FINITE AND INFINITE BASIS IN P AND NP

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1. Abstract

This article provide new approach to solve P vs NP problem by using cardinality of bases function. About NP-Complete problems, we can divide to infinite disjunction of P-Complete problems. These P-Complete problems are independent of each other in disjunction. That is, NP-Complete problem is in infinite dimension function space that bases are P-Complete. The other hand, any P-Complete problem have at most a finite number of P-Complete basis. The reason is that each P problems have at most finite number of Least fixed point operator. Therefore, we cannot describe NP-Complete problems in P. We can also prove this result from incompleteness of P.

2. Difference of basis between P and NP

By using SAT and these verification, we prove that some NP-Complete problems have infinite basis of P-Complete problems.

Definition 1. We will use the term "$v_i \in V$" as problem which verify formula with special valuation i.

That is, if

$t \in SAT$

then

$v_i(t) = \top \leftrightarrow t(i) = \top$

Theorem 2. *$v_i \in P - Complete$*

Proof. First, we show that $v_i \in P$. A Polynomial DTM can verify valuation i to a given formula f and accept if $f(i) = \top$.

Next, we show that $CIRCUIT - VALUE \leq_L v_i$. $CIRCUIT - VALUE \in P - Complete$[1], therefore if $CIRCUIT - VALUE \leq_L v_i \in P$ then $v_i \in P - Complete$. If we modify $C \to C'$ to match $x \to i$, v_i compute C' as $\langle C, x\rangle$. We can modify $C \to C'$ to negate some C variables that x mismatch i. This modification can compute in L.

Therefore, $CIRCUIT - VALUE \leq_L v_i \in P$ and $v_i \in P - Complete$. □

Theorem 3. *V is basis of SAT*

Proof. To think about relation between SAT and $v_i \in V$, SAT is disjunction of V, i.e.

$SAT = \bigcup V = \bigvee_{i=0}^{\infty} v_i$

Each v_i is independent of each other in disjunction because every input p have another input q that change only v_i output.

$\forall p \exists q \left(\left(v_0(p), \cdots, v_i(p), \cdots \right) \to \left(v_0(q) = v_0(p), \cdots, v_i(q) = \neg v_i(p), \cdots \right) \right)$

If $v_i(p) = \top$ then $q = p \wedge (\neg i)$
else if $v_i(p) = \bot$ then $q = p \vee (i)$
That is, $V \setminus \{v_i\}$ cannot compute SAT problems.
Therefore V is basis of SAT. □

From descriptive complexity, $P = FO + LFP$[1, 2, 3]. This means that every P problem have at most a finite number of LFP operators in finite first-order logic model. Therefore P problem have at most a finite number of P-Complete basis.

Theorem 4. *Any $p \in P$ have at most a finite number of P-Complete basis.*

Proof. To prove it by using reduction to absurdity. We assume that $p \in P$ have infinite number of basis of P-Complete. These basis independent of each other and have independent LFP operators. But $P = FO + LFP$ have at most finite number of LFP operators. Therefore we cannot describe p in finite length $FO + LFP$. □

Theorem 5. $P \neq NP$

Proof. Mentioned above 3, SAT have infinite P-Complete basis. But mentioned above 4, any $p \in P$ have finite P-Complete basis. Therefore SAT is not any $p \in P$. □

3. From view of countable and continuum

We show another proof from the view of completeness.

Theorem 6. $P \neq NP$

Proof. Let $\langle v, i\rangle$ be a code number of v_i. To assign this number after the decimal point, $0.\langle v, i\rangle$ correspond to number within $[0, 1]$, and $[0, 0.\langle v, i\rangle] + \bigcup V = [0, 0.\langle v, i\rangle] + \bigvee_{i=0}^{\infty} v_i$ correspond to Dedekind cut of P.

If $[0, 0.\langle v, i\rangle] + \bigvee_{i=0}^{\infty} v_i$ also P then P become isomorphic as real number and contradict that P is countable. Therefore $NP \ni \bigcup V \notin P$ and $P \neq NP$. □

References

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