Verification of Collatz conjecture for a positive integer px, where p is any prime number and x is an odd integer derived using Fermat's little theorem which is specific for each prime

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Abstract: Collatz conjecture states that starting with any positive integer n, we divide it by 2 if it is even or multiply it by 3 and add 1 if it is odd and repeat this algorithm on the answer always using the same odd or even rule, we will ultimately end up with an answer of 1. Here we prove this conjecture for a special integer which is the product of any prime number "p" greater than three with another positive odd integer "x" that has been derived by using the Fermat's little theorem and is therefore unique for each prime. Thus we prove Collatz's conjecture for a small fraction of positive integers "px" which would be expected to roughly represent the same proportion of integers as prime numbers.

Results:

For prime numbers 2 and 3, Collatz's conjecture is easily verified.

Let p be any prime larger than 3.

By Fermat's Little Theorem

$2^p-2=pa$, where a is a positive integer.

$2(2^{p-1}-1)=pa$ .....(I)

Since p is large prime it is odd and therefore it is of the form 2k+1 where k is a positive integer

Therefore left hand side can be written as

$2(2^{p-1}-1)= 2(2^{2k}-1)=2(2^k-1)(2^k+1)$

Since 2^k-1, 2^k,2^k+1 are three consecutive positive integers and 2^k is only divisible by 2 and therefore the product $(2^k-1)(2^k+1)$ must be divisible by 3.

Therefore $2(2^{p-1}-1)$ must be divisible by 6.

Therefore pa must be divisible by 6.

Since p is an odd prime greater than three, therefore a must be divisible by 6 and can be replaced as a=6x

Equation I can be rewritten as

$2(2^{p-1}-1)=6px$

$2^{p-1}-1=3px$

$2^{p-1}= 3px+1$

Since the Left hand side is even therefore 3px must be odd, 3,p, x all odd.

Therefore

$2^{p-1}= 3px+1$

$2^{p-1}=3(px)+1$

This resembles

$2^{p-1}=3(\text{odd integer})+1$

Therefore according to rules of Collatz's conjecture, all numbers of form "px" which is odd, would in the first step be multiplied by 3 to which 1 is added giving an even number 2^{p-1}, which is a power of 2 and therefore would be subjected to p-1 steps of even Collatz rules to give an answer of one.

Therefore all number of the form px would take exactly p steps to reduce to unity.