An interesting property of the primes congruent to 1 mod 45 and an ideea for a function

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Abstract. In this paper I show a certain property of the primes congruent to 1 mod 45 related to concatenation, namely the following one: concatenating two or three or more of these primes are often obtaied a certain kind of composites, id est composites of the form m*n, where m and n are not necessarily primes, having the property that m + n - 1 is a prime number. Plus, I present an ideea for a function which be interesting to study.

The primes congruent to 1 mod 45 seem to have the following interesting property: concatenating two or three or more of these primes are often obtaied a certain kind of composites, id est composites of the form m*n, where m and n are not necessarily primes, having the property that m + n - 1 is a prime number or a composite which conducts through the same operation to a prime). Let’s take the first 11 primes congruent to 1 mod 45 (sequence 142312 in OEIS):

181, 271, 541, 631, 811, 991, 1171, 1531, 1621, 1801, 2161.

We obtain, concatenating to the left the prime 181 with the following ten ones from the sequence above:

: 181271 =
17*10663 and 10663 + 17 - 1 = 59*181 and 181 + 59 - 1 = 239, prime;
: 181541 =
379*479 and 479 + 379 – 1 = 857, prime;
: 181631 =
1219*149 and 1219 + 149 – 1 = 1367, prime; also 181631 = 23*7897 and 23 + 7897 – 1 = 7919, prime;
: 181811 =
133*1367 and 133 + 1367 – 1 = 1499, prime;
: 181991 =
127*1433 and 127 + 1433 – 1 = 1559, prime;
: 1811171 =
563*3217 and 3217 + 562 = 3779, prime;
: 1811531 =
509*3559 and 3559 + 509 – 1 = 4067 = 49*83 and 49 + 83 – 1 = 131, prime;
: 1811621 =
7*258803 and 258803 + 7 – 1 = 258809, prime;

: 1811801 =
661*2741 and 661 + 2741 - 1 = 19*179 and 19 + 179 - 1 = 197, prime;
: 1812161 = 13*139397 and 13 + 139397 - 1 = 139409, prime.

We obtain, concatenating to the right the prime 181 with the following ten ones from the sequence above:

: 271181 prime;
: 541181 prime;
: 631181 prime;
: 811181 =
7*115883 and 7 + 115883 - 1 = 17*6817 and 17 + 6817 - 1 = 6833, prime;
: 991181 prime;
: 1171181 =
17*68893 and 17 + 68893 - 1 = 68909, prime; also 1171181 = 187*6263 and 187 + 6263 - 1 = 6449, prime;
: 1531181 prime;
: 1621181 =
41*39541 and 41 + 39541 - 1 = 39581, prime;
: 1801181 =
893*2017 and 893 + 2017 - 1 = 2909, prime;
: 2161181 = 53*40777 and 53 + 40777 - 1 = 40829, prime.

We obtain, concatenating to the left three consecutive primes from the sequence (first three cases):

: 181271541 =
11*16479231 and 11 + 16479231 - 1 = 16479241, prime; also 181271541 = 33*5493077 and 33 + 5493077 - 1 = 19*289111 and 19 + 289111 - 1 = 289129, prime;
: 271541631 =
53*5123427 and 53 + 5123427 - 1 = 5123479, prime;
: 541631811 =
2897*186963 and 2897 + 186963 - 1 = 189859, prime.

We obtain, concatenating to the left first four consecutive primes from the sequence:

: 181271541631 =
39*6630529 and 39 + 6630529 - 1 = 3*2210189 and 2210189 + 3 - 1 = 181*12211 and 181 + 12211 - 1 = 12391, prime.

We obtain, concatenating to the left first five consecutive primes from the sequence:

: 181271541631811 =
41*4421257112971 and 41 + 4421257112971 - 1 = 947*4668698113 and 947 + 4668698113 - 1 = 4668699059, prime.

We obtain, concatenating to the left first six consecutive primes from the sequence:

: 181271541631811991 = 3*60423847210603997 and 3 + 60423847210603997 – 1 = 229*263859594806131 and 229 + 263859594806131 – 1 = 263859594806359, prime.

An ideea for a function

Let mc(x) be the function defined on the set of odd positive integers with values in the set of primes in the following way:
: mc(x) = 1 for x = 1;
: mc(x) = x, for x prime;
: for x composite, mc(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),...,p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then mc(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),...,q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then mc(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of mc(x).

Example: let's calculate the value of mc(x) for a randomly selected set of five consecutive odd numbers:

: let x = 181811 = 7*19*1367;
7 + 19 + 1367 – 2 = 1391 = 13*107;
13 + 107 – 1 = 119 = 7*17;
7 + 17 – 1 = 23, prime, so mc(x) = 23;
: let x = 181813; this is prime, so mc(x) = x = 181813;
: let x = 181815 = 3*5*17*23*31;
3 + 5 + 17 + 23 + 31 - 4 = 75 = 3*5*5;
3 + 5 + 5 - 2 = 11, prime, so mc(x) = 11;
: let x = 181817 = 113*1609;
113 + 1609 – 1 = 1721, prime, so mc(x) = 1721;
: let x = 181819 = 11*16529;
11 + 16529 – 1 = 16539 = 3*37*149;
3 + 37 + 149 – 3 = 187 = 11*17;
11 + 17 – 1 = 27 = 3*3*3;
3 + 3 + 3 – 2 = 7, prime, so mc(x) = 7.

Note: it would be interesting to construct a much larger such sequence and study, for instance, the possible relations between the odd integers who share the same value for mc(x) or between those who share the same numbers of iterative steps until the value of mc(x) is found.