a reference MBS and S SBSs within distance T in eICIC scheme-A (for $R_m = 300$ m, $T = 100$ m, $S' = 200$, $\lambda_m = 15$, $\lambda_s = 3$). BS indices $[2, \cdots, 4]$ represent the S SBSs within distance T.

probability of the NU is higher for MBS than the nearest SBS. This is due to the fact that with same traffic load intensities in both tiers, CAA scheme follows the mechanism of RSP scheme. Thus, the NU is highly likely to associate with a high power MBS. On the other hand, when $\lambda_m \geq \lambda_s, S' = 500$, the association probability of the NU with the nearest SBS turns out to be greater than the MBS. Clearly, in this case, the NU has a higher association probability with SBSs within distance T due to high traffic load in the MBS. It is however important to note that, the association probability of the NU with an MBS turns out to be higher again if S' reduces to 200. This occurs due to small number of SBSs that are relatively far apart from NU compared to the case when $S' = 500$.

2) Association probabilities in m-ABS mode: In this mode, MBSs operate in ABS mode and the NU has only option to associate with one of the SBSs. Thus, the association probability of the NU with the s ranked SBS can be derived by conditioning on $h_{(s)}$ as follows:

$$p^{\mathrm{m-ABS}}_{(s)|h_{(s)}} = \prod_{\substack{n=1\\n\neq s}}^{S} F_{h_{(n)}}(h_{(s)}). \tag{37}$$

The unconditional $p^{\mathrm{m-ABS}}_{(s)}$ can then be derived by averaging over the distribution of $h_{(s)}$ as follows:

$$p^{\mathrm{m-ABS}}_{(s)} = \int_0^\infty \prod_{\substack{n=1\\n\neq s}}^{S} F_{h_{(n)}}(h_{(s)}) f_{h_{(s)}}(h_{(s)}) dh_{(s)}. \tag{38}$$

Substituting $F_{h_{(s)}}(h_{(s)})$, and $f_{h_{(s)}}(h_{(s)})$ given in (29) and (27), respectively, solution to the (38) can be obtained using `MATHEMATICA` and `MAPLE`.

Fig. 4 compares the analytical and simulation results for the association probabilities of the NU with a reference MBS and S SBSs in eICIC scheme-A considering (i) $\rho_m = 0.2$ (ii) $\rho_m = 0.8$. For $\rho_m = 0.2$, the association probability of the NU with the nearest SBS is slightly higher than the MBS which is different from the case of no interference coordination in Fig. 3. For $\rho_m = 0.8$, the association probability with MBS further decreases. This is because, in the m-ABS mode, the NU has only option to select one of SBSs. Therefore, the higher ρ_m the higher would be the chance to associate with SBSs. Since the transmit power of all SBSs are same, the CAA scheme behaves like the RSP scheme during m-ABS mode. This becomes more evident when $\rho_m = 0.8$.

B. Statistics of Received Signal Powers

1) Association with MBS: Since the NU can associate with an MBS in the m-NABS mode, the received signal power at the NU from MBS γ_m is defined in (3). The MGF of γ_m can be derived as [21]:

$$\mathcal{M}_{\gamma_m}(t) = {}_2F_1\left[\kappa_\chi, -\frac{2}{\beta}, 1-\frac{2}{\beta}, \frac{-P_m t \Theta_\chi}{R_m^\beta}\right] - \frac{\Gamma(\kappa\chi + \frac{2}{\beta})\Gamma(1-\frac{2}{\beta})}{R_m^2 \Gamma(\kappa_\chi)(P_m t \Theta_\chi)^{\frac{2}{\beta}}}. \tag{39}$$

2) Association with SBS: The NU can associate with the s^{th} ranked SBS within distance T in both m-NABS mode and m-ABS mode. When NU associates with the s ranked SBS, the received signal power at the NU is defined as:

$$\gamma_{(s)} = P_s r_{(s)}^{-\beta} \zeta_s, \qquad \forall\ s = 1, ..., S, \tag{40}$$

where $r_{(s)}$ represents the distance between the NU and s ranked SBS. The PDF of $\gamma_{(s)}$ can be derived by conditioning on the distribution of $r_{(s)}$ in (40), doing transformation of RV, i.e., $f_{\gamma_{(s)}}(\gamma_{(s)}|r_{(s)}) = \frac{r_{(s)}^\beta}{P_s} f_\zeta\left(\frac{\gamma_{(s)} r_{(s)}^\beta}{P_s}\right)$ and averaging over the truncated PDF of $r_{(s)}$ as follows:

$$f_{\gamma_{(s)}}(\gamma_{(s)}) = \int_0^T \frac{r_{(s)}^\beta}{P_s} f_\zeta\left(\frac{\gamma_{(s)} r_{(s)}^\beta}{P_s}\right) \tilde{f}_{r_{(s)}}(r_{(s)}) dr_{(s)}. \tag{41}$$

Given that $\zeta \sim \mathrm{Gamma}(\kappa_\zeta, \Theta_\zeta)$, substituting (23) and solving the integral using [18, 3.381/8], the closed-form PDF of $\gamma_{(s)}$

can be derived as follows:

$$f_{\gamma_{(s)}}(\gamma_{(s)}) = \sum_{n=0}^{S-s} K_n \frac{\Gamma_l\left(\frac{\beta\kappa_\zeta-2n+2S}{\beta}, \frac{T^\beta\gamma_{(s)}}{P_s\Theta_\zeta}\right)}{\left(\frac{\gamma_{(s)}}{P_s\Theta_\zeta}\right)^{\frac{2(S-n)}{\beta}}\beta\,\gamma_{(s)}\Gamma(\kappa_\zeta)}. \quad (42)$$

Using the definition of MGF and substituting (42), the closed form MGF of $\gamma_{(s)}$ can be derived as:

$$\mathcal{M}_{\gamma_{(s)}}(t) = \sum_{n=0}^{S-s} \frac{K_n\,{}_2F_1[1,\kappa_\zeta, \frac{\beta\kappa_\zeta-2n+2S+\beta}{\beta}, \frac{T^\beta}{T^\beta+P_s\Theta_\zeta\, t}]}{(\beta\kappa_\zeta-2n+2S)(T^\beta+P_s\Theta_\zeta t)^{\kappa_\zeta}T^{\beta\kappa_\zeta-2n+2S}}. \quad (43)$$

C. Statistics of Received Interference

Approximation Since NU is highly vulnerable to the interference caused by nearby SBSs, we approximate the cumulative interference of all SBSs with the cumulative interference of all SBSs that are located within distance T excluding the one the NU is associated with. The effect of SBSs beyond T is nearly negligible because of low transmission power of an SBS as illustrated in Fig. 5.

1) Association with MBS: When the NU associates with an MBS, the cumulative interference at the NU is given by:

$$I_m^{\mathrm{m-NABS}} = I_m^{\mathrm{co}} + I_m^{\mathrm{cr}}, \quad (44)$$

where I_m^{co} and I_m^{cr} denote the co-tier and cross-tier interferences received at the NU when it is associated with an MBS in non-ABS mode. The MGF of $I_m^{\mathrm{m-NABS}}$ is given by:

$$\mathcal{M}_{I_m^{\mathrm{m-NABS}}}(t) = \mathcal{M}_{I_m^{\mathrm{co}}}(t)\, \mathcal{M}_{I_m^{\mathrm{cr}}}(t). \quad (45)$$

The co-tier interference from the neighboring MBSs is defined as:

$$I_m^{\mathrm{co}} = \sum_{l=1,l\neq m}^{M} X_l, \quad (46)$$

where X_l is defined in (4). The distribution of the distance r_l is given in [21, Eq. 5]. Note that, $X_l\ \forall l$ are correlated random variables. To avoid the complexity of correlation due to the locations of interfering MBSs, we use the approximate approach presented in [21]. We approximate r_l with $r_{w,z}$ such that $r_l \approx r_{z,w} = \sqrt{r_z^2 + D_l^2 - 2r_z\ D_l\cos(\theta_l - \theta_w)}$, where (r_z, θ_w) represents the polar coordinate of the NU's location from reference MBS. The detailed approximation procedure can be found in [21].

Conditioned on $r_{z,w}$, the MGF of X_l can then be written as $\mathcal{M}_{X_l|r_{z,w}}(t) = 1 + \frac{t\ P_l}{r_{z,w}^\beta \lambda}$, where $\mathcal{Z}$ represents the number of circular zones of equal width and $\mathcal{W}$ represents the equal angular intervals. Consequently, the MGF of I_m^{co} can then be derived as follows:

$$\mathcal{M}_{I_m^{\mathrm{co}}}(t) = \sum_{z=1}^{\mathcal{Z}}\sum_{w=1}^{\mathcal{W}} \frac{1}{\mathcal{Z}\mathcal{W}} \prod_{l=1,l\neq m}^{M} \mathcal{M}_{X_l|r_{z,w}}(t). \quad (47)$$

A comparison of the CDF of cumulative cross-tier interference I_m^{cr} from S and S' SBSs considering (i) the exact location of the NU; (ii) NU is located at origin (assumption), is demonstrated in Fig. 5. It is observed that the approximation turns out to be quite accurate for small values of S' because SBSs

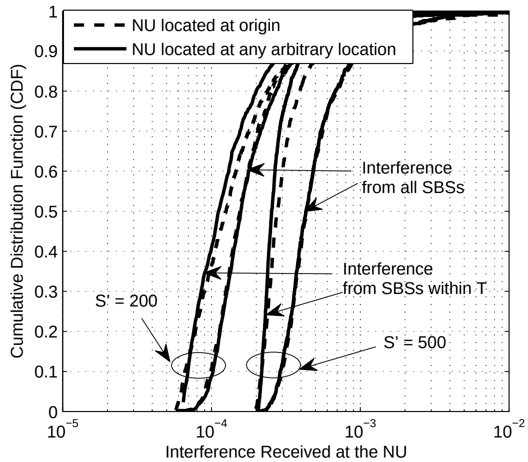

Fig. 5. Comparison of the CDF of the cumulative interference from S and S' SBSs considering (i) the exact location of the NU; (ii) NU is located at origin (assumption) (for $T = 100$, $P_m = 10W, P_s = 0.1W, \beta = 2.0$).

are highly likely to be located far apart and thus the effect of interference received from SBSs beyond T is negligible. On the other hand, with increasing S', the impact of interference received from $S' - S$ SBSs becomes dominant.

The MGF of I_m^{cr} from S SBSs within distance T can then be derived as follows:

$$\mathcal{M}_{I_m^{\mathrm{cr}}}(t) = \prod_{s=1}^{S} \mathcal{M}_{\gamma_{(s)}}(t), \quad (48)$$

where $\mathcal{M}_{\gamma_{(s)}}(t)$ is given in (43).

2) Association with SBS: The NU can associate with an SBS in both m-NABS and m-ABS modes. Therefore, we derive the MGF of cumulative received interference at the NU in both modes, respectively.

When the NU associates with the s^{th} ranked SBS (in terms of distance) out of S SBSs in m-NABS mode, the cumulative interference can be defined as $I_{(s)}^{\mathrm{m-NABS}} = I_{(s)}^{\mathrm{co}} + I_{(s)}^{\mathrm{cr}}$. The MGF of $I_{(s)}^{\mathrm{m-NABS}}$ can therefore be given as follows:

$$\mathcal{M}_{I_{(s)}^{\mathrm{m-NABS}}}(t) = \mathcal{M}_{I_{(s)}^{\mathrm{co}}}(t)\, \mathcal{M}_{I_{(s)}^{\mathrm{cr}}}(t). \quad (49)$$

The MGF of $I_{(s)}^{\mathrm{co}}(t)$ can be derived as:

$$\mathcal{M}_{I_{(s)}^{\mathrm{co}}}(t) = \prod_{n=1,n\neq s}^{S} \mathcal{M}_{\gamma_{(n)}}(t), \quad (50)$$

where $\mathcal{M}_{\gamma_{(s)}}(t)$ is given in (43). The MGF of the corresponding cross tier interference can therefore be given as:

$$\mathcal{M}_{I_{(s)}^{\mathrm{cr}}}(t) = \mathcal{M}_{I_m^{\mathrm{co}}}(t)\mathcal{M}_{\gamma_m}(t), \quad (51)$$

where $\mathcal{M}_{I_m^{\mathrm{co}}}$ and $\mathcal{M}_{\gamma_m}$ are given in (47) and (39), respectively.

When the NU associates with the s^{th} ranked SBS out of S SBSs within distance T, which is in m-ABS mode, the MGF of cumulative received interference at NU can be defined as:

$$\mathcal{M}_{I_{(s)}^{\mathrm{m-ABS}}} = \mathcal{M}_{I_{(s)}^{\mathrm{co}}}, \quad (52)$$

where $\mathcal{M}_{I^{\text{co}}_{(s)}}$ is given in (50).

V. Association Probabilities, Statistics of Signal and Interference Powers: eICIC Scheme-B

In eICIC scheme-B, S'_1 SBSs out of S' SBSs operate in s-ABS mode for ρ_s and s-NABS mode for $(1-\rho_s)$ proportion of subframes. First we evaluate the association probabilities of the NU with MBS ($p_m^{\text{s}-\text{NABS}}$) as well as SBSs ($p_{(s)}^{\text{s}-\text{NABS}}$) in s-NABS mode. Then, we derive the association probabilities of the NU with MBS ($p_m^{\text{s}-\text{ABS}}$) and active SBSs ($p_{(s)}^{\text{s}-\text{ABS}}$) in s-ABS mode. Finally, we derive the statistics of the received signal and interference at the NU when it is associated with MBS or SBS in s-NABS and s-ABS modes.

A. Association Probabilities

1) Association Probabilities in s-NABS mode: In this mode, all SBSs operate normally and the NU has option to associate with either an MBS or an SBS depending on the association criterion. This is similar to m-NABS mode of eICIC scheme-A. The association probabilities of the NU with an MBS and an SBS in s-NABS mode can therefore be given as $p_m^{\text{s}-\text{NABS}} = p_m^{\text{m}-\text{NABS}}$ and $p_{(s)}^{\text{s}-\text{NABS}} = p_{(s)}^{\text{m}-\text{NABS}}$.

2) Association probabilities in s-ABS mode: In the s-ABS mode, S'_1 SBSs are allowed to be in ABS mode. Thus, the NU can associate either with an MBS, or an active SBS within distance T. The association probability of the NU with the MBS can be derived by conditioning on h_m as follows:

$$p_{m|h_m}^{\text{s}-\text{ABS}} = \Pr\Big(\underset{q'=1,2,\ldots,Q'}{h_{(q')}} \leq h_m\Big) = \prod_{q'=1}^{Q'} F_{h_{(q')}}(h_m). \quad (53)$$

The unconditional $p_m^{\text{s}-\text{ABS}}$ can then be derived by averaging over the distribution of Q' and h_m, respectively, as follows:

$$p_m^{\text{s}-\text{ABS}} = \sum_{Q'=1}^{S}\Big(\int_0^\infty \prod_{q'=1}^{Q'} F_{h_{(q')}}(h_m) f_{h_m}(h_m)dh_m\Big)p_{Q'}(Q'). \quad (54)$$

Substituting $F_{h_{(q')}}(h_{(q')})$ and $f_{h_m}(h_m)$, (54) can be solved using standard mathematical softwares.

Similarly, the association probability of the NU with the q' ranked SBS can be derived by conditioning on $h_{(q')}$ as follows:

$$\begin{aligned} p_{(q')|h_{(q')}}^{\text{s}-\text{ABS}} &= \Pr\Big(\underset{\substack{n'=1,2,\ldots,Q'\\ n'\neq q'}}{h_{(n')}}, h_m \leq h_{(q')}\Big) \\ &= \prod_{\substack{n'=1\\ n'\neq q'}}^{Q'} F_{h_{(n')}}(h_{(q')})F_{h_m}(h_{(q')}). \quad (55)\end{aligned}$$

The unconditional $p_{(q')}^{\text{s}-\text{ABS}}$ can be derived by averaging over the distribution of Q' and $h_{(q')}$, respectively, as follows:

$$\begin{aligned} p_{(q')}^{\text{s}-\text{ABS}} = \sum_{Q'=1}^{S}\Big(\int_0^\infty \prod_{\substack{n'=1\\ n'\neq q'}}^{Q'} F_{h_{(n')}}(h_{(q')})F_{h_m}(h_{(q')})\times \\ f_{h_{(q')}}(h_{(q')})dh_{(q')}\Big)p_{Q'}(Q'). \quad (56)\end{aligned}$$

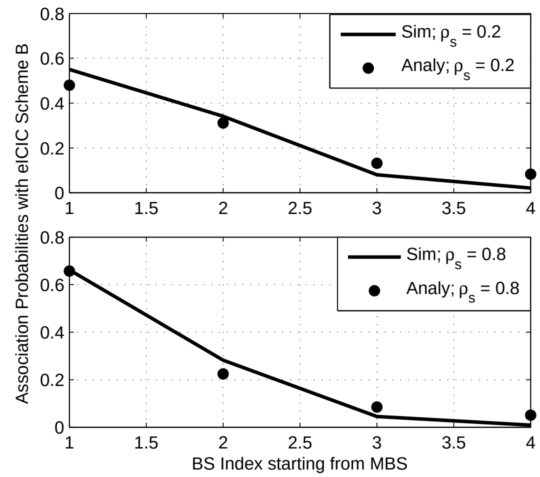

Fig. 6. Association probabilities of the NU with reference MBS and Q' active SBSs in eICIC scheme B (for $R_m = 300$ m, $T = 100$ m, $S' = 200$, $S'_1 = 100$, $\lambda_m = 15, \lambda_s = 3$). BS indices $[2, \cdots, 4]$ represent the active SBSs within distance T.

Substituting $F_{h_{(q')}}(h_{(q')})$, $F_{h_m}(h_m)$ and $f_{h_{(q')}}(h_{(q')})$, we can solve (55) using MATHEMATICA and MAPLE.

Fig. 6 compares the analytical and simulation results for the association probabilities of the NU with MBS and Q' active SBSs in eICIC scheme-B considering (i) $\rho_s = 0.2$ (ii) $\rho_s = 0.8$. As expected, the association probability of the NU with the MBS continues to increase with increasing ρ_s.

B. Statistics of Received Signal Powers

1) Association with MBS: When the NU is associated with an MBS in s-NABS mode or in s-ABS mode, the received signal power at the NU γ_m is defined in (3). The MGF of γ_m is same as given in (39).

2) Association with SBS: When the NU is associated with an SBS in s-NABS mode, the received signal power at the NU $\gamma_{(s)}$ is defined in (40). The PDF and MGF of $\gamma_{(s)}$ are same as defined in (42) and (43), respectively. When the NU associates with the $(q')^{\text{th}}$ ranked active SBS in s-ABS mode, the received signal power at the NU is defined as:

$$\gamma_{(q')} = P_{q'} r_{(q')}^{-\beta} \zeta_{q'}, \qquad \forall\ q' = 1, ..., S, \quad (57)$$

where $r_{(q')}$ represents the distance between the NU and q' ranked active SBS. The PDF and MGF of $\gamma_{(q')}$ can be obtained similarly by replacing s with q' in (42) and (43), respectively.

C. Statistics of Received Interference

1) Association with MBS: When the NU associates to an MBS in s-NABS mode, the MGF of the cumulative received interference at the NU is given by:

$$\mathcal{M}_{I_m^{\text{s}-\text{NABS}}}(t) = \mathcal{M}_{I_m^{\text{m}-\text{NABS}}}(t), \quad (58)$$

where $\mathcal{M}_{I_m^{\text{m}-\text{NABS}}}(t)$ is given in (45).

When the NU associates with an MBS in s-ABS mode, the cumulative interference at the NU is defined as $I_m^{\text{s}-\text{ABS}} = I_m^{\text{sco}} + I_m^{\text{scr}}$, where I_m^{sco} and I_m^{scr} represent the co-tier and cross-tier interference received at the NU, respectively. The MGF of

$I_m^{\text{s-ABS}}$ can then be given as:

$$\mathcal{M}_{I_m^{\text{s-ABS}}}(t) = \mathcal{M}_{I_m^{\text{sco}}}(t)\ \mathcal{M}_{I_m^{\text{scr}}}(t), \tag{59}$$

where

$$\mathcal{M}_{I_m^{\text{sco}}}(t) = \mathcal{M}_{I_m^{\text{co}}}(t), \tag{60}$$

$$\mathcal{M}_{I_m^{\text{scr}}}(t) = \sum_{Q'=1}^{S} \prod_{q'=1}^{Q'} \mathcal{M}_{\gamma_{(q')}}(t) p_{Q'}(Q'), \tag{61}$$

where $\mathcal{M}_{I_m^{\text{co}}}(t)$ is given in (47).

2) Association with SBS: When the NU associates with the s^{th} ranked (in terms of distance) SBS out of S SBSs in the s-NABS mode, the MGF of the cumulative interference at NU is given by:

$$\mathcal{M}_{I_{(s)}^{\text{s-NABS}}}(t) = \mathcal{M}_{I_{(s)}^{\text{m-NABS}}}(t), \tag{62}$$

where $\mathcal{M}_{I_{(s)}^{\text{m-NABS}}}(t)$ is given in (49).

When the NU associates with the $(q')^{\text{th}}$ ranked SBS out of Q' active SBSs within distance T, which is in s-ABS mode, the MGF of the cumulative interference at NU can be expressed as:

$$\mathcal{M}_{I_{(q')}^{\text{s-ABS}}}(t) = \mathcal{M}_{I_{(q')}^{\text{sco}}} \mathcal{M}_{I_{(q')}^{\text{scr}}}, \tag{63}$$

where

$$\mathcal{M}_{I_{(q')}^{\text{sco}}} = \sum_{Q'=1}^{S} \prod_{n=1, n\neq q'}^{Q'} \mathcal{M}_{\gamma_{(n)}}(t) p_{Q'}(Q'), \tag{64}$$

$$\mathcal{M}_{I_{(q')}^{\text{scr}}} = \mathcal{M}_{I_{(s)}^{\text{cr}}}, \tag{65}$$

where $\mathcal{M}_{I_{(s)}^{\text{cr}}}$ is given in (51).

VI. Spectral Efficiency of Downlink Transmission

A. Spectral Efficiency of Downlink Transmission to the NU: eICIC Scheme-A

In this subsection, we derive the SE of downlink transmission to the NU considering the m-NABS mode for the BS, ($\mathcal{C}^{\text{m-NABS}}$) and the m-ABS mode, ($\mathcal{C}^{\text{m-ABS}}$). Finally, we compute the SE ($\mathcal{C}^{\text{scheme-A}}$) for eICIC scheme-A.

1) SE in m-NABS mode: If the NU associates with an MBS, it accesses the transmission channel with probability $\frac{1}{U_m+1}$. Thus, averaging over the distribution of U_m, the SE of transmission to the NU associated with the MBS, which is in the non-ABS mode, can be calculated by using the lemma proposed in [23] as follows:

$$\mathcal{C}_m^{\text{m-NABS}} = \sum_{u_m=0}^{\infty} \frac{\lambda_m \int_0^{\infty} \frac{\mathcal{M}_{I_m^{\text{m-NABS}}}(t)(1-\mathcal{M}_{\gamma_m}(t))}{t} e^{-t\sigma^2} dt}{\ln 2\ u_m!\ (u_m+1) e^{\lambda_m}}, \tag{66}$$

where $\mathcal{M}_{\gamma_m}(t)$ and $\mathcal{M}_{I_m^{\text{m-NABS}}}(t)$ are given in (39) and (45), respectively. Similarly, if the NU associates with the s^{th} ranked SBS, it accesses the transmission channel with probability $\frac{1}{U_s+1}$. The SE of downlink transmission to the NU associated with the s^{th} ranked SBS, which is in m-NABS mode, can be calculated as follows:

$$\mathcal{C}_{(s)}^{\text{m-NABS}} = \sum_{u_s=0}^{\infty} \frac{\lambda_s \int_0^{\infty} \frac{\mathcal{M}_{I_{(s)}^{\text{m-NABS}}}(t)(1-\mathcal{M}_{\gamma_{(s)}}(t))}{t} e^{-t\sigma^2} dt}{\ln 2\ u_s!\ (u_s+1) e^{\lambda_s}}, \tag{67}$$

where $\mathcal{M}_{\gamma_{(s)}}(t)$ and $\mathcal{M}_{I_{(s)}^{\text{m-NABS}}}(t)$ are given in (43), and (49), respectively. The average SE in the m-NABS mode can be then derived as $\mathcal{C}^{\text{m-NABS}} = p_m^{\text{m-NABS}}\ \mathcal{C}_m^{\text{m-NABS}} + \sum_{s=1}^{S} p_{(s)}^{\text{m-NABS}} \mathcal{C}_{(s)}^{\text{m-NABS}}$.

2) SE m-ABS mode: The SE of downlink transmission to the NU when associated with the s^{th} ranked SBS in the m-ABS mode can be computed as follows:

$$\mathcal{C}_{(s)}^{\text{m-ABS}} = \sum_{u_s=0}^{\infty} \frac{\lambda_s \int_0^{\infty} \frac{\mathcal{M}_{I_{(s)}^{\text{m-ABS}}}(t)(1-\mathcal{M}_{\gamma_{(s)}}(t))}{t} e^{-t\sigma^2} dt}{\ln 2\ u_s!\ (u_s+1) e^{\lambda_s}}. \tag{68}$$

The SE in m-ABS mode can therefore be given as: $\mathcal{C}^{\text{m-ABS}} = \sum_{s=1}^{S} \mathcal{C}_{(s)}^{\text{m-ABS}} p_{(s)}^{\text{m-ABS}}$.

Finally, the SE with eICIC scheme-A can be computed as follows:

$$\mathcal{C}^{\text{scheme-A}} = \rho_m\ \mathcal{C}^{\text{m-ABS}} + (1-\rho_m)\ \mathcal{C}^{\text{m-NABS}}. \tag{69}$$

B. Spectral Efficiency of Downlink Transmission to the NU: eICIC Scheme-B

In this subsection, we derive the SE considering the s-NABS mode ($\mathcal{C}^{\text{s-NABS}}$). Then, we derive the SE considering the s-ABS mode ($\mathcal{C}^{\text{s-ABS}}$). Finally, we derive SE ($\mathcal{C}^{\text{Scheme-B}}$) for eICIC scheme-B.

1) SE in s-NABS mode: This is same as in m-NABS mode, i.e., $\mathcal{C}^{\text{s-NABS}} = \mathcal{C}^{\text{m-NABS}}$.

2) SE in s-ABS mode: The SE of downlink transmission to the NU when associated with an MBS in s-ABS mode can be computed as:

$$\mathcal{C}_m^{\text{s-ABS}} = \sum_{u_m=0}^{\infty} \frac{\lambda_m\ e^{-\lambda_m} \int_0^{\infty} \frac{\mathcal{M}_{I_m^{\text{s-ABS}}}(t)(1-\mathcal{M}_{\gamma_m}(t))}{t} e^{-t\sigma^2} dt}{\ln 2\ u_m!\ (u_m+1)}. \tag{70}$$

Similarly, the SE of NU when associated to $(q')^{\text{th}}$ ranked active SBS in s-ABS mode can be computed as:

$$\mathcal{C}_{(q')}^{\text{s-ABS}} = \sum_{u_{q'}=0}^{\infty} \frac{\lambda_{q'}\ e^{-\lambda_{q'}} \int_0^{\infty} \frac{\mathcal{M}_{I_{(q')}^{\text{s-ABS}}}(t)(1-\mathcal{M}_{\gamma_{(q')}}(t))}{t} e^{-t\sigma^2} dt}{\ln 2\ u_{q'}!\ (u_{q'}+1)}. \tag{71}$$

The SE for the s-ABS mode can therefore be derived as: $\mathcal{C}^{\text{s-ABS}} = p_m^{\text{s-ABS}}\ \mathcal{C}_m^{\text{s-ABS}} + \sum_{Q'=1}^{S} \sum_{q'=1}^{Q'} \mathcal{C}_{(q')}^{\text{s-ABS}} p_{(q')}^{\text{s-ABS}} p_{Q'}(Q')$.

Finally, the SE with eICIC scheme-B can be computed as follows:

$$\mathcal{C}^{\text{scheme-B}} = \rho_s\ \mathcal{C}^{\text{s-ABS}} + (1-\rho_s)\ \mathcal{C}^{\text{s-NABS}}. \tag{72}$$

VII. Numerical Results and Discussion

This section presents numerical results on SE considering different user association and interference coordination

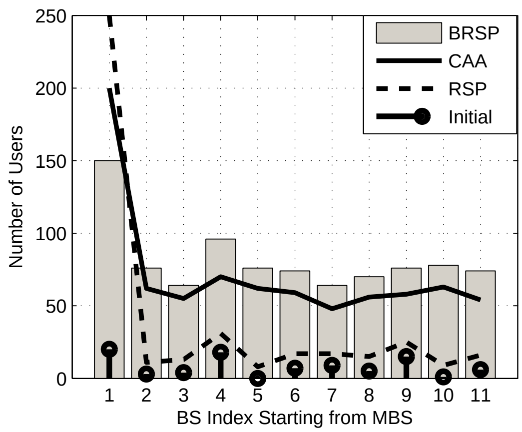

Fig. 7. Traffic load balancing after arrival of 250 NUs in the two-tier smallcell network considering different association schemes (for $\lambda_m = 15$, $\lambda_s = 3$, $\rho_m = \rho_s = 0$).

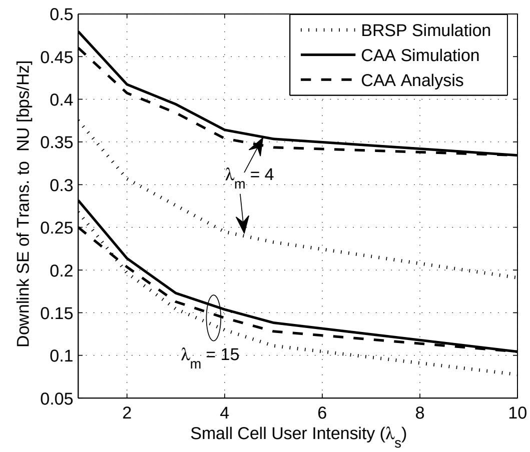

Fig. 8. Downlink SE of transmission to the NU with increase in smallcell user intensity (λ_s) for different user association schemes (for $\rho_m = \rho_s = 0$).

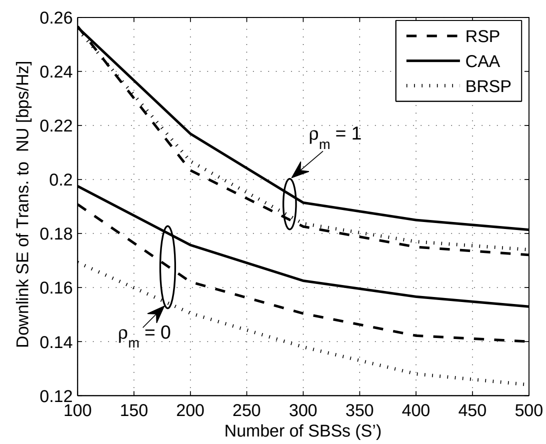

Fig. 9. Downlink SE of transmission to the NU as a function of S' considering different user association schemes (for $\lambda_m = 15, \lambda_s = 3$).

schemes. We start with the no interference coordination scenarios and gradually move on to analyze the impact of adopting eICIC scheme-A and eICIC scheme-B on the SE performance for the NU, macrocell user (MU), and smallcell user (SU) as a function of network design parameters.

In Monte-Carlo simulations, we consider 7 circular macrocells, i.e., $M = 7$ and $S' = 200$ smallcells arbitrarily deployed in $R_m + D_l$ region. We consider generalized-$\mathcal{K}$ composite shadowing and fading, i.e., $f_\chi(\chi) \approx \mathrm{Gamma}(2/3, 3/2)$ and $f_\zeta(\zeta) \approx \mathrm{Gamma}(1/2, 2)$. The coverage radii of an MBS and an SBS are $R_m = 300$ m and $R_s = 50$ m, respectively. We set $T = 100$ m, path-loss exponent $\beta = 2.0$, the thermal noise power density $\sigma^2 = 1 \times 10^{-10}$ W/Hz, the transmission powers of an MBS and an SBS as $P_m = 10$ W and $P_s = 0.1$ W, respectively, and bias $b = 13$ dBW.

A. Results: No eICIC

1) Traffic load balancing: Fig. 7 considers a two-tier cellular network that comprises of ten smallcells. First, the initial traffic loads of different BSs are generated. A large number of NUs (250 NUs) are then assumed to be entering into the system who become associated to different BSs depending on the user association schemes (i.e., CAA, RSP, and BRSP). The impact of different user association schemes on the network traffic load is then analyzed by characterizing the final traffic load of each BS. It can be observed that RSP-based association allows more users to associate with the MBS due to its high transmission power. On the other hand, the BRSP-based association tends to balance the traffic loads of SBSs and MBS. Finally, the traffic load balancing accomplished by CAA scheme is observed to be in between the two extremes of traditional BRSP and RSP-based association schemes.

2) SE as a function of λ_m and λ_s: Fig. 8 demonstrates the degradation in SE with increasing in λ_s and λ_m. This degradation is due to decrease in channel access probability with MBS and SBSs. For small values of λ_m, the gains with CAA association are significantly higher than those with BRSP-based association because the NU can select an MBS due to its high received signal power and channel access probability. But with BRSP-based association, the NU is forcefully pushed to low-power SBSs and the presence of strong macro interference further deteriorates the SE performance. For high λ_m, the performance gains with CAA over BRSP are still evident. This is due to the fact that CAA allows the NU to associate with an SBS having high channel access probability as well as high received signal power. On the other hand, BRSP-based association does not distinguish different smallcells based on their traffic loads. The derived expressions closely follow the Monte-Carlo simulations and the impact of approximation is also observed to be minimal.

B. Results: eICIC Scheme-A

1) Impact of ρ_m on the SE: Fig. 9 illustrates the impact of increasing the number of SBSs (S') on the SE. Note that with increasing S' more SBSs tend to fall within distance T. This provides more options for the NU to select lightly-loaded SBSs at smaller distances. However, the increase in co-tier interference turns out to be more dominant. For this reason,

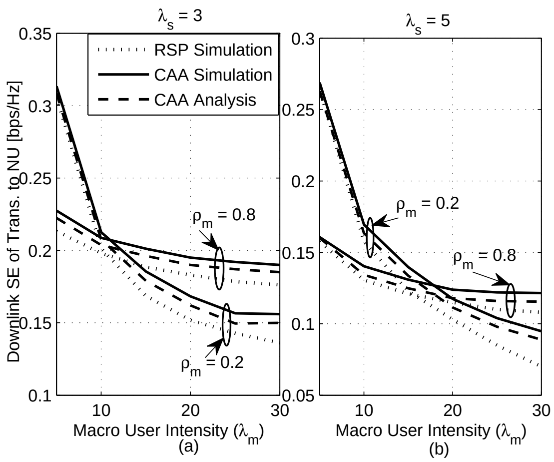

Fig. 10. Downlink SE of transmission to NU as a function of λ_m considering RSP and CAA user association schemes (for (i) $\lambda_s = 3$ for (a) $\rho_m = 0.2$, (b) $\rho_m = 0.8$, (ii) $\lambda_s = 5$ for (a) $\rho_m = 0.2$, (b) $\rho_m = 0.8$).

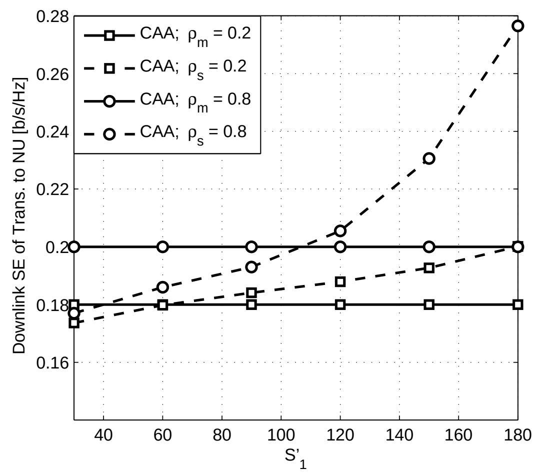

Fig. 11. Downlink SE of transmission to the NU as a function of S_1' (for $\lambda_m = 15$, $\lambda_s = 3$).

the SE is observed to degrade with increasing S'. In Fig. 9, we also comparatively analyze the impact of CAA, RSP, and BRSP-based user association schemes considering $\rho_m = 1$ and $\rho_m = 0$. With $\rho_m = 0$, the performance gains of CAA over RSP and BRSP are significant. This is due to the fact that, the RSP scheme tends to always select the MBS due to high transmit power and the BRSP scheme selects SBSs without considering their traffic load conditions. However, with $\rho_m = 1$, RSP performs similar to BRSP since RSP allows the NU to associate with a low-power BS irrespective of the traffic loads, which is similar to the BRSP mechanism. The CAA scheme outperforms both the RSP and BRSP schemes.

2) Selecting the proportion of macro ABS as a function of traffic load intensities: Fig. 10 (a)-(b) depict the effect of increasing λ_m on the SE of downlink transmission to the NU. The SE decreases as λ_m increases due to reduced channel access probability with the MBS. For small values of λ_m, the higher SE gains can be obtained with small values of ρ_m. This is due to the fact that the benefit achieved from MBS (i.e., high channel access probability) outweighs the need for interference mitigation. On the other hand, for large values of λ_m, high SE gains can be achieved by selecting large values of ρ_m. It happens because the NU is highly likely to associate with an SBS due to high channel access probability and thus reduced cross-tier interference is crucial. As such, the fraction of macro ABS needs to be designed carefully according to the traffic load intensities of different tiers λ_m and λ_s. Note that the cross-over point shifts to right with increasing λ_s. Thus, setting low values of ABS at MBS become more beneficial for the NU as the traffic loads of the SBSs increase and approach the traffic load of an MBS.

C. Results: eICIC Scheme-B

1) eICIC scheme-A vs. eICIC scheme-B: Fig. 11 demonstrates the downlink SE of transmission to the NU as a function of the number of smallcells in ABS mode, S_1'. In the eICIC scheme-B, the SE increases with increasing S_1'; however, the network operators may likely support only small values of S_1' to ensure reduced coverage holes for large number of SUs. Interestingly, for small values of S_1', the eICIC scheme-B is observed to fall behind eICIC scheme-A. In particular, it seems better to switch-off MBS and let the NU to associate with an SBS (with significantly reduced macro-tier interference) rather than switch-off few SBSs and allow association with an MBS or an SBS with significant co-tier and cross-tier interferences. Nonetheless, for high values of S_1', the eICIC scheme-B outperforms eICIC scheme-A due to extremely reduced interference from SBSs.

For the eICIC scheme-A and eICIC scheme-B, the SE increases with increasing ρ_m and ρ_s, respectively. Increase in ρ_m reduces macro-tier interference at the NU and SUs significantly at the cost of few MUs. On the other hand, the SE gains are not significant with increasing ρ_s for small values of S_1'. However, significant SE gains are observed with increasing ρ_s for large values of S_1'. Interestingly, it is also observed that, compared to large values of ρ_s, with small values of ρ_s, the eICIC scheme-B surpasses the eICIC scheme-A for a wide range of S_1'.

Fig. 12(a) illustrates the SE of downlink transmissions to the MU and SU as a function of S_1'. As expected, the SE for an MU increases with S_1' in eICIC scheme-B due to reduction in cross-tier interference from the SBSs. On the other hand, the SE for an SU decreases with increasing S_1' in eICIC scheme-B due to the coverage holes. Fig. 12(b) illustrates the impact of increasing ρ_m and ρ_s on the SE for any arbitrary MU and SU. The performance of MU (SU) increases (decreases) with increasing ρ_s, and vice versa with increasing ρ_m. To provide fairness between MU and SU with respect to the SE gains, selecting a suitable value of S_1', ρ_m, and ρ_s will be crucial while employing eICIC scheme-A or B.

VIII. Conclusion

For two-tier macrocell-smallcell networks, we propose a low-complexity channel access aware (CAA) downlink user association scheme that enhances the spectral efficiency (SE) and also balances the traffic loads among different BSs.

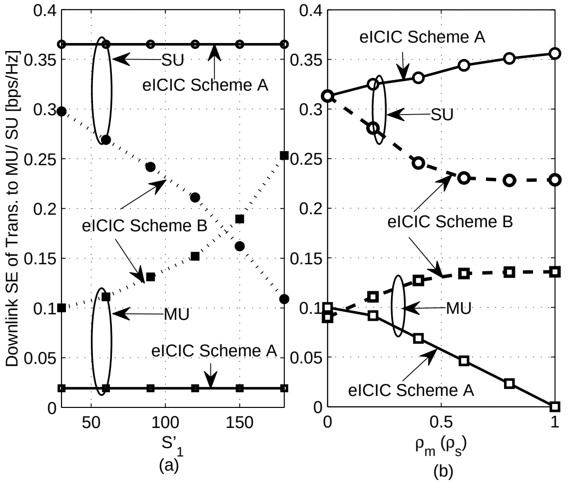

Fig. 12. Downlink spectral efficiency of transmission to MU/ SU as a function of (a) S_1' (for $\lambda_m = 15$, $\lambda_s = 3$, $\rho_m = \rho_s = 0.8$), (b) $\rho_m(\rho_s)$ (for $\lambda_m = 15$, $\lambda_s = 3$, $S_1' = 100$).

We develop a tractable mathematical framework to characterize the SE of downlink transmission to a user who associates with a BS using the proposed CAA scheme. The framework characterizes the impact of state-of-the-art almost blank subframe (ABS)-based interference coordination (namely, eICIC scheme-A and eICIC scheme-B) in macrocell-tier and smallcell-tier on the SE performance of the proposed CAA scheme. The fraction of ABS (ρ_m or ρ_s) needs to be designed carefully according to the traffic load intensities λ_m and λ_s of the two network tiers. For low values of λ_m compared to λ_s, small values of ρ_m are feasible. On the other hand, the reverse is true when λ_m is higher than λ_s. Interestingly, it is also observed that, compared to large values of ρ_s, with small values of ρ_s, the eICIC scheme-B outperforms the eICIC scheme-A for a wide range of S_1'. Thus small values of ρ_s are favorable in the eICIC scheme-B. Finally, exercising interference coordination schemes imposes a trade-off on the SE gains for an SU and an MU. The fairness between the SE performances for MU and SU thus needs be achieved by selecting suitable values of S_1', ρ_m, and ρ_s.

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