Longitudinal Waves in Scalar, Three-Vector Gravity

Kenneth Dalton

email: kxdalton@yahoo.com

Abstract

The linear field equations are solved for the metrical component g_{00}. The solution is applied to the question of gravitational energy transport. The Hulse-Taylor binary pulsar is treated in terms of the new theory. Finally, the detection of gravitational waves is discussed.

1. Introduction

The founders of the theory of special relativity did not use 4-vectors in their work. Lorentz, Poincare, Einstein, Planck and others made use of scalars (time interval, energy, scalar potential) and 3-vectors (spatial displacement, momentum, vector potential).[1] [1] The distinction is seen clearly in the definition of energy and momentum

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} = \frac{E}{c^2}\,\mathbf{v} \tag{1}$$

as well as in the power formula

$$\frac{dE}{ds} = \mathbf{v} \cdot \frac{d\mathbf{p}}{ds} \tag{2}$$

Energy has no directional character whatsoever.

The interval

$$ds^2 = c^2 dt^2 - d\mathbf{r}^2 \tag{3}$$

is invariant under a Lorentz transformation. At any point P, the vector $d\mathbf{r}$ is projected onto an orthonormal 3-frame: $\mathbf{i} \cdot d\mathbf{r}$, $\mathbf{j} \cdot d\mathbf{r}$, $\mathbf{k} \cdot d\mathbf{r}$. These projections, together with the time interval dt, are then transformed into new values, which are observed in a relatively moving 3-frame. No system of coordinates is involved with this procedure. Such frame transformations may take place in the presence of gravitation.

2. Scalar, three-vector gravity [2]

Gravitation is described by means of the structure in a coordinate system $\{x^\mu\}$. To this end, displacements in time and space are expressed in the form

$$c\,dt = e_0(x)dx^0 \qquad d\mathbf{r} = \mathbf{e}_i(x)dx^i \tag{4}$$

where $e_\mu = (e_0,\, \mathbf{e}_i)$ is a scalar, 3-vector basis. Substitution into (3) gives

$$\begin{aligned} ds^2 &= (e_0 dx^0)^2 - \mathbf{e}_i \cdot \mathbf{e}_j \, dx^i dx^j \\ &= g_{\mu\nu} dx^\mu dx^\nu \end{aligned} \tag{5}$$

[1] Minkowski invented the 4-vector in 1909, long after the completion of special relativity.

where

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij} & \\ 0 & & & \end{pmatrix} \tag{6}$$

is the scalar, 3-vector metric. An observer is free to introduce new coordinates $\{x^{\mu'}\}$. In order to retain the distinction between scalars and 3-vectors, the coordinate transformations are restricted to the form

$$x^{0'} = x^{0'}(x^0) \qquad\qquad x^{i'} = x^{i'}(x^j) \tag{7}$$

Displacements (4) will then be invariant, while the metric transforms as a tensor

$$g_{0'0'} = \frac{\partial x^0}{\partial x^{0'}}\frac{\partial x^0}{\partial x^{0'}}\, g_{00} \qquad\qquad g_{i'j'} = \frac{\partial x^m}{\partial x^{i'}}\frac{\partial x^n}{\partial x^{j'}}\, g_{mn} \tag{8}$$

The Christofel coefficients

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\rho} \left(\partial_\lambda g_{\nu\rho} + \partial_\nu g_{\rho\lambda} - \partial_\rho g_{\nu\lambda}\right) \tag{9}$$

yield the Ricci tensor

$$R_{\mu\nu} = \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\lambda_{\rho\nu}\Gamma^\rho_{\mu\lambda} - \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} \tag{10}$$

The gravitational field equations

$$\frac{c^4}{8\pi G}\Big(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\Big) + T^{(m)}_{\mu\nu} = 0 \tag{11}$$

derive from the Einstein-Hilbert action

$$\delta \int \frac{c^4}{16\pi G}\, g^{\mu\nu} R_{\mu\nu} \sqrt{-g}\, d^4x + \delta \int L^{(m)} \sqrt{-g}\, d^4x = 0 \tag{12}$$

There are seven field equations, corresponding to the seven variations $\delta g^{\mu\nu} = (\delta g^{00}, \delta g^{ij})$. Components R_{0i} and $T^{(m)}_{0i}$ do not appear.[2]

[2]The Birkhoff theorem in general relativity follows from the three equations involving R_{0i}. [3] These equations do not exist in the new theory, and the theorem is no longer relevant. This will have important consequences for gravitational radiation.

3. The gravitational field strength tensor

The structure of the basis system is expressed by the formula

$$\nabla_\nu e_\mu = e_\lambda Q^\lambda_{\mu\nu} \tag{13}$$

Define the conditions $Q^0_{j\nu} = Q^i_{0\nu} \equiv 0$, in order to separate (13) into scalar and 3-vector parts

$$\nabla_\nu e_0 = e_0 Q^0_{0\nu} \tag{14}$$

$$\nabla_\nu \mathbf{e}_i = \mathbf{e}_j Q^j_{i\nu} \tag{15}$$

In terms of the metrical functions (6),

$$\partial_\lambda g_{00} = 2 g_{00} Q^0_{0\lambda} \tag{16}$$

$$\partial_0 g_{ij} = g_{in} Q^n_{j0} + g_{jn} Q^n_{i0} \tag{17}$$

$$\partial_k g_{ij} = g_{in} Q^n_{jk} + g_{jn} Q^n_{ik} \tag{18}$$

If $Q^i_{jk} = Q^i_{kj}$ and if the two terms in (17) are assumed to be equal, then

$$Q^0_{0\lambda} = \Gamma^0_{0\lambda} = \frac{1}{2} g^{00} \partial_\lambda g_{00} \tag{19}$$

$$Q^i_{j0} = \Gamma^i_{j0} = \frac{1}{2} g^{in} \partial_0 g_{nj} \tag{20}$$

$$Q^i_{jk} = \Gamma^i_{jk} = \frac{1}{2} g^{in} \left(\partial_k g_{jn} + \partial_j g_{nk} - \partial_n g_{jk} \right) \tag{21}$$

Together, they comprise the formula

$$Q^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} + g^{\mu\rho} g_{\lambda\eta} Q^\eta_{[\nu\rho]} \tag{22}$$

where

$$Q^\mu_{[\nu\lambda]} \equiv Q^\mu_{\nu\lambda} - Q^\mu_{\lambda\nu} \tag{23}$$

The non-zero components of $Q^\mu_{[\nu\lambda]}$ are

$$Q^0_{[0i]} = Q^0_{0i} = \frac{1}{2} g^{00} \partial_i g_{00} \qquad Q^i_{[j0]} = Q^i_{j0} = \frac{1}{2} g^{in} \partial_0 g_{nj} \tag{24}$$

They transform as tensor components

$$Q^{0'}_{[0'i']} = \frac{\partial x^n}{\partial x^{i'}}\, Q^0_{[0n]} \qquad\qquad Q^{i'}_{[j'0']} = \frac{\partial x^{i'}}{\partial x^m}\frac{\partial x^n}{\partial x^{j'}}\frac{\partial x^0}{\partial x^{0'}}\, Q^m_{[n0]} \tag{25}$$

This field strength tensor serves to define the gravitational energy tensor

$$T^{(g)}_{\mu\nu} = \frac{c^4}{8\pi G}\left\{Q^\rho_{[\lambda\mu]}Q^\lambda_{[\rho\nu]} + Q_\mu Q_\nu - \frac{1}{2}g_{\mu\nu}g^{\eta\tau}(Q^\rho_{[\lambda\eta]}Q^\lambda_{[\rho\tau]} + Q_\eta Q_\tau)\right\} \tag{26}$$

where $Q_\mu = Q^\rho_{[\rho\mu]}$. For a static Newtonian potential ψ

$$g_{00} = 1 + \frac{2}{c^2}\psi \tag{27}$$

so that $Q^\mu_{[\nu\lambda]}$ is given by

$$Q^0_{[0i]} = \frac{1}{c^2}\,\partial_i\psi \qquad\qquad Q^i_{[j0]} = 0 \tag{28}$$

It follows that

$$T^{(g)}_{00} = \frac{1}{8\pi G}(\nabla\psi)^2 \tag{29}$$

$$T^{(g)}_{0i} = 0 \tag{30}$$

$$T^{(g)}_{ij} = \frac{1}{4\pi G}\left\{\partial_i\psi\,\partial_j\psi - \frac{1}{2}\delta_{ij}(\nabla\psi)^2\right\} \tag{31}$$

which is the Newtonian stress-energy tensor.

The field strength tensor also plays a crucial role in particle dynamics. The planetary equations of motion

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda}u^\nu u^\lambda = 0 \tag{32}$$

follow from the variation

$$\delta\int\sqrt{g_{\mu\nu}u^\mu u^\nu}\,ds = 0 \tag{33}$$

where $u^\mu = dx^\mu/ds$. Gravitational force and power are calculated by expressing the energy and momentum (1) in terms of coordinates

$$E = mc^2 e_0 u^0 \qquad\qquad \mathbf{p} = mc\;\mathbf{e}_i u^i \tag{34}$$

The rate of change of $e_\mu u^\mu$ is

$$\frac{d(e_\mu u^\mu)}{ds} = e_\mu \frac{du^\mu}{ds} + \frac{de_\mu}{ds} u^\mu = e_\mu \Big\{ \frac{du^\mu}{ds} + Q^\mu_{\nu\lambda} u^\nu u^\lambda \Big\} \tag{35}$$

where $de_\mu = e_\lambda Q^\lambda_{\mu\nu} dx^\nu$. Substitute (22) and then make use of the equation of motion (32) to obtain

$$\frac{d(e_\mu u^\mu)}{ds} = e^\mu g_{\lambda\eta} Q^\eta_{[\nu\mu]} u^\nu u^\lambda \tag{36}$$

Separate this formula into scalar and 3-vector parts, then substitute the tensor components (24) to find that the energy and momentum change as follows:

$$\frac{dE}{ds} = e^0 \frac{mc^2}{2} \Big\{ -\partial_n g_{00}\, u^n u^0 + \partial_0 g_{mn}\, u^m u^n \Big\} \tag{37}$$

$$\frac{d\mathbf{p}}{ds} = \mathbf{e}^i \frac{mc}{2} \Big\{ \partial_i g_{00}\, u^0 u^0 - \partial_0 g_{in}\, u^0 u^n \Big\} \tag{38}$$

These equations are invariant under the coordinate transformations (7). They express the power and force which are exerted by the gravitational field. In the Newtonian limit (27), $u^0 = 1$ and $u^n = v^n/c$ so that

$$\frac{dE}{dt} = -m\, \nabla\psi \cdot \mathbf{v} \qquad\qquad \frac{d\mathbf{p}}{dt} = -m\, \nabla\psi \tag{39}$$

4. The linear field equations

If the coordinate system is nearly rectangular, then the metric tensor may be expanded

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{40}$$

where the absolute values of $h_{\mu\nu}$ are small compared with unity. The largest terms in the Ricci tensor (10) are

$$R_{\mu\nu} = \frac{1}{2} \Big\{ \eta^{\lambda\rho} \partial_\lambda \partial_\rho h_{\mu\nu} + \partial_\mu \partial_\nu h^\lambda_{\ \lambda} - \partial_\mu \partial_\lambda h^\lambda_{\ \nu} - \partial_\nu \partial_\lambda h^\lambda_{\ \mu} \Big\} \tag{41}$$

with time and space components

$$R_{00} = \frac{1}{2}\Big\{\partial^n\partial_n\, h_{00} + \partial_0\partial_0\, h^n_n\Big\} \tag{42}$$

$$R_{ij} = \frac{1}{2}\Big\{\eta^{\lambda\rho}\partial_\lambda\partial_\rho h_{ij} + \partial_i\partial_j(h^0_0 + h^n_n) - \partial_i\partial_n h^n_j - \partial_j\partial_n h^n_i\Big\} \tag{43}$$

The single condition

$$h^0_0 = h^n_n \tag{44}$$

gives

$$R^0_0 = \frac{1}{2}\,\partial^\lambda\partial_\lambda h^0_0 \tag{45}$$

Rewrite the field equations in the form

$$R^\nu_\mu = -\frac{8\pi G}{c^4}\left(T^{(m)\nu}_\mu - \frac{1}{2}\,\delta^\nu_\mu\, T^{(m)}\right) \tag{46}$$

in order to obtain

$$\partial^\lambda\partial_\lambda h^0_0 = -\frac{8\pi G}{c^4}\left(T^{(m)0}_0 - T^{(m)n}_n\right) \tag{47}$$

This equation is solved by

$$h^0_0(\mathbf{x}, t) = -\frac{2G}{c^4}\int \frac{(T^{(m)0}_0 - T^{(m)n}_n)}{|\mathbf{x} - \mathbf{x}'|}\,|_{\rm ret}\, d^3x' \tag{48}$$

where the retarded solution is chosen.

If the material energy tensor is $T^{(m)\nu}_\mu = \rho c^2 u_\mu u^\nu$, then $T^{(m)} = \rho c^2$ and $(T^{(m)0}_0 - T^{(m)n}_n) = (\rho c^2 - 2T^{(m)n}_n)$. In regions very far from the source, (48) takes the form

$$h^0_0(\mathbf{x}, t) = -\frac{2G}{c^4 r}\int (\rho c^2 - 2T^{(m)n}_n)\,|_{\rm ret}\, d^3x' \tag{49}$$

The first integral is the rest energy, while the second may be transformed by means of the identity [4]

$$\int T^{ij}\, d^3x = \frac{1}{2}\int x^i x^j\, \partial_k\partial_l T^{kl}\, d^3x \tag{50}$$

The conservation law $\partial_\mu T^{\mu\nu} = 0$ gives $\partial_k \partial_l T^{kl} = \partial_0 \partial_0 T^{00}$, and it follows that[3]

$$\begin{aligned} \int T^{(m)n}_n \, d^3x &= -\frac{1}{2} \int r^2 \, \partial_0 \partial_0 T^{(m)00} \, d^3x \\ &= -\frac{1}{2} \frac{d^2}{dt^2} \int r^2 \rho \, d^3x = -\frac{1}{2} \frac{d^2 I}{dt^2} \end{aligned} \tag{51}$$

Expression (49) becomes

$$h^0_0(\mathbf{x}, t) = -\frac{2G}{c^4 r} \Big(Mc^2 + \frac{d^2 I}{dt^2} |_{\text{ret}} \Big) \tag{52}$$

In regions far from the source, the equations $R_{\mu\nu} = 0$ yield plane wave solutions. It was shown in [2] that along the x^3-axis, the following components satisfy the wave equation

$$h^0_0 = h^3_3 \quad h^1_1 = -h^2_2 \quad h^1_2 = h^2_1 \tag{53}$$

while $h^2_3 = h^3_1 = 0$. The gravitational energy current is given by (26)

$$\begin{aligned} T^{(g)}_{0i} &= \frac{c^4}{8\pi G} \Big(Q^0_{[0n]} Q^n_{[i0]} + Q^n_{[n0]} Q^0_{[0i]} \Big) \\ &= \frac{c^4}{32\pi G} \Big(\partial_n h^0_0 \, \partial_0 h^n_i + \partial_i h^0_0 \, \partial_0 h^n_n \Big) \end{aligned} \tag{54}$$

The presence of g_{00} is especially significant: there can be no flux of gravitational energy without a spatially dependent component g_{00}. For the plane waves

$$T^{(g)}_{03} = \frac{c^4}{16\pi G} \, \partial_3 h^0_0 \, \partial_0 h^3_3 = -\frac{c^4}{16\pi G} (\partial_0 h^0_0)^2 \qquad (k_3 = -k_0) \tag{55}$$

Therefore, the longitudinal field accounts for *all* of the energy transport in the wave zone. In formula (52), the rate of change of rest mass is negligible, leaving

$$\partial_0 h^0_0 = -\frac{2G}{c^5 r} \frac{d^3 I}{dt^3} \tag{56}$$

---

[3] $T^{(g)}_{\mu\nu}$ may be ignored here, because it is second order in the $h_{\mu\nu}$ and is much smaller than $T^{(m)}_{\mu\nu}$.

Substitution into (55) gives

$$\frac{1}{r^2}\frac{dP}{d\Omega} = \frac{c}{2}T_0^{(g)3} = \frac{G}{8\pi r^2 c^5}\left(\frac{d^3 I}{dt^3}\right)^2 \tag{57}$$

and the total power

$$\frac{dE}{dt} = \frac{G}{2c^5}\left(\frac{d^3 I}{dt^3}\right)^2 \tag{58}$$

In a binary system, d^3I/dt^3 is given by [4]

$$\frac{d^3 I}{dt^3} = -\frac{2m_1 m_2}{a(1-e^2)}\, e\,\sin\theta\,\dot{\theta} \tag{59}$$

Substitution into (58) gives the energy loss

$$-\frac{dE}{dt} = \frac{2Gm_1^2 m_2^2}{c^5 a^2 (1-e^2)^2}\, e^2 \sin^2\theta\,\dot{\theta}^2 \tag{60}$$

while the average over one period is

$$\left\langle -\frac{dE}{dt}\right\rangle = \frac{Gm_1^2 m_2^2 (m_1+m_2)}{c^5 a^5}\, e^2 (1+\frac{e^2}{4})(1-e^2)^{-7/2} \tag{61}$$

Inserting the stated parameter values for the Hulse-Taylor pulsar, this formula yields a rate $\dot{T}/T$ which is smaller than the observed rate by a factor of 30. The disparity could be due to uncertainty in the orbital parameters. However, another physical process might contribute to the energy loss, such as the emission of cosmic rays. In this regard, the Crab pulsar was recently found to emit gamma rays, with far greater energy than previously thought possible. [5]

5. Concluding remarks

A supernova explosion should generate an intense burst of spherical gravitational waves. According to (38), the radiation field (53) will produce a force given by

$$\frac{d\mathbf{p}}{dt} = -\frac{mc^2}{2}\Big\{\mathbf{i}_1\Big(\partial_0 h^1_{\ 1}\,\frac{v^1}{c} + \partial_0 h^1_{\ 2}\,\frac{v^2}{c}\Big) + \mathbf{i}_2\Big(\partial_0 h^2_{\ 1}\,\frac{v^1}{c} + \partial_0 h^2_{\ 2}\,\frac{v^2}{c}\Big) + \mathbf{i}_3\Big(\partial_3 h^0_{\ 0} + \partial_0 h^3_{\ 3}\,\frac{v^3}{c}\Big)\Big\} \tag{62}$$

The transverse forces are velocity-dependent. Therefore, a detector which is at rest can only respond to the longitudinal wave. Its acceleration will be along the direction of propagation

$$\frac{d^2x^3}{dt^2} = -\frac{c^2}{2}\,\partial_3 h^0_{\ 0} = -\frac{G}{c^3 r}\,\frac{d^3 I}{dt^3}\Big|_{\rm ret} \tag{63}$$

where I is the supernova's moment of inertia (see (52)). In view of what has been learned during the experiments at LIGO, GEO and VIRGO, it would be desirable to conduct a search for longitudinal waves.

References

1. E.T. Whittaker, *A History of the Theories of the Aether and Electricity* Vol. 2 (Dover, New York, 2nd ed. 1989).

2. K. Dalton, "Gravitational Waves," *Hadronic J.* **35**(2), 209-220 (2012).

3. C. Misner, K. Thorne, and J. Wheeler, *Gravitation*, (Freeman, New York, 1973) section 32.2.

4. N. Straumann, *General Relativity and Relativistic Astrophysics*, (Springer, 1984) sections 4.5, 5.6.

5. N. Otte et. al., "Detection of Pulsed Gamma Rays Above 100 GeV from the Crab Pulsar," *Science* **334**(6052), 69-72 (2011).