

Geometric Theory of Harmony

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Abstract

In the earlier article I introduced the notion of Hamiltonian cycle as a mathematical model for musical harmony and also proposed a connection with biology: motivations came from two observations. The number of icosahedral vertices is 12 and corresponds to the number of notes in 12-note system and the number of triangular faces of icosahedron is 20, the number of aminoacids. This led to a group theoretical model of genetic code and replacement of icosahedron with tetraicosahedron to explain also the 21st and 22nd amino-acid and solve the problem of simplest model due to the fact that the required Hamilton's cycle does not exist. This led also to the notion of bioharmony.

This article was meant to be a continuation to the mentioned article providing a proposal for a theory of harmony and detailed calculations. It however turned out that the proposed notion

of bioharmony was too restricted: all icosahedral Hamilton cycles with symmetries turned out to be possible rather than only the 3 cycles forced by the assumption that the polarity characteristics of the amino-acids correlate with the properties of the Hamiltonian cycle. In particular, it turned out that the symmetries of the Hamiltonian cycles are the icosahedral symmetries needed to predict the basic numbers of the genetic code and its extension to include also 21st and 22nd aminoacids. One also ends up with a proposal for what harmony is leading to non-trivial predictions both at DNA and amino-acid level.

1 Introduction

For some time ago I introduced the notion of Hamiltonian cycle as a mathematical model for musical harmony and also proposed a connection with biology: motivations came from two observations [L2], [K7, K8]. The number of icosahedral vertices is 12 and corresponds to the number of notes in 12-note system and the number of triangular faces of icosahedron is 20, the number of amino-acids and the number of basic chords for the proposed notion of harmony. This led to a group theoretical model of genetic code and replacement of icosahedron with tetra-icosahedron to explain also the 21st and 22nd amino-acid and solve the problem of simplest model due to the fact that the required Hamilton's cycle does not exist.

This article was meant to be a continuation to the mentioned article providing a proposal for a theory of harmony and detailed calculations. It however turned out that the proposed notion of bio-harmony was too restricted: all isosahedral Hamilton cycles with symmetries turned out to be possible rather than only the 3 cycles forced by the assumption that the polarity characteristics of the amino-acids correlate with the properties of the Hamiltonian cycle. This working hypothesis had to be given up. The fuel of the minirevolution was the observation the symmetries of the Hamiltonian cycles (Z_6, Z_4, Z_2) are nothing but the icosahedral symmetries needed to predict the basic numbers of the genetic code and its extension to include also 12st and 22nd amino-acids. Thus icosahedral Hamiltonian cycles predict genetic code without further assumptions.

One also ends up with a proposal for what harmony is leading to non-trivial predictions both at DNA and amino-acid level.

1. 3-adicity and also 2-adicity are essential concepts allowing to understand the basic facts about harmony. The notion of harmony at the level of chords is suggested to reduce to the notion of closeness in the 3-adic metric using as distance the distance between notes measures as the minimal number of quints allowing to connect them along the Hamilton's cycle. In ideal case, harmonic progressions correspond to paths connecting vertex or edge neighbors of the triangular faces of icosahedron.
2. An extension of icosahedral harmony to tetra-icosahedral harmony was proposed as an extension of harmony allowing to solve some issues of icosahedral harmony relying on quint identified as rational frequency scaling by factor $3/2$.

This extension is kept also now. One must however give up the idea about correlation between polarity characteristics of proteins and properties of Hamilton cycles. One must allow *all* 11 icosahedral harmonies with symmetries as bio-harmonies: their symmetry groups Z_6, Z_4, Z_2 can be identified as the symmetry groups defined the decomposition of 60 DNA codons to 20+20+20 codons in the model of the genetic code. The 4 remaining DNAs and amino-acids can be assigned to both tetra-icosahedron and tetrahedron and icosahedron regarded as defining separate genetic codes. This explains why stopping codons can code for the 21st and 22nd amino-acid under some circumstances.

Tetrahedral code is second member in the hierarchy of genetic codes [K3] inspired by the notion of Combinatorial Hierarchy $M(n+1) = M_{M(n)} = 2^{M(n)} - 1$ giving the numbers 2, 4, 7, 64, $2^{126}, \dots$ as numbers of DNA codons. The fourth member would correspond to what I called "memetic code" allowing representation of codons as sequences of 21 DNAs. It is not known whether the Combinatorial Hierarchy of Mersenne primes continues as Hilbert conjectured.

3. The notion of bio-harmony is partially characterized by the triplet $n = (n_0, n_1, n_2)$, characterizing the numbers of 0-, 1-, and 2-quint chords which in turn correspond to DNA codons in

consistency with the observation that codons indeed correspond to triplets of nucleotides. n -quint chord corresponds to a triangle (face of icosahedron) containing n edges of the Hamiltonian. Particular bio-harmony requires a selection of a specific Hamiltonian cycle from each class of cycles (1 Z_6 symmetric cycle having $n = (2, 12, 6)$, 2 Z_4 symmetric cycles $n \in \{(0, 16, 4), (4, 8, 8)\}$, 3 $Z_2 = Z_2^{rot}$ with $n \in \{(0, 16, 4), 1(2, 12, 6), (4, 8, 8)\}$ and 5 $Z_2 = Z_2^{refl}$ symmetric cycles with $(n \in \{(2, 12, 6), (4, 8, 8)\})$. Note that there are only three different triplets n .

4. The original idea was that the rules of bio-harmony could be applied to amino-acid sequences interpreted as sequences of basic 3-chords. DNA would have represented the notes of the music. For *given choice of harmony* as Hamiltonian cycle meaning selection of 4, 5 or 10 amino-acids coded by the 20 DNAs in question, the hypothesis had to be modified by replacing amino-acid sequences with DNA sequences.

These DNA sequences however define also amino-acid sequences identifiable as specific triangle at the orbit of Z_n defining the DNA codons assigned to that amino-acid (there is a singular fiber space structure). Together the three 20-plets of DNAs define an amino-acid harmony with $(4+5+10 = 19)$ chords with tetrahedral extension defining a harmony with 22 chords/amino-acids). Hence both DNA sequences and amino-acid sequences define “bio-music”.

5. The assumption that harmonic transitions between chords (DNA codons) minimize the distance between chords defined by quint-metric leads to highly non-trivial and testable predictions about both DNA sequences and amino-acid sequences. Negentropy Maximization Principle (NMP) [K5] suggests that evolution favors the generation of harmony which should thus increase in the proposed sense for DNA sequences defining particular genes or other functional units of DNA during evolution. Large quint-distances between subsequent codons/chords would tend to be polished out under evolutionary pressures.
6. Could icosahedron, tetrahedron, and tetra-icosahedron have direct physical counterparts in living matter? For instance, water molecules form icosahedral clusters and the chathrates associated with synaptic contacts have icosahedral symmetries. Tetra-icosahedron has 13 vertices with the added vertex representing one note- say E- in C-key as note with slightly different frequency to resolve the basic problem of rational number based 12-note scale (12 quints give slightly more than 7 octaves). Intriguingly, microtubules consist of basic structures consisting of 13 tubulins with 2 states defining bit: could these bit sequences define representation for the 3-chords and thus representation of sequence of DNA codons and realization of genetic code.
7. Music is language of emotions and peptides are molecules of emotion as Candace Pert [?] expressed it. Could bio-harmonies serve as direct correlates for emotions? What is bio-music? A natural TGD inspired guess is that sounds can be replaced with $h_{eff} = n \times h$ dark photons with low frequencies and having energies in the range of bio-photons (visible and UV range maximally effective biologically) as proposed on basis of some physical facts and theoretical ideas [K7]. The frequency spectrum of dark cyclotron photons along magnetic flux tubes would define bio-music as “music of dark light” and bio-harmonies would correlate with emotions and moods.

If one can find various icosahedral Hamilton’s cycles one can immediately deduce corresponding harmonies. This would require computer program and a considerable amount of analysis. My luck was that all this has been done. One can find material about icosahedral Hamilton’s cycles in web, in particular the list of all 1024 Hamilton’s cycles with one edge fixed [A1, A2] (this has no relevance since only shape matters). If one identifies cycles with opposite internal orientations, there are only 512 cycles. If the cycle is identified as a representation of quint cycle giving representation of 12 note scale, one cannot make this identification since quint is mapped to fourth when orientation is reversed. The earlier article about icosahedral Hamiltonian cycles as representations of different notions of harmony is helpful [L2].

The tables listing the 20 3-chords associated with a given Hamilton’s cycle make it possible for anyone with needed computer facilities and music generator to test whether the proposed rules

produce aesthetically appealing harmonies for the icosahedral Hamiltonian cycles. Biologist with access to DNA sequences could experiment with DNA codons to see whether they are harmonious in the sense that the distance between subsequent chords assignable to DNA codons tend to be small in quint metric. Note that DNA decomposes to pieces corresponding to different Hamiltonian cycles (harmonies) so that the comparison is not quite straightforward.

2 What could be the basic principles of harmony?

It indeed seems that the idea about definition of notion of harmony in terms of Hamiltonian cycles makes sense.

2.1 Icosahedral harmonies

1. Chords (major and minor) are labeled by their basic tones and comes either as major or minor. Harmony in classical sense requires that the transitions from key to another take place by a small number of fifths and that the piece does not wander too far from the major key, say C.

If quint corresponds to a step along the edge of the cycle in the direction of its orientation, the notion of tonal closeness corresponds to the closeness in the metric of icosahedron. For instance C, F, and G are commonly used keys in same piece and correspond to 3 subsequent points along Hamiltonian cycle. Note that the number of \sharp s of the key increases by one unit in standard direction and the number of \flat s by one unit in opposite direction.

2. It turns out that major and minor 3-chords are mapped to each other in the orientation reversal for icosahedral path so that basic moods “happy” and “sad” in music have this orientation as a geometric correlate. The effect of orientation reversal does not actually depend on the icosahedral representation but is implied by quint cycle representation alone. C and half-octave $F\sharp$ defining the triton interval are the fixed points of the orientation reversal. Orientation reversal induces pairings ($C \leftrightarrow C, F\sharp \leftrightarrow F\sharp, G \leftrightarrow F, D \leftrightarrow B\flat, A \leftrightarrow D\sharp, E \leftrightarrow G\sharp, H \leftrightarrow C\sharp$). Fifths of cycle correspond to the fourths of oppositely oriented cycle so that majors and minors are mapped to each other and one can say that the moods “happy” and “sad” have geometric correlates in the sense that majors and minors are transformed to each other in the reversal of orientation of the cycle.

The notion of harmony can be characterized in terms of numbers of basic 3-chords identified as faces of the icosahedron and their neighborhood relationship telling when corresponding chords are near to each other or vertex or face neighbours. The wall neighbours assignable to given edge are expected to be in very special relationship harmonically since they possess a common quint.

The basic classification is according to the number $n = 0, 1, 2$ of edges of cycle contained by them and the triplet $n = (n_0, n_1, n_2)$ for the numbers of faces of various kinds gives the first rough classification. 2-quint chords have common edge and thus two common notes with two 1-quint chords and are therefore natural intermediates in transitions between them. 0-quint chords are tonal loners having no edge neighbours turns out that they involve dissonances since they consists of three notes spanning length of 1 or $3/2$ steps (say $EFG, EF\sharp G$ or $D\sharp EF$). Maximally symmetric harmony is an exception: 0-quint chords correspond to augmented chords of type $CEG\sharp$ with two major thirds.

The numbers of three different kinds of face neighbor pairs for the 12 edges of the path serve as an additional classification criterion in terms of the $p = (p_{1,1}, p_{1,2}, p_{2,2})$ for the numbers $p_{i,j}$ of different kind of edges. Note that the neighbor faces of an edge correspond to 3-chords, which possess two common notes and are in this sense close to each other. These numbers characterize the most natural transitions between the chords of the harmony. A further criterion is the distribution of these neighbor pairs along the cycle.

2.2 Why fifths are near to each other harmonically?

The naive expectation would be that frequencies near to each other (using half-note as unit) are close to each other. This is not true. Their simultaneous presence is experienced as dissonance.

This probably has a neurophysiological correlate: in ear the hair cell groups detecting notes which are near to each other in frequency space are overlapping. This explanation does not however tell why the conscious experience is dissonance.

The distance measure for notes could be formulated in terms of distance defined as the number of quints connecting them. For quint the distance would be minimal. This measure applies also to chords and allows to understand the basic rule of classical harmony stating that harmonic transitions take place the chords related by quint shift of the basic note (adding either one \sharp or one \flat to the scale). Also the key changes can be understood using the same rule: consider the changes $C \rightarrow G$ and $C \rightarrow F$ as examples. Note that in this case the chords have common note.

One could of course question the assumption that it is possible to choose the shortest route. The notes obtained by quint scaling are not quite same in the two directions and means that \sharp is the inverse of \flat in well tempered scale only. Could it be that people with absolute ear are able to distinguish between the two slightly differing scales and experience notes of quint C-G as harmonically close when 1 quint connects them but as harmonically distant 11 quints in opposite direction connects them?

If cognition is p-adic, one can ask whether the notion of harmony can be formulated in terms of p-adic distance concept.

1. By octave equivalence the scaling by power of two means nothing so that the scalings by $3/2$ are equivalent with scalings by 3 and the distance defined by 3-adic norm having values 3^k , where k is the number of quints makes sense. The distance defined as quints could be identified the absolute value of k along the quint cycle in the direction in which the distance is shorter. If so, the maximal distance is 6 units.
2. 3-adic measure of distance seems to be rather realistic. Quint corresponds to 1 unit distance. Half step corresponds to a distance of 5 units and 6 units defines the largest distance and corresponds to the tritonus interval which was forbidden by catholic church. Fourth (C-F) corresponds to 1- step in opposite direction and 11 steps in standard direction.
3. There is also a problem. Second (C-D) corresponds to 3 quints but third (C-E) corresponds to 4 quints and small third to 3 quints in opposite direction. Major third would thus correspond to a longer harmonic distance than second. This is a genuine problem, whose solution might be provided by the extension of icosahedral scale to icosatetrahedral one bringing in one additional note which is very near to one of the icosahedral notes and is major or minor third of icosahedral note.
4. Could one use the number of icosahedral edges as distance between notes but not as a minimal distance along the Hamiltonian cycle but along a minimal edge path along icosahedron? The icosahedral measure of distance would be analogous to a distance between points of object along shortest route in space that it inhabits and depends on harmony characterized by the shape of icosahedral cycle. C and E (and also C and F \sharp !) could be close to each other in some harmony and distant from each other in some other harmony. Icosahedral geometry would become an active determinant of the harmony.

To sum up, music seems to have both 2-adic (octave equivalence) and 3-adic (12-note scale by quint scalings) characters. The principle of tonal unity for classical music stating that modulations of key should not lead too many quints away from the basic chord would have 3-adic interpretation.

2.3 What could be the rules for building a harmony?

What guarantees good harmony when one has fixed the key/harmony/representation of particular Hamilton cycle?

1. One should pose conditions on the allowed transitions between chords. Are there principles would imply harmonic smoothness in geometric sense? Could the transitions occur only between chords with a common note? Or can one require a common pair of notes? Or can one require even a common quint. If so, 0-quint chords would become tonal hermits and could not be used at all. In practice their dissonant character has eliminated them in popular music and much of classical music too.

The standard quint and fourth transitions (say C to G and C to F) are basic examples in which there is only one common note between chords, and it seems that one cannot require more than this in the general case. Playing with the chords of bio-harmony however suggests that smooth bossa nova/jazz emotionally ambivalent mood is created if common pair of notes or even quint connects the neighboring chords. The rule is that only transitions between chords with same basic note are allowed. Obviously this is too stringent a condition.

2. Could 2-quint chords act as bridges between two 1-quint chords? For instance, for the maximally symmetric harmony consisting of disjoint groups of chords related by half-octave scaling the augmented chords ($F^{aug} = FAC\sharp$ and G^{aug} mapped to each other both by half-octave scaling and reversal of orientation could serve as mediating bridges.
3. Could harmonic transitions take place only between neighboring faces of icosahedron (see <http://en.wikipedia.org/wiki/Icosahedron>) or should it only tend to minimize the quint distance between subsequent chords (this distance vanishes if they have a common note)? For the 0-quint distance harmony, the harmonic movement could be seen as a path in dodecahedron which is dual of icosahedron. In the most general case the transition can take place to both wall and vertex neighbors, whose total number is $3+3=6$. In this geometric picture harmony and melody could be seen as duals of each other.

Dodecahedron is dual of icosahedron and one can ask whether the harmonic motion could correspond to a path at dodecahedron. The vertex of dodecahedron is pentagon and has 3 neighbours (see <http://en.wikipedia.org/wiki/Dodecahedron>). The above argument gives $3+3 > 3$ neighbors for the triangle of icosahedron. Are the wall neighbors of icosahedral triangle mapped to nearest neighbor vertices? If so then transitions between vertex neighbor triangles should correspond to longer steps at dodecahedron. By the duality triangles of icosahedron correspond to three pentagons associated with the vertex of dodecahedron. The rule that comes in mind is that steps can occur between vertices for which the 3-pentagons have one or 2 common pentagons.

Note that if the dodecahedral path is Hamiltonian cycle, it is unique apart from isometries of dodecahedron and would define a unique chord progression. One can - and of course must - allow self-intersecting harmonic paths. The condition that there exists a basic chord from which everything begins and to which everything ends implies that closed but in general self-intersecting path is in question.

4. An interesting test for the idea would a computerized generation of random chord sequences satisfying at least one common vertex rule and finding whether they are aesthetically appealing. Incidence matrix (see Appendix) for the icosahedral (and tetra-icosahedral) triangles whole element tells how many common vertices two chords have allows computational construction of the allowed chord sequences as random sequences.
5. For most harmonies 0-quint chords involve dissonances induced by three nearby notes (such as $CC\sharp D$) and spanning large number of quints (maximally symmetric harmony has 2 0-quint chords, which do not have dissonances and second harmony with 2 reflection symmetries has no 0-quint chords). Also $maj7_-$, $sus4_+$, and 6_- 1-quint chords have half-note dissonances. Dissonances as such are however not un-sesthetical. For instance, Bach used them to create a deeply melancholic feeling.

2.4 More general notion of harmony

The notion of harmony discussed in previous section is rather conservative and certainly too stringent.

1. 0-quint rule is too restrictive already in chord based music. For instance, the downwards progression Am, G, F, E appearing in Spanish music and music forms like Passacaglia would have chords with 1-quint distance. Hence one must consider also a weaker notion of harmonic chord progression according to which this distance is minimized and below some maximum value k_{max} . One quint would define the smallest non-vanishing maximal distance. One can define incidence matrices for chords with n -quint distance. The incidence matrices with

different values of k_{max} have disjoint sets of non-vanishing elements and the total incidence matrix is their sum.

2. Even this is not enough. The direction of step matters for scales (major-minor difference) and it seems to matter also for chord harmonies. The inverse E, F, G, Am of the above mentioned progression does not sound harmonic in the same Am key. The impression of achieving the goal/ending down to something dictated by fate is lost.

Instead of $EFGA$ one often has $EF\sharp G\sharp A$ as a melodic progression and with $E, B7, E7, Am$ as a chord progression having only 0-quint steps. The rule seems to be that 1-quint steps are possible only downwards in minor harmony, whereas upwards steps are 0-quint steps. Climbing slowly upwards by 0-quint steps and falling down by 1-quint steps! Could this “gravitational analogy” serve as a metaphor?

Also the number of n-quint steps between chords matters. The larger this number, the closer the chords are. Two 0-quint steps means that chords have two common notes, 1 0-quint step that they have single common note. The two 1-quint steps for downwards step $Am - G$ are between 3rd and 1st ($C \rightarrow G$) and 5th and 3rd ($E \rightarrow H$). For upwards 0-quint steps $E - H7$ 1-quint steps are between 5th and 5th ($H \rightarrow F\sharp$) and 1st and 1st ($E \rightarrow H$). For $H7 \rightarrow E$ the reversals of these steps occur. For $E7 \rightarrow Am$ one has 3 1-quint steps: (the reversals 1-quint steps $E \rightarrow A$ and $H \rightarrow E$ steps and 1 quint step $D \rightarrow A$. The last step seems to be the smallest one in a well-defined sense.

For G-F step the number of 1-quint steps is one ($C \rightarrow C$): same is true for F-E step (A and E).

Using geometry language, for chords connected by 1-quint step(s) the mutual orientation of corresponding triangles with shape defined by the intervals involved matters since the number of 1-quint steps depends on the orientation.

The notion of chord harmony does not apply as such to polyphonic music with several simultaneous melodies unless one can say that it involves definite chord sequence. One could try to apply the concept of harmony for melody also in this case. The challenge is to guess what harmony for melodies could mean.

1. A conjecture inspired by the genetic code is that the codons defining the allowed melody notes associated with a given chord are in one-one correspondence with the triangles at the orbit of the triangle associated with the chord under the group Z_6, Z_4 , or Z_2 characterizing the chord as a counterpart of amino-acid. In table 4.3 the Z_6 orbits are represented as groups of 6 similar chords (2 for 1-quint chords and 1 for 2-quint chords). In table 4.3 for Z_4 chords the groups consist of 4 similar chords and in the tables 4.3 and 4.3 for Z_2 harmony the chord groups consist of 2 similar chords.
2. The first guess is that the union of the notes of these chords could define the chords, whose notes are compatible with chord in the time scale shorter than the duration of the chord. Note that same triangle can appear at orbits of several chords since the orbits of each group span entire icosahedron.

If the note lasts for a duration of several chords, the notes must be consistent with all the chords involved. The rule would explain why fast chromatic sequences (in the scale of chord duration) sound harmonic but slow chromatic sequences do not.

For melodies in Am key $EFGA$ is rare and does sound harmonic being often replaced with $E, F\sharp, G\sharp, A$. As far as intervals are considered, this is the inversion $D\sharp, F, G, G\sharp$ of $AGFE$ shifted upwards by 5 quints. Could one regard progressions (say Am, G, F, E) breaking the strongest rule for chord harmony as polyphonic progressions satisfying the rules for polyphonic progressions.

To conclude whether the DNA inspired notion of harmonic is realistic, one should understand how the sub-groups Z_n , $n = 6, 4, 2$ of the isometries of the icosahedron and defining the genetic code act on the Hamiltonian cycles.

1. The simplest guess is that these groups are represented as subgroups of Z_{12} (also a subgroup of icosahedral group) representing quint cycle. Z_n generator would shift the basic note of the chord by $12/n$ - that is 2, 3, 6 quints.
2. Z_n maps chords of same type to chords of same type only if it is a *rotational* symmetry of the harmony. For instance, the action of Z_6 (see Fig. 1) on icosahedron allows doublet orbit consisting of *Xaug* type chords, since Z_3 maps 2 0-quint triangles in the middle of the figure to themselves and reflection group Z_2 permutes them. 6-element orbits consist of either minor or major chords. More generally, the inspection of the cycles shows that the cyclic orbits of triangle under Z_n correspond to the orbits of corresponding subgroups of icosahedral group.
3. Z_2^{refl} maps the shape of the chord to its mirror images and so that the character of the chord can vary along Z_4 orbits. The rules are $(M \leftrightarrow m), (6 \leftrightarrow 7)$. For other chords the character is unaffected.
4. Any subgroup of icosahedral isometry group $A_5 \times Z_2^{refl}$ having 120 elements must map chords to chords (faces to faces). In particular any Z_n even if it is not a symmetry of a particular harmony. The character of the chord is not preserved and the number of quints can change. Whether these maps have interpretation in terms of music remains unclear.

These considerations forced me to finally realize that the 3 groups Z_6 , Z_4 , and Z_2 that I had assigned to 20+20+20 DNA codons in the model of the genetic code are nothing but Z_6^- , Z_4^- , and Z_2 -symmetric Hamilton cycles! The numbers of amino-acids associated with various types would be 3+1=4, 5, and 10 (with empty amino-acid included). Tetrahedral extension based on gluing of tetrahedron at triangle corresponding to $X6$ type chord possessed by all Z_2^{refl} type harmonies would give 3 additional real amino-acids giving altogether real 22 amino-acids as required. This has implications.

1. All 11 Hamilton cycles are realized separately as DNA level harmonies. Amino-acid level harmonies would correspond to selection of three Hamiltonian cycles, one for each Z_n .
2. To get something one must give something away. Now one must give up the idea that (4, 8, 8) is special via the corresponding of n-quint property with polarity properties. This is a pity, since just taking this correspondence seriously led to the extension of the icosahedral cycles to tetra-icosahedral ones. Fortunately, the extension itself makes sense for all Hamiltonian cycles.

To understand the action of symmetries one must look how the groups Z_n act on C major chord.

1. Z_2 would induce half-octave shift and map $C = (C, E, G)$ to $F\sharp m = (F\sharp, B\flat, D\sharp)$. The assignment of $F\sharp$ -tritonus - with C note sounds strange in the ears of harmonic conservatives.
2. Z_4 would map $C = (C, E, G)$ to $A = (A, C\sharp, E)$, $F\sharp = (F\sharp, B\flat, C\sharp)$ and $D\sharp = (D\sharp, G, B\flat)$. These would span 8 notes since $E, G, B\flat, C\sharp$, appear twice. Note that C, E, G, A are the notes assignable to the tetrahedron in the extension of the scale and pentatonic scale corresponds to C, D, E, G, A . Z^4 orbit does not contain the notes $DFG\sharp H$ but the orbit of G chord does so. The orbit of C chord plus $G7$ chord alone define the notes of C major key.
3. Z_6 would map C and E to the same "impressionistic" 6-note scale consisting of 6 whole notes. Together with the Z_6 image of G one obtains all 12 notes of the scale.

3 Harmony and biology

3.1 Could harmonic principles be realized in biology?

The basic idea behind icosahedral harmony is the connection with biology suggested by the fact that the number of icosahedral basic chords is 20 which is also the number of amino-acids. Actually there are two additional amino-acids and one ends up to an extension of genetic code by attaching

to icosahedron a tetrahedron and thus adding one vertex more. The number of DNA codons increases from 60 for icosahedral code to 64 for the real code. The triangle along which icosahedral and tetrahedral amino-acids are attached together corresponds to punct coded by stopping codons.

Could the application of harmonic principles to biology make sense? The triangles of the icosahedron correspond to amino-acids or DNA codons for the amino-acids coded by 20 codons in question.

1. The strictest rule stating that there must be common edge of Hamiltonian cycle between the amino-acids/DNAs cannot be satisfied since 0-quint amino-acids/DNA codons would be total loners and effectively eliminated from biology.
2. The weaker “common edge or vertex” rule could however make sense. A given codon in the group of 20 codons/amino-acid could be followed only by 3+3 different nearest neighbor similar codons/amino-acids. If the first amino-acid is fixed there would be only 6^N N-amino-acid sequences instead of 20^N sequences. This kind of symmetry would have been probably observed if exact but one can ask whether harmonic pairs could more probable than completely random pairs.
3. A more plausible formulation is obtained by restricting the rule to the level of DNA sequences and generalizing it so that it applies also to transitions between harmonies with different symmetries so that a transition between corresponding amino-acids is induces.
4. An even weaker formulations states that the transitions occur with highest probabilities between codons/amino-acids having shortest quint distance.

A natural conjecture is that evolution favors the generation of harmony even in the very concrete sense that proteins defined by harmonious chord sequences for bio-harmony are emerge as what Darwinist would call the fittest ones.

3.1.1 Icosahedral water clusters made from tetrahedra

The obvious questions concern the concrete realization of the icosahedron - or more generally icosahedral symmetries. One should also understood what the attachment of tetrahedron to icosahedron means (note that tetra-icosahedron is not the the same thing as icosi-tetrahedron, which is Archimedean (not Platonic) solid (http://en.wikipedia.org/wiki/Pentagonal_icositetrahedron)). What comes in mind is attachment of an information molecule to the receptor of cell membrane.

Water molecules form icosahedral structures and - what is amazing to me - Plato regarded icosahedron as a symbol of water (<http://www.interferencetheory.com/Blog/files/4a3378c13bcad793a52213a325.html>)! The page “Water structure and science” of Martin Chaplin gives illustrations about the rather complex icosahedral structures. Icosahedral structures of size 3 nm can be formed from 20 14-molecule tetrahedral water molecule clusters containing 280 water molecules altogether. They can also consists of cyclic pentamers and tricyclo-decamers and also from bi-cyclo-octomers. The 20 tetrahedrons correspond to the faces of the icosahedron and tetra-icosahedron would be formed as tetrahedron is glued to the the icosahedron along one of the faces.

The bioharmonies could manifest themselves already in the structure of water molecules. Second - more plausible - option is that they differ only at the level of the magnetic body of the biomolecule. Bio-harmony suggests that 3 radial magnetic flux tubes or flux tube pairs emerge from each water tetrahedron. Hamilton’s cycle could be realized as a flux tube connecting the vertices of the icosahedron and assigning the quint cycle to the cyclotron frequencies (magnetic field strengths).

This scenario raises several questions related to the pairings between ordinary DNA/amino-acids, their icosahedral representations, and their representations as dark proton sequences.

Suppose that one takes seriously the idea that genetic code is represented as dark proton sequences with the states of dark protons formed from 3 quarks representing DNA and RNA codons, amino-acids, and even tRNA.

1. How dark proton sequences are realized? Could one regard them as icosahedral bound states of 20 dark protons? Or with a Hamiltonian cycle consisting of penta-quarks and representing dark nuclear string? Could the icosahedral representation as dark nucleus consisting of 20 dark protons and dodecahedral representation as dark nucleus consisting of 12 dark 5-proton

states be dual manners to interpret the state or are they different states related duality. Equivalence of the two pictures would require that dark protons are color excited and in an entangled state.

2. Could dark proton sequences correspond to sequences of icosahedrons connected by flux tubes connecting the dark protons assignable to the dark proton states assignable to the faces of the icosahedrons? These dark nuclei would be definitely different from those possibly associated with the Hamiltonian cycle.
3. What about the tetrahedral part of the genetic code in relation to dark protons sequences? What dark proton states could tetrahedral codons and amino-acids correspond? Are they associated with water tetrahedrons representing the faces of the water icosahedron? Note the amusing numerological co-incidence that the vertices of tetrahedron have 3 quarks associated with them and those of icosahedron 5 and that the quint for icosahedral edge is replaced with third for tetrahedral edge.
4. Could the chords correspond to triplets of cyclotron frequencies for quarks associated with the three flux tubes emanating from the each face of the icosahedron? Could the breaking of the rotational symmetry from $SO(3)$ to $SO(2)$ - now actually $Z_3 \subset SO(2)$ - assumed to occur for dark proton states correspond to the reduction forced by the triangular geometry?
5. How DNA -amino-acid correspondence is represented at the level of dark DNA? The correspondence should be realized in terms of magnetic flux tube triplets connecting dark DNA and dark amino-acid and resonance condition would be essential. When the chords at the orbits of Z_n are of same type, different DNAs correspond to the same chord but with different key. When Z_2^{refl} is involved, the two chords at the orbit are not of same type (note the analogy with left and right-handed biomolecules). The only manner to circumvent the problem is to assume that the chord associated with amino-acids magnetic body is that of DNA. Information is not actually lost in translation, it is only transformed to different kind of information perhaps representing correlates of emotions.
6. Could the non-representability of one of the Z_6 codons as amino-acid have an analog?

The fiber space having icosahedron as a base and 3 copies of icosahedron assigned with 3 regions of icosahedron corresponding to Z_n , $n = 6, 4, 2$, defines a formal geometric representation of genetic code. Could this space represented in terms of water icosahedra?

1. Perhaps one should first try to identify the function of water icosahedrons. The first guess is that they serve as local bridges between dark DNA/amino-acid sequences and ordinary DNA/amino-acid sequences. This would suggest that dark proton of dark DNA forms a flux tube connection with the face of water icosahedron dictated by the state of the dark proton : this would take place by flux tube reconnection and cyclotron resonance. Water icosahedron in turn couples with the DNA/amino-acid like DNA conjugate codon with codon so that kind of double helix is formed.
2. What about the pairing of ordinary DNA/amino-acids and water icosahedrons? Water icosahedron has size of about 3 nm. The size of single DNA codon is about 1 nm. Single codon corresponds to a twist of $3\pi/5=36$ degrees, an angle closely related to Golden Mean. If the radius of the helix consisting of water icosahedrons is above some minimal radius which is easy to estimate from an equation for the helix. There are 10 DNAs per $L(151) = 10$ nm and they correspond to a total twist of $3 \times 2\pi$. Therefore the twist angle is $\Delta\Phi = \pi/5 = 36$ degrees for single codon and corresponds to a distance of $L(151)/10 = 1$ nm). From this one has equation for DNA and icosahedron helices as $z = k\Phi$, $k = h/(6\pi)$, $h = L(151) = 10$ nm (radii are constant). Single codon corresponds to a distance $s = \sqrt{dz^2 + R^2d\phi^2}\Delta\Phi$ along the water icosahedron helix of radius R accompanying DNA helix. One must have $s \geq L = 3$ nm defining the size of water icosahedron in order to avoid overlap. $\Delta s \geq L = 3$ nm gives the condition $R \geq 10 \times \sqrt{2}/(3\pi)$ nm $\simeq 1.5$ nm.
3. If the representation of genetic code is possible, do the fiber icosahedrons correspond to subsets of faces of the icosahedron itself? Or do they correspond to faces the of icosahedrons in

some manner associated with the amino-acid icosahedron. Direct attachment is not possible but association could be achieved by connecting the icosahedrons by flux tubes with the tetrahedron at the ends of flux tubes identified as representation of the same amino-acid. This kind of structure with three icosahedra emanating from a given icosahedron could be iterated and one would obtain a fractal structure representing a binary tree. Could the water icosahedrons organize in this manner to form larger clusters?

What could be the physical correlates of Hamilton cycles representing harmonies?

1. Could Z_6 , Z_4 and Z_2 orbits associated with the Hamiltonian cycles be realized even in the structure of water icosahedrons? Could they be realized as structures formed by the water tetrahedra and correspond to three separate regions of these icosahedral structures? Could one assign to each of the three regions of icosahedron icosahedron such that the attached icosahedron decomposes to the orbits associated with that particular region? Could the hierarchy of the icosahedral symmetry breakings have a direct counterpart at the level of the icosahedral structures formed by water molecules? My intuitive feeling is that the answer to these questions is negative.
2. Could Hamiltonian cycles be realized only at the level of dark photons as quint cycles defined by closed flux tube giving rise to dark nucleus, that is in terms of 3-chords formed by dark photons propagating along flux tubes emanating from the icosahedron? If cyclotron frequencies of dark quarks are in question then the magnetic fields associated with the flux tubes would define the notes.
3. The breaking of Z_2^{refl} symmetry is of special interest since it could serve as a prebiotic analog of chiral selection and could relate to dark variant of weak physics with effectively massless weak bosons in nano-scales. This would require dark magnetic body. Half-octave scaling is second broken symmetry and would have also an analog in Z_2^{refl} variant of icosahedron. Note that 256 variants of the bio-harmony are predicted and could be realized for magnetic body naturally. The presence of electric fields at flux tubes is possible and if the electric and magnetic fields are non-orthogonal, U(1) instanton density is non-vanishing and induces parity breaking. Is this breaking associated with Z_2^{refl} only?

3.1.2 Clathrin molecules as icosahedral structures

Clathrin (<http://en.wikipedia.org/wiki/Clathrin> is a structure appearing at the ends of microtubules and necessary for the transmission of signals between the presynaptic and post-synaptic neurons. Clathrin consists of triskelions - kind of triangular structures with three spiral like legs and having as symmetries the rotational symmetry group Z_3 of equilateral triangle. Clathrins can form hexagonal planar lattices and pentagonal icosahedral lattices consisting of 12 pentagonal faces - the number of vertices of icosahedron . One can associate 3 triskelions with each pentagonal face: this makes $12 \times 3 = 36$ triskelions altogether. One can regard the centers of the 12 faces as vertices of icosahedron and assign to this structure 20 faces, which are triangles formed by 3 pentagons.

If proteins and other molecules attach to the faces of clathrin, one can ask whether each icosahedral triangle of this kind has an address formed by the three notes associated with it and serving as a password: only those molecules, which “know” this password can attach to the face. The realization would be in terms of three U-shaped magnetic flux tubes emerging from the 3 pentagonal faces representing the three notes as frequencies of dark $h_{eff} = n \times h$ cyclotron photons with ELF frequencies but energies of bio-photons (in visible and UV range). The binding of the molecule to the face triangle would be preceded by reconnection of U-shaped flux tubes of the clathrin and molecule, by a resonant interaction by dark cyclotron photons, and by an h_{eff} reducing phase transition bringing the molecule to the face.

3.1.3 Microtubules as music instruments?

It has become clear that microtubules have a central role in biology, neuroscience and perhaps also in consciousness theory and the evidence that they are quantum coherent systems is accumulating. Could music metaphor could help to understand microtubules?

1. Tetra-icosahedron has 13 vertices with the added vertex representing one note- say E- in C-key as note with slightly different frequency to resolve the basic problem of rational number based 12-note scale (12 quints give slightly more than 7 octaves). Intriguingly, microtubules consist of basic structures consisting of 13 tubulins with 2 states defining bit: could these bit sequences define representation for the 3-chords and thus representation of sequence of DNA codons and realization of genetic code.
2. The recent TGD inspired model of microtubules [L1], [K9] was inspired by the findings of the group of Bandyopadhyay (<https://www.youtube.com/watch?v=VQngptkPYE8>) [?], [?] relies on the general vision about bio-communications and control as being based on dark cyclotron photon radiation travelling along magnetic flux tubes.
 These dark photons have a universal energy spectrum in the range of bio-photons (visible and UV) to which they transform as the value of $h_{eff} = n \times h$ reduces to its standard value. Frequencies would span a wide energy range but EEG frequencies would be of special importance since they would also couple to acoustic vibrations. The precise value of the energy scale of cyclotron photons would be determined by the strength of the magnetic field at flux tube.
3. Frequency modulation would be the general manner to code information in living matter: “whale’s song” would be a good metaphor for it. This is assumed in the model for cell membrane as generalized Josephson junction: the modulation would be now induced by the variations of generalized Josephson frequency by variations of the membrane potential. Also microtubules have been proposed to base their communications on frequency modulation.
4. The first possibility coming in mind is that the continually varying microtubule length codes for the frequency [L1]. The change of the frequency by say octave would however require quite fast and large variations of microtubule length. Neither does this realization conform with the idea that the state of single tubulin corresponds to frequency. Microtubule length could also code for the length of the music piece represented by the microtubule serving as a music instrument or musician at the bio-molecular level. It would also the number of microtubular units and thus the size of the orchestra consisting of 13-units.
5. Another possibility inspired by the proposal is that magnetic flux tubes form an analog of 3-D grid ideal for communication purposes using 12-note (or actually 13-note) system as a code equivalent with genetic code. Also microtubules would involve three kinds of flux tubes [L1] defining coordinate grid of cylindrical coordinates: longitudinal, radial and those which rotate along the microtubule. Radial flux tubes would be ideal for communication using 13-note system as a realization of genetic code.
6. 13-note system as cyclotron frequency spectrum for given value of h_{eff} would be determined by the spectrum of the magnetic field strengths going transversally through the microtubule and each tubulin would correspond to one particular note represented as magnetic field strength. The system would be highly analogous to the system formed by hair cells in cochlear. Note would indeed characterize single tubulin molecule rather than entire microtubule as required if one wants to code chords using the two tubulin conformations as a bit. Tubulin conformation would determine whether the tubulin serves as a sending/receiving antenna or not.
7. Melody in 12-note system can be interpreted as a discretized version of frequency modulation with frequency being piece-wise constant in time. Obviously the 13 bit sequences defined by tubulin conformations code for the chords of rational 12-note scale involving a representation of one particular note (the third note of the Pythagorean scale) with two slightly different frequencies in order to avoid problems caused by the rational number ratios of frequencies. 13th bit could also serve as a kind of period. Also chords could be coded up to a chord with 13 notes so that microtubules would have quite a high representative power.

The is an objection against the model.

1. One could argue that a unit consisting of 13 tubulins allows only one octave to be represented. One can of course assume that the magnetic field strengths for subsequent units differ by octave. What makes this interesting is that microtubules allow two variants, called A and B. B type microtubules appear as 13-units since microtubular surface has a gap so that the helical symmetry is broken. For variant A, which is not found in vivo or in vitro, 13-units integrate to form longer helical units. This is assumed in Penrose-Hameroff model and the experimental absence of A type microtubules is one of the basic objections against Penrose-Hameroff hypothesis.
2. The TGD inspired proposal is that A type microtubules corresponds to a critical state having therefore an enhanced symmetry and long range correlations: criticality would explain their experimental absence. The experiments of the group of Bandyopadhyay support that the critical state is induced by a resonant excitation at specific AC frequencies [L1]. Long range correlations would mean enhance helical symmetry - that is fusion of several 13-units to form a longer helical structure. This structure would allow an interpretation as a structure with frequency spectrum of several octaves represented coherently in terms of magnetic field strength: the 10 octave span for hearing would mean the integration of 10 microtubule units meaning length scale of order micrometer assuming that tubulin size is of order 10 nm.
3. If the field strength for subsequent units differ by octave, one can argue that for variant B various octaves play their own music without knowing of each other and thus without coherence. In state A they would play together forming something analogous to orchestra or choir.

If the octave is same for all 13-units, the phase transition would involve octave scaling of the magnetic field strength at the flux tubes. The flux tube radius should suffer p-adic scaling by an integer number of half-octaves, which makes sense if one accepts p-adic length scale hypothesis. This kind of phase transition have been proposed as candidate for a basic step of energy metabolism since they can store or liberate cyclotron energy as metabolic energy.

4. Microtubules could directly couple with both DNA and clathrin molecules if they represent 12 note system as a resonant system able to receive the radiation with corresponding frequencies. 12-note system and the 3-chord system associated with it could define universal communication code allowing communications between DNA, proteins, and microtubules.

To sum up, 13-note extension of 12-note system could be seen as a realization of the genetic code in terms of frequencies. The existence of kind of realization was obvious from the beginning and I proposed it in the model of microtubules as quantum antennas during the first years of TGD inspired theory of consciousness [K6]. Discovering the precise realization of the proposal has however required time.

3.2 Could biology help in the understanding of musical harmony?

One can also ask whether biology could provide ideas about the notion of harmony. Could icosahedral harmony possessing additional 13th note very near to the fourth of basic major chord provide a better view about harmony?

1. The extension of the ideas about harmony to the case of isosatetrahedron is a non-trivial task. If one assumes that the extended Hamiltonian cycle is obtained by deforming tetrahedral Hamiltonian cycle according to the proposal made earlier, one ends up with a problem since the cycle makes a wedge while making a side track of two steps via the new vertex. The two steps must give one quint so that the new vertex must correspond to either minor or major third of note where it started from (and ended to). This would add to the scale a chord of type CGD a chord of type CEG or $CE\flat G$ (plus two other chords containing major or minor third. Depending on the orientation of the cycle one would obtain major or minor key. The remarkable feature of icosahedral harmonies is that they often lack a unique basic chord. Could it be that the addition of tetrahedron breaks the symmetry and fixes the key?
2. The added third could be slightly different from the icosahedral third and this could allow to resolve the problems due to the fact that quint cycle does not quite close ($(3/2)^{12} = 2^7$ does

not hold true exactly. The problems can be of course solved by introducing well-tempered scale defined in terms of powers of $2^{1/12}$: for this choices the topologically induced by these scalings is same as that induced by real topology in frequency space. Algebraically this means introduction of an algebraic extension of rationals. The problem is that persons with absolute ear prefer rational number based scale and experience tempered scale as unaesthetic.

The problem with 3-adic distance of notes was already described: the distance is 4 quints for major third (C-E) and 3 quints for minor third ($C - Eb$). A smaller distance is suggestive for major third.

1. The proposed extension of the scale would break symmetry by bringing a third which is indeed nearest neighbor of the basic note plus two other notes, which are in corners of a *1-quint* triangle in the biological realization. Thus chord CEG and and chord containing EG and third note would be introduced.
2. Using the general results one can readily find the possible extensions of harmony if one assumes that both major and parallel minor with same number of \sharp s or \flat s are obtained. The chord chosen for extension must be CGA , which can be seen as part of $C6$ or $Am7$. If the added vertex corresponds to E one obtains $C = CEG$, $Am = CEA$, and the GEA which is part of $C6/Am7$ as also the lost chord. In amino-acid analog CGA would become “empty” amino-acid, punct, and would be replaced with GEA contained also in $C6$. One can perform this kind of realization for all 11 harmonies and/or their mirror images. The modification induces symmetry breaking and defines a key which is otherwise not obvious for the icosahedral harmonies. Also half-octave symmetry is broken.
3. One can perform the modification also for the inverted harmony. The transformation to reverted harmony $X \rightarrow Y$ corresponds to $X7 \leftrightarrow Y6$ and vice versa so that the presence of $X7$ type chords in harmony guarantees the existence of the required type extension in the reverted harmony. One can of course define extension also using X^7 type chords. This would generate besides CEG two dissonant chords of type $GEE\flat$ and $CEE\flat$.
4. In maximally symmetric harmony (2,12,6) with 6-fold rotation symmetry, there are as many as 6 manners to perform this modification so that any note of the 6-note scale spanning “impressionistic” octave can define the key. The key is either F,G,A or $Dm, E, F\sharp m$. The harmony contains however no $X7$ type chords and since the transition to the reverted harmony acts as $X6 \leftrightarrow Y7$, it does not allow a modification generating both major and parallel minor. There are also other harmonies possessing no $X6$ type chords such as (2, 12, 6) and bio-harmony (4, 8, 8) with 2-fold rotational symmetry so that the extension in the simplest form can be performed only for their reversals.
5. For the two harmonies with 4-fold reflection symmetry there are 2 manners to perform the modification and modified chords are related by half-octave shift. With the conventions of Table ?? the modification introduces key which is either $A (F\sharp m)$ or $D\sharp (Cm)$ for both harmonies (second one is bio-harmony (4, 8, 8)).

3.3 About the interpretation of bioharmonies

3.3.1 How ideas about harmony evolved?

A brief summary about the evolution of the notion of bio-harmony is in order.

1. The first guess [L2] was that amino-acids could be understood as chords of icosahedral bio-harmony characterized by 3-tuples (3,10,7), where the integers tell the numbers of icosahedral triangles with 0,1, or 2 edges of the Hamiltonian cycle and identifiable as 3-chords with 0,1, or 2 quints. The interpretation was that 3 0-quint chords correspond to 3 basic polar amino-acids, 10 1-quint chords to the 10 non-polar amino-acids, and 7 2-quint triangles to the 7 polar and acidic polar amino-acids. It turned out however that (3,10,7) does not appear as Hamiltonian cycle although it satisfies the necessary conditions.

2. I introduced also a model of genetic code motivated by the properties of the code table suggesting that 60 DNA codons are grouped into 3 groups of 20 codons. The idea that DNA codons coding for a given amino-acid form an orbit of a subgroup of icosahedral group with order which is not smaller than the number of these DNAs and has the aminoacid at it. Three subgroups Z_6 , Z_4 , and Z_2 would predict 3 amino-acids coded by 6 codons and two amino-acids coded by 1 codon, 5 amino-acids coded by 4 codons, and 10 amino-acids coded by 2 codons. The total number of codons would be $3 \times 6 + 2 + 4 \times 5 + 10 \times 2 = 20 + 20 + 20 = 60$ rather than 64. The number of doublets is 10 instead of 9. Could one Z_2 orbit corresponds to punct coded by two stopping codons? But what about the codon triplet associated with Ile? Something is clearly missing.

There is also second problem: a really realistic model of genetic code should include also 21st and 22nd amino-acids (Pyl and Sec). Pyl or pyrrolysine is modification of Lys and is basic polar amino-acid so that the number 3 of basic polar amino-acids increases to 4. Contrary to the original naive extrapolation Sec (selenocystein) is acidic polar rather than non-polar so that the number 2-quint triangles increases from 7 to 8. For the properties of amino-acids see http://en.wikipedia.org/wiki/Amino_acid%7Bsharp%7DPhysicochemical_properties_of_amino_acids. The notion of hydrophobicity is discussed at http://en.wikipedia.org/wiki/Hydrophobicity_scales).

3. The solution of the problems came from the extension of icosahedral code with tetrahedral code bringing 4 additional codons and 3 amino-acids assigned with the external faces of the tetrahedron (Ile, Pyl, and some standard non-polar amino-acid), and increasing the number of stopping codons from 2 to 3. This gives $60+3+1=64$ codons but one should code also Pyl and Sec. The solution of the problem would be that stopping codons code also these under some conditions. Are DNA codons or their mRNA counterparts pairing with tRNAs - perhaps their magnetic body - modified somehow?

For instance, Pyl and Sec could correspond to icosahedral codons before fusion. After fusion they cease to be coded - most naturally because the group orbits containing punct are replaced with those associated with tetrahedron. The 3 ordinary amino-acids represented by tetrahedron are Ile, 1-quint amino-acid and 2-quint amino-acid. As fusion is broken temporarily Pyl and Sec are coded.

4. The geometric correlate for the fusion of the codes is gluing of tetrahedron to icosahedron along one face which corresponds to "empty" face identifiable as punct coded by stopping codons. The icosahedral Hamiltonian cycle (4,8,8), which exists as two variants, is extended to (4,10,8) with two new amino-acids.
5. The music analogy for the fusion of tetrahedron is symmetry breaking bringing in a definite key by introducing the major and minor chords as 1-quint chord (but with 2-edges since tetrahedral edges correspond to major and minor thirds).

3.3.2 Understanding the misunderstanding

This was the picture as I started to work again with the notion of bio-harmony. Just when I thought that I understand the notion, I realized that something very essential is missing and even wrong.

1. One could argue that the assumption about the correlation of forms of amino-acid polarity with character of Hamiltonian cycle leading to (4,4,8) identification is ad-hoc: why not allow all harmonies? One can also wonder whether the group structure behind the genetic code leading to the identification of sets of DNA codons coding for a given amino-acid as orbit of the corresponding triangle can be totally dependent on the group structure emerging from the construction of the Hamiltonian cycles.
2. The question whether the group structures associated with genetic code and with the Hamiltonian cycles might have something to do with each other leads to the realization of the obvious: the groups involved are the same: Z_6 , Z_4 , and Z_2 ! The symmetries of DNA are the symmetries of cycles. DNA code would be inherent to the Hamiltonian cycles, and the

triangles of the icosahedron representing the harmony would correspond to DNA codons! 20+20+20 icosahedral triangles to 60 genetic codons and 4 icosahedral triangles the remaining 4! The three 20-plets corresponds to 3+1 amino-acids coded by 6 (resp 2) codons, to 5 amino-acids coded by 4 codons, and to 10 amino-acids coded by two codons.

By direct inspection of the illustrations of the appendix one can indeed convince oneself that the groups in question map chords to chords of same type and one obtains appropriate number of orbits. This of course follows from group theory alone.

3. One must give up the assumption that the integers $n = (n_0, n_1, n_2)$ correspond to the numbers of the basic polar, non-polar, and polar and acidic polar implying that only $n = (4, 4, 8)$ would define bio-harmony. All Hamiltonian cycles with symmetries define bio-harmonies and both Z_2^{rot} and Z_2^{refl} define Z_2 type bio-harmonies assignable to 10 amino-acids coded by 2 codons. This is somewhat frustrating outcome, since just this correspondence served as guideline leading to the extension of the icosahedral code. The extension as such is however independent of this identification and needed in order to get the 4 missing DNA codons and to understand the coding of 21st and 22nd amino-acids Pyl and Sec.

What do the Hamiltonian triplets n then correspond? Harmonies correlate with moods in music: maybe they serve as mathematical correlates for emotions and moods.

4. Harmonies are not for amino-acids but for DNAs coding them. One can however identify amino-acids as specific triangles the orbits and the chords associated with the amino-acids define much more restricted notion of harmony involving one representative of each basic type of chord. Perhaps the additional chords correspond to modulations of the harmony.
5. The rules of harmony generalize as such to transitions between DNA codons regarded as chords. If chords are near to each other with respect to the distance measured as quints, the transition between the chords respects harmony. One must think that DNA codons form a singular fiber space such that the union of fibers for type n gives the space of 20 amino-acids. The "gauge group" Z_n acting in the fiber is different in the 3 regions of the amino-acid space and the number of elements in the fiber is factor of n actually equal to n for $n \neq 6$ and having values 6 and 2 for $n = 6$. Each choice for the 3 Hamilton cycles of type Z_n , $n = 6, 4, 2$ defines a variant of this fiber space. The distance along the fiber isomorphic to the space of amino-acids is measured as minimal quint distance.

Note that the DNA codons for two different variants of the fiber space need not define same kind of chord so that also given amino-acid can correspond to several different chords. It is enough that the notes of the chords are specified - as they indeed are. The Z_n , $n = 6, 4, 2$ in turn can correspond to any Hamilton cycle with symmetry Z_n so that for $n = 1, 4, 2$ one can have $1, 2, 3 + 5 = 8$ different fiber spaces. The hierarchy of Fibonacci numbers is involved. A hierarchy of symmetry breakings is highly suggestive and leads to increasingly richer harmonies.

Z_6 has maximal symmetry but Z_4 is not a subgroup of Z_6 so that only the symmetry breakings $Z_4 \rightarrow Z_2^{rot}$ and $Z_4 \rightarrow Z_2^{refl}$ can be said to occur. Note that transition between different realizations of the covering space has interpretation as a phase transition and that it could occur at RNA rather than DNA level. These phase transitions need not relate to the bio-chemistry but to serve as correlates for emotions and moods. Also the degeneracy due to the existence of several DNAs coding given amino-acid could have similar interpretation.

One can of course play with more stringent scenarios for the transitions between DNAs or RNAs). For instance, the assumption that transitions can occur between chords of same type, leads to contradiction since the *Xaug* chords of Z_6 harmony do not appear in any other harmony.

In any case, the quint-rule in its various forms is readily testable for DNA sequences.

6. An open question concerns the change of the key. The convention of the illustrations is that 1-2 edge corresponds to C-G quint. Should one allow the DNAs at various sheets of covering space to be in different keys? Change of the key could be identified as a rotation by some number of quints. It would change the graph representing icosahedron and change the

chords. Z_{12} would allow to realize all keys. Z_{12} is not however a subgroup of the icosahedral isometries (whereas $Z_6 = Z_3 \times Z_2^{rot}$ is) so that the transformation should be interpreted as a translation in quint space acting as coordinate transformation.

The active transformations induced by isometries of icosahedron do not change the graph and would map chords to new ones. The action of Z_6 is well-defined also for other harmonies than Z_6 symmetric ones. Could the modulations of the basic key correspond to Z_6 transformations. If so, one would have 6 keys. Unfortunately, the most common modulation by quint ($G \rightarrow G$) would be missing.

The change of key could correspond also the change of the chords defined by the extension to tetra-icosahedral harmony. One can choose the chord for extension in several manners for Z_2^{rot} and Z_2^{refl} and these choices could define the allowed modulations of the key.

7. What would be the correlates of different keys the level of DNA? An attractive assumption is that notes are realized in terms of dark photons, which could also transform to ordinary sound since living matter is piezo-electric system. The general hypothesis is that dark photons have universal energy spectrum, which is that of bio-photons. Change of key corresponds to a change of frequency scale and would correspond the change of either Planck constant or of magnetic field strength the flux tubes of the magnetic body associated with DNA codon (or amino-acid perhaps). This would mean that 12-note scale would correspond to 12-note scale for the magnetic fields strength to which cyclotron frequency is proportional or equivalently for the thickness of the flux tube since magnetic flux is quantized if monopole fluxes are in question. 12-note scale could mean in biology a standardization of frequencies used.

One must modify the extension of the icosahedral Hamiltonian cycles to tetra-icosahedral ones appropriately.

1. The Z_6 symmetric 20-plet contains 3 6-plets and 1 doublet and the Z_2 symmetric code contains 10 doublets so that here is one 11 DNA doublets in the icosahedral code. "Ordinary" amino-acids have only 9 doublets. The interpretation is that the Z_6 doublet corresponds to ile and the additional ile is coded by tetrahedral codon. The second surplus doublet can be identified as 2 codons coding for punct, "punct". This gives $4+5+10=19$ amino-acid if "punct" is counted.
2. What is lacking is one ile, met, trp, plus Pyl and Sec. Also 4 DNA codons are needed. One of them must code ile, one met, one for punct, and one for trp. The tetrahedral codons would thus correspond to orbits of Z_1 . This is actually the only possible subgroup since for the choices $Z_n = 2, 3, 4$ the numbers of codons and amino-acids are not correct. This exhausts all DNA codons.
3. The only manner to proceed is to assume that icosahedral and tetrahedral codes can appear also as unfused versions. This would naturally occur for Z_2^{refl} for which all cycles contain X6 type chord but can occur also for Z_2^{rot} if the completion is done for the inverse harmony and then mapped to the harmony back. The icosahedral code would be as already described. The "free" tetrahedral codes would correspond to Z_1 and the faces coding punct in the two codes would code for Pyl and Sec. The fusion of the tetrahedral and icosahedral codes codes gives just the ordinary genetic code so that the proposal is consistent with the proposal that dark proton sequences realize genetic code [K4].
4. Note that geometrically this extension means only that the amino-acid sheet of the fiber space is extended by tetrahedral sheet.

The challenge is to construct the covering space of the icosahedron representing amino-acids.

1. The has as a local fiber the orbit under Z_n associated with the amino-acid defining base point. The space of amino-acids decomposes to disjoint regions corresponding to the 20+20-20 DNA codons. Z_n is the analog of gauge group and by symmetry breaking is different from three different regions of amino-acid space. There are $1 \times 2 \times 8 = 16$ variants of this space due to existence of several harmonies for given symmetries. There are actually only three different options for n given by $n = (0, 16, 4)$, $(2, 12, 6)$, and $(4, 8, 8)$.

2. The Z_n orbits of the three disjoint amino-acid regions (containing 3+1=4, 5, resp. 10 amino-acids) intersect each other. The challenge is to choose the representative amino-acids from the orbits of Z_n in such a manner that the chosen amino-acids belong to the three disjoint regions. It remains to be proven that this is possible. One must also understand how uniquely this can be done.
3. One could think of choosing a set P_2 of 10 representatives from the 10 orbits of Z_2 related by 6-quint scaling along Hamiltonian cycle. The 3+1+5=9 amino-acids associated with Z_6 and Z_4 would belong to the mirror images $P(S)$ of this 10-element set. $P(S)$ decomposes into set P_6 of 3+1 triangles and set P_4 of 5 triangles and there are 2-element, 4-element and 6-element orbits connecting the elements of the sets P_2, P_4 , and P_6 .

The following observations lead to a rather detailed and surprisingly simple picture.

1. The key observation is that the construction of the covering space - that is identifications of amino-acids at the orbits of the groups involved - depends only on whether the choice of Z_2 as Z_2^{rot} or Z_2^{refl} ! Thus the two codes (ordinary one and code with Pyl and Sec coded by stop codons) are distinguished by different DNA-amino-acid covering spaces. The details of the Hamiltonian cycle do not matter. Only the structures and mutual relationships of the groups $Z_6 = Z_3 \times Z_2^{refl}$, $Z_4 = Z_2^{rot} \times Z_2^{refl}$ and Z_2^{rot} and Z_2^{refl} matter. Furthermore, the actions of the groups Z_2^{rot} , Z_3 and Z_2^{refl} determine also the actions of Z_6 and Z_4 . Only Z_2^{rot} and Z_3 are non-commuting actions.
2. One can decompose amino-acids to 10 pairs of Z_2^{refl} orbits and visualize the 20 codons involved as two layers on top of each other such that two on top of each other correspond to the same 2-orbit - 2 boxes on top of each other. The choice of the two layers is not unique since one can permute the members of any vertical box pair.
3. By a suitable choice of the members of vertical box pairs one can arrange that Z_3 and Z_2^{rot} act along the two layers horizontally. Z_2^{rot} orbits divide each layer to 5 pairs of horizontal boxes. One can also permute the vertical pairs horizontally in such a manner that the 5+5 Z_2^{rot} orbits correspond to neighboring horizontal boxes along upper and lower layer giving 2+2+2+2+2 decomposition. This still leaves the possibility to permute these 5+5 horizontal pairs defining 4-orbits of Z_4 horizontally with each other.

Simply by drawing one find that Z_3 orbits divide each layer to 3 triplets and 1 singlet and by a suitable choice Z_3 singlets correspond to the 10th box on the right for both layer. The Z_3 orbits and Z_2^{rot} orbits overlap in such a manner that the middle Z_3 orbit contains entire Z_2^{rot} orbit.

4. It is clear how to choose amino-acids from the orbits.
 - (a) Consider first the $Z_2 = Z_2^{refl}$ case. The lower layer corresponds to the 10 Z_2^{refl} amino-acids (punct included) coded by 2 codons. One must choose from each Z_4 orbit consisting of a square of 4 boxes one upper box to represent Z_4 amino-acid (ala, val, gly, pro, thr). Each 4-unit contains one free upper box to which one can assign 1 Z_6 amino-acid. One cannot however put two amino-acids on 3-orbit. There are 3+1 Z_6 amino-acids and 5 boxes so that one box remains unused. This must be the case. The used box must belong to either second or third horizontal Z_2^{rot} 2-box: if it were filled, the middle Z_3 3-orbit would contain 2 Z_6 amino-acids and the fiber space-structure would fail. Contrary to the original intuition, the unfilled box is *not* at the 2-orbit of Z_6 containing as Ile but at the middle upper 3-orbit, which would contain 2 amino-acids if filled. It is associated with one of the 10 amino-acids coded by two codons and is same for both Z_2^{rot} and Z_2^{refl} . One expects that this amino-acid is somehow special: maybe it is punct. Also the corresponding 6-amino-acid (Ser, Arg, or Leu) might be somehow special.
 - (b) $Z_2 = Z_2^{rot}$ can be treated similarly. The upper row of boxes is filled in the same manner as in the previous case. The horizontal box pairs in the lower row contain one Z_2^{rot} box and one Z_4 box. The difference to the previous case is that Z_2 boxes are now shared by the both rows: in the previous case they belonged to the lower row.

- The assignment of amino-acids to the orbits is not unique: for n similar orbits there are $n!$ different assignments. Inside orbit there is also some non-uniqueness.

The following table represent the two situations graphically.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|------|
| 4 | 6 | 4 | 6 | 4 | | 4 | 6 | 4 | 6(2) |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | | | | | | | | | |
| 2 | 6 | 2 | 6 | 2 | | 2 | 6 | 2 | 6(2) |
| 4 | 2 | 4 | 2 | 4 | 2 | 4 | 2 | 4 | 2 |

Table 1: The representations of the the associations of amino-acids to the orbits of of Z_n , $n = 6, 4, 2$ for $Z_2 = Z_2^{refl}$ (upper two rows) and $Z_2 = Z_2^{rot}$ (lower two rows). The integer n in box tells that the amino-acid associated with that box corresponds to Z_n type amino-acid. “(2)” tells that the Z_6 orbit in question consists of 2 codons.

3.3.3 Music and physical correlates of emotions

Peptides are regarded as molecules of emotion and also information and positive/negative coloring of emotions would naturally correlate with the increase/reduction of negentropic resources of the system as negentropy is transferred to or from it away or increases as a whole. Music induces and expresses emotions. Therefore the idea that music in generalized form - say represented by dark photons with ELF frequencies and having energy spectrum in visible and UV energy range of bio-photons- could be the fundamental correlate of emotions and whether tetra-icosahedral music could be in special role (note that one can associated Hamilton’s cycles and “music” with any graph).

There are 11 candidates for the icosahedral harmony and its extensions. The candidates have either Z_6 (Fig. 1, Z_4 reflection symmetry (Figs. 2, 3), or Z_2 rotation symmetry (Figs. 4, 5,6), and Z_2 reflection symmetry (Figs. 7, 8, 9, 10, 11). For the first case Z^2 reflection symmetry and for the second case Z_2 rotation symmetry are represented as as half-octave shift. Second reflection symmetry corresponds geometrically to reflection in horizontal direction. The extension assigns to them definite key and adds to 1-quint chords minor and major chords absent for the icosahedral bio-harmonies. The question is whether one of these harmonies is selected in biology or whether all three can appear and are perhaps realized at the level of magnetic bodies of amino-acids.

The reversal of the harmony differs from the original one and major-minor transformation takes place. Could it be that both “moods” are realized at the level of magnetic body and even serve as the physical correlates of moods and emotions? Could emotions be realized at the level of amino-acid magnetic bodies as phase transitions affecting parts of organism or even entire organisms and in this manner changing the mood. Peptides are regarded as molecules of emotion: could these phase transitions occur only for peptides and other information molecules involving proteins? Could peptides also serve as seeds of these phase transitions? Could even the Hamiltonian cycle be changed for the magnetic body of the entire organism and correspond to some importance two-valued characteristic of emotional profile?

Could orientation reversal relate to time reversal, which in Zero Energy Ontology (ZEO) corresponds to state function at opposite boundary of causal diamond (CD)? This reversal would occur in volitional acts: the subsequent reduction would not affect the quantum state in positive energy but in TGD framework they affect the state at opposite boundary CD and in this manner give rise to the experience flow of time.

The simplest extension of the harmony in the proposed form requires that harmony possesses X_6 chord. It does not exist for for the candidate with Z_2^{rot} symmetry but for its reversal 4 of them are present as images of $D7, E7$ and $G\sharp7, Bb7$ which are chords of type X^6 . One can however map the harmony to its reversal, perform the completion for it, and perform the reversal back to the original harmony. The reversal depends on what note remains invariant in the reversal. One can require that it is the basic note of the chord to itself: with this condition one would obtain $Dm, Em, G\sharp m, Bbm$ and major keys $C\sharp, F, A, H$. 4 different harmonies would result. Without the restriction the number of harmonies is different and each has different emotional characteristics.

3.3.4 Religious myths, music, and biology

These symmetries define a hierarchy of symmetry breakings. This hierarchy has amazing connections with the myths, which I believe to reflect deep facts about consciousness and biology at fundamental level expected if also consciousness is fractal. The story of genesis is a good representative in this respect.

1. The hierarchy of symmetry breakings proceeding from Z_6 down to Z_2^{refl} brings strongly in mind evolution as loss of innocence. For Z_6 one as 4 orbits. One orbit contains 2 triangles (chords, DNA codons assignable to ile). The other orbits correspond to six codons assignable to amino-acids ser, arg, and leu. The chords at the orbits are major chords and 7-chords, and minor chords and 6-chords for the inverse of the harmony.

There are no dissonant chords in 0-quint sector: dissonances appear only for the remaining groups as 0-quint chords. This is musical representation of paradize. This harmony is based on 6-note scale for the basic notes of the chords and used by impressionistic composers. Amino-acids correspond to selections of preferred chord from each orbit and there are only four different chords: this sub-harmony is very simple. Life in paradize is simple!

2. Next comes an intriguing observation. The number of amino-acids obtained as projections of the icosahedral DNA orbits is 19, not 20. Could it be impossible to have 20 amino-acids as projections of the orbits and that 19 is the maximum number? The reason for 19 is that the number of amino-acid of type Z_6 is $3 + 1 = 4$ rather than 5. Therefore there is one "non-playable" chord - located at some "paradize orbit" -, which does not correspond to any amino-acid.

The first guess for the non-playable chord is as one of the *aug* type chords (say $CEG\sharp$, which is the last breath in many finnish tangos telling about unhappy love end - it is something between happy CM and sad Am, "raueta" is finnish word for this manner to come to an end: "expire" might be the nearest english counterpart). This chord is located at the 2-chord orbit related to the other chord of the orbit by half-octave shift (chords could be $CEG\sharp$ and $F\sharp B\flat D$), the tritonus denied by church.

Unfortunately, this identification is not consistent with the argument identifying the amino-acid chords at Z_n orbits (see table 3.3.2) the non-playable chord must belong to an intersection of 6-orbit and 4-orbit and is not completely unique without further assumptions. It belongs to a 2-orbit of Z_2^{refl} : if it is somehow special, it could belong to the 2-orbit assignable to punct. If the chords at the 2-orbit have basic notes differing by tritonus, the inspection of the Table 4.3 shows that it is possible to find a unique chord pair having this property for all 5 Z_2^{refl} cycles.

One cannot avoid the associations between non-playable chord and the denied fruit hanging in the tree of good and bad knowledge in the story of Adam and Eve, and its analog in many fairy tales. The non-playable chord also brings in mind the hilarious story of Gödel-Escher-Bach about non-playable record (a truth unprovable in given axiom system).

3. The hierarchy of symmetry breakings leading from Z_6 to Z_2^{refl} encourages one to continue with the biblical analogies. Z_6 , Z_4 and Z_2^{rot} cycles have half-octave shift as a symmetry: good and evil do not exist in paradise, but dissonances are already there for Z_4 and Z_2 harmonies - the evil snake! These states correspond to the consciousness of animals, children, and saints. Note that bio-harmony corresponds to the presence of one sub-harmony of type Z_n , $n = 6, 4, 2$.
4. The banishing from the paradize takes place as Z_2^{refl} symmetric harmony replaces Z_2^{rot} harmony: half-octave shift is not a symmetry anymore, and one can tell between good and evil, and eventually church decides to deny tritonus as a symbol of evil! Paradise is left as icosahedral and tetrahedral code are fused to form the tetra-icosahedral code - the ordinary genetic code leading to the breaking of Z_2^{refl} symmetry.
5. In banishment punct ("empty" amino-acid) as a counterpart of chord shared by tetrahedron and icosahedron emerges and means stopping of the music piece altogether. Death of the sinner! For unfused codes this chord is playable as Sec/Pyl and the music piece is never-ending:

life is eternal in paradise! No notion of time, no sin, no death! Amusingly, impressionist music with 6-note scale is music of "now", attempt to catch this moment.

6. Also the holy trinity finds an analog as $Z_6 - Z_4 - Z_2$ trinity of the bio-harmony. Holy Spirit, Father, Son: perhaps in this order. Even more, Z_2^{rot} can be associated with Son in Heaven and Z_2^{refl} with Son at Earth as ordinary mortal!

3.3.5 What do DNAs/amino-acids sound like?

If DNA/amino-acid sequences correspond to chord sequences of tetra-icosahedral harmony, one can ask what they sound like. The best manner to study this question is to build concrete simulations of the DNA/amino-acid sequences.

1. This requires specification of harmony by selecting one Hamiltonian cycle from the cycles belonging to the groups of cycles with Z_n , $n = 6, 4, 2$ symmetry and decomposing amino-acids to 3 groups correspondingly (those coded by 6, 4, and 2 codons). One must include tetrahedral codons and amino-acids.
2. The basic rule of harmony would be the minimization of quint distance between initial and final chords of the transition. One can consider probabilistic versions of this rule or pose strict form of the rules stating in the most stringent form that only transitions with vanishing quint distance (between neighboring triangles) are possible.
3. The transitions between different amino-acid regions would be governed by this rule. Also the transitions between different variants of the DNA-amino-acid space defined by different choices of the Hamilton cycles would be governed by the same rule
4. The most plausible looking model considers only transitions between DNA codons since DNA sequences induce amino-acid sequences.

Appendix represents an example about randomly generated chord sequence assignable to bio-harmony defined as a composite of 3 harmonies - one from each symmetry type and $Z_2 = Z_2^{refl}$ involving tetra-icosahedral extension. Anyone having garage band skills in guitar playing can check what these chord sequences sound like and maybe try to build a melody on the background. One could also test the proposal that codons at the orbit of amino-acid define the melody by finding a concrete representation for the orbits and building random melodies defined by DNA sequences coding for the chord sequence.

4 Icosahedral harmonies

In the following the icosahedral harmonies are discussed in detail. This includes overall summary and tables giving the 20 3-chords of the harmonies and illustrations of the Hamiltonian cycles.

4.1 About symmetries of the icosahedral harmonies

Some words about the symmetries associated with the icosahedral harmonies and genetic code are in order.

There are 3 different kind of bio-harmonies characterized partially by the symmetry group which can be Z_6 , Z_4 or Z_2 which acts either as rotations or reflections.

1. The first variant as $Z_3^{rot} \times Z_2^{refl}$ subgroup of icosahedral group as symmetries and its orbits correspond to 3 6-plets and 1 2-plets for which Z_3 leaves the triangle invariant. The counterparts for the orbits are 3 DNA 6-plets and one 2-plet.
2. The second variant has Z_4 symmetry generated by two commuting reflection as symmetries as is obvious from figures 3,???: the reflections act on vertical and horizontal coordinates. The orbits are five 4-plets of chords. Vertical reflection induces half-octave shift and horizontal one permutes the note sequences $BbCDG\sharp F\sharp E$ and $D\sharp C\sharp HFGA$.

3. Z_2^{rot} or Z_2^{refl} acts as symmetries of the remaining 3+5 cycles. The covering space of 10 amino-acids involved defined by 20 DNA codons decomposes to 10 2-plets.

The 2-fold rotation symmetry of the Hamiltonian cycles is obvious from the illustration ??: it corresponds to 6-quint rotation and the chord sets must be invariant under this rotation. This rotation corresponds to the 1/2 octave shift realized as rotation. These symmetries are realized as “coordinate transformations” for the cycle - a curve in the “imbedding space” defined by icosahedron but induced from the “imbedding space symmetries” acting as isometries of icosahedron.

DNA codons have also almost exact Z_2 symmetry discussed in [?, K1, K2].

1. For the last codon the reflection A-T, C-G is an almost symmetry broken only for special cases. This approximate symmetry could be understood as following from the fact that the number of DNAs coding given amino-adic is even in most cases. The exceptions are ile, met, trp coded by odd number of DNA codons. By mapping DNAs to binary sequences one can order the situation so that the 6:th binary digit is the almost-symmetry digit.
2. What is trivial is that RNA has chosen the third bi-digit to be the almost symmetry digit with the ordering UCAG of the nucleotides so that a genuine physical symmetry is in question. An interesting question is how this symmetry relates to the model of genetic code based on tetra-icosahedral orbits.

The restriction of DNAs to 60 icosahedral DNAs demonstrates that this symmetry originates from the icosahedral Z_2 . The tetrahedral extension of the code breaks this symmetry by extending ile and punct multiples by one codon and introducing also 4 singlets met, trp, Pyl, and Sec.

The detailed correspondence between chords of the harmony and DNA codons is also a problem to be solved.

1. The correspondence matters in the proposed scenario since the chords at at the orbits are different and the gluing of tetrahedron breaks the symmetry in Z_2 sectors so that quint rule determining harmonic DNA sequences is different.
2. The common face of tetrahedron and icosahedron corresponds to punct so that the quint rule for different representations says something about the pairs of form codon-stop codon that is about the codon preceding the last codon of gene! This codon could allow to recognize what Hamiltonian cycle is in question. If C-major is one of the added chords, stop codons correspond to what was $C6 = CGA$ chord and its Z_2 image, which is $X7$ type chord. By the strongest form of the quint rule only the chords having common notes with these chords would correspond to DNA codons of Z_6 and Z_4 cycles which can precede stopping codon.
3. There are some restrictions on the correspondence. Z_2^{refl} symmetry would correspond to the flipping of the 6th bit for the bit representation defined by nucleotides representing 2-bits in the case of $Z^3 = Z_3 \times Z_2^{refl}$. $Z_4 = Z_2^{rot} \times Z_2^{refl}$. For $Z_2 = Z_2^{rot}$ the role of Z_2^{refl} must be taken by Z_2^{rot} . One can of course ask whether Z_2^{rot} cycles are realized at all. For Z_4 cycles Z_2^{rot} would correspond to symmetry permuting the AT, CG doublets for the first nucleotide. For Z_6 subgroup Z_3 would cyclically permute the 3 doublets with respect to third nucleotide. These constraints do not fix the correspondence completely.

To sum up, there is a connection between genetic code and the groups acting along the Hamiltonian cycle. The simplest option fixes the orbits of the triangles and therefore also the representation of genetic code.

4.2 Summary of the basic results

One can find the list of Hamiltonian cycles at <http://cs.smith.edu/~orourke/MathOverflow/hpaths.html>. The edge {1, 2} is fixed and cycles are oriented so that there are 1024 of them. All of them are relevant from the point of music interpretation and the change of orientation corresponds

to major-minor duality, albeit not in the simplest sense. Note that this duality does not affect the characteristics listed above.

The general following general results hold true as one can learn at <http://mathoverflow.net/questions/37788/why-are-there-1024-hamiltonian-cycles-on-an-icosahedron>. One can classify the cycles using their symmetries which can correspond to isometries of icosahedron leaving them fixed or to a reflection taking the vertex n at the cycle to vertex $12-n$. This symmetry is not same as change of orientation which is purely internal operation and cannot change the cycle.

One can even find images of the cycles possessing symmetries at <https://www.flickr.com/photos/edwynn/sets/72157625709580605/> and deduce the triplets n and p characterizing them by visual inspection. Also one can write explicitly the 3-chords defined by the three kinds of faces. I have deduced the triplets n and the 3-chords defining the harmony by the inspection of the images. “Bio-harmony” (4,8,8) forced by the model of extended genetic code involving also the 21st and 22nd amino-acids is of special interest. The classes of cycles with symmetries 6-fold rotational symmetry and two distinct reflection symmetries realize it.

Before continuing some terminology and notation is in order. Take C as the major key. Submediant or relative minor corresponds to Am , subdominant (sharp or flat) to F major (F) or F minor (Fm), dominant to G . The notation for chords is such that quints correspond to subsequent notes in the chord. For 1-quint chords this means that first two notes define the quint. The following table summarizes notation inspired by the popular music notation. The basic different is that the third is in most cases excluded so that the emotional character of the chord is not fixed.

$$\begin{aligned} CEG &\equiv C , & CD\sharp G &\equiv Cm , & CD\sharp F\sharp &\equiv C^o , & CEG\sharp &\equiv Caug , \\ CFG &\equiv C4 , & CF\sharp G &\equiv C4_+ , & CGG\sharp &\equiv C6_- , & CGA &\equiv C6 , \\ CGB\flat &\equiv C7 , & CGB &\equiv Cmaj7 , & CGC\sharp &\equiv C9_- , & CGD &\equiv C9 . \end{aligned} \quad (4.1)$$

Besides these notions it is convenient to introduce additional notations for various dissonant chords appearing as 0-quint chords.

$$\begin{aligned} CC\sharp D &\equiv Cex1 , & CC\sharp D\sharp &\equiv Cex2 , & CDD\sharp &\equiv Cex3 , & CDE &\equiv Cex4 , \\ CD\sharp E &\equiv Cex5 , & CC\sharp E &\equiv Cex6 , & CDF\sharp &\equiv Cex7 , & CDG\sharp &\equiv Cex8 . \end{aligned} \quad (4.2)$$

Clearly, the sets $\{ex1\}$, $\{ex2,ex3\}$, $\{ex4,ex5,ex6\}$, $\{ex7\}$, $\{ex8\}$, corresponds to the span of 2,3,4,6,8 half notes for the chord. The following summarizes the results. Note that $Cex7$ can be seen as part of $D7$ chord.

1. There are 6 collections of cycles without any symmetries containing 48 cycles each: these 48 cycle are mutually isometric so that one can say that there 6 different harmonies.
2. There is a collection with 6-fold rotational symmetry, $48/6=8$ examples. $n = (2, 12, 6)$. The chords of this scale define 6-note scale involving only total steps. CDF and its 6 translates by integer number of steps define 6 1-quint chords. $CE\flat G$ (Cm) and its 6 translates (they obviously correspond to the 6-fold rotational symmetry) define also 6 1-quint chords. The reflection transforms these series to those defined by $GB\flat G$ and its translate and by FAC (F major) and its translates. Impressionists like Debussy used 6-note scale of this kind. Half-octave shift is an exact symmetry. 1-chords lack the third so that one cannot assign to 3-chords any emotional quality. The extension to 4-chord can however bring either “happy” or “sad” quality. Clearly, these harmonies have “jazzy” character.

0-quint chords are $Faug \equiv FAC\sharp$ and $Gaug \equiv GHD\sharp$ are transformed to each other by both half-octave shift and inversion.

3. There are 2 collections with 2 distinct reflectional symmetries with $12=48/4$ representatives in each. Half-octave scaling is a symmetry of both these scales as one might guess.

The first cycle (see Fig. 2) has $n = (0, 16, 4)$ so that there are no 0-quint chords which in general are dissonant. Second cycle (see Fig. 3) realizes $n = (4, 8, 8)$ bio-harmony and deserves some comments. It will be discussed in detail later.

- (a) The 8 2-quint chords consist of $B\flat FG \equiv B\flat 9, C9, F9, G9$ and their half-octave scalings. Clearly, the simple four-note scale appears here.
 - (b) Using the popular notion introduced earlier 1-quint chords consist of two 4-plets $Dmaj7, E9_-, A7, A6$ and $G\sharp maj7, B\flat 9_-, D\sharp 7, D\sharp 6$ related by half-octave shift. The harmony contains no “simple” major or minor chord and only the extension to tetrahedral harmony can provide them. The same is true for the second bio-harmony.
 - (c) The 4 0-quint chords are $Cex3 \equiv CDD\sharp$ and $Eex2 \equiv EFG$ and their half-octave scalings $F\sharp ex3 \equiv F\sharp G\sharp A$ and $B\flat ex2 \equiv B\flat BC\sharp G$.
4. There are 3 collections with Z_2 rotational symmetry with $48/2 = 24$ representatives in each. The triplets n are (0, 16, 4) (see Fig. 4), (2, 12, 6) (see Fig. 5), and (4, 8, 8) (see Fig. 6). All these harmonies are symmetric with respect to half-octave shift (tritonus), which obviously corresponds to the Z_2 rotation. Tritonus would not have been tolerated by catholic church! This symmetry characterizes all 3 harmonies. Basic 3-chords do not contain pure minor and major chords. The reflection of the scale does not leave the collection of chords invariant but it is not clear whether this corresponds only to a change of scale, probably not.

Consider the (4,8,8) case (see Fig. 6).

- (a) The 8 2-quint chords appear as four-plet $H9, C\sharp 9, D\sharp 9, F9$ and its half octave shift (tritonus interval) acting as a symmetry of the harmony. 2-quint chords are always of type X^9 (note that the third is missing) but also 1-quint chord can be of form X^9 as explicit construction of chords demonstrates: I have denoted these 1-quint chords by symbol $X4$ (CDG is obviously equivalent with CDG).
- (b) Using the popular music notation introduced earlier, the 8 1-quint chords are $D7, Amaj7, A4_+, E7$ and their half-octave shifts $G\sharp 7, D\sharp 7, D\sharp 4_+, B\flat 7$.

No major and minor chords are included and only the extension to tetra-icosahedral harmony can provide them and also break the symmetry giving rise to well-defined key.

5. The four 0-quint chords appear in two types. $D\sharp ex2 \equiv D\sharp EF\sharp$ and its half-octave shift $Aex2 \equiv AB\flat C$ plus $Hex3 \equiv HC\sharp G$ and its half-octave shift $Fex3 \equiv FGC\sharp$. According to usual thinking these chords involve dissonances. This dissonance character is a rather general phenomenon for the harmonic loners and classical views about harmony would exclude them as asocial cases! In the case of maximally symmetric harmony the loners are diminished chords and thus not so dissonant. In some cases there are no 0-quint chords.

There are 5 collections with Z_2 reflection symmetry having 24 representatives in each (see Figs. 7, 8,9,10,11). The integer triplets n are (2, 12, 6), (2, 12, 6), (4, 10, 6), (2, 12, 6), (2, 12, 6). Bio-harmony has representative also in this class (see Fig. 9). The half-octave scaling symmetry is broken for these harmonies. I have not found simple characterization for the symmetry which corresponds to reflection in the direction of x-axis since it changes the interval structure of the chords.

Some comments (4, 8, 8) case are in order (see Fig. 9).

1. 2-quint chords appear as reflection related multiplets $C9, D9, H\sharp 9, D\sharp 9$ and $C\sharp 9, H9, F9, B\flat 9$.
2. 1-quint chords appear as symmetry related mutiplets $G, D7, Amaj7, E7$ and $C\sharp m, F\sharp 6, H6_-, E6$. Key G major and $C\sharp$ minor would be natural looking keys even without tetrahedral extension. For the mirror image $B\flat$ minor and E major would be the natural looking keys. For extension E major would be the key.

To sum up, half octave shift is a symmetry of all harmonies expected those having only Z_2 reflection symmetry, and fails thus also for the corresponding bio-harmonies.

4.3 Tables of basic 3-chords for the icosahedral harmonies with symmetries

The tables below give list for the three types of 3-chords for the 11 harmonies possessing symmetries. One must remember that the reversal of the orientation for the cycle induces the transformation $C \leftrightarrow C, F\sharp \leftrightarrow F\sharp, H \leftrightarrow C\sharp, F \leftrightarrow G, D \leftrightarrow B\flat, E \leftrightarrow G\sharp, A \leftrightarrow D\sharp$ and produces a new scale with minor type chords mapped to major type chords and vice versa. Also one must remember that all 3-chords except those which are simple majors or minors lack the third so that their emotional tone remains uncharacterized. For instance, $C6$ does could be replaced with $Cm6$ and $G7$ with $Gm7$. The reader can check the chords by direct inspection of the figures. The convention used is that vertex number one corresponds to C note.

| (n_0, n_1, n_2) | 0-chords | 1-chords | 2-chords |
|-------------------|----------------|--|--|
| (2,12,6) | $(Faug, Gaug)$ | $(Cm, Dm, Em, F\sharp m, G\sharp m, B\flat m)$, | $(C9, D9, E9, F\sharp 9, G\sharp 9, B\flat 9)$. |
| | | $(F6, G6, A6, B6, C\sharp 6, D\sharp 6)$. | |

Table 2. The table gives various types of 3-chords for harmonies with Z_6 rotational symmetry. Note that half-octave shift is an exact symmetry. Note that $G^{aug} = CEG\sharp, F^{aug}$ act as bridges between the groups related by half octave shift. The chords have been arranged so that they form orbits of Z_6 . “Amino-acid chords” correspond to preferred chords at the orbits.

| (n_0, n_1, n_2) | 0-chords | 1-chords | 2-chords |
|-------------------|---|---|------------------------------------|
| (0,16,4) | | $(D7, D6, G\sharp 7, G\sharp 6)$, | $(B\flat 9, B9, E9, F9)$. |
| | | $(G4+, A9-, C\sharp 4+, D\sharp 9-)$, | |
| | | $(Emaj7, Gmaj7, B\flat maj7, C\sharp maj7)$, | |
| | | $(C9-, A9-, F\sharp 9-, D\sharp 9-)$. | |
| (4,8,8) | $(Cex3, Eex2, F\sharp ex3, B\flat ex2)$. | $(Dmaj7, E9-, A7, A6)$, | $(B\flat 9, F9, C9, G9)$. |
| | | $(G\sharp maj7, B\flat 9-, D\sharp 7, D\sharp 6)$. | $(E9, B9, F\sharp 9, C\sharp 9)$. |

Table 3. The table gives various types of 3-chords for the two harmonies with $Z_4 = Z_2^{rot} \times Z_2^{refl}$ symmetry. 4-plets represent the orbits. First cycle has no harmonic loners. Second cycle gives rise to bio-harmony (4,8,8) for which 0-quint chords are dissonant. Both cycles have Z_2 rotation symmetry acting as a vertical reflection symmetry in figures and realized also as half-octave shift so that 4-plets contains chords and their half-octave shifts. The genuine reflection symmetry acts as a horizontal reflection symmetry in figures. The cycles correspond to figures 2, 3.

| (n_0, n_1, n_2) | 0-chords | 1-chords | 2-chords |
|-------------------|-------------------------------------|--|--------------------------------------|
| (0,16,4) | | $(Em, B\flat m), (Cm, F\sharp m)$, | $(D9, G\sharp 9)$, |
| | | $(G6, C\sharp 6), (A6, D\sharp 6)$, | $(E9, B\flat 9)$. |
| | | $(D4+, G\sharp 4+), (B4+, F4+)$, | |
| | | $(Cmaj7, F\sharp maj7), (G6-, C\sharp 6-)$. | |
| (2,12,6) | $(Aex4, D\sharp ex2)$. | $(Am, D\sharp m), (G9-, C\sharp 9-)$, | $(C9, F\sharp 9)$, |
| | | $(C4, F\sharp 4), (E4+, B\flat 4+)$, | $(A9, D\sharp 9)$, |
| | | $(Dmaj7, G\sharp maj7)$, | $(D9, G\sharp 9)$. |
| | | $(Bmaj7, Fmaj7)$. | |
| (4,8,8) | $(Aex2, Hex8, D\sharp ex2, Fex8)$. | $(D7, G\sharp 7), (Amaj7, D\sharp maj7)$, | $(G9, C\sharp 9), (A9, D\sharp 9)$, |
| | | $(A4+, D\sharp 4+), (E7, B\flat 7)$. | $(B9, F9), (E9, B\flat 9)$. |

Table 4. The table gives various types of 3-chords for harmonies with Z_2 rotation symmetry acting as half-octave shift. The doublets represent 2-chord orbits. The cycles correspond to figures 4, 5, and 6.

| (n_0, n_1, n_2) | 0-chords | 1-chords | 2-chords |
|-------------------|--|---|--|
| (2,12,6) | $(F\sharp ex3, Hex4)$, | $(Am, D\sharp), (A6, D\sharp7),$ $(D7, B\flat6), (G6-, Fmaj7),$ $(D4+, B\flat9-), (E9, G\sharp4+)$, | $(C9, F9), (B9, F\sharp9),$ $(E9, C\sharp9)$. |
| (2,12,6) | $(Dex4, Hex4)$. | $(F, Fm), (C6-, B\flatmaj7),$ $(D7, G\sharp6), (Gmaj7, D\sharp6-),$ $(C\sharp4-, A4+), (E4+, F\sharp6)$. | $(C9, D\sharp9),$ $(D\sharp9, C\sharp9),$ $(E9, B9)$. |
| (4,8,8) | $(Fex1, D\sharp ex3, G\sharp ex1, Aex2)$. | $(E7, E6), (Amaj7, B9-),$ $(G, C\sharp m), (D7, F\sharp6)$. | $(D9, B9), (C9, C\sharp9),$ $(F9, G\sharp9), (D\sharp9, B\flat9)$. |
| (2,12,6) | $(Hex3, Eex7)$. | $(D7, G\sharp6), (G, D\sharp m),$ $(F, Fm), (C6-, B\flatmaj7),$ $(A9-, C\sharp4+), (E7, F\sharp6)$. | $(C9, D\sharp9),$ $(D9, C\sharp9),$ $(E9, B9)$. |
| (2,12,6) | $(F\sharp ex2, Fex3)$. | $(F, B\flat m), (C7, G\sharp6),$ $(Amaj7, B9-), (E6, E7),$ $(G, C\sharp m), (D7, B6)$. | $(B\flat9, D\sharp9),$ $(C9, C\sharp9),$ $(D9, H9)$. |

Table 5. The table gives various types of 3-chords for harmonies with single reflection symmetry. The cycles correspond to figures 7, 8, 9,10, 11.

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5 Appendix

5.1 Chord tables for some harmonies and their inverses

The formula for inversion of the harmonic keeping note X as fixed can be represented as a product of translation taking X to C , inversion keeping C fixed, and translation taking C back to X . The inversion maps the chord having C as basic note to its mirror image so that the order of notes can change and basic note can change. For instance, the major chord $CM = CEG$ goes to minor chord $CG\sharp F = Fm$ so that $k = 0$ goes to $k \equiv \Delta k_{inv} = 11$. This delicacy must be taken into account. If X remains fixed inversion is just the transformation

$$k \rightarrow k_{inv} = (2 \times k(X) - \Delta k_{inv}) \text{ mod } 12 . \tag{5.1}$$

The following table gives the inversion of the scale leaving C (and also $F\sharp$) invariant:

| | | | | | | | | | | | |
|---|---|----|----|----|----|----|----|----|----|----|---|
| C | G | D | A | E | H | F+ | C+ | G+ | D+ | B- | F |
| C | F | Bb | D+ | G+ | C+ | F+ | H | E | A | D | G |

The inversion for the types of the chords does not depend on the basic note as is clear from the distance preserving character of the inversion. The following table gives the inversion of for the types of the chords leaving C fixed. The elements of the rows give the type of the chord and the number of quints k corresponding to it. For chords having C as basic note one has $k = 0$. It is easy to deduce the transformation formula in more general case from the table.

| | | | | | | | | | |
|------|------|--------|-------|-------|-------|-------|--------|---------|--------|
| M,0 | m,0 | sus4,0 | aug,0 | 4,0 | 9,0 | 4+,0 | 9-,0 | 6-,0 | maj7,0 |
| m,11 | M,11 | sus,0 | aug,0 | 4,0 | 9,10 | 9-,11 | 4+,11 | maj7,11 | 6-,11 |
| 6,0 | 7,0 | ex1,0 | ex2,0 | ex3,0 | ex4,0 | ex5,0 | ex6,0 | ex7,0 | ex8,0 |
| 7,11 | 6,11 | ex1,10 | ex3,3 | ex2,3 | ex4,8 | ex6,8 | ex5,80 | ex8,6 | ex7,6 |

Table 6: Table gives the transformation of inversion leaving C invariant on the basic chords having C as basic note.

The following tables give the chords and corresponding inverse chords for the 11 icosahedral harmonies.

| | | | | | | | |
|-------|--------|---------|-------|---------|--------|---------|---------|
| ro6 | iro6 | re41 | ire41 | re42 | ire42 | ro21 | iro21 |
| F.aug | F.aug | D.7 | A.6 | C.ex3 | A.ex2 | E.m | F.M |
| G.aug | D+.aug | D.6 | A.7 | E.ex2 | F.ex3 | B-.m | B.M |
| C.m | F.M | G+.7 | D+.6 | F+.ex3 | D+.ex2 | C.m | A.M |
| D.m | D+.M | G+.6 | D+.7 | B-.ex2 | B.ex3 | F+.m | D+.M |
| E.m | C+.M | G.4+ | E.9- | D.maj7 | B.6- | G.6 | D.7 |
| F+.m | B.M | A.9- | D.4+ | E.9- | A.4+ | C+.6 | G+.7 |
| G+.m | A.M | C+.4+ | B-.9- | A.7 | E.6 | A.6 | C.7 |
| B-.m | G.M | D+.9- | G+.4+ | A.6 | E.7 | D+.6 | F+.7 |
| F.6 | C.7 | E.maj7 | G.6- | G+.maj7 | F.6- | D.4+ | G.9- |
| G.6 | B-.7 | G.maj7 | E.6- | B-.9- | D+.4+ | G+.4+ | C+.9- |
| A.6 | G+.7 | B-.maj7 | C+.6- | D+.7 | B-.6 | B.4+ | B-.9- |
| B.6 | F+.7 | C+.maj7 | B-.6- | D+.6 | B-.7 | F.4+ | E.9- |
| C+.6 | E.7 | C.9- | B.4+ | F.9 | D+.9 | C.maj7 | A.6- |
| D+.6 | D.7 | A.9- | D.4+ | C.9 | G+.9 | F+.maj7 | D+.6- |
| C.9 | C.9 | F+.9- | F.4+ | G.9 | C+.9 | G.6- | D.maj7 |
| D.9 | B-.9 | D+.9- | G+.4+ | E.9 | E.9 | C+.6- | G+.maj7 |
| E.9 | G+.9 | B.9 | G.9 | B.9 | A.9 | D.9 | D.9 |
| F+.9 | F+.9 | E.9 | D.9 | F+.9 | D.9 | G+.9 | G+.9 |
| G+.9 | E.9 | F.9 | C+.9 | C+.9 | G.9 | E.9 | C.9 |
| B-.9 | D.9 | B-.9 | G+.9 | B-.9 | B-.9 | B-.9 | F+.9 |

Table 7. Pairs “X” and “iX” of columns give the chords of the bio-harmonies and their inverses depicted in figures 1,2, 3,4.

| ro22 | iro22 | ro23 | iro23 | re21 | ir21 | re22 | ir22 |
|---------|-------|---------|--------|--------|---------|---------|---------|
| A.ex4 | G.ex4 | A.ex2 | B-.ex3 | F+.ex3 | D+.ex2 | D.ex4 | E.ex4 |
| D+.ex2 | C.ex3 | H.ex8 | B-.ex7 | H.ex4 | B-.ex4 | H.ex4 | F+.ex4 |
| A.m | B-.M | D+.ex2 | E.ex3 | A.m | E.M | F.M | E.m |
| D+.m | E.M | F.ex8 | F.ex7 | D+.M | B-.m | F.m | E.M |
| G.9- | C.4+ | D.7 | A.6 | A.6 | E.7 | C.6- | A.maj7 |
| C+.9- | F+.4+ | G+.7 | D+.6 | D+.7 | B-.6 | B-.maj7 | B.6- |
| C.4 | C.4 | A.maj7 | D.6- | D.7 | B.6 | C.9- | A.4+ |
| F+.4 | F+.4 | D+.maj7 | G+.6- | B-.6 | D+.7 | D.7 | G.6 |
| E.4+ | D+.9- | A.4+ | D.9- | G.6- | F+.maj7 | G+.6 | C+.7 |
| B-.4+ | A.9- | D+.4+ | G+.9- | F.maj7 | G+.6- | G.maj7 | D.6- |
| D.maj7 | F.6- | E.7 | G.6 | D.4+ | B.9- | D+.6- | F+.maj7 |
| G+.maj7 | B.6- | B-.7 | C+.6 | B-.9- | D+.4+ | C+.4 | C+.4 |
| B.maj7 | G+.6- | B-.9 | G+.9 | G+.4+ | F.9- | A.4+ | C.9- |
| F.maj7 | D.6- | G.9 | B.9 | E.9- | A.4+ | E.4+ | F.9- |
| C.9 | D.9 | C+.9 | F.9 | C.9 | G+.9 | F+.6 | D+.7 |
| F+.9 | G+.9 | A.9 | A.9 | F.9 | D+.9 | D+.9 | C+.9 |
| A.9 | F.9 | B.9 | G.9 | B.9 | A.9 | C+.9 | D+.9 |
| D+.9 | B.9 | F.9 | C+.9 | F+.9 | D.9 | E.9 | C.9 |
| D.9 | C.9 | E.9 | D.9 | E.9 | E.9 | B.9 | F.9 |
| G+.9 | F+.9 | D+.9 | D+.9 | C+.9 | G.9 | D+.9 | C+.9 |

Table 8. Pairs “X” and “iX” of columns give the chords of the bio-harmonies and their inversions depicted in figures 5,6, 7,8.

| re23 | ire23 | re24 | ire24 | re25 | ire25 | | |
|--------|--------|---------|--------|--------|--------|--|--|
| F.ex1 | F.ex1 | H.ex3 | G.ex2 | F+.ex2 | F.ex3 | | |
| D+.ex3 | G+.ex2 | E.ex7 | F+.ex8 | F.ex3 | F+.ex2 | | |
| G+.ex1 | D.ex1 | D.7 | A.6 | F.M | B-.m | | |
| A.ex2 | D.ex3 | G+.6 | D+.7 | B-.m | F.M | | |
| E.7 | B.6 | G-.M | B.m | C.7 | D+.6 | | |
| E.6 | B.7 | D+.m | G+.M | G+.6 | G.7 | | |
| A.maj7 | F+.6- | F.M | F+.m | A.maj7 | F+.6- | | |
| B.9- | E.4+ | F.m | F+.M | B.9- | E.4+ | | |
| G.M | G+.m | C.6- | B.maj7 | E.6 | B.7 | | |
| C+.m | D.M | B-.maj7 | C+.6- | E.7 | B.6 | | |
| D.7 | C+.6 | A.9- | D.4+ | G.M | G+.m | | |
| F+.6 | A.7 | C+.4+ | B-.9- | C+.m | D.M | | |
| B-.9 | C.9 | E.7 | G.6 | D.7 | C+.6 | | |
| D.9 | G+.9 | F+.6 | F.7 | B.6 | E.7 | | |
| B.9 | B.9 | C.9 | F+.9 | D+.9 | G.9 | | |
| C.9 | B-.9 | D+.9 | D+.9 | C.9 | B-.9 | | |
| F.9 | F.9 | D.9 | E.9 | C+.9 | A.9 | | |
| G+.9 | D.9 | C+.9 | F.9 | B-.9 | C.9 | | |
| D+.9 | G.9 | E.9 | D.9 | D.9 | G+.9 | | |
| C+.9 | A.9 | B.9 | G.9 | H.9 | B-.9 | | |

Table 9. Pairs “X” and “iX” of columns give the chords of the bio-harmonies and their inversions depicted in figures 9,10, 11.

5.2 Calculation of incidence matrices

The most stringent definition of harmonic chord progression is as a chord sequence in which two subsequent chords have at least one common note: the distance between subsequent chords defined as the minimal distance between triangles representing them vanishes. Some general comments are in order.

1. Incidence matrices can be computed by using expressions of chords as sets of three notes (possible in Python) and just counting the number of common notes defining the value of the element of the incidence matrix. The quint distance between the chords vanishes if they have common notes. More general incidence matrices would correspond to a larger quint distance.
2. In the case of genetic code and amino-acids one Hamilton cycle from each class labelled by Z_n , $n \in \{6, 4, 2\}$ is involved.

- (a) There are $N = 1 \times 3 \times 8 = 24$ cycle combinations if one does not allow the inverse harmonies. Allowing them gives $N = 8 \times 24$ combinations. If transitions between all representations are possible, there are $M = N^2$ 20×20 -dimensional incidence matrices to be calculated for the icosahedral restriction of the code. Incidence matrices are symmetric so that only $D(D+1)/2 = 20(20+1)/2 = 210$ independent matrix elements need to be calculated for given 20×20 -D incidence matrix.
- (b) Equivalently, one can calculate the incidence matrix for a space with $N \times 20$ points which is Cartesian product of N amino-acid spaces with 20 points. N has values 24 and 8×24 . Remarkably, the magic number 24 of also stringy mathematics appears.
- (c) If the transitions can be restricted to single triplet of cycles, one must calculate 6 20×20 -dimensional incidence matrices. This situation could be realistic for portions of the genetic code if the transitions between different cycle triplets are analogous to phase transitions. The number of incidence matrices (one can also use single 60×60 incidence matrix) is still reasonably small and can be documented in written form. In a model for random chord sequences one must specify the probabilities for the transitions between chords with different n for Z_n . Simplest starting point assumption is that the probabilities are identical.
3. For the extended genetic code the most natural assumption is that the extension of the code to icoso-tetrahedral code take places place only in Z_2 sector meaning the extension of amino-acid space by 4 amino-acids and the increase of the number of DNA codons from 60 to 64. There are two kinds of transitions between icosahedral and tetrahedral codons. Tetrahedral codon can correspond to a codon, which is outside the icosahedron having at least one common vertex with the icosahedral codon: this allows 3+3 transitions. Tetrahedral codon can correspond also to punct. Unless the codon/amino-acid contains at least one of these notes, it cannot precede stopping codon. These chords extend the harmony by the counterparts of CM and Am and punct corresponds to $C6 = CGA$.
4. Also the situation in which tetrahedral and icosahedral codes are disjoint must be considered. In this case there are no transitions between tetrahedral and icosahedral sectors. In tetrahedral sector the distances between faces always vanish so that the calculation of this part of the incidence matrix is trivial. Icosa-tetrahedral part of the incidence matrix can be readily written. The difficult part of the calculation of incidence matrices reduces to that for the icosahedral case such that the common face corresponds to either punct or Sec/Pyl. This gives selection rules telling which codons/amino-acids can precede stopping codon/punct in given bio-harmony.

5.3 Simulation of harmonic DNA sequence

The following sequence represents a random harmonic sequence based on zero quint distance between neighboring chords (at least one common note). The harmony if combination 3 harmonies 1, 2, and 7 extended by adding chords Bb, Gm and $G7$ and associated $Bb6$ representing stopping codon and punct in tetra- icosahedral code and and Sec or Pyl in their unfused variants. These three harmonies correspond to groups of 20, 20, and 24 DNA codons at orbits of Z_6 , Z_4 , and Z_2 which is now taken to be Z_2^{refl} . To deduce DNA sequence one must assume detailed correspondence between the codons at the orbits and corresponding chords.

It is assumed that all transitions between neighboring DNAs occurs with the same probability and induce the transitions between amino-acids.

Faug, A6, Dm, G6, G6, G6, Em, G6, Cm, G6, F6, Faug, F+m, Dm, G6, G6,
 Gaug, G+m, Cm, F6, Dm, Dm, F+m, Dm, F6, F6, B-m, C+6, B-m, F6, Dm, G6,
 G6, G6, Gaug, G+m, Cm, Gaug, G6, Dm, B-m, F6, Faug, A6, G6, Gaug, G+m,
 Cm, F6, Faug, F6, Cm, F6, G6, Gaug, Gaug, B6, Gaug, G6, Gaug, Em, Gaug,
 Em, A6, F+m, B-m, F6, Cm, Gaug, Em, A6, Faug, B-m, B-m, Faug, F6, G6,
 G6, F6, Faug, F6, Dm, G6, F6, Dm, F+m, Dm, F+m, A6, Faug, F6, Faug, Dm,
 Dm, B-m, B-m, C+6, C+6, G+m, B6, A6, F+m, Faug, B-m, Dm, B-m, C+6, B-m,
 F+m, B6, Gaug, Cm, G+m, Cm, F6, F6, B-m, Dm, F6, F6, G6, Dm, G6, G6, Em,

A6, G6, Cm, Cm, G+m, B6, G+m, C+6, C+6, C+6, Faug, B-m, Dm, Dm, G6, Cm, Gaug, Cm, F6, Cm, G6, Gaug, G6, F6, Dm, F6, Faug, Faug, Faug, A6, Em, Em, G6, Dm, Faug, F6, B-m, F6, Cm, F6, B-m, F+m, Dm, G6, F6, F6, Cm, Cm, Em, G+m, Em, A6, Em, A6, F+m, B-m, B-m, B-m, F+m, B6, A6, Em, G+m, B6, B6, Em, G6, Dm, B-m, Dm, Dm, B-m, Dm, Faug, Faug, F6, Cm, G6, Gaug, B6, G+m, Em, G6, G6, Dm, Faug, Faug, F6, Cm, Gaug, G+m, Gaug, B6, F+m, A6, G6, Em, Cm, F6, Dm, Dm, Dm, G6, Em, Em, A6, Em, Gaug, Em, Cm, Cm, Gaug, G6, G6, Cm, F6, Dm, Faug, A6, Faug, A6, Faug, F+m, F+m, B-m, C+6, G+m, Em, Gaug, G6, Gaug, G6, G6, Dm, Dm, Dm, F6, B-m, F6, G6, Cm, G+m, Em, G+m, B6, G+m, Cm, Cm, F6, Faug, Faug, Faug, F6, Dm, G6, Dm, F+m, Faug, Faug, B-m, C+6, G+m, C+6, Faug, F+m, B-m, Faug, Faug, A6, G6, Em, Cm, F6, G6, Cm.

5.4 Illustrations of icosahedral Hamiltonian cycles with symmetries

The figures below illustrate the Hamiltonian cycles involved. Quite generally, the Z_n symmetry acts by a shift by $12/n$ quintus along the cycle and the orbits of chords consist of at most n chords of same type as the reader is encouraged to verify.

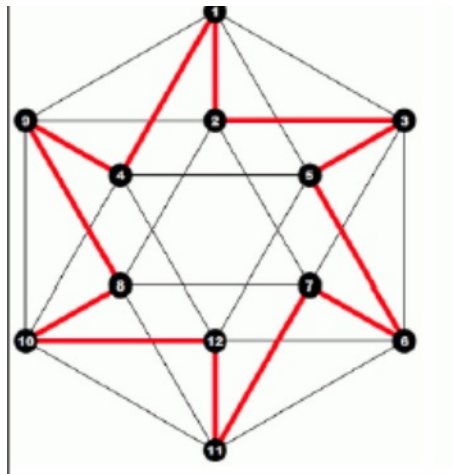


Figure 1: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 6-fold rotation symmetry acting shifts generated by a shift of 2 quintus.

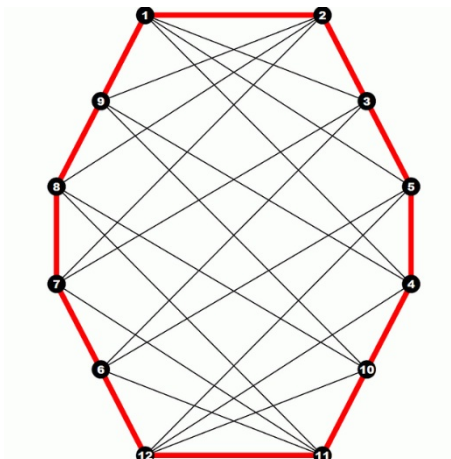


Figure 2: $(n_0, n_1, n_2) = (0, 16, 4)$ Hamiltonian cycle with 4 reflection symmetries generated by reflections in vertical and horizontal directions.

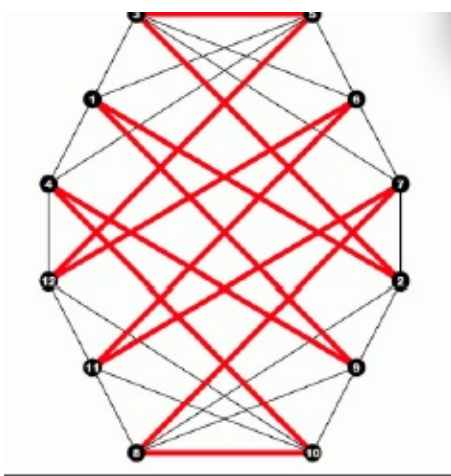


Figure 3: $(n_0, n_1, n_2) = (4, 8, 8)$ Hamiltonian cycle with 4 reflection symmetries.

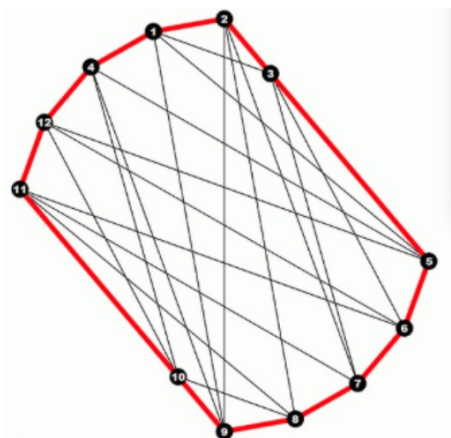


Figure 4: $(n_0, n_1, n_2) = (0, 16, 4)$ Hamiltonian cycle with 2-fold rotational symmetry realized as 6-quint shift along the cycle.

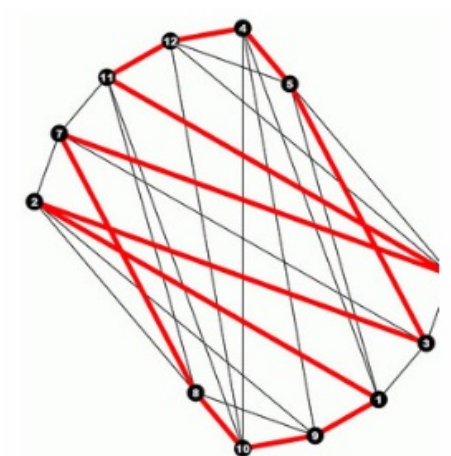


Figure 5: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 2-fold rotation symmetry.

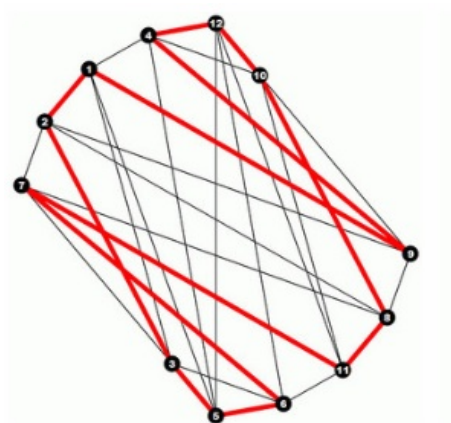


Figure 6: $(n_0, n_1, n_2) = (4, 8, 8)$ Hamiltonian cycle with 2-fold rotation symmetry.

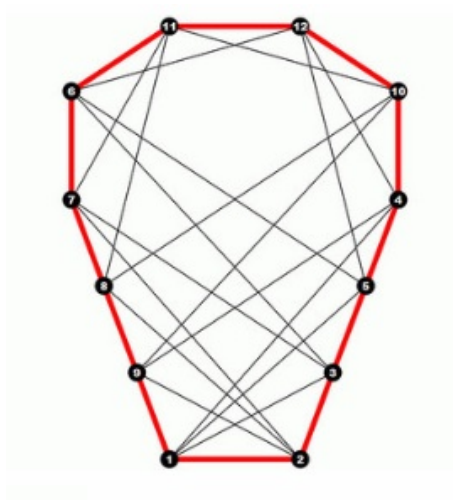


Figure 7: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 2-fold reflection symmetry realized as horizontal reflection

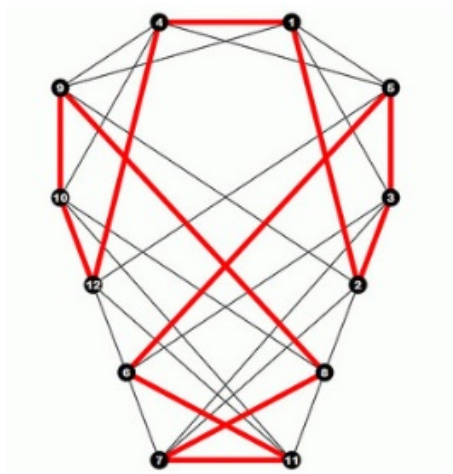


Figure 8: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 2-fold reflection symmetry.

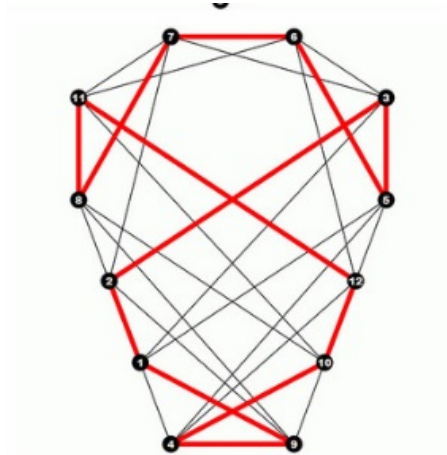


Figure 9: $(n_0, n_1, n_2) = (4, 8, 8)$ Hamiltonian cycle with 2-fold reflection symmetry.

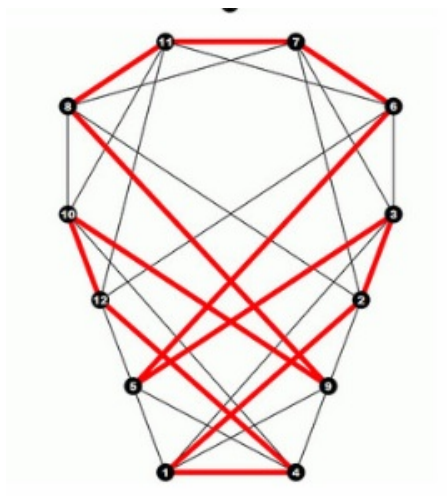


Figure 10: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 2-fold reflection symmetry.

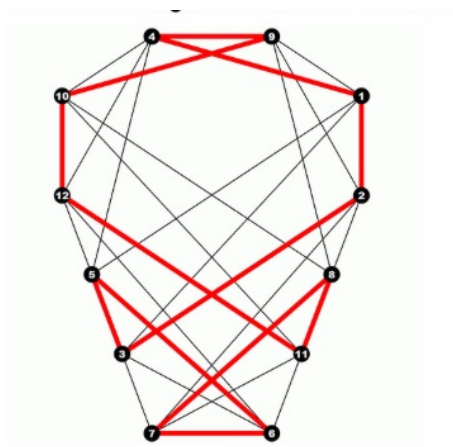


Figure 11: $(n_0, n_1, n_2) = (2, 12, 6)$ Hamiltonian cycle with 2-fold reflection symmetry.