

# Another Paradox of Special Relativity: Interlinked Clocks

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An original thought experiment “interlinking” time between relatively moving frames through belt drive clocks offers concrete evidence on the unviability of the time dilation predicted by the Special Relativity. It also shows that the Special Relativity length contraction gives contradictory time results.

## Introduction

Time dilation is one of the main predictions of the Special Relativity<sup>1</sup>. It simply says that when two inertial frames are in relative motion, time in the “traveling” frame is dilated with respect to an observer in the “stationary” frame. In other words, relative to the stationary frame observer, a clock in the traveling frame runs slower than an identical clock in the stationary frame. On the other hand, lengths in the traveling frame are contracted, in the relative motion direction, with respect to the stationary frame.

The relativistic time dilation and length contraction phenomena result in numerous paradoxes. Beside the time dilation twin paradox, most of the Special Relativity known paradoxes are related to length contraction (e.g., the barn-pole, bar and ring, Ehrenfest, and Bell’s spaceship paradoxes). A new time paradox is explored in this paper.

In the Special Relativity, clocks are the adopted means to measure time. i.e., judgments on whether time runs differently in moving frames are basically made through clocks (once synchronized) comparison. Therefore, if we can find a clock system that consistently provides time information in contradiction with what would be otherwise predicted by the Special Relativity, the viability of such a prediction would be challenged.

In this paper, a clock system “interlinking” time rigidly between relatively moving frames is designed to conduct a thought experiment, the results of which contradict the concept of the relativistic time dilation, as confirmed by the clocks.

## The Linked Clock Paradox

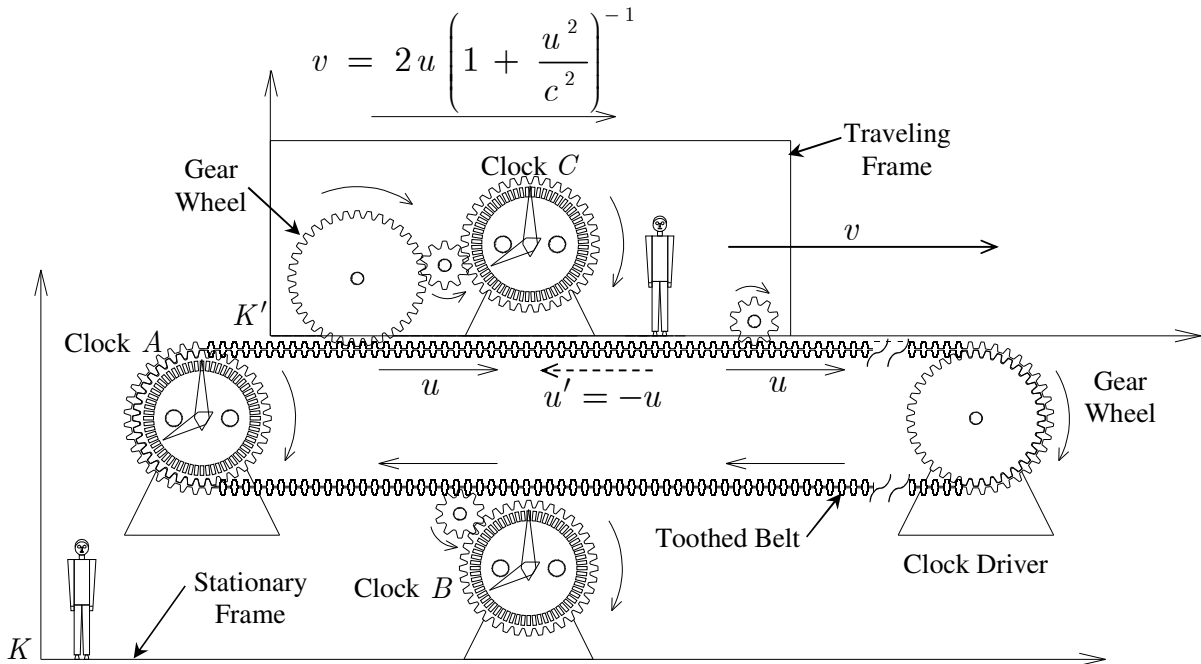
The setting of the experiment is shown in Fig.1. The arrangement may be thought of as a multi-clock system driven by a common mechanism. The clock driver is running at a known constant rotational speed. A linear uniform motion is created through a gear-belt arrangement. A geared clock mechanism is formed in such a way that when the gear is connected to the moving belt (through gears), the clock gear runs at the same driver’s speed (equal radii). The clock display is calibrated to show the actual elapsed time (e.g.,  $n$  revolutions correspond to a unit of time). Therefore, the number

of revolutions is translated into the elapsed time. When any clock made as such is connected through gears to the belt while the clock “sees” the belt running at the calibrated speed  $u$ , it will show the actual time.

It follows that clocks  $A$  and  $B$ , fixed in the stationary frame  $K$ , will evidently run at the same rate. The traveling frame rolls over the moving belt through its gear wheels, one of which is connected to clock  $C$ , identical to clocks  $A$  and  $B$ . The traveling frame is set to run relative to the stationary frame at a speed of

$$v = 2u \left( 1 + \frac{u^2}{c^2} \right)^{-1},$$

so that clock  $C$  in the traveling frame  $K'$  “sees” the speed  $u'$  of the belt equal to  $-u$  (using the relativistic speed addition formula).



**Fig. 1** Interlinked clock system arrangement

It follows that clock  $C$  should run at the same rate with respect to the  $K$  observer as the clocks in the stationary frame. However, if we consider the relativistic length contraction, the radii of the gears in the traveling frame would contract in the travel direction, so the perimeter of the clock gear will become smaller than the perimeter of the clocks’ in the stationary frame. Hence, to keep up with the constant linear belt speed ( $u' = -u$ ), clock  $C$  would run (rotate) faster than the clocks in the stationary frame, which is in contradiction with the SR prediction.

Consequently, the length contraction cannot be happening since it would yield a contradictory result, in the Special Relativity frame. On the other hand, in order to obtain time dilation, i.e. slower running of the clock, the perimeter of the clock gear in the traveling frame should increase (i.e., its

radius should expand), which would also contradict the Special Relativity prediction. If we hypothetically argue that the clock gear radius (perimeter) contracted, and the time displayed by the traveling clock was not the actual dilated time in the traveling frame, then due to time dilation the observer in the stationary frame must see the traveling clock gear rotating slower than the gear in their frame (each revolution of the gear corresponds to a fixed amount of time), which is physically impossible, since for a decreased gear perimeter while maintaining the same linear speed ( $u = -u'$ ), the rotational speed must increase. Hence, it would be impossible to maintain the physical coherence of the proposed clock-gear-belt system under the assumption of time dilation. Therefore, we're only left with one logical outcome: the clock in the traveling frame will show the same time as the clocks in the stationary frame, so the time is the same in both frames, as well as the object lengths.

## Conclusion

The proposed thought experiment clearly and rationally reveals, through “linking” the time rigidly between the relatively moving frames, that the relativistic time dilation and length contraction, as predicted by the Special Relativity, are unviable.

- 1 A. Einstein, "Zur elektrodynamik bewegter Körper," *Annalen der Physik* **322** (10), 891–921 (1905).