A Proof of the ABC Conjecture

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Introduction: The ABC conjecture was proposed by Joseph Oesterle in 1988 and David Masser in 1985. The conjecture states that for any infinitesimal quantity $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$, such that for any three relatively prime integers a, b and c satisfying $a + b = c$, the inequality $\max(|a|, |b|, |c|) \le C_\varepsilon \prod_{p|abc} p^{1+\varepsilon}$ holds water, where p/abc indicates that the product is over prime p which divide the product abc. This is an unsolved problem hitherto although somebody published papers on the internet claiming proved it.

Abstract

We first get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1. After that, expound relations between C and raf (ABC) by the symmetric law of odd numbers. Finally we have proven $C \le C_\varepsilon [\text{raf (ABC)}]^{1+\varepsilon}$ in which case A+B=C, where gcf (A, B, C) =1.

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Values of A, B and C in set A+B=C

For positive integers A, B and C, let raf (A, B, C) denotes the product of all distinct prime factors of A, B and C, e.g. if $A=11^2\times13$, $B=3^3$ and $C=2\times13\times61$, then raf (A, B, C) $=2\times3\times11\times13\times61$ =52338. In addition, let gcf (A, B, C) denotes greatest common factor of A, B and C.

The ABC conjecture states that given any real number $\varepsilon > 0$, there exists a constant $C_\varepsilon >0$ such that for every triple of positive integers A, B and C satisfying A+B=C, and gcf (A, B, C) =1, then we have $C\le C_\varepsilon\,[\mathrm{raf}\,(ABC)]^{1+\varepsilon}$.

Let us first get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1, as listed below.

1. If A, B and C all are positive odd numbers, then A+B is an even number, yet C is an odd number, evidently there is only $A+B\neq C$ according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A+B is an even number, C is an odd number, yet when A+B is an odd number, C is an even number, so there is only $A+B\neq C$ according to an odd number ≠ an even number.

3. If A, B and C all are positive even numbers, then they have at least a common prime factor 2, manifestly this and the given prerequisite of gcf (A, B, C) =1 are inconsistent, so A, B and C can not be three positive even numbers together.

Therefore we can only continue to have a kind of A+B=C, namely A, B and

C are two positive odd numbers and one positive even number. So let following two equalities add together to replace A+B=C in which case A, B and C are two positive odd numbers and one positive even number.

1. $A+B=2^X S$, where A, B and S are three relatively prime positive odd numbers, and X is a positive integer.

2. $A+2^Y V=C$, where A, V and C are three relatively prime positive odd numbers, and Y is a positive integer.

Consequently the proof for ABC conjecture, by now, it is exactly to prove the existence of following two inequalities.

(1). $2^X S \leq C_\varepsilon\, [\mathrm{raf}\,(A, B, 2^X S)]^{1+\varepsilon}$ in which case $A+B=2^X S$, where A, B and S are three relatively prime positive odd numbers, and X is a positive integer.

(2). $C \leq C_\varepsilon\, [\mathrm{raf}\,(A, 2^Y V, C)]^{1+\varepsilon}$ in which case $A+2^Y V=C$, where A, V and C are three relatively prime positive odd numbers, and Y is a positive integer.

Circumstances Relating to the Proof

Let us divide all positive odd numbers into two kinds of A and B, namely the form of A is 1+4n, and the form of B is 3+4n, where n is a positive integer or 0. From small to large odd numbers of A and of B are arranged as follows.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67…3+4n …

We list also from small to great natural numbers, well then you would discover that Permutations of seriate natural numbers show up a certain law.

1, 2^1, 3, 2^2, 5, $2^1\times3$, 7, 2^3, 9, $2^1\times5$, 11, $2^2\times3$, 13, $2^1\times7$, 15, 2^4, 17, $2^1\times9$, 19, $2^2\times5$, 21, $2^1\times11$, 23, $2^3\times3$, 25, $2^1\times13$, 27, $2^2\times7$, 29, $2^1\times15$, 31, 2^5, 33, $2^1\times17$, 35, $2^2\times9$, 37, $2^1\times19$, 39, $2^3\times5$, 41, $2^1\times21$, 43, $2^2\times11$, 45, $2^1\times23$, 47, $2^4\times3$, 49, $2^1\times25$, 51, $2^2\times13$, 53, $2^1\times27$, 55, $2^3\times7$, 57, $2^1\times29$, 59, $2^2\times15$, 61, $2^1\times31$, 63, 2^6, 65, $2^1\times33$, 67, $2^2\times17$, 69, $2^1\times35$, 71, $2^3\times9$, 73, $2^1\times37$, 75, $2^2\times19$, 77, $2^1\times39$, 79, $2^4\times5$, 81, $2^1\times41$, 83, $2^2\times21$, 85, $2^1\times43$, 87, $2^3\times11$, 89, $2^1\times45$, 91, $2^2\times23$, 93, $2^1\times47$, 95, $2^5\times3$, 97, $2^1\times49$, 99, $2^2\times25$, 101, $2^1\times51$, 103 …→

Evidently even numbers contain prime factor 2, yet others are odd numbers in the sequence of natural numbers above-listed.

After each of odd numbers in the sequence of natural numbers is replaced by self-belongingness, the sequence of natural numbers is changed into the following forms.

A, 2^1, B, 2^2, A, $2^1\times3$, B, 2^3, A, $2^1\times5$, B, $2^2\times3$, A, $2^1\times7$, B, 2^4, A, $2^1\times9$, B, $2^2\times5$ A, $2^1\times11$, B, $2^3\times3$, A, $2^1\times13$, B, $2^2\times7$, A, $2^1\times15$, B, 2^5, A, $2^1\times17$, B, $2^2\times9$, A $2^1\times19$, B, $2^3\times5$, A, $2^1\times21$, B, $2^2\times11$, A, $2^1\times23$, B, $2^4\times3$, A, $2^1\times25$, B, $2^2\times13$, A $2^1\times27$, B, $2^3\times7$, A, $2^1\times29$, B, $2^2\times15$, A, $2^1\times31$, B, 2^6, A, $2^1\times33$, B, $2^2\times17$, A $2^1\times35$, B, $2^3\times9$, A, $2^1\times37$, B, $2^2\times19$, A, $2^1\times39$, B, $2^4\times5$, A, $2^1\times41$, B, $2^2\times21$, A $2^1\times43$, B, $2^3\times11$, A, $2^1\times45$, B, $2^2\times23$, A, $2^1\times47$, B, $2^5\times3$, A, $2^1\times49$, B, $2^2\times25$, A, $2^1\times51$, B …→

Thus it can be seen, leave from any given even number >2, there are finitely many cycles of (B, A) leftwards until (B=3, A=1), and there are infinitely many cycles of (A, B) rightwards.

If we regard an even number on the sequence of natural numbers as a symmetric center of odd numbers, then two odd numbers of every bilateral symmetry are A and B always, and a sum of bilateral symmetric A and B is surely the double of the even number. For example, odd numbers 23(B) and 25(A), 21(A) and 27(B), 19(B) and 29(A) etc are bilateral symmetries whereby even number $2^3 \times 3$ to act as the center of the symmetry, and there are $23+25=2^4 \times 3$, $21+27=2^4 \times 3$, $19+29=2^4 \times 3$ etc. For another example, odd numbers 49(A) and 51(B), 47(B) and 53(A), 45(A) and 55(B) etc are bilateral symmetries whereby even number 2×25 to act as the center of the symmetry, and there are $49+51=2^2 \times 25$, $21+27=2^2 \times 25$, $19+29=2^2 \times 25$ etc. Again give an example, 63(B) and 65(A), 61(A) and 67(B), 59(B) and 69(A) etc are bilateral symmetries whereby even number 2^6 to act as the center of the symmetry, and there are $63+65=2^7$, $61+67=2^7$, $59+69=2^7$ etc.

Overall, if A and B are two bilateral symmetric odd numbers whereby $2^X S$ to act as the center of the symmetry, then there is $A+B=2^{X+1}S$.

The number of A plus B on the left of $2^X S$ is exactly the number of pairs of bilateral symmetric A and B. If we regard any finite-great even number $2^X S$ as a symmetric center, then there are merely finitely more pairs of bilateral symmetric A and B, namely the number of pairs of A and B which express $2^{X+1}S$ as the sum is finite. That is to say, the number of pairs of bilateral symmetric A and B for symmetric center $2^X S$ is $2^{X-1}S$, where $S \geq 1$.

On the supposition that A and B are bilateral symmetric odd numbers whereby $2^X S$ to act as the center of the symmetry, then $A+B=2^{X+1}S$. By now, let A plus $2^{X+1}S$ makes $A+2^{X+1}S$, then B and $A+2^{X+1}S$ are still bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $B+(A+2^{X+1}S) =(A+B)+2^{X+1}S =2^{X+1}S+2^{X+1}S =2^{X+2}S$.

If substitute B for A, let B plus $2^{X+1}S$ makes $B+2^{X+1}S$, then A and $B+2^{X+1}S$ are too bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $A+ (B+2^{X+1}S) =2^{X+2}S$.

Provided both let A plus $2^{X+1}S$ makes $A+2^{X+1}S$, and let B plus $2^{X+1}S$ makes $B+2^{X+1}S$, then $A+2^{X+1}S$ and $B+2^{X+1}S$ are likewise bilateral symmetry whereby $3\times2^X S$ to act as the center of the symmetry, and $(A+2^{X+1}S)+(B+2^{X+1}S) =3\times2^{X+1}S$.

Since there are merely A and B at two odd places of each and every bilateral symmetry on two sides of an even number as the center of the symmetry, then aforementioned $B+(A+2^{X+1}S)=2^{X+2}S$ and $A+(B+2^{X+1}S)=2^{X+2}S$ are exactly $A+B=2^{X+2}S$ respectively, and write $(A+2^{X+1}S)+(B+2^{X+1}S)=3\times2^{X+1}S$ down $A+B=3\times2^{X+1}S= 2^{X+1}S_t$, where S_t is an odd number ≥3.

Do it like this, not only equalities like as $A+B=2^{X+1}S$ are proven to continue the existence, one by one, but also they are getting more and more along with which X is getting greater and greater, up to exist infinitely more equalities like as $A+B=2^{X+1}S$ when X expresses every natural number.

In other words, added to a positive even number on two sides of $A+B=2^X S$, then we get still such an equality like as $A+B=2^X S$.

Whereas no matter how great a concrete even number $2^X S$ as the center of the symmetry, there are merely finitely more pairs of A and B which express $2^{X+1}S$ as the sum.

If X is defined as a concrete positive integer, then there are only a part of $A+B=2^X S$ to satisfy gcf $(A, B, 2^X S) =1$. For example, when $2^X S=18$, there are merely 1+17=18, 5+13=18 and 7+11=18 to satisfy gcf $(A, B, 2^X S) =1$, yet 3+15=18 and 9+9=18 suit not because they have common prime factor 3.

If add or subtract a positive odd number on two sides of $A+B=2^X S$, then we get another equality like as $A+2^Y V=C$. That is to say, equalities like as $A+2^Y V=C$ can come from $A+B=2^{X+1}S$ so as add or subtract a positive odd number on two sides of $A+B=2^{X+1}S$.

Therefore, on the one hand, equalities like as $A+2^Y V=C$ are getting more and more along with which equalities like as $A+B=2^{X+1}S$ are getting more and more, up to infinite more equalities like as $A+2^Y V=C$ exist along with which infinite more equalities like as $A+B=2^{X+1}S$ appear.

Certainly we can likewise transform $A+2^Y V=C$ into $A+B=2^X S$ so as add or subtract a positive odd number on the two sides of $A+2^Y V=C$.

On the other hand, if C is only defined as a concrete positive odd number, then there is merely finitely more pairs of A and $2^Y V$ which express C as the sum. But also, there is probably a part of $A+2^Y V=C$ to satisfy gcf $(A, 2^Y V, C) =1$. For example, when C=25, there are merely 1+24=25, 3+22=25, 7+18=25, 9+16=25, 11+14=25and 13+12=25 to satisfy gcf $(A, 2^Y V, C) =1$, yet

5+20=25 and 15+10=25 suit not because they have common prime factor 5.

After factorizations of A, B, S, V and C in $A+B=2^{X+1}S$ plus $A+2^{Y}V=C$, if part prime factors have greater exponents, then there are both $2^{X+1}S \geq$ raf (A, B, $2^{X+1}S$) in which case $A+B=2^{X+1}S$ satisfying gcf (A, B, $2^{X+1}S$) =1, and $C \geq$ raf (A, $2^{Y}V$, C) in which case $A+2^{Y}V=C$ satisfying gcf (A, $2^{Y}V$, C) =1. For examples, $2^{7} >$ raf (3, 5^{3}, 2^{7}) for $3+5^{3}=2^{7}$; and $3^{10} >$ raf (5^{6}, $2^{5}\times 23\times 59$, 3^{10}) for $5^{6}+2^{5}\times 23\times 59=3^{10}$.

On the contrary, there are both $2^{X+1}S \leq$ raf (A, B, $2^{X+1}S$) in which case $A+B=2^{X+1}S$ satisfying gcf (A, B, $2^{X+1}S$) =1, and $C \leq$ raf (A, $2^{Y}V$, C) in which case $A+2^{Y}V=C$ satisfying gcf (A, $2^{Y}V$, C) =1. For examples, $2^{2}\times 7 <$ raf (13, 3×5, $2^{2}\times 7$) for $13+3\times 5=2^{2}\times 7$; and $3^{4} <$ raf (11×7, 2^{2}, 3^{4}) for $11\times 7+2^{2} = 3^{4}$.

Since either A or B in $A+B=2^{X+1}S$ plus an even number is still an odd number, and $2^{X+1}S$ plus the even number is still an even number, thereby we can use $A+B=2^{X+1}S$ to express every equality which plus an even number on two sides of $A+B=2^{X+1}S$ makes.

Consequently, there are infinitely more $2^{X+1}S \geq$ raf (A, B, $2^{X+1}S$) plus $2^{X+1}S \leq$ raf (A, B, $2^{X+1}S$) in which case $A+B=2^{X+1}S$.

Likewise, either $2^{Y}V$ plus an even number is still an even number, or A plus an even number is still an odd number, and C plus the even number is still an odd number, so we can use equality $A+2^{Y}V=C$ to express every equality which plus an even number on two sides of $A+2^{Y}V=C$ makes.

Consequently, there are infinitely more $C \geq$ raf (A, $2^{Y}V$, C) plus $C \leq$ raf (A,

2^YV, C) in which case A+2^YV = C.

But, if let $2^{X+1}S \geq$ raf (A, B, $2^{X+1}S$) and $2^{X+1}S \leq$ raf (A, B, $2^{X+1}S$) separate, and let C $\geq$ raf (A, 2^YV, C) and C $\leq$ raf (A, 2^YV, C) separate, then for inequalities like as each kind of them, we conclude not out whether they are still infinitely more.

However, what deserve to be affirmed is that there are $2^{X+1}S \geq$ raf (A, B, $2^{X+1}S$) and $2^{X+1}S \leq$ raf (A, B, $2^{X+1}S$) in which case A+B=$2^{X+1}S$ satisfying gcf (A, B, $2^{X+1}S$) =1, and there are C $\geq$ raf (A, 2^YV, C) and C $\leq$ raf (A, 2^YV, C) in which case A+2^YV =C satisfying gcf (A, 2^YV, C) =1, according to the preceding illustration with examples.

Proving $C \leq C_\varepsilon$ [raf (A, B, C)] $^{1+\varepsilon}$

Hereinbefore, we have deduced that both there are $2^{X+1}S \leq$ raf (A, B, $2^{X+1}S$) and $2^{X+1}S \geq$ raf (A, B, $2^{X+1}S$) in which case A+B=2^XS satisfying gcf (A, B, $2^{X+1}S$) =1, and there are C $\leq$ raf (A, 2^YV, C) and C $\geq$ raf (A, 2^YV, C) in which case A+2^YV =C satisfying gcf (A, 2^YV, C) =1, whether each kind of them is infinitely more, or is finitely more.

First let us expound a set of identical substitution as the follows. If an even number on the right side of each of above-mentioned four inequalities added to a smaller non-negative real number such as R $\geq$ 0, then the result is both equivalent to multiply the even number by another very small real number, and equivalent to increase a tiny real number such as $\varepsilon \geq 0$ to the exponent of the even number, i.e. form a new exponent $1+\varepsilon$, but when R= 0, the multiplied real number is 1, yet $\varepsilon = 0$.

Actually, aforementioned three ways of doing, all are in order to increase an identical even number into a value and the same.

Such being the case the identical substitution between each other, then we set about proving aforesaid four inequalities, one by one, thereinafter.

(1). For inequality $2^{X+1}S \leq \mathrm{raf}(A, B, 2^{X+1}S)$, $2^{X+1}S$ divided by raf $(A, B, 2^{X+1}S)$ is equal to $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$ as a true fraction, where $S_1 \sim S_n$ express all distinct prime factors of S; t-1~m-1 are respectively exponents of prime factors $S_1 \sim S_n$ orderly; A_{raf} expresses the product of all distinct prime factors of A; and B_{raf} expresses the product of all distinct prime factors of B.

After that, even number raf $(A, B, 2^{X+1}S)$ added to a smaller non-negative real number such as $R \geq 0$ to turn the even number itself into $[\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Undoubtedly there is $2^{X+1}S \leq [\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ successively.

By now, multiply $[\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ by $2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, then it has still $2^{X+1}S \leq 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}[\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Also let $C_\varepsilon = 2^X S_1^{t-1} \sim S_n^{m-1}/A_{raf}B_{raf}$, we get $2^{X+1}S \leq C_\varepsilon [\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Manifestly when R= 0, it has $\varepsilon = 0$, and $2^{X+1}S = C_\varepsilon [\mathrm{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

(2). For inequality $C \leq \mathrm{raf}(A, 2^Y V, C)$, C divided by raf $(A, 2^Y V, C)$ is equal to $C_1^{j-1} \sim C_e^{f-1}/2A_{raf}V_{raf}$ as a true fraction, where $C_1 \sim C_e$ express all distinct prime factors of C; j-1~f-1 are respectively exponents of prime factors C_1~C_e orderly; A_{raf} expresses the product of all distinct prime factors of A; and V_{raf} expresses the product of all distinct prime factors of V.

After that, even number raf (A, 2^YV, C) added to a smaller non-negative real number such as $R \geq 0$ to turn the even number itself into $[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

Undoubtedly there is $C \leq [\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$ successively.

By now, multiply $[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$ by C_1^{j-1}~$C_e^{f-1}/2A_{raf}V_{raf}$, then it has still $C \leq C_1^{j-1}$~$C_e^{f-1}/2A_{raf}V_{raf}\,[\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

Also let $C_\varepsilon = C_1^{j-1}$~$C_e^{f-1}/2A_{raf}V_{raf}$, we get $C \leq C_\varepsilon [\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

Manifestly when R= 0, it has $\varepsilon = 0$, and $C = C_\varepsilon [\text{raf}(A, 2^Y V, C)]^{1+\varepsilon}$.

(3). For inequality $2^{X+1}S \geq$ raf (A, B, 2^{X+1}S), 2^{X+1}S divided by raf (A, B, 2^{X+1}S) is equal to $2^X S_1^{t-1}$~$S_n^{m-1}/A_{raf}B_{raf}$ as a false fraction, where S_1~S_n express all distinct prime factors of S; t-1~m-1 are respectively exponents of prime factors S_1~S_n orderly; A_{raf} expresses the product of all distinct prime factors of A; and B_{raf} expresses the product of all distinct prime factors of B.

Evidently $2^X S_1^{t-1}$~$S_n^{m-1}/A_{raf}B_{raf}$ as the false fraction is greater than 1.

Then, even number raf (A, B, 2^{X+1}S) added to a smaller non-negative real number such as $R \geq 0$ to turn the even number itself into $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

After that, multiply $[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$ by $2^X S_1^{t-1}$~$S_n^{m-1}/A_{raf}B_{raf}$, then it has $2^{X+1}S \leq 2^X S_1^{t-1}$~$S_n^{m-1}/A_{raf}B_{raf}\,[\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Let $C_\varepsilon = 2^X S_1^{t-1}$~$S_n^{m-1}/A_{raf}B_{raf}$, we get $2^{X+1}S \leq C_\varepsilon [\text{raf}(A, B, 2^{X+1}S)]^{1+\varepsilon}$.

Manifestly when R= 0, it has ε = 0, and $2^{X+1}S = C_{\varepsilon}$ [raf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$.

(4). For inequality C ≥ raf (A, $2^{Y}V$, C), C divided by raf (A, $2^{Y}V$, C) is equal to $C_1^{j-1}\sim C_e^{f-1}/2A_{raf}V_{raf}$ as a false fraction, where $C_1\sim C_e$ express all distinct prime factors of C; j-1~f-1 are respectively exponents of prime factors $C_1\sim C_e$ orderly; A_{raf} expresses the product of all distinct prime factors of A; and V_{raf} expresses the product of all distinct prime factors of V.

Evidently $C_1^{j-1}\sim C_e^{f-1}/2A_gV_q$ as the false fraction is greater than 1.

Then, even number raf (A, $2^{Y}V$, C) added to a smaller non-negative real number such as R ≥ 0 to turn the even number itself into [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$.

After that, multiply [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$ by $C_1^{j-1}\sim C_e^{f-1}/2A_{raf}V_{raf}$, then it has $C\le C_1^{j-1}\sim C_e^{f-1}/2A_{raf}V_{raf}$ [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$.

Let $C_{\varepsilon}= C_1^{j-1}\sim C_e^{f-1}/2A_{raf}V_{raf}$, we get $C \le C_{\varepsilon}$ [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$.

Manifestly when R= 0, it has ε = 0, and $C = C_{\varepsilon}$ [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$.

We have concluded $C_{\varepsilon}=2^{X}S_1^{t-1}\sim S_n^{m-1}/A_{raf}B_{raf}$ and $C_{\varepsilon}=C_1^{j-1}\sim C_e^{f-1}/2A_{raf}V_{raf}$ in preceding proofs, evidently each and every C_{ε} is a constant because it consists of known numbers.

Besides, for a smaller non-negative real number R ≥ 0, actually, it is merely comparatively speaking, if raf (A, B, $2^{X+1}S$) or raf (A, $2^{Y}V$, C) is very great a positive even number such as 2×11×13×99991×99989×99961×99929×99923 ×87641×72223×8117×12347, then even if R=2015.11223√2, it is also a smaller non-negative real number. Since raf (A, B, $2^{X+1}S$) or raf (A, $2^{Y}V$, C) may be infinity, so R may tend to infinity.

Taken one with another, we have proven that there are both infinitely more $2^{X+1}S \le C_{\varepsilon}$ [raf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$ when X is each and every natural number, and infinitely more $C \le C_{\varepsilon}$ [raf (A, $2^{Y}V$, C)] $^{1+\varepsilon}$ when C is each and every positive odd number ≥ 1.

But then, when X is a concrete natural number, even if the concrete natural number tends to infinity, there also are merely finitely more $2^{X+1}S \le C_{\varepsilon}$ [raf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ in which case $A+B=2^{X+1}S$.

When C is a concrete positive odd number, even if the concrete positive odd number tends to infinity, there also are merely finitely more $C \le C_{\varepsilon}$ [raf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$ in which case $A+2^{Y}V=C$.

To sum up, the proof is completed by now. Consequently the ABC conjecture does hold water.