Polymer Physics, Electrical Conductivity of Metals and Cosmology

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ABSTRACT – Flory's model of a polymer deals with three characteristics lengths, namely: the monomer's length, the chain's length and the length of the radius of gyration. A first analogy can be established between this model and that of the electrical conductivity of good metals at the room temperature, by taking the Compton's length, the electron mean free path and the Fermi's length. A second analogy is considered, thinking the universe world line as a segmented chain, where we have the Planck's length, the radius of the observable universe and the length associated to the cosmological constant. A characteristic time for the evolution of the universe is obtained (not the Hubble's time), and it is compared with a possible proton's decay time.

1 – Introduction

As was pointed out by deGennes [1], a brilliant scheme for computing the radius of gyration of a polymer was devised by Flory[2]long time ago, for the three dimensional case. Later on, Fisher [3] extended Flory's results to other dimensions. The radius of gyration of a polymer is equivalent to the end-to-end distance of a self-avoiding random walk (SARW) (please see Raposo et al [4]).

In the present work we intend to explore the analogies with the polymer physics as a means to investigate some interesting features of the electrical conductivity of metals as well the cosmology.

One of the subjects to be pursued is to look at the universe world line, making a connection of a possible end point of it with the proton decaying (please see Narlikar [5]).

2 – Some preliminaries

Let us to consider a chain of radius R and immersed in a d-dimensional space composed by N monomers of length a. The Flory's free energy of this chain reads (please see references [1,6])

$$F = N^2 a^d / R^d + R^2 / (Na^2). \tag{1}$$

The first term of (1) corresponds to certain repulsive energy in the chain due to monomer-monomer interactions. The second term of (1) is the entropic contribution to the Flory's free energy, and it comes from a Gaussian distribution for R tied to the random walk character present in the growing of the polymer chain [1].

Minimizing (1) relative to R, we obtain Flory's result for the radius of gyration R_F which scales as

$$R_F \sim N^{\upsilon}, \tag{2}$$

where υ is given by

$$\upsilon = 3 / (d+2). \tag{3}$$

3 – Various radius of gyration

Let us pay attention to the dimension d=4: three space plus one time dimension. It is possible to think about a hierarchy of chains such that the first one is composed by N_1 monomers of length a. Then, we have the first radius of gyration R_1 given by

$$R_1 = \sqrt{N_1}\, a. \tag{4}$$

Next, we construct a second chain with N_2 monomers of length equal to R_1. The second radius of gyration reads

$$R_2 = \sqrt{N_2}\, R_1. \tag{5}$$

The third chain with N_3 monomers with length R_2, have the third radius of gyration given by

$$R_3 = \sqrt{N_3}\, R_2. \tag{6}$$

In the case where $N_1 = N_2 = N_3 = N$, we have

$$R_1 = \sqrt{N}\, a, \tag{7}$$

$$R_2 = N\, a, \tag{8}$$

$$R_3 = N^{3/2}\, a. \tag{9}$$

We observe that the second radius of gyration R_2 corresponds in this case to the length of the first chain.

4 – Polymer physics and the electrical conductivity of metals

It would be interesting to make analogy between the ideas we just developed and the electrical conductivity of good metals at the room temperature (copper, silver and gold, for instance), by estimating some parameters tied to that physical property.

By taking the monomer length to be equal to the reduced Compton wavelength (λ_C) of the electron, the first radius of gyration being equal to the characteristic Fermi wavelength (λ_F) of a good metal, and the second radius of gyration equal its electron mean free path (l), we can write

$$a \equiv \lambda_C = \hbar/(mc), \qquad R_1 \equiv \lambda_F = \sqrt{N}\,\lambda_C, \qquad R_2 \equiv \mathit{l} = N\,\lambda_C. \tag{10}$$

Besides this, we have

$$\mathit{l} = v_F\,\tau = p_F\,\tau/m = (h\,\tau)/(m\,\lambda_F). \tag{11}$$

Solving (11) for τ and by using (10), we obtain

$$2\pi\, c\tau = (N^{3/2}\,\hbar)/(mc) \equiv R_3. \tag{12}$$

Therefore, by taking the reduced Compton wavelength of the electron as the monomer length, the first radius of gyration o the chain gives the Fermi wavelength, the second radius of gyration represents the electron mean free path, and the third radius of gyration is the perimeter of a circle, which radius is the speed of light in vacuum times the characteristic time (τ) between collisions.

In order to get a better feeling of the subject, it is convenient to put numbers in the relations we just evaluated. By taking $\sqrt{N}=322$, we get

$$\lambda_F = 1.24\ \text{Å}, \qquad \mathit{l} \cong 400\ \text{Å}, \qquad \tau = 6.8 \times 10^{-15}\ \text{s}. \tag{13}$$

We also have

$$E_F = p_F^2/(2m) = \hbar^2/(2m\lambda_F^2) \cong 2.5 \text{ eV}. \quad (14)$$

As we can see, the numerical results displayed in (13) and (14) are representative of those known parameters of good metals at the room temperature (please see references [7,8]).

5 – Cosmology and polymer physics

In reference [6], some basic concepts of polymer physics were used in order to evaluate the cosmological constant parameter. There, the universe world line was represented by a segmented line as a lattice model of a polymer (SARW).

The lattice spacing was taken as the Planck's length (λ). Therefore let us take

$$\lambda = (\hbar G/c^3)^{1/2}. \quad (15)$$

The extension of the chain in this cosmologic analogy will be given by

$$R_2 \equiv L = N\,\lambda, \quad (16)$$

where N is the number of monomers composing the chain. As was done in [6], L is here interpreted as the radius of the observable universe and according to the present nomenclature it also corresponds to the second radius of gyration of the problem, namely R_2.

The first radius of gyration (R_1), is related to the cosmological constant scale as was reported in [6].

Making analogies with the metallic conductivity, we can also define a Fermi velocity v_F and a characteristic time τ, now tied to the evolution of the universe. We have

$$L = v_F\,\tau = (h\,\tau)/(m\,\lambda_F) = (h\,\tau)/(m\,R_1). \qquad (17)$$

We notice that in (17) we have made the identification between the Fermi length and the first radius of gyration of the polymer chain.

After some little algebra we also can write

$$2\pi\,c\tau = N^{3/2}\,\lambda \equiv R_3. \qquad (18)$$

Numerical estimates, by considering only order of magnitude of the involved quantities, and using $\lambda \sim 10^{-35}$ m, and $L \sim 10^{26}$ m, gives

$$N \sim 10^{61}, \qquad R_3 = \sqrt{10} \times 10^{56}\ \text{m}, \qquad \tau \sim 10^{47}\ \text{s}. \qquad (18A)$$

It is also possible write

$$2\pi c\tau = N^{1/2}\,v_F\tau, \qquad (19)$$

which implies that

$$2\pi c = N^{1/2}\,v_F. \qquad (20)$$

Order of magnitude estimates gives for v_F, the value

$$v_F \sim 10^{-21}\ \mathrm{m/s}. \tag{21}$$

It is also interesting to verify that the present treatment permit us to write two Hubble's-like relations, namely

$$R_2\, H_0 \equiv L\, H_0 = c, \tag{22}$$

$$2\pi\, N^{1/2}\, \lambda\, H_0 = 2\pi\, R_1\, H_0 = v_F. \tag{23}$$

In (22) and (23), H_0 stands for the Hubble's constant.

Next we give interpretations for eqs. (22) and (23). Two galaxies separated by a curved path of length $L \equiv R_2$, where L represents the radius of the observable universe, are receding relative to each other with the velocity c, the speed of light in vacuum (eq. (22)). On the other hand, if we consider the straight line of length R_1 connecting the end -to-end points of the polymer chain, the modified Hubble's law (eq. (23)) gives a recession velocity equal to $v_F/(2\pi)$.

Meanwhile, the analogy with the electrical conductivity of metals, permit us to interpret eq.(17) as a certain test particle travelling with speed v_F, that will strike the surface horizon of radius L, after a time interval τ and starting from the origin. Here we are considering the observable universe as a black hole.

6 – Proton decay time

Hydrogen is the most abundant chemical element of the universe, and although hadronic matter contributes in a small amount to the mass-energy of it (remember dark energy and dark matter give by far more important contributions), proton decay time may indicate a possible scale for the universe lifetime. But in evaluating a proton decay time having a possible connection with the universe's extinction time, the energy scale where we have a matching between the gravitational and the electromagnetic couplings perhaps plays its role.

Let us imagine a boson of mass M_B, having mass-energy equal to the energy scale of the matching between the gravitational and electromagnetic couplings. We can write

$$G\, M_B^{\,2} = \alpha\, \hbar\, c = \alpha\, G\, M_{Pl}^{\,2}. \qquad (24)$$

In (24), G is the Newton's gravitational constant and M_{Pl} is the Planck's mass.

If we assume that the proton decay process is intermediated by this boson of mass M_B, we can imagine that this boson plays a similar role of that played by the boson M_X in the grand unification theory (GUT:SU(5)) (please see the book by J. V. Narlikar [5].

As a means to evaluate the proton decay time τ_p, we consider the relations

$$m_p\, c^2 = h\, \upsilon, \qquad (25)$$

$$f = (m_p / M_B)^{\,4}. \qquad (26)$$

Here f is a geometric factor, namely if we take in the momenta space a four dimensional cube of a four-volume $(M_B\, c)^{\,4}$, the fraction f of this cube occupied by a small cube which edge is given by the characteristic momentum of the proton, we obtain relation (26). Next we write

$$\Gamma = \upsilon\, f = [(m_p\, c^2) / h]\, (m_p / M_B)^{\,4}. \qquad (27)$$

Finally we get for the proton decay time

$$\tau_p = 1/\Gamma = [h/(m_p\, c^2)]\, (M_B/m_p)^4 = [(h\alpha^2)/(m_p\, c^2)]\, (M_{Pl}/m_p)^4. \qquad (28)$$

Putting numbers in (28), we find the proton decay time, namely

$$\tau_p = 6.7 \text{ x } 10^{48} \text{ s}. \qquad (29)$$

Therefore if we consider that the boson which mediates the process leading to a possible proton decay has a mass-energy equal to the scale of matching the gravitational and the electromagnetic interactions, the proton decay time estimated in this way is two orders of magnitude greater than the time of "extinction" of the universe (please see (18A)).

We would like to point out that the way we have used to estimate the proton decay time in the present paper, is analogous to the analysis we have done in references [9,10].

References

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