Induced Second Quantization

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Abstract

The notion of induced second quantization is introduced as an unavoidable aspect of the induction procedure for metric and spinor connection, which is the key element of TGD. Induced second quantization provides insights about the QFT limit, about generalizes Feynman diagrammatics, and about TGD counterpart of second quantization of strings which appear in TGD as emergent objects. Zero energy ontology (ZEO) naturally restricts the anti-commutation relations inside causal diamonds defining quantum coherence regions so that the counterintuitive implication that all identical particles of the Universe are in totally symmetric/antisymmetric state is avoided. The relation of statistics to negentropic entanglement and the new view about position measurement provided by ZEO are discussed.

1 Introduction

Theory building is continual fight against self-censorship. Some unpleasant question pops up again and again during morning hours but one manages to forget them before the morning coffee. Particle statistics and its relationship to Zero Energy Ontology (ZEO), causal diamonds (CDs), and negentropic entanglement (NE) involve this kind of suppressed questions. Also the relation between second quantized induced spinor fields and the modes of imbedding space spinor fields defining ground states of super-conformal representations and defining the fields at the QFT limit of the theory involves this kind of unpleasant questions. In the following I try to overcome the internal censorship and articulate the problems as clearly as possible and try to answer them.

The main new notion introduced below is that of induced second quantization. I should have introduced this notion for decades ago as an unavoidable aspect of the induction procedure for metric and spinor connection, which is the key element of TGD but for some strange reason remained unaware of this concept.

Induced second quantization provides insights about the QFT limit, about generalizes Feynman diagrammatics, and about TGD counterpart of second quantization of strings which appear in TGD as emergent objects. Zero energy ontology (ZEO) naturally restricts the anti-commutation relations inside causal diamonds defining quantum coherence regions so that the counterintuitive implication that all identical particles of the Universe are in totally symmetric/antisymmetric state is avoided. The relation of statistics to negentropic entanglement and the new view about position measurement provided by ZEO are discussed.

2 Localization of fermions at string world sheets and induced second quantization

The original picture about connection between WCW spinor geometry and fermionic statistics [K1, K4] was following.

1. At the level of WCW anti-commutation relations of complexified gamma matrices can be assigned as local anti-commutation relations for induced spinor fields for space-like 3-surface considered and by holography for space-time surface connecting the space-like 3-surfaces at the ends of causal diamond (CD). Standard local anti-commutations are expected to hold only for the either end of CD.
2. For given point of WCW (3-surface) WCW gamma matrices are linear combinations of fermionic oscillator operators. Ordinary gamma matrices are linear combinations of the same flat space gamma matrices and this should hold true now. Flat space gamma matrices would naturally correspond to oscillator operators assignable to the induced spinor fields.

The situation is not however so simple as it looks first.

1. In TGD fundamental fermions are localized at 2-D string world sheets from the well-definedness of em charge [K3]. Fundamental spinor fields are second quantized at string world sheets and imbedding space gamma matrices are expressible in terms of the oscillator operators so that statistics is geometrized.

   This raises questions. Should one assume that the oscillator operators assignable to different string world sheets anti-commute or do they anti-commute at the string associated with the boundaries of CD? What about induced spinors associated with disjoint space-time surfaces inside CD or associated with disjoint CDs? What determines the detailed anti-commutations for induced spinor fields?
2. At QFT limit one has QFT in imbedding space $H = M^4 \times CP_2$ or inside CD and in good approximation in M^4 should satisfactorily describe the physics predicted by TGD, at least at the limit when the size of CD becomes infinite. How the anti-commutation relations restricted to the string worlds sheets and their intersection points with partonic 2-surfaces can be consistent with QFT anti-commutation relations? Are the QFT anti-commutations over-generalization and should they restricted to quantum coherence regions for fields defined by space-time sheets at space-time level and CDs at imbedding space level in TGD framework? Should the stringy anti-commutations follow from something implying also QFT anti-commutations?

Notice that I restrict the consideration to anti-commutation relations because bosons and all elementary particles emerge in TGD framework from fundamental fermions.

2.1 Induced second quantization

The questions posed above obviously relate to what one means with second quantization of first quantized string theory.

1. The modes of massless imbedding space spinor fields define ground states of super-conformal representations. They also correspond naturally to modes of massless QFT in imbedding space expected to have something to do with QFT limit of TGD. Could second quantization of fermionic strings be somehow induced from second quantization of free imbedding space spinor fields?
2. If this the case, do the QFT anti-commutation relations hold true for free spinor fields in H (massless in 8-D sense) or just inside given CD? Are they consistent with the anti-commutation relations for the induced spinor fields? The restriction of anti-commutations of H spinor fields inside given CD is rather attractive since in this manner one would avoid the rather un-intuitive conclusion that all fermions of the Universe are entangled by statistics. This would hold true only at boundary of CD.

This raises the question what the induction for second quantized spinor field of imbedding space (or CD to get discrete basis) could mean.

1. Are the analogs of flat space gamma matrices for WCW expressible in terms of the oscillator operators of free "massless" spinor fields of H or CD (quarks and leptons corresponding to the two H-chiralities)? This would imply a generalization of induction procedure to the level of quantum fields. The oscillator operators for induced spinor fields would be expressible as linear combinations of H (CD) oscillator operators.

2. The formula is easy to guess. Generalize the induction procedure for metric and spinor connection from imbedding space to space-time surface to a procedure assigning to imbedding space spinor field induced spinor field at string world sheets.

   In other words, take the second quantized free massless spinor field in H or CD having expansion $\Psi = \sum a_n Psi_n$, where a_n is oscillator operator. Replace positive or negative energy part of Ψ_n (thus with a well-defined quark or lepton number) with its projection to the spinor basis Φ_α defined at string world sheets. Using Dirac's bra-ket notation very schematically one has

   $$\Psi_n \rightarrow \sum_\alpha \langle \Phi_\alpha, \Psi_n \rangle \Phi_\alpha \equiv \sum_\alpha c_{\alpha,n} \Psi_\alpha$$

   so that one has projection of quantized spinor field:

   $$\Psi \rightarrow \Phi = \sum_\alpha a_\alpha \Phi_\alpha \quad , \quad a_\alpha = \sum_n \langle \Phi_\alpha, \Psi_n \rangle a_n \quad .$$

   One can think that either spinor modes of H are projected to the string world sheet or that the oscillator operator of string world sheet are expressed in terms of imbedding space oscillator operators.

   This procedure acts induces oscillator operator algebra rather than trying naively to express induced spinor fields in terms of imbedding space spinor fields, which is of course impossible if spinor fields are second quantized spinor fields.

3. The definition of the "inner product" - that is quadratic form $c_{\alpha,n}$ must be consistent with generalize chiral symmetry implying H-masslessness and separate conservation of baryon and lepton numbers and therefore be non-vanishing only between spinor modes of same H-chirality. Thus $c_{\alpha,n}$ must integral of a current $\overline{\Psi}_\alpha D \Psi_n$.

   (a) The integration over 2-D string world sheet would force the choice $O = D = \Gamma^\alpha D_\alpha$, where D is Kähler-Dirac operator annihilating the spinor modes Ψ_α so that the outcome would vanish identically. Note that Γ^α is Kähler-Dirac gamma matrix which is contraction of canonical momentum current $T^{\alpha k}$ associated with Kähler action with imbedding space gamma matrices γ_k.

   (b) The second option is just 1-D integral over the string world sheet at either space-like boundary of CD for the fermion current component in the direction normal to the string world sheet so that one has $O = \Gamma^n$.

   (c) The proposal for generalized Feynman diagrammatics assumes that induced spinor modes are eigenstates of Chern-Simons Dirac operator assignable to the light-like boundaries of string world sheets at light-like orbits of partonic 2-surfaces. The generalized eigenvalues are $p^k \gamma_k$ where p_k is massless incoming four-momentum. This suggest the identification $O = p^k \gamma_k$ and integration over the light-like portion of the boundary of stringy curve. The component of the induced metric vanishes at this curve by light-likeness but the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices would not be vanishing in general, and would define a non-vanishing integration measure $dV = \sqrt{g_1} dx$. This would lead directly to the TGD counterpart of twistorial formula in which the inverse of the massless fermion propagator replaces fermion propagator as an outcome of residue integral over the virtual four-momentum associated with the fermion line. Virtual fermion is massless and on mass shel but has non-physical helicity.

The two identifications of $c_{\alpha,n}$ would represent dual and equivalent approaches if strong form of general coordinate invariance implying that the space-like 3-surfaces and light-like partonic orbits are physically equivalent choices holds true.

One must answer several questions.

1. Does the induction procedure give rise to anti-commutation relations of conformal field theory at string world sheets and partonic 2-surfaces as it should? If so then H-massless theory in H (or in CD) would map to conformal theory at string world sheet level and 4-D conformal invariance (masslessness) of low energy QFT limit could be understood.

2. Does the induction for the oscillator operators imply that induced spinor fields assignable to separate string world sheets inside CD anti-commute. If so one would have a justification for the intuitive expectation that QFT anti-commutations remain true for given CD but not necessary for entire imbedding space or sub-CDs. CD would define a quantum coherence region at imbedding space level. Each CD would have its own discrete spinor basis depending on the scale of CD. This would allow to get rid of some paradoxical looking consequences of standard anti-commutation relations.

3. The induced spinor field can be localized at 2-surfaces only for Kähler-Dirac action since non-vanishing Kähler-Dirac gamma matrices can space 2-D space. Does induction procedure makes sense only for Kähler action and for Kähler-Dirac equation?

2.2 Generalized Feynman diagrams and induced second quantization

Can induced second quantization allow to say something interesting about generalized Feynman diagrams? The intuitive picture that suggets itself is the following.

1. At the incoming partonic 2-surfaces imbedding space spinor modes with well-defined four-momentum and color quantum numbers are transformed to modes propagating along string world sheets. The matrix $c_{\alpha,n}$ characterizes this transition. Virtual fermions propagate like in twistor diagrams as massless on massless particles but with non-physical helicity. Bosonic propagation reduces at basic level to the propagation of fermion and (usually) anti-fermion at opposite throats of wormhole contact.

   Physical particles are actually more complex: since wormhole throats carry Kähler magnetic charge, the wormhole contacts appear as pairs and string at the end of string world sheet connects the wormhole throats to each other.

2. Partonic 2-surfaces defined the vertices at which incoming partonic 2-surfaces meet. At the level of fermions the situation reduces to the description for what happens to fermion lines. Basic vertex is 4-fermion vertex in which fermions scatter. This vertex corresponds to stringy aspect of generalized Feynman diagrams. The basic interaction is defined by wormhole contact and characterized by stringy propagator $1/L_0$ between two fermions.

   Induced second quantization suggests the simplest vertices would be defined by 4-point correlation functions for world sheet fermions at partonic 2-surfaces defining the vertices. Of course, also 6-point and higher n-point functions are possible. These n-point functions can be calculated directly by induced second quantization or if conformal field theory applies, using conformal theory.

3. The conservation of total four-momentum should emerge as Fourier transform involving integral over cm position of CD. The conservation of four-momentum at vertices is not quite so clear but should emerge through the integral over the position of the partonic 2-surface defining the vertex.

2.3 Braid statistics as induced statistics?

Braid statistics in which particle exchange is homotopy is possible in dimension $D = 2$ for 2-surfaces containing punctures - now the punctures correspond to the ends of light-like curves at partonic 2-surfaces of equivalently to the intersection points of light-like and space-like portions of string world sheet boundaries, that is edges of the string world sheet boundary.

1. Quantum computation relies on unitary entanglement matrix between initial and final states. The corresponding density matrix is proportional to unit matrix so that NE is in question. General unitary entanglement is not consistent with Fermi statistics in general which fixes the entanglement matrix to direct sum of 2×2 permutation symbols. Braid statistics however is and could apply at the string world sheets and partonic 2-surfaces.

2. Could braid statistics at string world sheets or partonic 2-surfaces be relevant for the quantization of the induced spinor fields since fundamental fermions are localized at string world sheets and would bring in the quantum phase defining quantum groups directly into the fundamental TGD as the hyper-finite factor of type II_1 property for the algebra spanned by WCW gamma matrices suggests?

   Could braid statistics might emerge directly from the proposed induced second quantization of spinor fields so that there would be not need to put it in by hand? Tee notion of measurement resolution essential and the inclusion of HFFs describe it. Quantum spaces would be essentially the spaces obtained when included HFF factor is divided away: the states obtained from each other by applying included factor are identified so that one has the analog of gauge symmetry.

3. One can consider also a stringy generalization of braid statistic. Braiding and knotting takes places for the paths of point like particles in dimension $D = 3$. 2-braiding and 2-knotting takes places for closed strings in dimension $D = 4$. Braid group is defined for flows permuting particles. Does the notion of braid group make sense for flows permuting closed strings?

2.4 Statistics and negentropic entanglement

The relationship of statistics to negentropic entanglement is highly interesting.

1. Total symmetrization/anti-symmetrization for fermions defined by single Slater determinant implies that any subsystem formed by some number of particles is negentropically entangled with its complement: density matrix for this kind of pair is indeed proportional to unit matrix. Two-particle state is obvious example of this and n-fermion state defined in terms of n-dimensional permutation symbol too.

2. The rather weird conclusion seems to be that all identical particles of the Universe are entangled. This does not however imply that all identical particles of the Universe are *negentropically entangled*. The density matrix is essentially direct sum of N $n_i \times n_i$ identity matrices multiplied by p_i/n_i: $\sum p_i = 1$ so that the single particle density matrix is not pure. One can thus have a superposition of negentropically n-particle states satisfying statistics labelled by additional label i characterizing the states in superposition. This would be analogous to a sum of Slater determinants appearing in wave mechanics. Negentropy Maximization Principle (NMP) [K2] would force the reduction to one of these states and NE.

3. One can however seriously ask whether the assumption that all fermions of the Universe can form single totally antisymmetric state is too strong or whether the antisymmetrization should make sense only inside quantum coherence regions. CD is the basic structural unit of ZEO, and one can ask whether the anti-symmetrization occurs only in the scale of given CD defining quantum coherence region at the level of imbedding space in ZEO.

3 What does position measurement really mean?

The identity for particles implies that one cannot provide them with labels or order them by putting them in row. What does the measurement of particle position then really mean? If one particle is localized and others are not, can one say that any of the n identical particles appears in the localized state. What does position measurement really mean in TGD framework and in zero energy ontology (ZEO)?

1. The identification of position measurement as fixing of the position of fundamental fermion does not seem a reasonable option. Imbedding space spinor modes correspond in TGD to ground states of super-conformal representations assignable to CDs and can be assigned with the moduli degrees of freedom assignable with CDs. Hence position measurement should correspond to a measurement of position and other moduli characterizing CD. The tip (either of them) defines position of CD. Besides this there are other moduli characterizing CD.

2. An attractive hypothesis inspired by number theoretical universality is that the proper time distance a between the tips of CD is quantized as integer multiples of CP_2 time $a = a_n = nT_{CP_2}$. One must however be cautious: this hypothesis might hold true only in the intersection of realities and p-adicities. In any case, one would have a hierarchy of CDs. Furthermore the position of second tip is at hyperboloid $a = a_n$ and the same argument suggests that it the position corresponds to a discrete subspace defined by some subgroup of Lorentz group SL(2,C) inducing a tesselation of the hyperboloid with "lattice cell" identifiable as hyperbolic manifold.

3. Since the tip of CD seems to play the role of position coordinate, the assumption that quantum states are associated with single CD only looks too simplistic. The only reasonable quantum view is that WCW decomposes to sub-WCWs corresponding to different CDs and that one has wave function in the space of sub-WCWs corresponding to different CDs. If second boundary of CD is fixed so that it belongs to a boundary of future or past directed light-cone then one can have wave function for the second boundary in the discrete space having the values of a and points of the hyperbolic lattice as coordinates.

   The first state function reduction at say "lower" boundary of CD implies localization of the lower boundary and quantum measurement of various observables at it. The subsequent reductions do not change the localization nor the parts of zero energy states at this boundary. Eventually comes the reduction which by NMP forces the localization of the opposite boundary of CD.

4. Position measurement would therefore have nothing to do with the measurement of coordinate appearing as argument of induced spinor field. Of course, already in QFT it is clear that the argument of quantum field has nothing to do with the position operator of wave mechanics. The induction of second quantized spinor fields from those of imbedding space however implies fermionic statistics also in the moduli space of CDs.

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