Observation on the paper:
Logical independence and quantum randomness

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17th October 2014

Abstract I comment on the background meaning, beneath Boolean encodings, used in the paper by Tomasz Paterek et al.

Keywords foundations of quantum theory, quantum mechanics, quantum randomness, quantum indeterminacy, quantum information, prepared state, measured state, unitary, orthogonal, scalar product, mathematical logic, logical independence, mathematical undecidability.

Introduction

In *classical physics*, dice-throwing and coin-tossing experiments are *deterministic,* in the sense that, a perfect knowledge of initial physical conditions would render an outcome perfectly predictable — and that, in repeated experiments, statistical 'randomness' stems from the degree of ignorance of that *physical information*.

In diametrical contrast, in the case of *quantum physics* and *quantum randomness*, the theorems of Kocken and Specker [4], the inequalities of John Bell [3], and experimental evidence of Alain Aspect [1,2] and others [6,7,8], all indicate, no such *physical information* exists.

In response, Tomasz Paterek, et al, offer a *'non-physical'* explanation by providing evidence that quantum randomness originates in *mathematical information*. In their experiments [5] they demonstrate a link connecting quantum randomness with logical independence. Specifically, in experiments measuring photon polarisation, the Paterek research demonstrates statistics, correlating *predictable* outcomes with logical dependence, in the system algebra, encoded via Boolean propositions — and *random* outcomes with logical independence.

Observation

The substance of the Paterek logical independence lies ultimately with the scalar product. On the face of it, the system's algebra is $\mathsf{su}(2)$, the algebra of the Pauli operators. But hidden beneath the Boolean encodings is the fact that not every photon measurement, precisely and faithfully, demands $\mathsf{su}(2)$. In a pair of operators representing the sequence: *state preparation* followed by *state measurement*, when the pair is encoded *complementary (orthogonal)*, information asserted by their product, is *involutory*[1] AND *unitary*. But when encoded *same (parallel)*, their product asserts involutory information only – and unitarity is redundant. Every measurement implies the involutory component of the algebra; but the unitary component is implied only in the non-parallel case. For parallel measurement, unitarity may freely switch off with no contradiction. This is because any 2×2 matrix, whose square is the identity matrix, can represent this measurement. For the parallel measurement, the algebra is free to flip out of the $\mathsf{su}(2)$ symmetry. This freedom affects the *logical form* of the theory, but in standard quantum theory, where unitarity is imposed *by Postulate*, this freedom is blocked.

The involutory information is logically independent of the unitary information

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[1] An involutory operator is one whose square is the identity operator. eg. $\mathbf{aa} = \mathbf{1}$.

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