A Proof of the Beal's Conjecture

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Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still both unproved and un-negated a conjecture hitherto.

Abstract

First we classify A, B and C according to their respective odevity, and ret rid of two kinds from $A^X+B^Y=C^Z$. Then affirm $A^X+B^Y=C^Z$ in which case A, B and C have a common prime factor by concrete examples. After that, prove $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by the mathematical induction with the aid of the symmetric law of odd numbers after the decomposition of the inequality. Finally, we have proven that the Beal's conjecture holds water after the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

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The Proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider the limits of values of above-mentioned A, B, C, X, Y and Z as given requirements for hinder concerned equalities and inequalities.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C, all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, so there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number, yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

Thus we continue to have merely two kinds of $A^X+B^Y=C^Z$ under the given requirements, as listed below.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification, in fact, it has many sets of solutions with A, B and C which are positive integers. Let us instance two concrete equations respectively to explain two such propositions below.

When A, B and C, all are positive even numbers, if let A=B=C=2, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $2^3+2^3=2^4$. Evidently $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 2, 2 and 2, and A, B and C have common even prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $162^3+162^3=54^4$. Evidently $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 162, 162 and 54, and A, B and C have two common prime factors, i.e. even 2 and odd 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3, and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $3^3+6^3=3^5$. Manifestly $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 3, 6 and 3, and A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7, and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $7^6+7^7=98^3$. Manifestly $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 7, 7 and 98, and A, B and C have common prime factor 7.

Consequently indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification does hold water, but A, B and C must have at least one common prime factor.

By now, if we can prove that there is only $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then we proved completely the conjecture.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not any common prime factor can only occur under the prerequisite that A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not any common prime factor, then any two of them have not any common prime factor either, because if any two have a common prime factor, namely A^X+B^Y or C^Z-A^X or C^Z-B^Y have a common prime factor, yet another has not the prime factor, then it would lead to $A^X+B^Y \neq C^Z$ or $C^Z-A^X \neq B^Y$ or $C^Z-B^Y \neq A^X$ surely according to the unique factorization theorem of natural number.

Since it is so, if we can prove $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then the Beal's conjecture is surely tenable, otherwise it will be negated.

Unquestionably, let following two inequalities add together, are able to replace completely $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C are two positive odd numbers and a positive even number without a common prime factor.

1. $A^X+B^Y \neq 2^Z G^Z$ under the given requirements plus the qualifications that

A and B are two positive odd numbers, and A, B and 2G have not any common prime factor.

2. $A^X+2^YD^Y \neq C^Z$ under the given requirements plus the qualifications that A and C are two positive odd numbers, and A, C and 2D have not any common prime factor.

For $A^X+B^Y \neq 2^ZG^Z$, when G=1, it is exactly $A^X+B^Y \neq 2^Z$. When G>1: if G is a positive odd number, then the inequality changes not, namely it is still $A^X+B^Y \neq 2^ZG^Z$; if G is a positive even number, then either the inequality can express as $A^X+B^Y \neq 2^W$, or it can express as $A^X+B^Y \neq 2^WH^Z$, where W=Z+NZ, N≥1, and H is an odd number ≥3.

Undoubtedly $A^X+B^Y \neq 2^W$ can represent $A^X+B^Y \neq 2^Z$, and $A^X+B^Y \neq 2^WH^Z$ can represent $A^X+B^Y \neq 2^ZG^Z$, where W=Z+NZ, N≥1, and H is an odd number ≥3. So express $A^X+B^Y \neq 2^ZG^Z$ into two inequalities as the follows.

(1) $A^X+B^Y \neq 2^W$, where A and B are positive odd numbers without a common prime factor, and X, Y and W are integers ≥3.

(2) $A^X+B^Y \neq 2^W H^Z$, where A, B and H are positive odd numbers without a common prime factor, X, Y and Z are integers ≥3, W=Z+NZ, N≥1, H ≥3.

For $A^X+2^YD^Y \neq C^Z$, when D=1, it is exactly $A^X+2^Y \neq C^Z$. When D>1: if D is a positive odd number, then the inequality changes not, namely it is still $A^X+2^YD^Y \neq C^Z$; if D is a positive even number, then either the inequality can express as $A^X+2^W \neq C^Z$, or can express as $A^X+2^WR^Y \neq C^Z$, where

$W=Y+NY$, $N\geq 1$, and R is an odd number ≥ 3.

Undoubtedly $A^X+2^W\neq C^Z$ can represent $A^X+2^Y\neq C^Z$, and $A^X+2^WR^Y\neq C^Z$ can represent $A^X+2^YD^Y\neq C^Z$, where $W=Y+NY$, $N\geq 1$, and R is an odd number ≥ 3. So express $A^X+2^YD^Y\neq C^Z$ into two inequalities as the follows.

(3) $A^X+2^W\neq C^Z$, where A and C are positive odd numbers without a common prime factor, and X, W and Z are integers ≥ 3.

(4) $A^X+2^WR^Y\neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, X, Y and Z are integers ≥ 3, $W=Y+NY$, $N\geq 1$, $R\geq 3$.

We regard values of A, B, C, H, R, X, Y, Z and W in aforementioned four inequalities, added to their co-prime relation in each inequalities as known requirements for hinder concerned inequalities and equalities.

So proving $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the above-listed four inequalities under the known requirements.

Before prove four such inequalities, we must expound the circumstances relating to these proofs, so as to understand these proofs easier.

Let us first divide all positive odd numbers into two kinds of A plus B, namely the form of A is $1+4n$, and the form of B is $3+4n$, where $n\geq 0$.

Odd numbers of A plus B from small to great arrange respectively below.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Then again divide all odd numbers of A into two kinds, i.e. A_1 and A_2, and again divide all odd numbers of B into two kinds, i.e. B_1 and B_2.

Or rather, the form of A_1 is 1+8n; the form of B_1 is 3+8n; the form of A_2 is 5+8n, and the form of B_2 is 7+8n, where n≥0.

Four kinds of odd numbers are arranged as follows respectively.

A_1: 1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105…1+8n …

B_1: 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107…3+8n …

A_2: 5, 13, 21, 29, 37, 45, 53, 61, 69, 77, 85, 93, 101, 109…5+8n …

B_2: 7, 15, 23, 31, 39, 47, 55, 63, 71, 79, 87, 95, 103, 111…7+8n …

We list from small to great seriate positive odd numbers and label a belongingness of each of them alongside, then you would discover that permutations of four kinds of odd numbers are possessed of a certain law.

1^k, A_1; 3, B_1; 5, A_2; 7, B_2; (2^3);9, A_1; 11, B_1; 13, A_2; 15, B_2; (2^4);

17, A_1; 19, B_1; 21, A_2; 23, B_2; 25, A_1; 3^3, B_1; 29, A_2; 31, B_2; (2^5);

33, A_1; 35, B_1; 37, A_2; 39, B_2; 41, A_1; 43, B_1; 45, A_2; 47, B_2;

49, A_1; 51, B_1; 53, A_2; 55, B_2; 57, A_1; 59, B_1; 61, A_2; 63, B_2; (2^6);

65, A_1; 67, B_1; 69, A_2; 71, B_2; 73, A_1; 75, B_1; 77, A_2; 79, B_2;

3^4, A_1; 83, B_1; 85, A_2; 87, B_2; 89, A_1; 91, B_1; 93, A_2; 95, B_2;

97, A_1; 99, B_1; 101, A_2; 103, B_2; 105, A_1; 107, B_1; 109, A_2; 111, B_2;

113, A_1; 115, B_1; 117, A_2; 119, B_2; 121, A_1; 123, B_1; 5^3, A_2; 127, B_2; (2^7);

129, A_1; 131, B_1; 133, A_2; 135, B_2; 137, A_1; 139, B_1; 141, A_2; 143, B_2;

145, A_1; 147, B_1; 149, A_2; 151, B_2; 153, A_1; 155, B_1; 157, A_2; 159, B_2;

161, A_1; 163, B_1; 165, A_2; 167, B_2; 169, A_1; 171, B_1; 173, A_2; 175, B_2;

177, A_1; 179, B_1; 181, A_2; 183, B_2; 185, A_1; 187, B_1; 189, A_2; 191, B_2;

193, A_1; 195, B_1; 197, A_2; 199, B_2; 201, A_1; 203, B_1; 205, A_2; 207, B_2;

209, A_1; 211, B_1; 213, A_2; 215, B_2; 217, A_1; 219, B_1; 221, A_2; 223, B_2;

225, A_1; 227, B_1; 229, A_2; 231, B_2; 233, A_1; 235, B_1; 237, A_2; 239, B_2;

241, A_1; 3^5, B_1; 245, A_2; 247, B_2; 249, A_1; 251, B_1; 253, A_2; 255, B_2; (2^8);

257, A_1; 259, B_1; 261, A_2; 263, B_2; 265, A_1; 267, B_1; 269, A_2; 271, B_2; …

From the above-listed sequence of odd numbers, we can see that permutations of seriate positive odd numbers from small to great are infinitely many cycles of $A_1B_1A_2B_2$.

To wit: $A_1B_1A_2B_2$ $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$…

By now, let us list seriate kinds of odd numbers which have a common odd base number, and label a belongingness of each of them alongside.

1^1, A_1;	3^1=3, B_1;	5^1=5, A_2;	7^1=7, B_2; (2^3);
1^2, A_1;	3^2=9, A_1;	5^2=25, A_1;	7^2=49, A_1;
1^3, A_1;	3^3=27, B_1;	5^3=125, A_2;	7^3=343, B_2;
1^4, A_1;	3^4=81, A_1;	5^4=625, A_1;	7^4=2481, A_1;
1^5, A_1;	3^5=243, B_1;	5^5=3125, A_2;	7^5=16807, B_2;
1^6, A_1;	3^6=729, A_1;	5^6=15625, A_1;	7^6=117609, A_1;
…	…	…	…
9^1=9, A_1;	11^1=11, B_1;	13^1=13, A_2;	15^1=15, B_2; (2^4);
9^2=81, A_1;	11^2=121, A_1;	13^2=169, A_1;	15^2=225, A_1;

9^3=729, A_1; 11^3=1331, B_1; 13^3=2197, A_2; 15^3=3375, B_2;

9^4=6561, A_1; 11^4=14641, A_1; 13^4=28561, A_1; 15^4=50625, A_1;

9^5=59049, A_1; 11^5=161051, B_1; 13^5=371293, A_2; 15^5=759375, B_2;

9^6=531441, A_1; 11^6=1771561, A_1; 13^6=4826809, A_1; 15^6=11390625, A_1;

… … … …

17^1=17, A_1; 19^1=19, B_1; 21^1=21, A_2; 23^1=23; B_2…

17^2=289, A_1; 19^2=361, A_1; 21^2=441, A_1; 23^2=529; A_1…

17^3=4193, A_1; 19^3=6859, B_1; 21^3=9261, A_2; 23^3=12167; B_2…

17^4=83521, A_1; 19^4=130321, A_1; 21^4=194481, A_1; 23^4=279841; A_1…

17^5=1419857, A_1; 19^5=2476099, B_1; 21^5=4084101, A_2; 23^5=6436343, B_2…

17^6=24137569,A_1;19^6=47045881,A_1;21^6=85766121, A_1;23^6=148035889, A_1..

… … … …

From above-listed kinds of odd numbers which have a common odd base number, we are not difficult to see, on the one hand, all odd numbers whereby A_1 to act as a base number belong still within A_1; All odd numbers whereby B_1 to act as a base number belong within B_1 plus A_1, and one of B_1 alternates with one of A_1; All odd numbers whereby A_2 to act as a base number belong within A_2 plus A_1, and one of A_2 alternates with one of A_1; And all odd numbers whereby B_2 to act as a base number belong within B_2 plus A_1, and one of B_2 alternates with one of A_1.

On the other hand, we classify them into set four kinds of odd numbers according to their respective belongingness, well then, all odd numbers of even exponents and odd numbers 1+8n of odd exponents belong within A_1; Odd numbers 3+8n of odd exponents belong within B_1; Odd numbers 5+8n of odd exponents belong within A_2; And odd numbers 7+8n of odd exponents belong within B_2, where $n \geq 0$.

Excepting common odd base number 1, two adjacent odd numbers which have a common odd base number >1 are an even number apart, but also such even numbers are getting greater and greater along which exponents of the adjacent odd numbers are getting greater and greater.

At all events, whether odd numbers of odd exponents or odd numbers of even exponents, all of them are included and dispersed within aforementioned four kinds of odd numbers, thus they conform to the symmetric law of odd numbers we shall define later.

We add 2^{W-1}, 2^W, $2^{W-1}H^Z$ and 2^WH^Z among the sequence of odd numbers, and regard each of them as a center of symmetry of odd numbers. Well then, odd numbers on the left side of the center and partial odd numbers on the right side of the center are one-to-one bilateral symmetries. For example, regard 2^{W-1} as a symmetric center, then $2^{W-1}-1 \in B_2$ and $2^{W-1}+1 \in A_1$, $2^{W-1}-3 \in A_2$ and $2^{W-1}+3 \in B_1$, $2^{W-1}-5 \in B_1$ and $2^{W-1}+5 \in A_2$, $2^{W-1}-7 \in A_1$ and $2^{W-1}+7 \in B_2$ etc are one-to-one bilateral symmetry. See also their symmetric permutation as follows.

$A_1B_1A_2B_2 \ldots A_1B_1A_2B_2\,(2^{W-1})\,A_1B_1A_2B_2A_1B_1A_2B_2 \ldots A_1B_1A_2B_2$

We consider such symmetric permutations of odd numbers for symmetric center $2^{W-1}H^Z$ as a symmetric law of odd numbers at the sequence of natural numbers, or as the symmetric law of odd numbers for short, where W, Z and H are positive integers, W≥3, and Z≥3.

Pursuant to preceding basic concepts, we set to prove aforementioned four inequalities, one by one. Of course, what we need first is to prove $A^X+B^Y\neq 2^W$ under the known requirements.

Firstly, Prove $A^X+B^Y\neq 2^W$ under the known requirements.

After regard 2^{W-1} as a symmetric center, leave from 2^{W-1}, both there are finitely many cycles of $B_2A_2B_1A_1$ leftwards until (B_2=7, A_2=5, B_1=3, A_1=1), and there are infinitely many cycles of $A_1B_1A_2B_2$ rightwards.

According to the symmetric law of odd numbers, two distances from the symmetric center to symmetric two odd numbers are equal length.

Consequently, on the one hand, a sum of every two symmetric odd numbers is equal to the numerical double of the symmetric center. On the other hand, a sum of any two non-symmetric odd numbers is unequal to the numerical double of the symmetric center absolutely.

Moreover, odd numbers on an identical distance which departs from 2^{W-1} on the either side of 2^{W-1}, all belong to a kind and the same, where W-1 is equal to each and every integer ≥ 3.

A and B in A+B $=2^W$ are bilateral symmetric odd numbers whereby 2^{W-1} to act as the center of the symmetry, yet here so-called bilateral symmetric odd numbers A and B, what concretely pointed is four kinds, i.e. B_2 and A_1 where $B_2<A_1$; A_2 and B_1 where $A_2<B_1$; B_1 and A_2 where $B_1<A_2$; and A_1 and B_2 where $A_1<B_2$.

Besides before making the proof, we give a stipulation that for an integer, if its exponent is greater than or equal to 3, then the integer is called an integer of the greater exponent; if its exponent is equal to 1 or 2, then the integer is called an integer of the smaller exponent, thereinafter.

By now, we set to prove $A^X+B^Y\neq 2^W$ under the known requirements by the mathematical induction.

(1) When W-1=3, each other's symmetric odd numbers on two sides of 2^3 are listed below.

1^3, 3, 5, 7, (2^3), 9, 11, 13, 15

To wit: $A_1B_1A_2$ B_2 (2^3) A_1 B_1 A_2 B_2

It is clear at a glance, that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^3. So we get $A^X+B^Y\neq 2^4$.

When W-1=4, each other's symmetric odd numbers on two sides of 2^4 are listed below.

1^4, 3, 5, 7, 9, 11, 13, 15, (2^4) 17, 19, 21, 23, 25, 3^3, 29, 31

To wit: $A_1B_1A_2$ B_2 $A_1B_1A_2$ B_2 (2^4) $A_1B_1A_2$ B_2 $A_1B_1A_2$ B_2

Evidently there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^4. So we get $A^X+B^Y\neq 2^5$.

When W-1=5 and W-1=6, each other's symmetric odd numbers on two sides of 2^6 including 2^5 are listed below.

1^6, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

To wit: $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$ (2^5) $A_1B_1A_2B_2A_1B_1A_2B_2$ $A_1B_1A_2B_2$ $A_1B_1A_2$ B_2 (2^6) $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2$ B_2 $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$

Likewise there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^6 or 2^5. So we get $A^X+B^Y\neq2^7$ or $A^X+B^Y\neq2^6$.

(2) Suppose that when W-1=K and K≥6, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^K. So we get $A^X+B^Y\neq2^{K+1}$ under the known requirements, where K≥6.

(3) Prove that when W-1=K+1, there are not two odd numbers of the greater exponents altogether either on two odd places of every bilateral symmetry for symmetric center 2^{K+1}. Namely this needs us prove $A^X+B^Y\neq2^{K+2}$ under the known requirements.

Proof * We have known that permutations of odd numbers on two sides of 2^{W-1} including 2^K plus 2^{K+1} conform to the symmetric law of odd numbers, please, see odd numbers on two sides of 2^K and of 2^{K+1}:

$A_1B_1A_2B_2 \ldots B_1A_2B_2A_1B_1A_2B_2$ (2^K) $A_1B_1A_2B_2A_1B_1A_2 \ldots A_1B_1A_2B_2$

$A_1B_1A_2B_2 \ldots B_1A_2B_2A_1B_1A_2B_2$ (2^{K+1}) $A_1B_1A_2B_2A_1B_1A_2 \ldots A_1B_1A_2B_2$

Now that bilateral symmetric odd numbers on two sides of 2^{W-1} at the sequence of natural numbers for symmetric center 2^{W-1} are A and B, then each other's-symmetry's A_1 and B_2 away from 2^{W-1} are 1+8n; each other's- symmetry's B_1 and A_2 away from 2^{W-1} are 3+8n; each other's-symmetry's A_2 and B_1 away from 2^{W-1} are 5+8n; and each other's-symmetry's B_2 and A_1 away from 2^{W-1} are 7+8n, where $n \geq 0$.

There are same symmetric permutations concerning the four kinds of odd numbers for symmetric center 2^{W-1} and 2^{K+1}, where $W-1 \leq K$. Yet all odd numbers of bilateral symmetries for symmetric center 2^K are turned into all odd numbers on the left side of 2^{K+1}.

Thus for odd numbers of bilateral symmetries whereby 2^{K+1} to act as the center of symmetry, a half of them retains still original places, and the half lies on the left side of 2^{K+1}, while another half on the right side of 2^{K+1} is formed from 2^{K+1} plus each and every left odd number.

Suppose that A^X and B^Y are any a pair of bilateral symmetric odd numbers whereby 2^K to act as the center of the symmetry. Since there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^K according to second step of the mathematical induction, so we let A^X as an odd number of the greater exponent, and let B^Y as an odd number of the smaller exponent, i.e. let $X \geq 3$ and $Y<3$.

By now, let B^Y plus 2^{K+1} makes B^Y+2^{K+1}. Since A^X and B^Y are bilateral symmetric odd numbers for symmetric center 2^K, additionally 0 and 2^{K+1} are bilateral symmetry for symmetric center 2^K too, therefore the distance from B^Y to 2^{K+1} is equal to the distance from 0 to A^X, then the distance from A^X to 2^{K+1} is equal to B^Y due to $2^{K+1}-A^X=A^X\sim B^Y+B^Y\sim 2^{K+1}$ $=A^X\sim B^Y+A^X=B^Y$.

In addition, the distance from 2^{K+1} to B^Y+2^{K+1} is equal to B^Y due to $(B^Y+2^{K+1})-2^{K+1}=B^Y$.

Now that from A^X to 2^{K+1} is equal to B^Y, and from 2^{K+1} to B^Y+2^{K+1} is equal to B^Y too, so A^X and B^Y+2^{K+1} are bilateral symmetry for symmetric center 2^{K+1}, and thus get $A^X+ (B^Y+2^{K+1}) =2^{K+2}$, where $X\geq 3$ and $Y<3$.

After regard 2^{K+1} as the symmetric center, 0 and 2^{K+2} are bilateral symmetry, so A^X and $2^{K+2}-A^X$ are bilateral symmetry.

Now that A^X and B^Y+2^{K+1} are bilateral symmetry, and A^X and $2^{K+2}-A^X$ are bilateral symmetry for symmetric center 2^{K+1}, consequently, we get $B^Y+2^{K+1}=2^{K+2}-A^X$, where $X\geq 3$ and $Y<3$.

Please, see a simple illustration at the number axis as follows.

					A^X+2^{K+1}		B^Y+2^{K+1}	
1, 3...	A^X	2^K	B^Y	2^{K+1}	$2^{K+2}-B^Y$	3×2^K	$2^{K+2}-A^X$	2^{K+2}

Since there is $B^Y+2^{K+1}=2^{K+2}-A^X$ when $X\geq 3$ and $Y<3$, then B^Y+2^{K+1} when $X\geq 3$ and $Y\geq 3$ must lies on the right side of $2^{K+2}-A^X$ in which case B is a constant value. Without doubt B^Y+2^{K+1} where $Y\geq3$ is greater than B^Y+2^{K+1} where $Y<3$, of course B^Y+2^{K+1} where $Y\geq3$ is greater than $2^{K+2}-A^X$ too.

Since A^X where $X\geq3$ and B^Y+2^{K+1} where $Y<3$ are bilateral symmetry for symmetric center 2^{K+1}, and $A^X + (B^Y+2^{K+1}) =2^{K+2}$, then A^X where $X\geq3$ and B^Y+2^{K+1} where $Y\geq3$ are not bilateral symmetry for symmetric center 2^{K+1}, so $A^X + (B^Y+2^{K+1}) \neq 2^{K+2}$ according to the preceding conclusion got.

Since bilateral symmetric odd numbers for symmetric center 2^{K+1} are only A and B, but also $A + B=2^{K+2}$ and 2^{K+2} can only be equal to the sum which A plus B makes, thus $A^X+(B^Y+2^{K+1}) =2^{K+2}$ is exactly $A^X+B^Y=2^{K+2}$ where $X\geq3$ and $Y<3$, naturally $A^X+ (B^Y+2^{K+1}) \neq 2^{K+2}$ is exactly $A^X+B^Y \neq 2^{K+2}$, where $X\geq3$ and $Y\geq3$. Of course, A^X belongs within either A or B, yet (B^Y+2^{K+1}) belongs within remainder one.

In reality, we can also directly deduce $B^Y+2^{K+1}=2^{K+2}-A^X$ from $A^X+B^Y=2^{K+1}$ due to $(A^X+B^Y) +2^{K+1}-A^X= (2^{K+1}) +2^{K+1}-A^X$, i.e. $B^Y+2^{K+1}=2^{K+2}-A^X$, where $X\geq3$ and $Y<3$.

Since $A^X+B^Y=2^{K+1}$ where $X\geq3$ and $Y<3$, then $B^Y+2^{K+1}=A^X+2B^Y$, and $A^X+2B^Y=2^{K+1}+B^Y=2^{K+1}+2^{K+1}-A^X=2^{K+2}-A^X$, so there is $B^Y+2^{K+1}=2^{K+2}-A^X$, where $X\geq3$ and $Y<3$.

Since A^X and $2^{K+2}-A^X$ i.e. B^Y+2^{K+1} i.e. A^X+2B^Y are bilateral symmetric odd numbers for symmetric center 2^{K+1}, thus $A^X+[A^X+2B^Y] =2[A^X+B^Y] =2^{K+2}$, where $X\geq3$ and $Y<3$.

But then, there is $A^X+B^Y\neq2^{K+1}$ where $X \geq3$ and $Y \geq3$, thus $A^X + [A^X+2B^Y]$

$= 2[A^X+B^Y] \neq 2^{K+2}$ where $X \geq 3$ and $Y \geq 3$.

Since bilateral symmetric odd numbers for symmetric center 2^{K+1} are only A and B, but also $A+B=2^{K+2}$ and 2^{K+2} can only be equal to the sum which A plus B makes, thus $A^X+[A^X+2B^Y]=2^{K+2}$ is exactly $A^X+B^Y=2^{K+2}$ where $X\geq 3$ and $Y<3$, naturally $A^X+[A^X+2B^Y] \neq 2^{K+2}$ is exactly $A^X+B^Y \neq 2^{K+2}$ where $X \geq 3$ and $Y \geq 3$. Of course, A^X belongs within either A or B, yet A^X+2B^Y belongs within remainder one.

Consequently we have proven $A^X+B^Y \neq 2^{K+2}$ where $X \geq 3$ and $Y \geq 3$ on above-mentioned two aspects.

In other words, A^X and B^Y+2^{K+1} are bilateral symmetric odd numbers for symmetric center 2^{K+1}, but B^Y+2^{K+1} is an odd number of the smaller exponent, though A^X is still an odd number of the greater exponent.

If exchange evaluations between the exponent of B and the exponent of A, i.e. let $X<3$ and $Y\geq 3$, then getting a conclusion via the inference like the above is just the same with the preceding conclusion.

If A^X and B^Y are two odd numbers of the smaller exponents, then after either A^X or B^Y plus 2^{K+1} makes another odd number, whether another odd number has a greater exponent or has a smaller exponent, it and un-incremental one in A^X and B^Y are too bilateral symmetry for symmetric center 2^{K+1}, but however there are not two odd numbers of the greater exponents altogether on two odd places of the bilateral symmetry.

To sum up, we have proven that when $W-1=K+1$, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^{K+1}. That is to say, we have proven $A^X+B^Y\neq 2^{K+2}$ under the known requirements, and K≥6.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3, … up to every integer ≥ 2, there are $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$, … up to $A^X+B^Y\neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and here point out emphatically H≥3 in them.

By now, we set to prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements by the mathematical induction thereinafter.

(1) When H=1, $2^{W-1}H^Z$ to wit 2^{W-1}, we have proven $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section. Namely there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^{W-1} to act as the center of the symmetry.

(2) When H=J, $2^{W-1}H^Z$ to wit $2^{W-1}J^Z$, suppose $A^X+B^Y\neq 2^W J^Z$ under the known requirements, where J is an odd number ≥1. Namely suppose that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}J^Z$ to act as the center of the symmetry, where J is an odd number ≥1.

(3) When H=K, $2^{W-1}H^Z$ to wit $2^{W-1}K^Z$, prove $A^X+B^Y\neq 2^W K^Z$ under the known requirements, where K=J+2. Namely prove that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}K^Z$ to act as the center of the symmetry, where K=J+2.

Since odd numbers on the left side of $2^{W-1}J^Z$ and part odd numbers on the right side of $2^{W-1}J^Z$ are one-to-one bilateral symmetry for symmetric center $2^{W-1}J^Z$, and the sum of every two symmetric odd numbers is equal to $2^W J^Z$.

Moreover there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center $2^{W-1}J^Z$ according to step 2 of the mathematical induction.

Thus we suppose that A^X and B^Y are any a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$, then either get $A^X+B^Y=2^W J^Z$, where X<3 and Y≥3, or get $A^X+B^Y\neq 2^W J^Z$, where X≥3 and Y≥3.

Regard $2^{W-1}K^Z$ as a symmetric center, then 0 and $2^W K^Z$, B^Y and $2^W K^Z-B^Y$ are bilateral symmetry respectively, also get $B^Y+(2^W K^Z-B^Y)=2^W K^Z$.

By now, let A^X plus $2^W(K^Z-J^Z)$ makes $A^X+2^W(K^Z-J^Z)$, then $A^X+2^W(K^Z-J^Z)$ $=A^X+2^W K^Z-2^W J^Z=2^W K^Z-(2^W J^Z-A^X)=2^W K^Z-B^Y$ due to $A^X+B^Y=2^W J^Z$, where X<3 and Y≥3.

Now that $A^X+2^W(K^Z-J^Z)=2^W K^Z-B^Y$, also B^Y and $2^W K^Z-B^Y$ are bilateral symmetry, then B^Y and $A^X+2^W(K^Z-J^Z)$ are bilateral symmetry for symmetric center $2^{W-1}K^Z$ too, so we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^W K^Z$, where X<3 and Y≥3.

Since $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$, moreover has supposed $A^X+B^Y\neq 2^W J^Z$, so get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W K^Z-2^W J^Z\neq 2^W K^Z$, where $X \geq 3$ and $Y \geq 3$.

On the other, since B^Y and $A^X+2^W(K^Z-J^Z)$ where $X<3$ and $Y\geq3$ are bilateral symmetry for symmetric center $2^{W-1}K^Z$, then B^Y and $A^X+2^W(K^Z-J^Z)$ where $X\geq3$ and $Y\geq3$ are not bilateral symmetry for symmetric center $2^{W-1}K^Z$, so $B^Y+[A^X+2^W(K^Z-J^Z)] \neq 2^WK^Z$ according to the preceding conclusion got.

Since bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ are only A and B, but also $A + B=2^WK^Z$ and 2^WK^Z can only be equal to the sum which A plus B makes, thus $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ is exactly $A^X+B^Y=2^WK^Z$ where $X<3$ and $Y\geq3$, naturally $B^Y+[A^X+2^W(K^Z-J^Z)] \neq 2^WK^Z$ is exactly $A^X+B^Y\neq2^WK^Z$ too where $X \geq3$ and $Y \geq3$. Of course, B^Y belongs within either A or B, yet $A^X+2^W(K^Z-J^Z)$ belongs within remainder one.

Taken one with another, we have proven $A^X+B^Y\neq2^WK^Z$ under the known requirements, where $X \geq3$, $Y \geq3$ and $K=J+2$.

That is to say, B^Y and $A^X+2^W(K^Z-J^Z)$ are bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, but $A^X+2^W(K^Z-J^Z)$ is an odd number of the smaller exponent, though B^Y is an odd number of the greater exponent.

If exchange the evaluations between the exponent of B and the exponent of A, i.e. let $X\geq3$ and $Y<3$, then getting a conclusion via the inference like the above is just the same with the preceding conclusion.

If A^X and B^Y are two bilateral symmetric odd numbers of the smaller exponents for symmetric center $2^{W-1}J^Z$, well then, after either A^X or B^Y plus $2^{W-1}(K^Z-J^Z)$ makes another odd number, another odd number has either a greater exponent or a smaller exponent, it and un-incremental one in A^X and B^Y are bilateral symmetry for symmetric center $2^{W-1}K^Z$ too, but however there are not two odd numbers of the greater exponents altogether on two odd places of the bilateral symmetry.

To sum up, we have proven $A^X+B^Y \neq 2^W K^Z$ under the known requirements, where K=J+2. Namely when H=J+2, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}(J+2)^Z$ to act as the center of the symmetry.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6… up to every odd number ≥1, there are $A^X+B^Y \neq 2^W(J+4)^Z$, $A^X+B^Y \neq 2^W(J+6)^Z$… up to $A^X+B^Y \neq 2^W H^Z$ under the known requirements, and here point out emphatically H≥3 in them.

Thirdly, we shall proceed to prove $A^X+2^W \neq C^Z$ under the known requirements below.

Since we have proven $A^X+B^Y \neq 2^W$ under the known requirements, herefrom can affirm $E^P+C^Z \neq 2^M$, where E and C are positive odd numbers without a common prime factor, P, Z and M are integers ≥3.

Since E and C have not a common prime factor, then get $E^P \neq C^Z$ accord to the unique factorization theorem of natural number, so let $C^Z > E^P$.

Since there is $2^M = 2^{M-1}+2^{M-1}$, then we deduce $E^P+C^Z > 2^{M-1}+2^{M-1}$ or $E^P+C^Z < 2^{M-1}+2^{M-1}$ from $E^P+C^Z \neq 2^M$.

Namely there is $C^Z-2^{M-1} > 2^{M-1}-E^P$ or $C^Z-2^{M-1} < 2^{M-1}-E^P$.

Besides, $A^X+E^P\neq 2^{M-1}$ exists objectively according to proven $A^X+B^Y\neq 2^W$ under the known requirements, where A and E are positive odd numbers without a common prime factor, and X, P and M-1 are integers ≥3.

Thus we deduce $2^{M-1}-E^P>A^X$ or $2^{M-1}-E^P<A^X$ from $A^X+E^P\neq 2^{M-1}$.

Therefore there is $C^Z-2^{M-1}>2^{M-1}-E^P>A^X$ or $C^Z-2^{M-1}<2^{M-1}-E^P<A^X$.

Consequently there is $C^Z-2^{M-1}>A^X$ or $C^Z-2^{M-1}<A^X$.

In a word, there is $C^Z-2^{M-1}\neq A^X$, i.e. $A^X+2^{M-1}\neq C^Z$.

For $A^X+2^{M-1}\neq C^Z$, let $2^{M-1}=2^W$, we obtain $A^X+2^W\neq C^Z$ under the known requirements.

Fourthly, let us last prove $A^X+2^WR^Y\neq C^Z$ under the known requirements, and here point out emphatically R≥3 in them.

Since we have proven $A^X+B^Y\neq 2^WH^Z$ under the known requirements, of course can too get $F^S+C^Z\neq 2^NR^Y$, where F, C and R are positive odd numbers without a common prime factor, S, Z and Y are integers ≥3, N=Y+PY, P≥1, and R≥3.

Since F and C have not any common prime factor, so get $F^S\neq C^Z$ accord to the unique factorization theorem of natural number, and let $C^Z>F^S$.

Since $2^NR^Y=2^{N-1}R^Y+2^{N-1}R^Y$, then deduce $F^S+C^Z>2^{N-1}R^Y+2^{N-1}R^Y$ or $F^S+C^Z<2^{N-1}R^Y+2^{N-1}R^Y$ from $F^S+C^Z\neq 2^NR^Y$.

Namely there is $C^Z-2^{N-1}R^Y>2^{N-1}R^Y-F^S$ or $C^Z-2^{N-1}R^Y<2^{N-1}R^Y-F^S$.

In addition, according to proven $A^X+B^Y\neq 2^WH^Z$ under the known requirements, we can get $A^X+F^S\neq 2^{N-1}R^Y$, where A, F and R are positive odd numbers without a common prime factor, X, S and Y are integers ≥ 3, $N-1=Y+DY$, $D\geq1$, and $R\geq3$.

So we deduce $2^{N-1}R^Y-F^S > A^X$ or $2^{N-1}R^Y-F^S < A^X$ from $A^X+F^S \neq 2^{N-1}R^Y$.

Thus there is $C^Z-2^{N-1}R^Y > 2^{N-1}R^Y-F^S > A^X$ or $C^Z-2^{N-1}R^Y < 2^{N-1}R^Y-F^S < A^X$.

Consequently there is $C^Z-2^{N-1}R^Y > A^X$ or $C^Z-2^{N-1}R^Y < A^X$.

In a word, there is $C^Z-2^{N-1}R^Y \neq A^X$, i.e. $A^X+2^{N-1}R^Y \neq C^Z$.

For $A^X+2^{N-1}R^Y\neq C^Z$, let $2^{N-1}=2^W$, we obtain $A^X+2^WR^Y\neq C^Z$ under the known requirements, and here point out emphatically $R\geq3$ in them.

To sun up, we have proven every kind of $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition, previous, we have proven $A^X+B^Y=C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has certain sets of solutions with A, B and C which are positive integers.

After pass the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements, we have reached inevitably the conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal conjecture does hold water.