Frequency gauged clocks on a free fall grid and some gravitational phenomena

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Abstract

Using frequency gauged clocks on a free fall grid we look at gravitational phenomena as they appear for observers on a stationary grid in a central field of gravity. With an approach based on Special Relativity, the Weak Equivalence Principle and Newton's gravitational potential we derive first order correct expressions for the gravitational red shift of stationary clocks and of satellites. We also derive first order correct expressions for the geodetic precession, the Shapiro delay basis and the gravitational index of refraction, so phenomena connected to the curvature of the metric. Our approach is pragmatic and inherently limited but, due to its simplicity, it might be useful as an intermediate in between SR and GR.

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I. USING FREQUENCY GAUGED CLOCKS ON A FREE FALL GRID

A. Some basic assumptions used in the paper

The most basis assumption made in this paper is the existence of frequency gauged clocks that emit frequency gauged photons. The frequency gauged clocks use the atomic standard of time that is based on a transition between two energy levels of an atom. The frequency gauged photon is emitted in this transition process. All clocks and photons in the paper are assumed to be frequency gauged using an equivalent of the 2006 SI standard:

> The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. It follows that the hyperfine splitting in the ground state of the caesium 133 atom is exactly 9 192 631 770 hertz, (hfs Cs) = 9 192 631 770 Hz. [1]

The highest relative accuracy of atomic clocks at the time has a fractional frequency inaccuracy of 10^{-17} [2]. Theoretically, the SI procedure could be applied at this relative accuracy, so all our clocks and photons can in principle be frequency gauged to this accuracy. And because the whole paper is about frequencies, the clocks do not have to be gauged to an absolute flow of time. Only time differences or intervals matter.

The second basic assumption used in this paper is the universal validity of the Weak Equivalence Principle (WEP). According to Ohanian, "the weak principle asserts that *in a given gravitational field all test particles of the same initial velocity fall with the same acceleration*". [3] In such a free-fall, like in an Einstein elevator, we assume the following:

> Inside a freely falling elevator, as long as the field is uniform (locally), they would be subject to zero total force, which is equivalent to being inside an elevator at rest in empty space (or moving with uniform velocity), in which case there would be no frequency shift. [4]

This implies that an atomic clock placed in an Einstein Elevator at rest in infinity and then set on a free fall trajectory towards a central mass M would all the way down to M remains at the same initial rest system frequency. And according to Will, tests or the WEP have reached accuracies comparable to the accuracy of atomic clocks:

> The Eöt-Wash experiments carried out at the University of Washington used a sophisticated torsion balance tray to compare the accelerations of various materials toward local topography on Earth, movable laboratory masses, the Sun and the galaxy, and have reached levels of $3 \cdot 10^{-13}$. [5]

So we will assume that the atomic clocks in free fall "Einstein Elevators" can in principle remain frequency gauged to a relative accuracy of around 10^{-13}. In this way we can, in principle, establish a free fall grid of highly accurate frequency gauged clocks capable of emitting highly accurate frequency gauged photons. As a consequence of the weak equivalence principle, the laws of Special Relativity remain valid in the Einstein Elevator on a free fall trajectory from infinity towards a central mass M.

In this paper we will not use the Strong Equivalence Principle (SEP), understood as the equivalence between being at rest in a field of gravity and being in a state of constant acceleration in free space. In Schild's short 1960 expression, SEP means that "acceleration is equivalent to a gravitational field". [6] The fact that we do not use it doesn't imply that we criticize it, it only means that we don't need it in this paper.

Before we use a grid of frequency gauged clocks on a free fall system we have to look at the principles of using a frequency gauged grid of relativistic clocks in a non-gravity Minkowskian space-time environment.

B. A grid of frequency gauged clocks

Assume that we have a grid of frequency gauged clocks on the one hand and a set of separate clocks A and B that are not on this grid on the other hand. We do not know how the other clocks A and B are ticking relative to each other but we have a procedure of how to relate the frequencies of the individual other clocks A and B to our grid. Then we are also able to compare the other clocks relative to each other. Let the frequency of all the clocks on our grid be ν_g and let there be two clocks A and B that are not on our grid and have frequencies ν_a and ν_b. Suppose we know how to relate the frequency of clocks on the grid to the frequencies of clocks A and B as in

$$\frac{\Delta\nu_{ag}}{\nu_a} = \frac{\nu_a - \nu_g}{\nu_a} = 1 - \frac{\nu_g}{\nu_a} \tag{1}$$

and

$$\frac{\Delta\nu_{bg}}{\nu_b} = \frac{\nu_b - \nu_g}{\nu_b} = 1 - \frac{\nu_g}{\nu_b}. \tag{2}$$

Then we also know how to relate the frequency of clock A directly to the one of clock B. We have

$$\begin{aligned}\frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b} &= \frac{\nu_a - \nu_g}{\nu_a} - \frac{\nu_b - \nu_g}{\nu_b} = \frac{\nu_b(\nu_a - \nu_g) - \nu_a(\nu_b - \nu_g)}{\nu_a\nu_b} \\ &= \frac{\nu_b\nu_a - \nu_b\nu_g - \nu_a\nu_b + \nu_a\nu_g}{\nu_a\nu_b} = \frac{\nu_a - \nu_b}{\nu_a} \cdot \frac{\nu_g}{\nu_b} = \frac{\Delta\nu_{ab}}{\nu_a} \cdot \frac{\nu_g}{\nu_b}, \end{aligned} \tag{3}$$

leading to the general formula to relate the frequency shifts of two clocks relative to each other using the frequency shifts of each of these clocks relative to the frequency of clocks on a grid of frequency gauged clocks:

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b}}{\frac{\nu_g}{\nu_b}}. \tag{4}$$

Next, imagine that we know how to Lorentz boost clocks from the grid to a position at rest right next to the off-the-grid clocks A and B with frequencies ν_a and ν_b. From Special Relativity we have

$$\nu_a = \frac{1}{\gamma_a}\nu_g \tag{5}$$

with the Lorentz boost factor

$$\gamma_a = \frac{1}{\sqrt{1 - \frac{v_a^2}{c^2}}}. \tag{6}$$

leading to

$$\frac{\Delta\nu_{ag}}{\nu_a} = \frac{\nu_a - \nu_g}{\nu_a} = 1 - \frac{\nu_g}{\nu_a} = 1 - \gamma_a. \tag{7}$$

For the clock B we have equal equations relative to our grid. We insert this Lorentz boost knowledge into Eqn.(4) to get

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b}}{\frac{\nu_g}{\nu_b}} = \frac{(1-\gamma_a) - (1-\gamma_b)}{\gamma_b} = \frac{\gamma_b - \gamma_a}{\gamma_b}. \tag{8}$$

Of course Eqn.(5) for clocks A and B can be used to arrive faster at the same result

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\nu_a - \nu_b}{\nu_a} = \frac{\frac{1}{\gamma_a}\nu_g - \frac{1}{\gamma_b}\nu_g}{\frac{1}{\gamma_a}\nu_g} = \frac{\frac{1}{\gamma_a} - \frac{1}{\gamma_b}}{\frac{1}{\gamma_a}} = \frac{\gamma_b - \gamma_a}{\gamma_b}. \tag{9}$$

If, from the perspective of the grid, we know the relativistic kinetic energy of clocks A and B, then the Lorentz boost connection between these clocks and the grid is also known through the SR definition

$$U_k = (\gamma - 1)U_0 \tag{10}$$

with rest energy $U_0 = m_0c^2$. Then it is not to difficult to calculate the frequency shift between clocks A and B.

C. From Free Fall Grid to Stationary Grid

We use the universality of free fall and the weak equivalence principle to define a grid of frequency gauged clocks on a free fall grid around a central mass M starting at rest in infinity and stretching all the way to just above the surface of this central mass. On this grid we have an infinite number of Einstein elevators in perfect free fall and small enough as to represent local Lorentz frames of reference. All the clocks at rest in the Einstein Elevators on this free fall grid (FFG) have rest system frequency $\nu_g = \nu_0 = \nu_\infty$. The free fall of all the elevators on our free fall grid started at rest in infinity, ensuring a shared starting frequency ν_0. Due to the experimental fact of the weak equivalence principle, our clocks on free fall trajectories towards M do not feel any acceleration and thus remain all the way down frequency gauged to the clocks at rest in infinity, because without acceleration there is no Lorentz boost and without Lorentz boost the clock frequency will not change. Thus our FFG of figure (1) constitutes a perfect example of a grid of frequency gauged clocks.

This free fall grid however is not very practical in performing scientific experiments. We quote Rohrlich, who started a 1963 paper on the principle of equivalence with the words:

> Unfortunately, laboratory experiments are not usually performed in falling elevators. (Footnote 1) They are carried out in reference frames which are not inertial, but which are supported in a static gravitational field. Footnote 1: According to general relativity, the special theory of relativity is valid only locally in freely falling reference frames. [11]

Most scientific experiments are carried out on a stationary grid in a static gravitational field. We position clocks A and B in Einstein Elevators on such a Stationary Grid (SG), so at rest at a definite radial distance from the center of M. The observers A and B feel the pull of gravity induced acceleration on their clocks. Using the conservation of energy and Special Relativity we can relate the clocks A and B on the SG to clocks on the FFG. Relative to the stationary clocks the Einstein Elevator in free fall has relativistic kinetic $U_k = (\gamma - 1)U_0$ and potential energy $U_\phi = m_0\Phi$ which, due to energy conservation, relate as $U_k = -U_\phi$ because

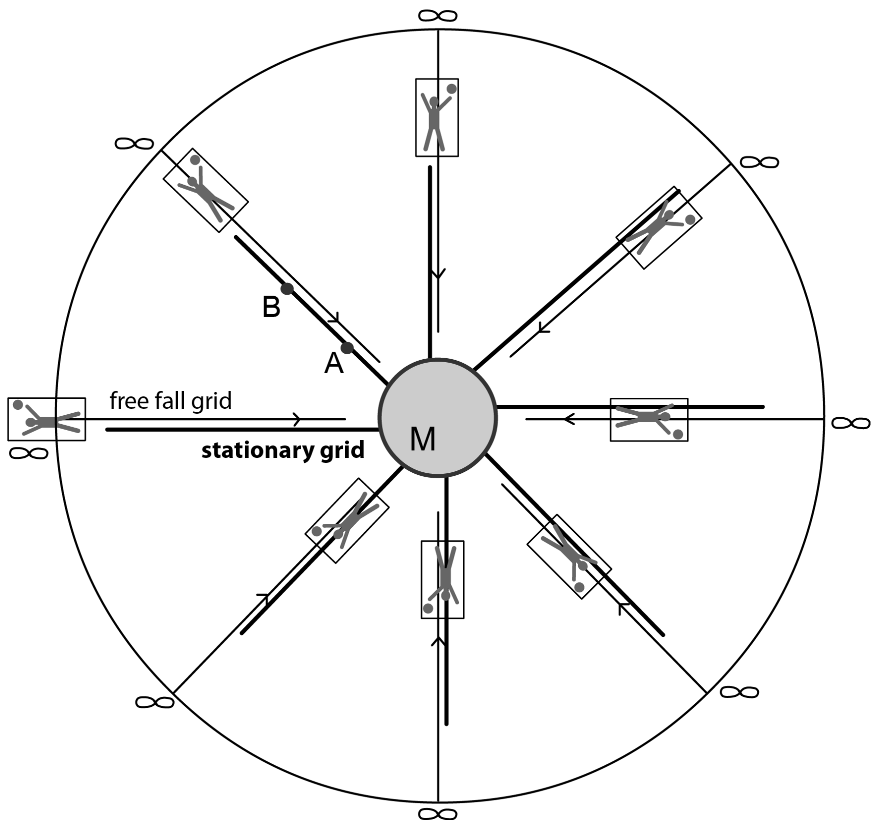

FIG. 1. Elevators in free fall on he FFG and stationary points A and B on the SG, relative to a central mass M on the one hand and elevators at rest at infinity in free space on the other hand.

the free fall started at infinity. This results in the Lorentz boost connection between a locally passing by elevator on the FFG and a clock on the SG as

$$\gamma_\phi = 1 - \frac{\Phi}{c^2} \tag{11}$$

A passing by elevator on the FFG can always position or launch a clock from his elevator next to a clock on the SG by such a Lorentz boost. This assures the relations between clocks on the SG and clocks on the FFG, with the latter functioning as a grid of frequency gauged clocks.

D. Relative redshift of clocks on the SG

A clock at rest in infinity, so on the FFG, then has a frequency shift relative to the clock A on the SG given by

$$\frac{\Delta\nu_{ag}}{\nu_a} = 1 - \gamma_a = \frac{\Phi_a}{c^2} = -\frac{GM}{R_a c^2} < 0 \tag{12}$$

so clock A stationary on the SG at A has a lower frequency than a clock stationary on the FFG at infinity. But being on the FFG at infinity is equivalent to being at the SG at infinity, both are at rest in a zero gravity environment.

The clock at B, located higher in the field, has a similar frequency shift relative to the FFG, with R_b replacing R_a, and thus the frequency shift of clock B relative to clock A is given by

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\gamma_b - \gamma_a}{\gamma_b} = \frac{(1-\frac{\Phi_b}{c^2}) - (1-\frac{\Phi_a}{c^2})}{1-\frac{\Phi_b}{c^2}} = \frac{\frac{\Phi_a}{c^2} - \frac{\Phi_b}{c^2}}{1-\frac{\Phi_b}{c^2}} = \frac{\frac{\Delta\Phi_{ab}}{c^2}}{1-\frac{\Phi_b}{c^2}} \approx \frac{\Delta\Phi_{ab}}{c^2}. \tag{13}$$

If we insert Newton's expression for the gravitational potential in Φ we get

$$\Delta\Phi_{ab} = \Phi_a - \Phi_b = -\frac{GM}{R_a} + \frac{GM}{R_b} = GM\left(\frac{R_a - R_b}{R_a R_b}\right) = -\frac{GM}{R_a R_b}h \tag{14}$$

When clocks A and B are close to each other relative to the distance R to the center of M, we get

$$\frac{\Delta\nu_{ab}}{\nu_a} \approx \frac{\Delta\Phi_{ab}}{c^2} \approx -\frac{gh}{c^2} < 0 \tag{15}$$

with h as the magnitude of the distance between the clocks A and B. As a result, the frequency of the clock at A will be lower than the frequency of the clock B who is positioned less deep in the gravitational field than clock A.

E. Relative redshift of photons exchanged on the SG

Now this has been formulated relative to the FFG and using clocks only. If clocks A and B exchange photons, these photons will travel, in the perspective of the observers on the FFG, with velocity c through gravity free space, due to free fall, in between the clocks A and B. According to the FFG observers, the frequency of these photons will not change during the voyage through the space in between A and B and the perceived relative frequency shift is solely due to the different rates of the clocks used to send, absorb or observe these photons.

In the perspective of the stationary observers A and B on the SG however, things look like these photons travel into, from B to A, or out of, from A to B, the field of gravity. The occurring relative frequency shift of the exchanged photons can be interpreted in terms of energies using Planck's constant in $U = h\nu$ and the apparent Compton mass of photons $m_c c^2 = h\nu$ as

$$\frac{h\Delta\nu_{ab}}{h\nu_a} \approx \frac{\Delta m_c \Delta\Phi_{ab}}{m_a c^2} \approx \frac{gy\Delta m}{m_a c^2} \tag{16}$$

or as

$$h\Delta\nu_{ab} = gy\Delta m, \tag{17}$$

with the relative high now given by the variable y. So photons traveling out of the field from A to B gain gravitational energy and lose photonic energy in the same rate, with, in absolute terms, $\Delta U_\phi = \Delta U_\nu$. For the SG observers it is as if the field of gravity is performing work upon the apparent Compton mass of the photons traveling between A and B, blue-shifting them while falling into the field from B to A and red-shifting them when moving out of the field from A to B.

This interpretation of the field of gravity performing work on the photons moving in or out of the field has always been a matter of controversy. If one accepts that the clocks sending out these photons are frequency shifted themselves, and accepts the fact that the photons arrive with exactly this clock frequency shift at a higher location in the field, then it is the clocks and not the field that produces the effect on the photons. But a bundle of photons send from A towards B and then moving on uninterrupted towards C higher in the field will be shifted in between B and C with exactly the apparent gravitational energy loss of the photon's Compton mass determined by the high of C relative to B. And in between B and C, the photon has not been in contact with the clock in A. So for local stationary observers on the SG, the gravitational redshift of photons seems to be a localized effect of the gravitational potential on the photons.

Dicke in 1960 concluded that there might be two red shift effects.

> One would be interpreted in the usual way as a light propagation effect. The other, if it exists, would be interpreted as resulting from an intrinsic change in an atom with gravitational potential. [7]

In 1986, Clifford Will states the same dilemma as

> Do the intrinsic rates of the emitter and the receiver or of the clocks change, or is it the light signal that changes frequency during its flight? The answer is that it doesn't matter. Both descriptions are physically equivalent. Put differently there is no operational way to distinguish between the two descriptions. [8]

About one and a half decades later, Okun, Selivanov and Telegdi express the opinion that only one of the descriptions is the right on, and that the other one is the wrong explanation of the red shift of the photon

On the one hand, the phenomenon is explained through the behavior of clocks which run faster the higher they are located in the potential, whereas the energy and frequency of the propagating photon do not change with height. The light thus appears to be red-shifted relative to the frequency of the clock. On the other hand, the phenomenon is alternatively discussed (even in some authoritative texts) in terms of an energy loss of a photon as it overcomes the gravitational attraction of the massive body. This second approach operates with notions such as the "gravitational mass" or the "potential energy" of a photon and we assert that it is misleading. [9]

So, from 1960 to 2000, the 'normal' explanation of the red shift of photons has shifted almost 180 degrees. In our perspective, the two interpretations are not either or related. On the FFG the photons are not influenced by gravity, only the clocks are. On the SG, the influence of the gravity potential on the frequency of photons seems the most natural interpretation of the photon redshift. But on the SG, things will become more complicated as we focus on the velocity of the photons in the perspective of the SG observers.

F. The FFG link between an orbiting satellite and Earth

In [10] we used hyperbolic relativity to derive the Lorentz boost connection between an observer locally passing by on a free fall grid and an orbiting satellite. We showed that two successive boosts could launch a satellite from the FFG elevator in a stable orbit around M. The first boost gave the satellite an escape amount of kinetic energy U_{esc} relative to the free fall elevator and the second boost gave it an orbital kinetic energy U_{orb}. Using relativistic kinetic energy U_k and the conservation of energy and the energy formulation of the virial theorem we get

$$\gamma_{esc} = 1 - \frac{\Phi}{c^2} \tag{18}$$

from the conservation of energy and from the virial theorem we get

$$\gamma_{orb} = 1 - \frac{\Phi}{2c^2}. \tag{19}$$

Under the condition that the two boosts are perpendicular relative to each other this results in the Lorentz boost connection between the FFG and the satellite as

$$\gamma_{sat} = \gamma_{esc}\gamma_{orb} = \left(1 - \frac{\Phi}{c^2}\right)\left(1 - \frac{\Phi}{2c^2}\right) = 1 - \frac{3\Phi}{2c^2} + \frac{\Phi^2}{2c^4} \tag{20}$$

The relative frequency shift between a stationary earth clock's ν_e and a satellite clock's ν_s is then given by

$$\frac{\Delta\nu_{es}}{\nu_e} = \frac{\gamma_s - \gamma_e}{\gamma_s} = \frac{(1 - \frac{3\Phi_s}{2c^2} + \frac{\Phi_s^2}{2c^4}) - (1 - \frac{\Phi_e}{c^2})}{1 - \frac{3\Phi_s}{2c^2} + \frac{\Phi_s^2}{2c^4}} = \frac{\frac{\Phi_e}{c^2} - \frac{3\Phi_s}{2c^2} + \frac{\Phi_s^2}{2c^4}}{1 - \frac{3\Phi_s}{2c^2} + \frac{\Phi_s^2}{2c^4}} \approx \frac{\frac{\Phi_e}{c^2} - \frac{3\Phi_s}{2c^2}}{1 - \frac{3\Phi_s}{2c^2}} \tag{21}$$

We can use the further approximation $1 - \frac{3\Phi_s}{2c^2} \approx 1$ to get

$$\frac{\Delta\nu_{es}}{\nu_e} \approx \frac{\Phi_e}{c^2} - \frac{3\Phi_s}{2c^2} = \frac{\Phi_e}{c^2} - \frac{\Phi_s}{c^2} - \frac{\Phi_s}{2c^2} = \frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2} = \frac{\Delta U_{\phi,s}}{U_0} + \frac{U_{k,s}}{U_0} \tag{22}$$

where in the last step we inserted the rest mass of the satellite and used the expressions for the potential energy and the relativistic kinetic energy of the satellite relative to the stationary Earth observers.

G. Limitations of the FFG approach

Our approach based on the use of the free fall grid as a grid of perfectly frequency gauged clocks is not a fundamental theory but a semi-phenomenological approach with inherent limitations. The approach isn't stronger than the assumptions on which it is constructed.

We can make a prediction based on our approach that we expect to be falsified, that should be falsified if General Relativity is correct. According to our analysis we should have a gravitational redshift between two stationary clocks at different heights of

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\gamma_b - \gamma_a}{\gamma_b} = \frac{\frac{\Phi_a}{c^2} - \frac{\Phi_b}{c^2}}{1 - \frac{\Phi_b}{c^2}} = \frac{\frac{\Delta\Phi_{ab}}{c^2}}{1 - \frac{\Phi_b}{c^2}} \approx \left(1 + \frac{\Phi_b}{c^2}\right)\frac{\Delta\Phi_{ab}}{c^2} = (1+\alpha)\,\frac{\Delta\Phi_{ab}}{c^2}. \tag{23}$$

But according to General Relativity, the factor α in the last expression should be identical zero. In the Earth bound situation, $\alpha \approx 10^{-10}$ according to our analysis. Present day accuracy of this redshift goes to $\alpha < 10^{-6}$. Within some decades the accuracy of the stationary redshift measurements should reach the 10^{-10} relative accuracy and we expect the limitations of one or more of our assumptions to become apparent.

For the redshift of a satellite in orbit relative to a stationary ground station, we have the interesting

$$\frac{\Delta\nu_{es}}{\nu_e} \approx \frac{\frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2}}{1 - \frac{3\Phi_s}{2c^2}} \approx \left(1 + \frac{3\Phi_s}{2c^2}\right)\left(\frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2}\right) = (1+\alpha)\left(\frac{\Delta U_{\phi,s}}{U_0} + \frac{U_{k,s}}{U_0}\right) \tag{24}$$

and in this case the α term could be interpreted as the de Sitter correction term in the redshift due to the curvature of the orbit of the satellite. In paper [10] we derived the

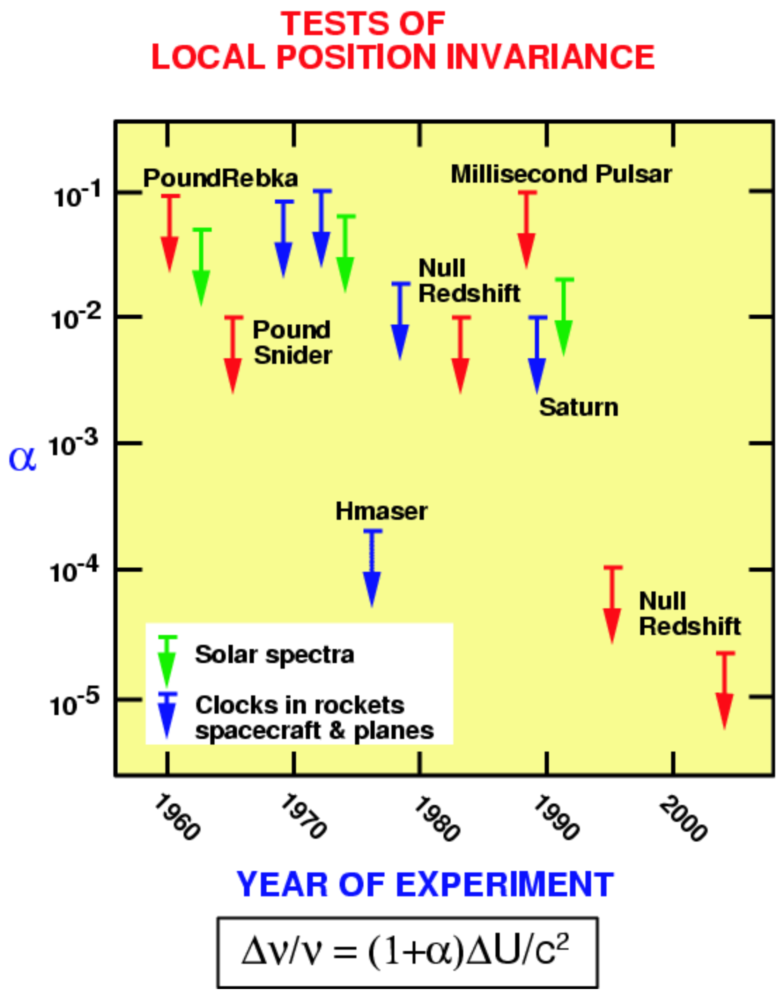

FIG. 2. Selected tests of local position invariance via gravitational redshift experiments, showing bounds on , which measures degree of deviation of redshift from the formula $\frac{\Delta\nu}{\nu} = \frac{\Delta\Phi}{c^2}$. In null redshift experiments, the bound is on the difference in between different kinds of clocks. From: www.livingreviews.org [5].

geodetic precession or the de Sitter precession using the free fall grid approach as

$$\Omega_G = (\gamma_s - 1)\Omega_s \approx -\frac{3\Phi_s}{2c^2}\Omega_s \tag{25}$$

which makes the interpretation of the α term as a de Sitter correction for a satellite redshift consistent.

As for our assumptions, we used the weak equivalence principle and the related universality of free fall principle. This has been experimentally tested with a 10^{-12} relative accuracy. We also used the kinetic energy in its Special Relativity formulation as $U_k = (\gamma - 1)U_0$. Then we assumed the gravitational mass to be equal to the rest mass in our energy considerations, by using $U_\phi = m_0\Phi$, thus ignoring all gravitational self energy complications. We used Newton's gravitational scalar potential, where in more complex situations tensor potentials, metric tensors, are needed. We assumed the virial theorem for a satellite to remain valid in relativistic contexts, by keeping its energy formulation classical. Somewhere on the line towards higher accuracy, higher velocities or stronger fields of gravity parts of these assumptions have to fail. It should be weird if they wouldn't fail. Nevertheless, our pragmatic approach has the value of simplicity.

II. THE VELOCITY OF LIGHT ON THE STATIONARY GRID AND SPACE CURVATURE

A. The velocity of light on the free fall grid

The velocity of light on the free fall grid is by definition the Special Relativity velocity of light c_0 because the Einstein Elevator has started its free fall from an at rest in Minkowskian free space in infinity position. During the free fall, all the laws of SR remain valid within the local area of the Einstein Elevator because it starts as and remains an inertial system.

This means that a photon bouncing between the ceiling and the floor of the Einstein Elevator on the free fall grid will be observed by persons in the elevator as moving with constant velocity of light c_0.

From this FFG perspective, photons moving in free space in or out a field of gravity do this without being affected by this field. It is sufficient to imagine two small holes in the ceiling and the floor of the elevator, through which photons pass and move through the elevator while other photons in the elevator move between a mirror on the ceiling and the floor. There should be no difference in their velocities, the bouncing photon and the passing through photon should travel at the same speed in the elevator. This coïncides with Okun's viewpoint in the matter: photons do not fall under the influence of gravity [9]. A photon is not an apple.

B. The velocity of the bouncing photon in the perspective of the stationary grid observer

What will be the outcome when an observer on the stationary grid determines the velocity of the bouncing photon on the locally passing by free fall elevator on the FFG? Well, on the elevator the velocity of the bouncing photons is determined by photon travel time interval and elevator length as

$$c_0 = \frac{\Delta L_0}{\Delta T_0} \tag{26}$$

For the stationary observer, the passing by free fall elevator has Lorentz boost factor γ_ϕ and will be perceived with the usual Lorentz contraction as having contracted length

$$\Delta L_\phi = \frac{1}{\gamma_\phi}\Delta L_0. \tag{27}$$

In a Special Relativity context, the clocks on the elevator would be slowed down by the same Lorentz boost factor and the stationary clock would run faster than the clock on the moving by elevator. In Minkowski space-time we would have for the stationary observer

$$\Delta T_\phi = \frac{1}{\gamma_\phi}\Delta T_0. \tag{28}$$

so a time that would seem contracted relative to the passing by elevator time, resulting in a velocity of light observed by the stationary guy as

$$c_\phi = \frac{\frac{1}{\gamma_\phi}\Delta L_0}{\frac{1}{\gamma_\phi}\Delta T_0} = \frac{\Delta L_0}{\Delta T_0} = c_0. \tag{29}$$

But now gravity destroys the symmetry and it is the clock on the stationary grid that is moving slower relative to the clock on elevator falling on the free fall grid. See figure (3). Gravitational time dilation breaks the Minkowskian Lorentz symmetry of time dilation and length contraction, resulting in an apparent velocity of light as perceived by the observer on the stationary grid as

$$c_\phi = \frac{\frac{1}{\gamma_\phi}\Delta L_0}{\gamma_\phi \Delta T_0} = \frac{1}{\gamma_\phi^2}\frac{\Delta L_0}{\Delta T_0} = \frac{1}{\gamma_\phi^2}c_0. \tag{30}$$

And with

$$\gamma_\phi = 1 - \frac{\Phi}{c^2} \tag{31}$$

we get

$$\gamma_\phi^2 = \left(1 - \frac{\Phi}{c^2}\right)^2 = 1 - \frac{2\Phi}{c^2} + \frac{\Phi^2}{c^4} \approx 1 - \frac{2\Phi}{c^2} \tag{32}$$

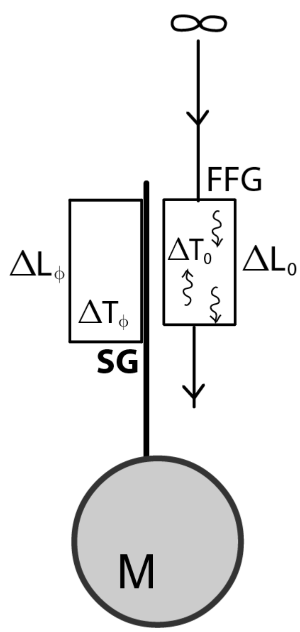

FIG. 3. A bouncing photon in the free fall elevator observed by a stationary grid observer

and

$$\frac{1}{\gamma_\phi^2} \approx \frac{1}{1-\frac{2\Phi}{c^2}} \approx 1+\frac{2\Phi}{c^2} = 1-\frac{2GM}{Rc^2} \tag{33}$$

resulting in an apparent velocity of light for the observer on the stationary grid as

$$c_\phi = \frac{1}{\gamma_\phi^2} c_0 \approx \left(1+\frac{2\Phi}{c^2}\right) c_0 = \left(1-\frac{2GM}{Rc^2}\right) c_0 < c_0 \tag{34}$$

This leads to the Shapiro delay and to the gravitational index of refraction. The last is then given by

$$n_\phi = \frac{c_0}{c_\phi} = \gamma_\phi^2 \approx 1+\frac{2GM}{Rc^2} > 1 \tag{35}$$

explaining the bending of light rays that pass by close to the sun.

So the apparent velocity of light produces real effects, subsequently ascribed to space-time curvature. But in the perspective of the observer in the elevator on the free fall grid, light will not be bend by the sun, nor will it experience a Shapiro delay. The question might be, which observer has the better access to the real world, the one on the free fall grid or the one on the stationary grid? Neither might be the correct answer: both perspectives are useful but decisions concerning reality claims are beyond our reach.

III. CONCLUSION

With our approach based on Special Relativity, the Weak Equivalence Principle and Newton's gravitational potential we could derive first order correct expressions for the gravitational red shift of stationary clocks and of satellites. We could also derive first order correct expressions for the geodetic precession, the Shapiro delay and the gravitational index of refraction, so phenomena connected to the curvature of the metric.

We did not derive an expression for the Lense-Thirring precession or drag of the metric by a rotating central mass M. Our free fall grid and the stationary grid were constructed around a stationary central mass M so we already excluded the Lense-Thirring effect in the construction phase of our model.

Our approach leads to the same first order results as the derivations based on the Schwartzschild solution of the Einstein Equations. But we did not formulate a fundamental theory of gravity. Our approach was opportunistic and ad-hoc because based upon a set of assumptions with limited reach. It would be interesting to find out at what point our approach will start to fail, so when our assumptions are no longer valid. It is most likely that our assumptions will not fail all at once and a detailed analysis of its actual falsification would be interesting.

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