The MC function and three Smarandache type sequences, diophantine analysis

Abstract. In two of my previous papers, namely "An interesting property of the primes congruent to 1 mod 45 and an ideea for a function" respectively "On the sum of three consecutive values of the MC function", I defined the MC function. In this paper I present new interesting properties of three Smarandache type sequences analyzed through the MC function.

As I mentioned in abstract, I already defined the MC function in previous papers, but, in order to be, this paper, self-contained, I shall define the MC function again, as the function defined on the set of odd positive integers with values in the set of primes such that: MC(x) = 1 for x = 1; MC(x) = x, for x prime; for x composite, MC(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),..., p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then MC(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),..., q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then MC(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of MC(x).

1. The concatenated odd sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n – 1).

The first ten terms of the sequence (A019519 in OEIS):

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

Notes:

Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence. The terms of this sequence are primes for the following values of n: 2, 10, 16, 34, 49, 2570 (the term corresponding to n = 2570 is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.

Analysis through MC function:

Another interesting property of the terms of the concatenated odd sequence could be the following one: seems that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of three consecutive terms of this sequence. To exemplify, MC(x), where x = 1 + 13 + 135, is found immediately (in one step), because MC(x) = x = 149, a prime number.

Examples:

Let's calculate MC(135791113 + 13579111315 + 1357911131517) = MC(1371626033945):
: 1371626033945 = 5*31*33769*262051;
: 5 + 31 + 33769 + 262051 – 1 = 295853, prime, so is the value of MC(x), obtained in just two steps.

Let's calculate MC(13579111315 + 1357911131517 + 135791113151719) = MC(137162603394551):
: 137162603394551 = 7*23*1171*727533421;
: 7 + 23 + 1171 + 727533421 – 3 = 727534619 = 7*103933517;
: 7 + 103933517 – 1 = 103933523, prime, so is the value of MC(x), obtained in just three steps.

2. The concatenated prime sequence

Definition:
S_n is defined as the sequence obtained through the concatenation of the first n primes.

The first ten terms of the sequence (A019518 in OEIS):
2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

Notes:
The terms of this sequence are known as Smarandache-Wellin numbers. Also, the Smarandache-Wellin numbers which are primes are named Smarandache-Wellin primes. The first three such numbers are 2, 23 şi 2357; the fourth is a number with 355 digits and there are known only 8 such primes. The 8 known values of n for which through the concatenation of the first n primes we obtain a prime number are 1, 2, 4, 128, 174, 342, 435, 1429. The computer programs not yet found, until n = 10^4, another such a prime. F.S. conjectured that there exist an infinity of prime terms of this sequence.

Analysis through MC function:
Another interesting property of the terms of the concatenated prime sequence could be the following one: seems that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of three consecutive terms of this sequence.

Example:

Let’s calculate MC(235711131719 + 23571113171923 + 2357111317192329) = MC(2380918141495971):

: 2380918141495971 = 3*3*7*7*11*11*11*149*4337*6227;

: 3 + 3 + 7 + 7 + 11 + 11 + 11 + 149 + 4337 + 6227 – 9 = 10807 = 101*107;

: 101 + 107 – 1 = 207 = 3*3*23;

: 3 + 3 + 23 – 2 = 27 = 3*3*3;

: 3 + 3 + 3 – 2 = 7, prime, so is the value of MC(x), obtained in just five steps.

3. The pierced chain sequence

Definition:

The sequence obtained in the following way: the first term of the sequence is 101 and every next term is obtained through concatenation of the previous term with the group of digits 0101.

The first seven terms of the sequence (A031982 in OEIS):

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, 10101010101010101010101, 101010101010101010101010101.

Notes:

Kenichiro Kashihara proved that there are no primes obtained through the division of the terms of the sequence by 101 (because, of course, all of them are divisible by 101).

Analysis through MC function:

Another interesting property of the terms of the pierced chain sequence could be the following one: probably that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the terms of this sequence, because of the fact that these terms are products of many distinct prime factors (indeed, 1010101 is a product of 3 distinct prime factors, 10101010101 of 6, 101010101010101 of 5 such factors, 1010101010101010101 of 6 such factors, 10101010101010101010101 of 9 and so on). This fact will increase the chances to find easier the value of the function MC(x), usually a prime much smaller than the number x.