Prior to my retirement from the service I had been teaching mathematics at the graduate level for 40 odd years, in Rajaram College Kolhapur, which is run by the Govt. of Maharashtra.

My specialization is **Pure Mathematics**. Naturally, the branches which I use to teach were; calculus, advance real analysis, topology, modern algebra and set theory.

It is well known that, we learn more while teaching a subject rather than while studying it. I seldom took anything for granted simply because it is printed in a standard book.

Almost all books, which I use to refer during the tenure of my service have categorized the **"brotherhood relation"** as non-reflexive, non-symmetric but transitive. I couldn't agree with this. I have penned down my thoughts in this regard in the following pages which the reader will find in my own handwriting.

I propose to upload this on many other sites also as I believe that, whenever any new thought stickers a mind of some human being it should become a property of all mankind.

Thank you.

Prof. Shrikrishna Jayraj Kalgaonkar "Swarangan", Plot No. 310, Society No 5, R.K. Nagar, Morewadi, Kolhapur MS India Phone +91.231.2639684 / +91.98814.68643 Mail : sjkalgaonkar@yahoo.in

Χ PAGE NO. antroduction: DATE: 031 08-1/6 Page Nō. and the discussion that follows it is \_ with expected that the reader is fumilias 17. elementary concepts of Set Theory such sets, elements of a set, the will bracket set & un of fintersection operations, notation, the incomplement of a set. With these clamendary i deas we set start the objection of a topic that will take us to study "Binary Relations" Order Pair In a set the only important information "which are the elements of it 2' and nothing else. The order of elements in a set is not important The sets \$3,47 and {4,3] are equal or in these woods they are "Came Sets r But incomany instances we require the elements taken in specific order eg. In an aring parade when the soldiers have standing in rows the person in the 4th row is not same 324 the 4th person in the 3rd row order to pachieve this we introduce the following definition 1 a and b are two elements ther - month Efra where set defined and a a } 29.67% 15 alled as CongratulationsbTake pridentiou are using a Nameet ecobudity Perioduct bit studing no O

"Shirtistrandy Namah M Ŧ ù WWW.navneet.com DATE: 0 31 0811 DATE: 03 08-1 10 the ordered part of and and burth 25 N 64 . The understanding that a is the an An first chement and b' is the second todand This will be deuted may (9,6) Thuc ... 52-22.0.72 ~ . . (9,6) = 1293 29,671 It\_ easy to see and to prove that this concept certainly enjoys the following two proposities CETEX - S SA (r)(9, b) = (b, a) unless as b and (ii) (9,2) = (C,d) iff 9= and b= d (1.36 th 2. 2. 1. 1. 3 As (i) says that if at bothen (915) = (5,a) it Justifies the hand to it as an ordered pair given Cartessian Product of two sets. If A and Brase too hon-emply then the Cartessian Product of Sats B is defined and wa A with (x, Y) / x E A and ] YEB - i - i - an is dended on AXB Thus 朴マを IREA <u>\* 4 3</u> 8 8. and 7 2 (24,7) AXB= S SYSE sints that be verified that AX B + BXA nuless lof course A= NEN Also if A has monuber of efforme has been annuber of selective B 1:13) à 18 S Congratulationst Take Pytide: Organist a general a static and the staticand the static and the staticand the st

2 PAGE NO. RAGE NO. 🗳 108-110 1 8015 5 3TAO DATE:03/08/ 6 Swith the AxiB has more elements mit. But the nature of elements of AXB Sthe his quite different from those of A or B. second If A = 1 4, 43 and B = 31, 2, 33 )\\_\_\_\_ then AXB= { (x, 1), (x, 2), (x, 3) (4,1) (4,2)(3) 4.15 and BxA- \$ (1, 2) (1, 1) (2, 2) (2, 4) (3, 2) (3, 7) 25 ne that Binary Relations. In everyday tipe we use statements the of the following type . . Arjun & the father of Abhimanyy. 1902 or Laxman "is a brother of" Rama and b=d or Usha is taller than & Rahul . etc. then In Mathematics we use fimillar statements the hand eg. 4'is greater than 3 pair" mor the tis parallel to be 22 Mil Sound -AABC" is simillar to" A POR etc. 05 In all the above examples the phrase n-emply that of put in the invested commas is called a relation which relates the  $1 > 1 \in \mathbb{N}$ 5.5 chement before the phrase with the element that follows the that phrase . ्रेको धर्ने eg. In the statement " Arjun is the father of Abhimanyy" the relation considered is " is the father of " and it relates Arjun to Abhimanya. course A=B Simillarly in the statement "4" is greater of element than " 3" the relation considered is chement Congra Congratulationst Jake pride You are using a Navneet ecobuddy Pproduct is utsignoo

DATE: 03108110 DATE: 031 081 'is agreated than' audit relates ENtry ( with 3 . 3A1 1 8 Please see that when 4 is related 5 3 under the real relation 'is greater than 1; \$3 is not related to 4 although 3 is may be No considered related to 4 under different relation ' is less than' but certainly not under " is greater than " Consider the following example Let S= { 2, 3, 4, 6, 8 } and the relation being considered is 'is a factor of ! The set of true statements formed in this case are : \* \* ~ · · 2 is a factor of 2 22.1140.8 2. 2 is a factor got 12 2- is a factor of 6 9.14 2. is a factor of 8 313 is a factor of 3 3 is a factor of 6 4 is a factor of 4 (ist a) 3 4 is a factor of 8 and the 6 is a factor of 6 reservice a factor of 8 moder th greeter Stranditoria ant air yealling the relation considered is Marine 2" Congratulationstidate pridection are using a Navneet ecobeddy productits luts on O

PAGE NO. 6 PAGE NO. 5 108110 1-80180 : stad DATE: 031 081 10 relates to avoid a peatedly writting the phrase 1.4 is a "factor of" we can say that related shet us dende this complete, parase lation by Rolthe first letter in the word relation) rof 1 So now in this context R always mean smay be " is a factor of . Indos The above set of the statements can (88) (non) be reproduced as than ! is greater 2R2 2R4 2R6 2R8 3R3 3R6-4R4 4R8 6R6 18 R8 R S Here is another way to look at the problem. Every statement of seclation 1 ic requires the elements. One is the element Which is related and the other in the element to which the first related forme Henre such relations are called the binary good relations in mar m It is interesting to see that not 14 only every statement in corporates يا الديني a pair of elements but the order In Which these elements deense 20 in also important. For in the 1995 - See - S abone messaurille 2 R4 but 4 R 2 Here the Statement in a binary en <sup>1</sup> Belation not only invokes la pair? ~\*\*ੈ but in involves 'an ordered pair.' at to be to be **.** A 17 If it is stipulated that in the topic 1.1 of binary relation (a, 6) will mean Conge Congratulationsl Take pride: You are using a Navneet ecobuiddy Deproduct is luts 1900

PAGE NO. 6 DATE: 0310-8-110 DATE: 031 0811 925, Ad the mistrelated to Be blows that ine in Ater words the related and element will be whitten intothe mitris first place anoth the the man above & statement will now appear Strange Strange on -1×3 (2,2) (2,4) (2,6)-(2,8), (3,3) (3,6) (4,4) (4,8) (6,6) (8,8) When this set of true statement 21's relaced with the set bracket the above relation will now took matike promisions wears and days 1,2,2) (2,4) (2,6) (2,8) (3,3) (3,6) (4,4) (4,8) (6,6) This set will be called the relation Set R here have here  $R = \overline{f(2,2)}(2,6)(2,8)(3,3)(3,6),(4,4)(4,8)(6,6)(8,8)$ IP can be easily seen that R CAXA (Although here flere the relation is first AXA & R introduced with meaningfulliwords like sis a factor of and vis developed to a subset of an a subset of Cartessian Product This development suggests in the depinition of a binas Sigot but selation and follows - 21 11-1 2 denses a claries (a, b) will be and Congratulationsi (Take pride: You: ane using a Navneet ecobuddy Ward duct is luterorio)

08110 DATE: 0 31 08-1 10 1-26 10 3 : 3TH2 Def IF A singer to many in on - surgery in g sets then a non-emply subset ated R of AXA is called a binarry 5 Ac relation in A he This A = g, R C AXA and R = g appear If or, y CA such that (20, 4) ER We shall say that x is related to y in A by R This is same thing as 66 (88) wonthing z Ry fitiment  $Thm (2C, Y) \in R = 2CRY$ " chet millock In the earlier example we have seen that 2 is a factor of 4 but 4 is not a factor of 2 7 (4, 8), (6,0) i.e. 2R4 by 4R2 or (2,4) c R but (4,2) & R relation This the elements here can not be interchanged without affectioning the (6,6) (8,8) touth value of the statements. We see that the elements 2 4 are not synctrically placed on both sides of R Z-R and indeed we are going to admit ods like the that this relation " is a factor of " is not synctric . al to Considering this and such other possibility set of possibilities we define certain relopenent Types of Binary Relations J-a binary V Reflexive Relation. Dethan A binary relation R in a bon-emply Conge Congratulationsb Take pride: You are using a Navneet ecobuddy y productits lot store of

CH BEAS PAGE NO. 8 1-20 12 0 :374C DATE: 061 081 10 set A missisaid to be a reflexing relation if xRx for every se f A. fire wif every chement of A is related to itself by R. The property that X + 2 eA > 2 Ra is called the reflexivily collis a factor of relation in any subset of integers is reflexive as every integer is a factor of itself. i) (ii) If parallelism of lines is defined as 1 two lines 418 12 are said to be parallel if they are co-planar and Lintz = p then parallelism can be not be meflexive for an Decause for any in line Lathough L&L are co-planar DALE Le and it can not be empty- 3 a three the state (Tim) But if pasallelism is defined as the two lines Lights in a planease said to be parallel if their slopes are equal them for any line L Slope of L = slope of L ang we shall have to accept that a time is always parallel with itself Heren the selation will be reflessive The above example will show that whether Congratulations/Take"gittection are using a Manneet octobed by "produced a long and the second secon

savnestcom Salli±: Fvå5t[V]	<mark>π<sup>n</sup> Γ<sup>§</sup>≪4π \</mark> ◀	ŢĴΨĢāe∰ġāŢā⇔G <sup>n</sup> , o↑Δ╤♦±g■♠i♥ <sup>n</sup> f@ <sup>n</sup> , o↑Δ╤ <b>ķ</b> ±g <b>u</b> ♠i♥ <sup>f</sup> @♥₽ <b>Ħ</b> <sup>~</sup> L©∘T <b>A</b>
F 08+ 10		DATE: 051 08110
Perire		a relation is reflexing on not depends
JoceA.	-	Stictly on the main definition of that
is related	stand and	relation. i.e. it depends upon the
2 that		meaning we atsibute to the selation.
		The reader is should keep this point
	<u><u></u>#2</u>	in his hind to understand the firsther
any		discussion management and subject
·ine		(ir) In many books on Set Theory
or of		Sotto-lesno Algebra the relation
		1 is a brother of is considered
defined		in the set of humans. As the statement
id to	1	solutions a britter of and and
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Abe	· · · · ·	because the main purpose of this writting
	1	is to establish the reflexivity of
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ine L		main to be a symetric relation of
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ON BEAS PAGE NO. 10 DATE: # 51 - 81 DATE: 05108110 church P The relation Misra fuctor of is certainly not symetric as the have seen that 2 is a factor 14 that 4 is in A a factor of 2 There 2 R.4 1but 4 K2 .... (ii) In the set of all triangles in a plane the congruency of triangles is symetric Because and AABC = APQR >> DPQR = DABC The botherhand relation is at us In almost all books sithe britterhood relation is declared to be non-symetric min the set of all human beings 2 sand This is soll specause the state in that se is a brother of y does ist mot necessarily mean that y istig further of 200 about some minstances Y'may be a sister of 200. But this imediately suggests that to in the set of all male human beings the brother hand relation will bei 2.1 symetric we will turn to this. Selation in detail later 3 Transitive Relations A bring selation R in A is said sta bled transitive relation if som x Ry and y Rzuito so Riz for all CongratulationsbTake prider You are using advanced ecobeddy V productins using a congratulations of the second sec

PAGE NC. 12 PAGE NO. 11 08110 DATE: 061 081 10 · ( ) · ) · staa 1 1.5 The property is called the transitivity. It can be described in me woods an stactor 14 that if there are two statements of relation token in specific order such 9. V 1 that the second statement begins - AN 11 s in a where the first ends then the relation is transited. In xRy and yRz, noungles the intermediate element y is sometimes looked upon as the median ABC through which R gets transited from as not rt || et z. By (i) The relation 'is a factor of hood 10n-symutric is transitive IS Transitive. If xRy and YRz then in this eingo. context it will mean that ment x is a factor of y i.e. y= more does for some integer m ista and y is a factor z i.e z=ny Hance for some integer n . + this By substituition z = n(mx) in the ings i.e. Z= (nm) > an mm is certainly an integer we "ber this get that so is a factor of Z i.e. SCKZ (i) an a book shop if we say that for the books ocly or Ry if ₹ţ**ı** s. said the box difference in their prices is less that 10 Rs. they this relation -yril will be reflexive, symetric but not зA÷ - Aიციბ CongratulationshTake/pride: You are using a Navnest ecobuddy % product is lutations)

IL ON BOAS 12 AGE NO. DATE:06108110 120 120 STAD winter transitive for a) price 1 x - price of x = 0 < 10Rs. and the stand xRx (Reflexinly D If xRy then China de /price g x - price g y/ 2 loks.  $r < 2^{n-1}$ ABON LOUD D But /price of x - price of y /= /price of y - price of x -. I price of y-price of af 2 loks. Red Sections have yR= Csynetoy ( and the second s But we can have a case of the following type x, y, z are books g. t could bee price of se = 100 Rs price of your 108 Rs. - 11 price of z = 116 Rs cleasty here scry and yrz But / price of x - price of z = 16 \$10 X R Z o) In almost all books the brotherhoo relation is considered trainitive 7-23:14 probably because the following statements is felt to be obvious If is a brother of y and monthe lar with y us it a brother of Z the 130 tid not in 2 birdtherig Z Congratulations/ Jake pride: You are wsing a Navneet ecobuiddy " productits luterand O

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, , , , , , , , , , , , , , , , , , , ,	Vo.st.	Now here is the main important emplysis
10Rs.		of this oproblem out minor ha
12. e 14		In the definition of transitivity the
		condition 2RX & YRZ => 2RZI
Rs.		for all x, y, z & A merit is not
		at all mentioned that it, 2, 2, 5 hould
2 y-price gx	<u>- 197</u>	be different from each ther. The
		condition must be satisfied for call x, y, z
oRg.		whether equal or unequal . Hance a
XCAR 1		situation can arise of the following type.
e of the	. And	Supposed that and y are byother of
	<u> </u>	each othersin. Then we swill have
•		. Mais Right and yi Rixin But want
	<u>/&gt;</u>	The first statement wends on yourse
	š	the second begins. I, therefore becomes
		the median Non if we are to
Ro		agree that R is transisive their pr Ry and
1-Z	*	YRe mont give up seRoe ie we
<u> </u>		will have to carept that a person is
*	gh that	brother of himself i.e. brotherhood is
	in the second	reflessive for If we dond agree about
e brotherhood		the reflexibility then because of the above
mitire	3.3 5 B-	situation we have to say that the relation
lowing		isi not transitive another as an
· obvious		This brotherhood relation can be
ud for		either reflexive range transitive or it
then	· · · ·	can be non- reflexive and non-trainsitive.
		Truly speaking there is no the reason
Congre		Congratulationst Take pride to an using a Navneet ecobuildy productits lutsion of

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	тос. тееплеп. мих- <u>E</u> [ си зеля: <u>1163130: 3740</u> DATE: 06108110
	DATE: 06108110
Siehryo	Wity mibrothishood should not be accepted
	as reflexive. The main trouble hes in
he	the fact that but the term brotherhood
	is nA defined here and it is taken
-1.0	on if a known thing. If at all we decid
5	define the term then our definition
٨. و	wills obviously be of the following type.
<u> </u>	Der 200 200 is sand to be a brother of y
<u>^&gt;</u>	13 yen 20 is a wale human being and
-yoe	ABARITOR and y share same parents.
	Ontred this definistion is given; whenever
	pres 1 conside a mate hundre being 2
· soal	then the statements that shoth 2 & 2
econes	have offersametriposients becomes a
The second se	iend statement i paid we get sold .
	brahechood instation becomes reflexing at
	Caul there fore transitive also.
21 100-	We now consider this problem from
	anerlas angle in order to again establish
- (BP)	the reflexinity of this brother hood relation
bore	2 At p Equivalence Relationer and
relation	A binary relation Ruin Aris said to be
	au equivalence setation if R is
<u> </u>	main my replexing disallored and
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<u>(5%)</u>	Truly sheeking there is no the real
	CongratulationstoTake goodatoou and using a Navneet ecobuddy Wardout is lutation
	A CONTRACT OF A CO

ivneet.c PAGE NO. 15 108110 · 25150:3TA DATE: 67 08110 accepted Before establishing various properties of equivalence relation us will first le hes in Aherhood show that in the above define all the is taken three conditions are essential and ho 1 we decide twe of them can imply the third. Lefinition This become clear from the following esamples. A villaria surviver in ing type (i) Gramphe & on page 12 shows in Ther y y s and that a relation can be reflexive symetric but not transitive. This with . whenever reflexinity & symetry -> transitivity ering 2 ii) If we consider the relation oth x & x Tis a factor of "lathen it becomes reflexive, transitive but not symetric ses a xRoc. This is in figure to and the und the replexing & transitivity & symetry rescine at (ii) The above too concerced usadily accepted but many students (and from some teachers (anthors) feel that the nstablish condition of reflexibly is a redundancy. d relation It is really not necessary ; it is follows from symetry & transitivity together, 2 AN is said to be that it is only mentioned in the defn. ica of equivalence relation on a formality just on the closure properly is mentioned in  $\sim 4$ the define of a group This thinking is 14.1.5 based on the following logic and 6.55 د د دمو ک Let R. be a Symetric & transitive relation ~ 1  $= \sqrt{2\pi r_{\rm s}} = 1.517$ Congr Congratulationst Take pride. You are using a Navneet ecobuddy Spröduct (1510/31000

21 CHERN PAGE NO. 16 180 15: STAC DATE: 67108110 with a set A ..... the non xRy - yRoc (by symetry As the relation is transitine we get 17 . I. M. in school and replexivity. Frank But this arguement is wrong. It only shows that in a symetric & transitin relation if some an elament is related The some Feloment of the set is related to itself. But if plandter show that might an etement not related it any other dement then - will it be selated to itself? The anne is on over necessarily, This suggests the construction of the following the eparaple surtices is Let A = 4 2, 7, 2 } Let R= 1 (22, 22) (2, 2) (7, 2) (7, 7) supporte reader is asked to verify that constitution is symetric & transitive Part is it replexive? No. Because · rijoh (8,2) ER D At SEA and yet Thur synatry & transitisty ai bander and in Der the for seffering 21 - These examples show that all the three Conditions developferinity symetry and mittained in the deformer sequined in the defor Congratulationships and and are using an and a cobundar the address and a cobundar the address and a construction of a c

16 FAGE NO. PAGE NO. 1 20 160: 3TAG DATE: 07108110 The equivalence selation direction synetry The concept of equivalence selation has it's roots in the concept of equality. Irrespective of the nature of objects we certain we get have in our wind that ng. It - y) 2 = 2 for any 2 x ic & tranitive i) x=Y => Y=2 for all x, Y is related and iii) x= y and y= z => x= z for all x, y, z. then it There properties are generalised and are haved as reflexive, symetric and 1- does - hamitire sespectively This cleanly ent Ir vent then indicates that equality is indeed an ? The example of equivalence relation. This Some other examples following i) Let I denotes that set of all integers ·X· i.e. I = 2 -3 -2 -3 +2 -----We fix some positive integer say 5 Once fixed this will not be changed now. In I we define a relation on follows --y that Form x, y EI wersay that xRy withine ! if 5/ yex. This is used as "fine Because divides y-x" meaning that y-x APOE is divisible by 5 i.e. y-x has ¢ R A factor 5 or yig = 20= 5, m, for some mEI We will first that this is an equivalence the three Selation in Iran donne mor 225 32 netry ang a) detra , el Fre and han a state Ke defri of Their 12, -27 = 0 = -9.5 + 0.000 01 and xRix which mean R is 1 - 5/3 - 2ronoa Congratulationsl Take pulder of our are using a Navneet ecobaid by Biproductis luis 1000

CI LON BOAR PAGE NO. Ð DATE: 0.71 0 81 DATE:071081 10 reflexiverations substantings set 6) Let DC, Y. F. I mpsith DCRY and the Lipe to to 5 / 9 2 3 M millions property of the second of the second Idanaas Zu ···· y-x= 5-m for some mET x-y= 5. (-m) Where - m EI × 3-12-4 Samp in the the We way from the 200 pater marine y Rocard as deriver when is Richburg Symetricities and provide Alet and y to en with serving yRz maked دال ال 5/ - - - and 5/2- - -1735 gunned tene 10 Y-317.5 m, and z-y=54 cropetr 11. Where 2 marsh Establish I to fi . . - Adding these two legns! X-2)+ (2-9)=5m+5n .... The changed now March Jrxed Have (mtm) & saisting if (mtm) 9-1 · T A Now mot not I When Manie I " x - Y - abis  $V = \neg A A$ x-Y, in xRZ (A) 2015 15 dovis166 Is my marst Raise trainitive to a ration in nodecut her we is an regulation ce relation We are very much (and mainly) Interested in the brotherhood selation We investigate this indetail 25 and xP2 Which incluse 57 St - X Congratulationsk Take pride i doutater using a Navneet acobuddy W product is supprod

21 PAGE NO. PAGE NO. 19 180.15 S. 3TAG 3 DATE:08108110 For the time being we restrict ourselve to the set of all male human beings mindet M= fall male humans & Let R mean " is a brother of " 11 - mersty Since we are analyzing this relation 1.28 now strictly Mathematically it necessary for us to define the term الله بين مالا مركن الله بين مالا مركن "brother' formally, As mentioned earlier we define Defty a is said to be a brother of y tifme za her make and both sed ay 5 acg simpler have same passinte a) Let n 2 GAM2 ( ) ) hand and a mis on male mound the with x and x share same parents A. T 2 R x mand R- is reflexive D Let x, y EM B. t. xRy Now x & y are males and x and retroit have same parents - Henre Y is a male and young & have save parents in a - . . . . Re and Ris symetric C. S. Same O Let 2, 4, 7 EM & t xRy and YRz. Here x, y, Z ase all males. xaudy have same parents) and (y and z have same parents) x and 7 have same parent ~Rz aud R is transitive Congratulationst Take pride: Your amusing a Navneet ecobudity "product iniuration of

19 108 110 PAGE NO. 20 11 SUIPO : atta DATE 081087 10 - owselve This Ris an equivalence relation 09/05/10 Of course, for the sales of asgnement, one bernon mz may say that if in the definition of 'brother' we introduce a condition that of 17 elations x + y then reflexisity con be impossible. i.e. If we say that x is a brither of 7 - 75 if x is a male, x = y, and both aly the term ed earlier have same parents then brotherhood will not be replexive (and of come it will no-Thes of y be transitive also) But 9 think it will be more natural och they to anoth reflexivity than trying to semone it by putting-sach additional condition. The reason for mysthinking on these lines lies rte in the effect of equivalence selation on a set and the effect of a partition in 9 Set we proceed to study this in detail. Equivalence classes off a set . of Let R be an equivalence selation in o x and y a set Suchet a es. and x Then & the set of all etoments of & S Thick aso solated to any is called as metric the equivalence class of a in Sunder R. - It is dented an Eat (readras 14.2 class a) Thus have a Ray. IAT = JX ESTXRay rud or La J= JxES / a Rx } (Because of synctory arents) ent - Charles - ARand from hor clearly IgJ C tive. engratuisiuis via temperate via via temperate and temperat

www.navnaet.co OF 3645 21 PAGE NO. VAN & GETAG DATE: 09108110 In the servariable - Xor on page 17 . 8 M & T Ame 1. Consider OF I withen 796 2 'n. 25 Cod (1, 1)(DO] = JYEI  $\infty R$ 25 ° 194 ð ¥. V Stervis 12.3 . : d` DEFI in the second second **-** 5 0  $-\infty$ 5 (5A)1 Y Ő жe I 5 < i.) a 1 15 ي ژ  $\sqrt{2}$ الم بالمجاهد والمواصد 1 Bart ZEI have a) is a mutiple  $\mathcal{X}$ 124 Ē 5. O. t. S. A 22. LAN 34 34 TO Emile hatroal 5,0,5,10215, 35 50 -/0 2-15 20 weren it millarly 3 ft المجلولية ما ال when a selfering the 200 i 🖄 Did the Asso 4 19 6 9 11 16 6767 <u>c</u> 1 The rea this with 6 Alle 3 Equivaleuploxita how See ø <u>\_</u>\_\_\_\_ CC V mary hete Fect. \$ \* 12573 properties bortan eterni, M Clay en eguira dan is never en 2. . . ewhyty ٠, 4.14  $\mathcal{A} \supset \hat{\mathcal{N}}^{+}$  $C_{2}$ とりれた llow di imediately AS Si 23.1 Mis na requirateous relation of . ; <u>s</u> reflex) Hem Hem for any accuse must 12.000 ñ a Ka 2  $\sim 10$ hame Ó х Ias **2**%: Thus elemen the ways al belone 52.53 1 estriva 9 mar not W. mar etoments name X  $\overline{\mathcal{D}}$  $\mathcal{L}$   $\mathfrak{q}$ Sec vor by - **6** - 147 Congratulationsl Take pride: You are using a Navneet ecobuddy & product its lutarpood

0 PACE NO. PAGE NO. 21 1108110 120 123 : 349 DATE:09108110 cestainly have a it will Fine Property 2 Any no equivalence classes are either disjoint or equal . 20 23 S Proof: - Let R be an equivalence relation · d' în S. -1° 3 Let a, b ES and consider I a] & Ib] 1. M. 1. C. 1. obviomly [9] ( [6] - \$ or [9] ( [5] + \$ Conij' If IAT AIST = of them the nuttiple two classes are disjoint and there is nothing to prove . 20 --anii) If EgINT 57 + g then there must exist at least one xES 11; 16 --B-t. XELAINE67 · see [a7 and osce [6] Aler 20 Ra and RRb (by defnig ee how classes a Ric and Rib (By symetry ster by transitinity aRb 24 Now for any yETAT is never -we have y Rains R Bint since aRb is already astablishe AS AS we get yRa & aRb + 15 we must Di insupple (by transitish \$ > · YELGT ‡ \$ . . . . . · . YESA] => YEE5 belone C Ib [a] may Stor. and s This result is closingly from the statemen nto but े । Congratulationsl Take pride: You amusing a Navneet ecobuddy % product relations) ປລາວກ່ວນ

يد :een مرا مر 23 CM BEAG PAGE NO. 180100:300 DATE: 09 087 10 ARD DOMENT CLASS 1873 But by symetry app abo gives m. bRa Herne we can show that i da da ta ta ta [6] C [a If it is needed to show this explicitly Į., then NO LE YELD > YRL • Trong But bRg is given YRB and on ٤. (by framitiril SAX  $(\gamma)$ ELAT 1. YELD YELA]. a. 14 . 63 Eblor Eat i) mege curs -<u>E67</u> conjusteme da FL7 cither their are disjoint or equaland do at all this can be described as also m two espinaleure claner either have hopping in common or ham everything in common for an equivalence relation Rins may contruct the set of all we distincts equivalence clanes of No way a under R and the suck a set as Congratulationshilake prider doutarousing a Navneet ecobuiday & products stategee?

ی رو 24 OM BOAS PAGE NO. 18.0 180 BM DATE: 091 081 10 ( Rozificaj this LAT AT ac Store ocursonly once FR and studies nes the membress of Then no egnal are heme in property they an be disjoint. wit ex phicstely Consider La7 الأرور في في مراجع 1-3 1. A.  $c^{c}$ Faje 105 - 1 1 trat Rife altifal (d or => Deel Some CARE INELL :Ko 0.554 clanes [a] a Equally bage 100 SARE SAT h Xar tonit the set Ea] JE Eat Edje P The lanes Ferer as clanes) such that Subsets new are disjoint and all together two faronrite language gine S. To m 21 m among Mathematicians these classes er ther and the second are smithally desposing and collectively han ganstin is Broto sub a splitting set in different motually disjond ins <u>a</u> subaretson is has called - q partition all Hene when the deal the day >  $\sqrt{}$ 160 phing det [ Si be the tron femply as uerono Congratulations/Take/pride:d/gpace using a planaest ecoburity Psprickschitsiutsigno0

PAGE NO. 25 24 PAGE NO. DATE: OQI OSTI DATE: 09108110 set P & subsets of Salts said to be a pastition in of S if Brep ep au 1.5 8 6 => AOB = B Cierta 2 A same ari the devent of the called as the members of partities With this defn and the earlier discuss 1 mon vest conclude that an equaleur selation we in Son introduces ta partition in S where in equivalence clases are The members the partition on page 17 The reader × example must have found tomat 3 are here we get only fine schietingt clanes havely Fold <del>15-10-5,0,5,10,15</del> <u>Ta</u>T 4-1, 6, 11, 16 16 59 2,7,12,17and TTS THAT! 4.9, 14, 19, -5 ATTX ் ே -personal found there we will 100 Drishe slow same mesil atagin au diagain as Form ્ર જે ગ Ω. to) The and 21 roztom to 2 avit- ar B. in duced pardition 1) introduced in Tis Henry the 的了了了 7-10-27.5 Congratulations/ Take Printer date using a plannest ecoburity "tom date bisluterone O

26 VIC ON YOUR PAGE NO. 08110 V 2 O M STARK: DATE: 11 108110 Et ~ Inst an an equivalence solution induces a partition it is equally time that a <u>i je</u> partition induces an equivalence selation in such a manner that the members ياتي ا of the partition become the equivalence classes. Here is the proof of this. + J partition 2 discussion Let D be a partition of a non-engly selation set J. Hence S is split into different montually S where in disjoint subsets which are members of rembers  $\mathcal{P}^{\widetilde{a}}$  denotes the state of the set Define a relation R in S as follows. eader For 26, YES we say that we xRy iff EbAth x & y belong to , havely same member of the paytition. ; - - -? then get We a) for any sees, , since S - MAROT AED 3-18 -- - 7 2CEA for some AEP. 19-5-1 ۱. ا But then see A and x = A (+=> +Ap www. 17 get A of P. Nº as ... R is reflexive. 32. obirn I is x, yES such that Ry By The dept of R. this means tool there 43 4 Congra Congratulationsl/Take pride: You are using a Navneet ecobuddy product talutageo

PAGE NO. 27  $\mathcal{O}$ OH BEAS DATE OF IND STAC DATE: 11 108-1 10 is some A & P such that bach 22  $Y \in A$ i.e. x c A and y c A -5 **(** , 5) - 5 YEA and ZEA (PA90) 914 673314 y 2 2 belong to same men namely A of P y Rx by defn Anathy is symetric Jaw Marin - OLET x, Y, Z E S such that z Ry & y Rz XRY = = AEP such the x e A and y e A 2-2- all al e 20 Z 4 1 1 6-1-3 6. 2 St JAN YRZ BOJERBER Such that 1. Q. emore B and ZEB M. Contraction sitten of and Pisa partition B or ANB= \$ YEA and YEB Brit YEADB Star 1 ABJØ • •  $\mathcal{P}$ Calling B as A now we can write YGA and YEA and ZEA …<sup>1</sup>≯−¢ کړ نړ xef  $\underline{x}, \underline{y}, \underline{z} \in$ particular 2 & 2 belong to same meurbier A. marin 2RZ المشينة. منه ي 1.97 fransitive Ìς Congratulationstidate projection are using a Navneet ecobuldoy "oproduct is loter projection

27	[	PAGE NO. 28
108110		DATE: // 1087 /0
Г bAb	p	Ris this an equivalence solution in
		Sinduced by the partition p.
		If we consider any element a of S
2 = 91 /		then as ~ S= UA we will
member	5 ela	AEP ALAN
		get some A in P s. t. GEA
	· · · · · · · · · · · · · · · · · · ·	
		Now by EaJ mean the equivalence
		class of a minus minder R.
eRy & yRz		Eag = 12xeS/2cRay
that		and the second second
<u>A-</u>		As aFA., xRa will mean xFA.
		Herine
that	<u>, 1. 18 % (</u>	$Ta] = \frac{1}{2} \frac{z}{z} \frac{z}{z$
≘ B .		when you the time and the manual
2 portition		i.e. [a] = A M ( down
	and a start of	Thus this equivalence selation induced
	- Jan La Carl	by the partition P is such that it's
		equivalence classes are exactly the
· · · · · · · · · · · · · · · · · · ·	¥.	members of the partition -
		We have seen now that with an egindence
an write		solation there corresponds a partition
4 and ZEA		and with a partition there corresponds
	<u></u>	an equivalence relation such that
	<u> </u>	equivalence clanes are same as the
same		members of the partition Meme we can
	<u>}</u>	say that an equivalence selation and
	<u>v : 30</u>	partition are but so two sides of the
		same coin.
lengriso (		Congratulationsi Take pridet Yourare using a Navneet ecobuiddy #productisiurang no D

savaga.c ON BOAS PAGE NO. DATE: 11/08/10 PATE: 11 1 0ST ai we they our attention once again to the set of male human beings ite M = Lall male humans 6 7 1 ARA PULA MAN ARATINGON in MURUch MA geto 9 12 22 12/08/10 Supose we introduce a split in this set by saying "all brothers should form their groups in The set M will be broken into different subsets, such that each subject will composise of men who are And brothers of each other. If al costain male humanbeing is the only son of his parents them he will form a single for subset in this split. These different subsets of M is would be mutually session disjoint and wif the union of all of them The taken the we will get our M back But this means that such a split forms a partition of M. As we have seen earlier a partition induces an equivalence relation So this part tion 2000 gran will also give in an equivalence Selation in M such that the equivalence classes will be the members of the 12) partition. In this equivalence relation propries equivalence class will be comprised 21 grannen wither are brothing of each other CongratulationskTake pride. You are using a Navneet ecologida WpToduct state and

1avneet.com		PAGE NO. 30
29 108/10		DATE: 12-108-110
e Jagain		Since the members Jone quivalence class
zings.	· 3.	are all related to each other this
		relation that we get is nothing but
		the brotherhood relation! The very fact
Marshal		that we are getting this as an equivalence
2		relation anarcon indicates that it is
, this set		reflexive. If a person & is the only
form their		Son of his parents ( thus forming a singleton
ken into -		set in this perpition) then also we get
zach		$\infty Rz$
who are	· · · · · · · · · · · · · · · · · · ·	In view of all that is said so far about
costain		equivalence relation and particition it
on of his		is obvious that the brotherhood selection
sight for		should be accepted as reflexive.
hifterent		Extension to all humanbeings.
nally	<u></u>	Massage and Anader A. Musicon to
all of them		The difference between the words
ns M back/		"brother", and 'sister' is only of gender.
split	an an Anna Anna Anna Anna Anna Anna Anna	A The remaining part of the condition
2 have		the of having same parents is common
Inces		to bak of them. Here to it is
is part tion		sngested that this gender difference should
iralence		be abondoned. In many instances we
equivalence		do this. & for example, when we talk
is of the	1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	about 'international brotherhood' or
relation		when we say that the peaceful progress
comprised	بالمراجع والمراجع	of mankind is possible if we behave like
each other.		brothers irrespective of our race seligion or
Signe		Congratulationst Take pride You are using a Navrieet ecobuddy sproduct is tutation of
		and a serie france affe and an an an and a france and an at the analysis of the analysis of the second s

ъč. Cas Revis 31 PAGE NO. N.Q. 5 N. 27 344 DATE: 12-108110 or nationality do we exclude all female humanbeing from our consideration? 4 N Certainly nof 1:00 Neglecting this grender difference we identify the words brother Car and sister to mean Samething . when this is done it is no longer he complete argnemen 动物的 only to con s No in Series humanbeings. 10 This 1f & is any human being · < 'is the brother of and R means conclude then we The Rocard Marsh W is a brAler of 2 ૼૺૺ૾ૺ or x is reflexive S OF AS <u>\_R</u> is no intention whatsoeve Now there 0810 14 - connecting Mathematics & Meta-Physics Nor I am trying to analyse in Ssimad Bhogwadgee ranything said such an attempty will be beyond rol shirt e an when in the my capacity. Yet concluding part of this epilonge my hundle opinion that mentioned ala Xulta Sa brother relation should be 3 315 auchter and seflexive, I was d of a Simillar statement Securin do 18.3 Congratulationst Take procedure are using a Navneet ecobudity productive lusing a

PAGE NO. 32 N BEA -108110 DATE: 14108110 : BTAO in Spinad Bhagwadgeda somewhere. 9 de all verdesation ? seasched and found that in chapter 6 2 - 2 - <u>2</u> Stanza 5 Lord Krishna has said to ne we Arjuna that rother ! आत्मेव ह्यात्मनो वंधुः / hing . meaning that 'we (and only we) are brother of ousselves ! longer Because of the word la comming only in zirrid ( this is a combination estend So for of 311 (1) the statement can be symbolically Noitten as follows. If R means 'x is a brother of y' being then for any men aly (XRX and (x = y =) x x y Since pAq => p ad always the above statement certainly given in whatsoever oc R oc Let me make this clear that the reta-Physics context of the above statement lyse <u>ا</u>ن ogwadgeela quite different. It is cited here to only because the similarity was <u>se beyond</u> falt interesting. the // Broom Sold con Sein arty 11 longe inion that (Shriksishnaspanmastu) Meaning that whatever is here worthen med done be offered to Lord krishna. 10 e and Ren !! patement-នោទ្ធគេេំ Congratulations/Take pride/You are using so Navneet ecobaddy #product/islutang.no.0