Current-Field Equations Including Charge Creation-Annihilation Fields and Derivation of Klein-Gordon and Schrödinger Equations and Gauge Transformation

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We found new current-field equations including charge creation-annihilation fields. Although it is difficult to treat creation and annihilation of charge pairs for Maxwell's equations, the new equations easily treat them. The equations cause the confinement of charge creation and annihilation centers, which means the charge conservation for this model. The equations can treat not only electromagnetic field but also weak and strong force fields. Weak gravitational field can be also treated by the equations, where four current means energy and momentum. It is shown that Klein-Gordon and Schrödinger equations and gauge transformation can be directly derived from the equations, where the wave function is defined as complex exponential function of the energy creation-annihilation field.

KEYWORDS: Maxwell's equations, Schrödinger equation, Klein-Gordon equation, gauge transformation, gravitational field

I. INTRODUCTION

Maxwell's equations have been used for analyses of electromagnetic field since J. C. Maxwell found the equations in 1865[1]. A. Einstein found special relativity in 1905[2, 3] to modify Newton's kinetics and general relativity in 1915[4] to treat gravitational field. E. Schrödinger found an equation for quantum mechanics in 1926[5], where a wave function is introduced to obtain energy and momentum from boundary conditions with potential energy. In early 1930s, E. Fermi proposed modified electromagnetic field model for quantum electro-dynamics (QED)[6–9], where he assumed that Lagrangian density includes a gauge fixing term. Gupta and Bleuler gave subsidiary conditions to Fermi's model in 1950[10, 11]. In 1960s, Nakanishi and Lautrup proposed the auxiliary field called Nakanishi-Lautrup (NL) field[12–15] to describe Lorentz covariant electromagnetic field model for QED. It is now included in the model of QED and quantum chromo dynamics (QCD)[16–19]. Recently, we found that the electromagnetic field model including a Lorentz scalar field, which is equivalent to NL field with Feynman gauge, can easily treat creation and annihilation of positive and negative charge pairs, although it is difficult for Maxwell's equations to treat them[20–22]. The equations for the above model, which are more symmetric than Maxwell's equations, can also give the relation between four current and four field for weak, strong, and gravitational forces. We found that Schrödinger and Klein-Gordon equations and gauge transformation can be derived from the current-field equations.

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Maxwell's equations are given by

$$\mathbf{J} = \frac{1}{\mu}\nabla \times \mathbf{B} - \varepsilon\frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\rho = \varepsilon\nabla\mathbf{E}, \tag{2}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{3}$$

$$\nabla\mathbf{B} = 0, \tag{4}$$

where $\mathbf{J}$ and ρ are current and charge density, ε and μ are permittivity and permeability, $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic field, respectively. Eqs. (1) and (2) directly give the following equation of the charge conservation,

$$\nabla\mathbf{J} + \frac{\partial \rho}{\partial t} = 0. \tag{5}$$

The creation and annihilation of positive and negative charge pairs are ordinarily described by the following equation, which is given by semiconductor physics[23–25],

$$\nabla\mathbf{J}_p + \frac{\partial \rho_p}{\partial t} = -\nabla\mathbf{J}_n - \frac{\partial \rho_n}{\partial t} = G, \tag{6}$$

where ρ_p and ρ_n are positive and negative charge concentration, $\mathbf{J}_p$ and $\mathbf{J}_n$ are positive and negative charge current density, and G is charge creation-annihilation rate. Since Maxwell's equations satisfy the principle of superposition[26], positive and negative charges must individually satisfy Eqs. (1) and (2). Therefore, positive charges satisfy

$$\mathbf{J}_p = \frac{1}{\mu}\nabla \times \mathbf{B}_p - \varepsilon\frac{\partial \mathbf{E}_p}{\partial t}, \tag{7}$$

and

$$\rho_p = \varepsilon \nabla \mathbf{E}_p, \tag{8}$$

where $\mathbf{E}_p$ and $\mathbf{B}_p$ denote electric and magnetic field induced by positive charges, respectively. Eqs. (7) and (8) directly give

$$\nabla \mathbf{J}_p + \frac{\partial \rho_p}{\partial t} = 0, \tag{9}$$

which contradicts (6) in the case of $G \neq 0$. Since this situation is same for negative charges, it is difficult for Maxwell's equations to treat creation and annihilation of charge pairs.

II. EXTENSION OF MAXWELL'S EQUATIONS

In order to solve the above problem, we introduce a gauge parameter α and a scalar field $\mathcal{N}$, which is equivalent to Nakanishi-Lautrup field except that its D' Alambertian is not always zero. The Lagrangian density of the electromagnetic field $\mathcal{L}_{EM}$ is given by[15]

$$\mathcal{L}_{EM} = -\frac{1}{4} F^{\nu\lambda} F_{\nu\lambda} + \mathcal{N} \partial^\nu A_\nu + \frac{1}{2} \alpha \mathcal{N}^2 - \mu J^\nu A_\nu, \tag{10}$$

where J^ν and A^ν denote four current $(c\rho, \mathbf{J})$ and four vector potential $(\psi/c, \mathbf{A})$, respectively, and $F^{\nu\lambda}$ is given by

$$F^{\nu\lambda} = \partial^\nu A^\lambda - \partial^\lambda A^\nu. \tag{11}$$

The above Lagrangian density gives the following equations.

$$\mu J_\nu = \Box A_\nu - \partial_\nu \partial^\lambda A_\lambda - \partial_\nu \mathcal{N}, \tag{12}$$

$$\partial^\nu A_\nu + \alpha \mathcal{N} = 0, \tag{13}$$

$$\pi^\nu = \frac{\partial \mathcal{L}_{EM}}{\partial(\partial_0 A_\nu)} = (\mathcal{N}, -\mathbf{E}/c), \tag{14}$$

where π^ν denotes four canonical momentum density and $\Box$ is d'Alembertian defined by $\Box \equiv \partial_0^2 - \nabla^2$.

Since $\mathbf{E}$ and $\mathbf{B}$ are written by

$$\mathbf{E} = -\nabla\psi - \frac{\partial \mathbf{A}}{\partial t}, \tag{15}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{16}$$

Eqs. (1) and (2) are rewritten by Eqs. (12), (15) and (16) as

$$\mathbf{J} = \frac{1}{\mu} \nabla \times \mathbf{B} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\mu} \nabla \mathcal{N}, \tag{17}$$

$$\rho = \varepsilon \nabla \mathbf{E} - \varepsilon \frac{\partial \mathcal{N}}{\partial t}. \tag{18}$$

Then, the charge creation-annihilation rate is given by

$$G = \nabla \mathbf{J} + \frac{\partial \rho}{\partial t} = -\frac{1}{\mu} \Box \mathcal{N}. \tag{19}$$

The above relation enable us to treat creation and annihilation of positive and negative charge pairs. Since $\mathcal{N}$ is connected with charge creation-annihilation rate G, we call $\mathcal{N}$ charge creation-annihilation field in this paper. It should be noticed that $G = 0$ needs not $\mathcal{N} = 0$ but $\Box \mathcal{N} = 0$. The above equations can be simply written as follows by using the complex electromagnetic field $\mathbf{E}/c - i\mathbf{B}$. Eqs. (3), (4), (13), (15), (16), (17), and (18) can be written by using four complex field as

$$\begin{pmatrix} E_1/c - iB_1 \\ E_2/c - iB_2 \\ E_3/c - iB_3 \\ i\alpha\mathcal{N} \end{pmatrix} = \begin{pmatrix} -\partial_0 & i\partial_3 & -i\partial_2 & i\partial_1 \\ -i\partial_3 & -\partial_0 & i\partial_1 & i\partial_2 \\ i\partial_2 & -i\partial_1 & -\partial_0 & i\partial_3 \\ -i\partial_1 & -i\partial_2 & -i\partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ i\psi/c \end{pmatrix}, \tag{20}$$

$$\mu \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ ic\rho \end{pmatrix} = \begin{pmatrix} -\partial_0 & -i\partial_3 & i\partial_2 & -i\partial_1 \\ i\partial_3 & -\partial_0 & -i\partial_1 & -i\partial_2 \\ -i\partial_2 & i\partial_1 & -\partial_0 & -i\partial_3 \\ i\partial_1 & i\partial_2 & i\partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} E_1/c - iB_1 \\ E_2/c - iB_2 \\ E_3/c - iB_3 \\ i\mathcal{N} \end{pmatrix}. \tag{21}$$

Eqs. (20) and (21) are the natural extension of Maxwell's equations[27]. Because Maxwell's equations are given by

$$\begin{pmatrix} E_1/c - iB_1 \\ E_2/c - iB_2 \\ E_3/c - iB_3 \end{pmatrix} = \begin{pmatrix} -\partial_0 & i\partial_3 & -i\partial_2 & i\partial_1 \\ -i\partial_3 & -\partial_0 & i\partial_1 & i\partial_2 \\ i\partial_2 & -i\partial_1 & -\partial_0 & i\partial_3 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ i\psi/c \end{pmatrix}, \tag{22}$$

$$\mu \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ ic\rho \end{pmatrix} = \begin{pmatrix} -\partial_0 & -i\partial_3 & i\partial_2 \\ i\partial_3 & -\partial_0 & -i\partial_1 \\ -i\partial_2 & i\partial_1 & -\partial_0 \\ i\partial_1 & i\partial_2 & i\partial_3 \end{pmatrix} \begin{pmatrix} E_1/c - iB_1 \\ E_2/c - iB_2 \\ E_3/c - iB_3 \end{pmatrix}. \tag{23}$$

It should be noticed that Eq. (21) includes not only Eqs. (17) and (18) but also Eqs. (3) and (4).

When the coordinate system has velocity v along x-axis, the Lorentz transformation of $\mathbf{A}, \psi, \mathbf{B}, \mathbf{E}, \mathcal{N}, \mathbf{J}$, and ρ are given by

$$\begin{pmatrix} A'_1 \\ A'_2 \\ A'_3 \\ i\psi'/c \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & 0 & 0 & \frac{i\beta}{\sqrt{1-\beta^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-i\beta}{\sqrt{1-\beta^2}} & 0 & 0 & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ i\psi/c \end{pmatrix}, \tag{24}$$

$$\begin{pmatrix} E'_1/c - iB'_1 \\ E'_2/c - iB'_2 \\ E'_3/c - iB'_3 \\ i\mathcal{N}' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-\beta^2}} & \frac{-i\beta}{\sqrt{1-\beta^2}} & 0 \\ 0 & \frac{i\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_1/c - iB_1 \\ E_2/c - iB_2 \\ E_3/c - iB_3 \\ i\mathcal{N} \end{pmatrix}, \tag{25}$$

$$\begin{pmatrix} J_1' \\ J_2' \\ J_3' \\ ic\rho' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & 0 & 0 & \frac{i\beta}{\sqrt{1-\beta^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-i\beta}{\sqrt{1-\beta^2}} & 0 & 0 & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ ic\rho \end{pmatrix}, \quad (26)$$

where β denotes v/c. Therefore, $\mathbf{A}, \psi, \mathbf{B}, \mathbf{E}, J$, and ρ have same transformation as original Maxwell's equations, and $\mathcal{N}$ is not changed by Lorentz transformation.

Although Eq. (25) does not satisfy the gauge invariance, if a scalar function χ satisfies $\Box\chi = 0$, $\mathbf{E}$, $\mathbf{H}$, and $\mathcal{N}$ are not changed by the transformation of

$$\mathbf{A}' = \mathbf{A} + \nabla\chi, \quad (27)$$

$$\psi' = \psi - \frac{\partial\chi}{\partial t}. \quad (28)$$

Next we consider about the electromagnetic field energy including the charge creation-annihilation field $\mathcal{N}$. By using Eqs. (3), (4), (14), (17) and (18), $cJ^\nu\pi_\nu$ is written by

$$cJ^\nu\pi_\nu = \mathbf{JE} + c^2\rho\mathcal{N} = -\frac{1}{\mu}\nabla\left(\mathbf{E}\times\mathbf{B} - \mathcal{N}\mathbf{E}\right) - \frac{1}{\mu}\frac{\partial}{\partial t}\left(\frac{E^2}{2c^2} + \frac{B^2}{2} + \frac{\mathcal{N}^2}{2}\right). \quad (29)$$

Since the above equation is regarded as the continuity equation for energy density, $\mathbf{JE} + c^2\rho\mathcal{N}$ is energy annihilation rate, $\varepsilon(\mathbf{E}\times\mathbf{B} - \mathcal{N}\mathbf{E})$ is momentum density, and $(E^2/c^2 + B^2 + \mathcal{N}^2)/2\mu$ is energy density. The charge creation-annihilation field $\mathcal{N}$ induces the additional energy density of $\mathcal{N}^2/2\mu$.

III. COMPARISON OF CURRENT VALUES BETWEEN MAXWELL'S AND THE NEW EQUATIONS

Now we compare the calculation results given by Maxwell's and the new equations including charge creation-annihilation field, using a simple structure. Fig. 1 shows an example structure consisting of a silicon sphere with radius R surrounded by SiO_2 under illumination or in a heating chamber, where

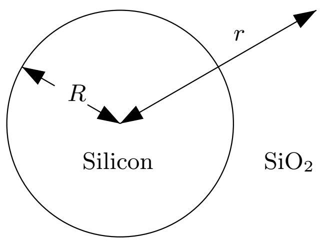

FIG. 1. A silicon sphere with radius R surrounded by SiO_2 under illumination or in a heating chamber.

$$\mathbf{J} = \mathbf{J}_p + \mathbf{J}_n = 0, \quad (30)$$

$$\rho = \rho_p + \rho_n = 0, \quad (31)$$

$$\mathbf{E} = \mathbf{E}_p + \mathbf{E}_n = 0, \quad (32)$$

$$\mathbf{B} = \mathbf{B}_p + \mathbf{B}_n = 0. \quad (33)$$

$\mathbf{E}_n$ and $\mathbf{B}_n$ denote electric and magnetic field induced by negative charges, respectively. Since this structure has spherical symmetry, the magnetic field does not exist[26],

$$\mathbf{B}_p = \mathbf{B}_n = 0. \quad (34)$$

Then, the charge creation-annihilation field $\mathcal{N}$ and scalar potential ψ also satisfy

$$\mathcal{N} = \mathcal{N}_p + \mathcal{N}_n = 0, \quad (35)$$

$$\psi = \psi_p + \psi_n = 0. \quad (36)$$

where $\mathcal{N}_p$ and ψ_p are induced by holes and $\mathcal{N}_n$ and ψ_n are induced by electrons. It is assumed that the hole and electron charge density ρ_p and ρ_n in the silicon generated by light or thermal energy increase linearly with time as

$$\rho_p = -\rho_n = \begin{cases} \rho_0(1+\frac{t}{\tau}) & (r \le R) \\ 0 & (r > R), \end{cases} \quad (37)$$

where the light or the heater is switched on at $t = 0$, ρ_0 is the charge density at $t = 0$, and the charge density increases with the charge creation rate of ρ_0/τ. Using spherical coordinate system and Gauss's law, the electric field has only radial component as

$$E_p = -E_n = \begin{cases} \frac{\rho_0 r}{3\varepsilon_{Si}}(1+\frac{t}{\tau}) & (r \le R) \\ \frac{\rho_0 R^3}{3\varepsilon_{ox} r^2}(1+\frac{t}{\tau}) & (r > R), \end{cases} \quad (38)$$

where ε_{si} and ε_{ox} are permittivity of silicon and SiO_2, respectively. Then ψ_p and ψ_n are given by

$$\psi_p = -\psi_n = \begin{cases} \frac{\rho_0\{(2\varepsilon_{Si}+\varepsilon_{ox})R^2 - \varepsilon_{ox}r^2\}}{6\varepsilon_{Si}\varepsilon_{ox}}(1+\frac{t}{\tau}) & (r \le R) \\ \frac{\rho_0 R^3}{3\varepsilon_{ox} r}(1+\frac{t}{\tau}) & (r > R). \end{cases} \quad (39)$$

In the case of Maxwell's equations, the radial component of the current J_p and J_n out of the sphere are needed by Eqs. (1), (34), and (38) as

$$J_p = -J_n = \begin{cases} -\frac{\rho_0 r}{3\tau} & (r \le R) \\ -\frac{\rho_0 R^3}{3\tau r^2} & (r > R). \end{cases} \quad (40)$$

The above result does not describe the real condition, because the hole and electron currents cannot exist in SiO_2.

Maxwell's equations cannot increase charge concentration without current because of the charge conservation of Eq. (5). If we consider the charge creation-annihilation field $\mathcal{N}_p$ and $\mathcal{N}_n$ for the charge pairs creation with assuming $\alpha = 1$ and $\nabla \mathbf{A} = 0$, they are given by

$$\mathcal{N}_p = -\mathcal{N}_n = \begin{cases} -\frac{\mu\rho_0\{(2\varepsilon_{Si}+\varepsilon_{ox})R^2-\varepsilon_{ox}r^2\}}{6\varepsilon_{ox}\tau} & (r \leq R) \\ -\frac{\mu\rho_0 R^3}{3\tau r} & (r > R). \end{cases} \tag{41}$$

Since the radial component of the gradient of $\mathcal{N}_p$ and $\mathcal{N}_n$ are given by

$$(\nabla\mathcal{N}_p)_r = -(\nabla\mathcal{N}_n)_r = \begin{cases} \frac{\mu\rho_0 r}{3\tau} & (r \leq R) \\ \frac{\mu\rho_0 R^3}{3\tau r^2} & (r > R), \end{cases} \tag{42}$$

the positive and negative charge current density J_p and J_n in and out of the sphere are given by Eqs. (17), (34), (38), and (40) as

$$J_p = -J_n = -\varepsilon\frac{\partial E_p}{\partial t} + \frac{1}{\mu}(\nabla\mathcal{N}_p)_r = \begin{cases} 0 & (r \leq R) \\ 0 & (r > R). \end{cases} \tag{43}$$

There is no current in and out of the sphere. Then the charge creation-annihiltion rate G is given by

$$G = -\frac{1}{\mu}\Box\mathcal{N}_p = \begin{cases} \frac{\rho_0}{\tau} & (r \leq R) \\ 0 & (r > R). \end{cases} \tag{44}$$

The electromagnetic field model including charge creation-annihilation field gives the reasonable result.

IV. CONFINEMENT OF CHARGE CREATION-ANNIHILATION CENTERS

Although the charge creation-annihilation field permits the existence of charge creation and annihilation centers by Eq. (19), they are confined by the energy to be proportional to the distance between them[28]. As shown by Eq. (41), the field $\mathcal{N}$ induced by a point charge creation or annihilation center is given by

$$\mathcal{N} = -\frac{\mu\sigma}{4\pi r}, \tag{45}$$

TABLE I. Force between A and B centers with creation and annihilation functions for positive and negative charges.

Charge-A	Function-A	Charge-B	Function-B	Force
positive	creation	positive	annihilation	attraction
positive	creation	negative	creation	attraction
negative	creation	negative	annihilation	attraction
positive	annihilation	negative	annihilation	attraction
positive	creation	positive	creation	repulsion
negative	creation	negative	creation	repulsion
positive	annihilation	positive	annihilation	repulsion
negative	annihilation	negative	annihilation	repulsion
positive	creation	negative	annihilation	repulsion
positive	annihilation	negative	creation	repulsion

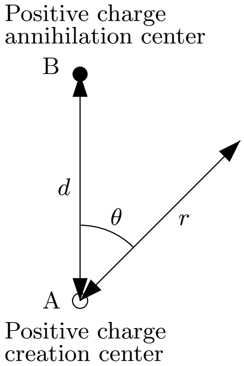

FIG. 2. Positive charge creation and annihilation centers with their distance d.

where σ denotes the creating charge per unit time. If the charge creation or annihilation center is isolated, the potential energy of the charge creation-annihilation field V_{CA} in a surrounding sphere with radius R is given by

$$V_{CA} = 4\pi\int_0^R \frac{\mathcal{N}^2}{2\mu}r^2 dr = \frac{\mu\sigma^2 R}{8\pi}. \tag{46}$$

Since the potential energy is proportional to R, an isolated charge creation or annihilation center cannot stably exist. However, some kinds of pairs of charge creation and annihilation centers can stably exist. Table I shows the force between two centers A and B that create or annihilate positive or negative charges, where the upper 4 cases induce attraction and the others induce repulsion. Only the upper 4 pairs can stably exist, because attractive force reduces the potential energy of charge creation-annihilation field. Fig. 2 shows the creation and annihilation centers for positive charges, where d denotes their distance. The total charge creation-annihilation field $\mathcal{N}_{pair}$ induced by the pair of creation and annihilation centers for positive charges is given by

$$\mathcal{N}_{pair} = -\frac{\mu\sigma}{4\pi}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+d^2-2rd\cos\theta}}\right), \tag{47}$$

where the charge creation and annihilation rates are assumed to be equal to σ, because the difference between the creation and annihilation rates induces similar potential energy as Eq. (46). If we assume the surrounding sphere radius R is enough larger than d ($R \gg d$), the creation-annihilation field potential energy of the pair

V_{CApair} in the sphere is given by

$$V_{CApair}(d) = 2\pi \int_0^R \left(\int_0^\pi \frac{\mathcal{N}_{pair}^2}{2\mu} \sin\theta d\theta \right) r^2 dr$$
$$= \frac{\mu\sigma^2}{8\pi} \left(\int_0^R 2dr - \int_0^R \int_0^\pi \frac{r\sin\theta}{\sqrt{r^2+d^2-2rd\cos\theta}} d\theta dr \right)$$
$$= \frac{\mu\sigma^2}{8\pi} \left(2R - \int_0^R \frac{d+r-|d-r|}{d} dr \right)$$
$$= \frac{\mu\sigma^2 d}{8\pi}. \quad (48)$$

Therefore the potential energy is proportional to the distance d and the attractive force between charge creation and annihilation centers is constant. It causes the confinement of charge creation-annihilation centers, which means the charge conservation in this model instead of Eq. (5). The above discussion does not depend on the gauge parameter α.

The quark confinement has been energetically studied[29–31] since Gell-Mann and Zweig introduced quarks in hadron's model in 1960s[32, 33]. Although duality model was proposed by Nambu, t'Hooft, and Mandelstam in 1970s[34–37], the theoretical explanation of the confinement has not succeeded yet. Since the potential dependence on the distance between creation and annihilation centers is same as the linear potential of quarks based on the spinning stick model for Regge trajectories[30], the quark confinement could be explained by the energy of charge creation-annihilation field.

V. DERIVATION OF KLEIN-GORDON AND SCHRÖDINGER EQUATIONS

If we define matrices M and M^* as

$$M \equiv \begin{pmatrix} -\partial_0 & -i\partial_3 & i\partial_2 & -i\partial_1 \\ i\partial_3 & -\partial_0 & -i\partial_1 & -i\partial_2 \\ -i\partial_2 & i\partial_1 & -\partial_0 & -i\partial_3 \\ i\partial_1 & i\partial_2 & i\partial_3 & -\partial_0 \end{pmatrix}, \quad (49)$$

$$M^* \equiv \begin{pmatrix} -\partial_0 & i\partial_3 & -i\partial_2 & i\partial_1 \\ -i\partial_3 & -\partial_0 & i\partial_1 & i\partial_2 \\ i\partial_2 & -i\partial_1 & -\partial_0 & i\partial_3 \\ -i\partial_1 & -i\partial_2 & -i\partial_3 & -\partial_0 \end{pmatrix}, \quad (50)$$

the relation among four current $\mathcal{J} \equiv (\mathbf{J}, ic\rho)$, four fields $\mathcal{F} \equiv (\mathbf{E}/c - i\mathbf{B}, i\mathcal{N})$ and $\bar{\mathcal{F}} \equiv (\mathbf{E}/c - i\mathbf{B}, i\alpha\mathcal{N})$, and four vector potential $\mathcal{A} \equiv (\mathbf{A}, i\psi/c)$ are given by

$$g\mathcal{J} = M\mathcal{F}, \quad (51)$$

$$\bar{\mathcal{F}} = M^*\mathcal{A}, \quad (52)$$

where g is the coupling constant, which is equal to μ in the case of electromagnetic field. The product of M^* and M is given by

$$M^*M = MM^* = \Box I, \quad (53)$$

where I denotes 4×4 unit matrix. Eqs. (51) and (52) can be regarded as geometry equations of Minkowski spacetime. Therefore, every Lorentz current vector J induces field vector $\mathcal{F}$, which consists of divergent and rotatory fields $\mathbf{E}$ and $\mathbf{B}$ and a scalar field $\mathcal{N}$. Furthermore, every field vector $\bar{\mathcal{F}}$ induces Lorentz vector $\mathcal{A}$, where Lorentz vectors and field vectors must satisfy Eqs. (24) and (25) for Lorentz boost transformation, respectively. If we ignore Faddeev-Popov ghost, Lagrangian density of Yang-Mills field $\mathcal{L}_{YM}$ is given by[18]

$$\mathcal{L}_{YM} = -\frac{1}{4}F^a_{\nu\lambda}F^{a\nu\lambda} + \mathcal{N}^a\partial^\nu A^a_\nu + \frac{1}{2}\alpha(\mathcal{N}^a)^2 - gJ^{a\nu}A^a_\nu, \quad (54)$$

$$F^a_{\nu\lambda} = \partial_\nu A^a_\lambda - \partial_\lambda A^a_\nu + gf_{abc}A^b_\nu A^c_\lambda, \quad (55)$$

where a, b, c denote numbers of 1, 2, ..., N^2-1 for $SU(N)$ group, and f_{abc} denotes structure constant. We define $\mathbf{E}^a/c$, $\mathbf{B}^a$, $\mathcal{F}^a$, and J^a as

$$\mathbf{E}^a/c \equiv -\partial_0\mathbf{A}^a - \nabla A^a_0 + gf_{abc}\mathbf{A}^b A^c_0, \quad (56)$$

$$\mathbf{B}^a \equiv \nabla \times \mathbf{A}^a - \frac{1}{2}gf_{abc}\mathbf{A}^b \times \mathbf{A}^c, \quad (57)$$

$$\mathcal{F}^a \equiv (\mathbf{E}^a/c - i\mathbf{B}^a, i\mathcal{N}^a), \quad (58)$$

$$\mathcal{J}^a \equiv (\mathbf{J}^a, iJ^a_0). \quad (59)$$

Eq. (54) gives

$$gJ^a_\nu = D^\lambda F^a_{\lambda\nu} - \partial_\nu B^a, \quad (60)$$

where

$$D_\nu \equiv \partial_\nu - igT^a A^a_\nu, \quad (61)$$

and T^a is a group generator. Then, we obtain

$$M_{YM} \equiv \begin{pmatrix} -D_0 & -iD_3 & iD_2 & -i\partial_1 \\ iD_3 & -D_0 & -iD_1 & -i\partial_2 \\ -iD_2 & iD_1 & -D_0 & -i\partial_3 \\ iD_1 & iD_2 & iD_3 & -\partial_0 \end{pmatrix}, \quad (62)$$

and

$$g\mathcal{J}^a = M_{YM}\mathcal{F}^a. \quad (63)$$

Therefore, electromagnetic, weak, and strong currents and fields satisfy Eqs. (62) and (63).

Gravitational field also satisfies Eqs. (49)-(52), when the field strength is enough small. Weak gravitational field is given by using a tensor $h_{\nu\lambda}$, which satisfies $|h_{\nu\lambda}| \ll 1$, as[38]

$$g_{\nu\lambda} \equiv \eta_{\nu\lambda} + h_{\nu\lambda}, \tag{64}$$

where $g_{\nu\lambda}$ is metric tensor and $\eta_{\nu\lambda}$ is a tensor defined by

$$\eta_{\nu\lambda} \equiv \begin{cases} 1 & (\nu = \lambda = 1,2,3) \\ 0 & (\nu \neq \lambda) \\ -1 & (\nu = \lambda = 0). \end{cases} \tag{65}$$

When we define $\bar{h}_{\nu\lambda}$ as

$$\bar{h}_{\nu\lambda} \equiv h_{\nu\lambda} - \frac{1}{2}\eta_{\nu\lambda} h^{\alpha}{}_{\alpha}, \tag{66}$$

in Lorentz gauge condition of $\bar{h}^{\nu\alpha}{}_{,\alpha} = 0$, we obtain

$$-\bar{h}_{\nu\lambda,\alpha}{}^{\alpha} = 2\kappa T_{\nu\lambda}, \tag{67}$$

where κ is Einstein's gravitational constant and $T_{\nu\lambda}$ is energy-momentum density tensor defined by

$$T_{\nu\lambda} = -\rho v_\nu v_\lambda, \tag{68}$$

where ρ is density of material and v_ν is four velocity. Since the energy-momentum density vector $\mathcal{J} \equiv (\rho\mathbf{v}, i\rho c)$ and metric vector potential $\mathcal{A} \equiv (\mathbf{A}, iA_0) \equiv (\bar{h}_{10}, \bar{h}_{20}, \bar{h}_{30}, i\bar{h}_{00})$ and $g = 2\kappa c$ satisfy

$$g\mathcal{J} = \Box\mathcal{A}, \tag{69}$$

gravitational field satisfies Eqs. (49)-(52), where

$$\mathbf{E}/c = -\partial_0\mathbf{A} - \nabla A_0, \tag{70}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{71}$$

$$\mathcal{N} = -\nabla\mathbf{A} - \partial_0 A_0. \tag{72}$$

Eqs. (69)-(72) show that particle momentum $\mathbf{P}$ and energy U and their inducing field also satisfy Eqs. (49)-(52), where the current vector $\mathcal{J}_G$, which is a Lorentz vector, and field $\mathcal{F}_G$ are given by

$$\mathcal{J}_G = (\mathbf{P}, iU/c), \tag{73}$$

$$\mathcal{F}_G = (\mathbf{E}_G/c - i\mathbf{B}_G, i\mathcal{N}_G). \tag{74}$$

$\mathbf{E}_G$ and $\mathbf{B}_G$ are divergent and rotatory gravitational fields, respectively and $\mathcal{N}_G$ is energy creation-annihilation field. In atomic scale, $|\mathbf{E}_G|/c$ and $|\mathbf{B}_G|$ can be assumed much smaller than $|\mathcal{N}_G|$. Then, $\mathbf{P}$ and U are directly obtained by $\mathcal{N}_G$ as

$$\mathbf{P} = \nabla\mathcal{N}_G, \tag{75}$$

$$U = -c\partial_0\mathcal{N}_G = -\frac{\partial}{\partial t}\mathcal{N}_G. \tag{76}$$

When we define a scalar function ϕ as

$$\phi \equiv \exp(\frac{i}{\hbar}\mathcal{N}_G), \tag{77}$$

we obtain

$$\nabla\phi = \frac{i}{\hbar}(\nabla\mathcal{N}_G)\phi = \frac{i}{\hbar}\mathbf{P}\phi, \tag{78}$$

and

$$\frac{\partial}{\partial t}\phi = \frac{i}{\hbar}(\frac{\partial}{\partial t}\mathcal{N}_G)\phi = -\frac{i}{\hbar}U\phi. \tag{79}$$

If the total energy is not changed, we can assume $\Box\mathcal{N}_G = 0$ and obtain

$$\Box\phi = \left(\frac{-U^2 + P^2c^2}{c^2\hbar^2} - \frac{i}{\hbar}\Box\mathcal{N}_G\right)\phi = -\frac{m^2c^2}{\hbar^2}\phi. \tag{80}$$

Eq. (80) is Klein-Gordon equation, which is equivalent to $\mathcal{J}_G = M\mathcal{F}_G$ under the condition of $E_G^2/c^2 + B_G^2 \ll \mathcal{N}_G^2$ and $\Box\mathcal{N}_G = 0$. In the case of $P \ll mc$, we obtain by using $U \approx mc^2 + P^2/2m$,

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\phi + mc^2\phi. \tag{81}$$

If we assume that V is potential energy and mc^2 is zero level of energy, we obtain Schrödinger equation as follows,

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\phi + V\phi. \tag{82}$$

Schrödinger equation is equivalent to $\mathcal{J}_G = M\mathcal{F}_G$ under the condition of $P \ll mc$ and $\nabla^2\mathcal{N}_G = 0$. Since this condition means $\partial_0{}^2\mathcal{N}_G = 0$, time dependent Schrödinger equation might have some problems caused by time dependence of energy creation-annihilation field $\mathcal{N}_G$. The wave function can be clearly defined as complex exponential function of the energy creation-annihilation field.

VI. DERIVATION OF GAUGE TRANSFORMATION

Since four vector potential $\mathcal{A}$ of electromagnetic field is Lorentz vector, it can induce four field vector

$$\mathcal{F}_A \equiv (\mathbf{E_A} - i\mathbf{B_A}, i\mathcal{N}_A), \tag{83}$$

as

$$\mathcal{A} = M\mathcal{F}_A. \tag{84}$$

Then,

$$\bar{\mathcal{F}} = M^*M\mathcal{F}_A = \Box\mathcal{F}_A. \tag{85}$$

Therefore, $\mathcal{N}_A$ changes not $\mathbf{E}$ and $\mathbf{B}$ but only $\mathcal{N}$. If the four momentum vector $\mathcal{P} \equiv (\mathbf{P}, iU/c)$ consists of electromagnetic component $\mathcal{P}_{EM}$ and the others $\mathcal{P}_{others}$, we obtain

$$\mathcal{P} = \mathcal{P}_{EM} + \mathcal{P}_{others}. \tag{86}$$

Since $\mathcal{P}_{EM}$ is given by the product of $\mathcal{A}$ and the particle charge q,

$$\mathcal{P}_{EM} = q\mathcal{A}, \tag{87}$$

$\mathcal{N}_G$ is given by

$$\mathcal{N}_G = q\mathcal{N}_A + \mathcal{N}_{others}, \tag{88}$$

because $\mathcal{P} = M\mathcal{F}_G$ and $\mathcal{A} = M\mathcal{F}_A$. When $\mathcal{N}_A$ changes to $\mathcal{N}_A + \chi$, **E** and **B** do not change, but $\mathcal{P}$ changes as

$$\mathcal{P}' = q\mathcal{A}' + \mathcal{P}_{others} = (\nabla(\mathcal{N}_G + q\chi), -i\partial_0(\mathcal{N}_G + q\chi)). \tag{89}$$

Therefore, the wave function ϕ changes to

$$\phi' = \exp[\frac{i}{\hbar}(\mathcal{N}_G + q\chi)] = \exp(\frac{iq\chi}{\hbar})\phi. \tag{90}$$

Eqs. (89) and (90) give gauge transformation as

$$\mathcal{A}'_\nu = \mathcal{A}_\nu + \partial_\nu\chi, \tag{91}$$

and

$$\phi' = \exp(\frac{iq\chi}{\hbar})\phi. \tag{92}$$

VII. CONCLUSION

The new current-field equations including charge creation-annihilation fields were found. They can easily treat creation and annihilation of charge pairs in electromagnetic field. It was found that the potential energy of charge creation-annihilation field for a pair of charge creation and annihilation centers is proportional to their distance, which causes the confinement of charge creation and annihilation centers. It means the charge conservation for this model. The current-field equations can treat not only electromagnetic field but also weak and strong force fields. Weak gravitational field can be also treated by the equations, where four current means energy and momentum. It was found that Klein-Gordon and Schrödinger equations and gauge transformation are directly derived from the equations, where the wave function is defined as complex exponential function of the energy creation-annihilation field.

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