

# Alternative Classical Mechanics IV

Alejandro A. Torassa

Creative Commons Attribution 3.0 License  
(2014) Buenos Aires, Argentina  
atorassa@gmail.com

- version 1 -

This paper presents an alternative classical mechanics which is invariant under transformations between reference frames and which can be applied in any reference frame without the necessity of introducing fictitious forces. Additionally, a new principle of conservation of energy is also presented.

## The Inertial Reference Frame

The inertial reference frame  $\hat{S}$  is a reference frame fixed to a system of particles, whose origin coincides with the center of mass of the system of particles. This system of particles (referred from now on as the free-system) is always free of external and internal forces.

The inertial position  $\hat{\mathbf{r}}_a$ , the inertial velocity  $\hat{\mathbf{v}}_a$  and the inertial acceleration  $\hat{\mathbf{a}}_a$  of a particle A relative to the inertial reference frame  $\hat{S}$ , are as follows:

$$\begin{aligned}\hat{\mathbf{r}}_a &\doteq (\mathbf{r}_a) \\ \hat{\mathbf{v}}_a &\doteq d(\mathbf{r}_a)/dt \\ \hat{\mathbf{a}}_a &\doteq d^2(\mathbf{r}_a)/dt^2\end{aligned}$$

where  $\mathbf{r}_a$  is the position of particle A relative to the inertial reference frame  $\hat{S}$ .

## The New Dynamics

[1] A force is always caused by the interaction between two particles.

[2] The resultant force  $\mathbf{F}_a$  acting on a particle A of mass  $m_a$  produces an inertial acceleration  $\hat{\mathbf{a}}_a$  according to the following equation:  $\hat{\mathbf{a}}_a = \mathbf{F}_a/m_a$

[3] This paper considers that not all forces obey Newton's third law (in its strong form or in its weak form)

## The Definitions

For a system of N particles, the following definitions are applicable:

$$\text{Mass} \quad M \doteq \sum_i m_i$$

$$\text{Linear Momentum} \quad \hat{\mathbf{P}} \doteq \sum_i m_i \hat{\mathbf{v}}_i$$

$$\text{Angular Momentum} \quad \hat{\mathbf{L}} \doteq \sum_i m_i \hat{\mathbf{r}}_i \times \hat{\mathbf{v}}_i$$

$$\text{Work} \quad \hat{W} \doteq \sum_i \int_1^2 \mathbf{F}_i \cdot d\hat{\mathbf{r}}_i = \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i)^2$$

$$\text{Kinetic Energy} \quad \Delta \hat{K} \doteq \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i)^2$$

$$\text{Potential Energy} \quad \Delta \hat{U} \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d\hat{\mathbf{r}}_i$$

$$\text{Lagrangian} \quad \hat{L} \doteq \hat{K} - \hat{U}$$

## The Principles of Conservation

The linear momentum  $\hat{\mathbf{P}}$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its weak form.

$$\hat{\mathbf{P}} = \text{constant} \quad [d(\hat{\mathbf{P}})/dt = \sum_i m_i \hat{\mathbf{a}}_i = \sum_i \mathbf{F}_i = 0]$$

The angular momentum  $\hat{\mathbf{L}}$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its strong form.

$$\hat{\mathbf{L}} = \text{constant} \quad [d(\hat{\mathbf{L}})/dt = \sum_i m_i \hat{\mathbf{r}}_i \times \hat{\mathbf{a}}_i = \sum_i \hat{\mathbf{r}}_i \times \mathbf{F}_i = 0]$$

The mechanical energy  $\hat{E}$  of a system of N particles remains constant if the system is only subject to conservative forces.

$$\hat{E} \doteq \hat{K} + \hat{U} = \text{constant} \quad [\Delta \hat{E} = \Delta \hat{K} + \Delta \hat{U} = 0]$$

## The Transformations

The inertial position  $\hat{\mathbf{r}}_a$ , the inertial velocity  $\hat{\mathbf{v}}_a$  and the inertial acceleration  $\hat{\mathbf{a}}_a$  of a particle A relative to a reference frame S, are given by:

$$\hat{\mathbf{r}}_a = \mathbf{r}_a - \mathbf{R}$$

$$\hat{\mathbf{v}}_a = \mathbf{v}_a - \boldsymbol{\omega} \times (\mathbf{r}_a - \mathbf{R}) - \mathbf{V}$$

$$\hat{\mathbf{a}}_a = \mathbf{a}_a - 2\boldsymbol{\omega} \times (\mathbf{v}_a - \mathbf{V}) + \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_a - \mathbf{R})] - \boldsymbol{\alpha} \times (\mathbf{r}_a - \mathbf{R}) - \mathbf{A}$$

where  $\mathbf{r}_a$ ,  $\mathbf{v}_a$  and  $\mathbf{a}_a$  are the position, the velocity and the acceleration of particle A relative to the reference frame S.  $\mathbf{R}$ ,  $\mathbf{V}$  and  $\mathbf{A}$  are the position, the velocity and the acceleration of the center of mass of the free-system relative to the reference frame S.  $\boldsymbol{\omega}$  and  $\boldsymbol{\alpha}$  are the angular velocity and the angular acceleration of the free-system relative to the reference frame S.

The position  $\mathbf{R}$ , the velocity  $\mathbf{V}$  and the acceleration  $\mathbf{A}$  of the center of mass of the free-system relative to the reference frame S, and the angular velocity  $\boldsymbol{\omega}$  and the angular acceleration  $\boldsymbol{\alpha}$  of the free-system relative to the reference frame S, are as follows:

$$M \doteq \sum_i^N m_i$$

$$\mathbf{R} \doteq M^{-1} \sum_i^N m_i \mathbf{r}_i$$

$$\mathbf{V} \doteq M^{-1} \sum_i^N m_i \mathbf{v}_i$$

$$\mathbf{A} \doteq M^{-1} \sum_i^N m_i \mathbf{a}_i$$

$$\boldsymbol{\omega} \doteq \mathbf{I}^{-1} \cdot \mathbf{L}$$

$$\boldsymbol{\alpha} \doteq d(\boldsymbol{\omega})/dt$$

$$\mathbf{I} \doteq \sum_i^N m_i [|\mathbf{r}_i - \mathbf{R}|^2 \mathbf{1} - (\mathbf{r}_i - \mathbf{R}) \otimes (\mathbf{r}_i - \mathbf{R})]$$

$$\mathbf{L} \doteq \sum_i^N m_i (\mathbf{r}_i - \mathbf{R}) \times (\mathbf{v}_i - \mathbf{V})$$

where  $M$  is the mass of the free-system,  $\mathbf{I}$  is the inertia tensor of the free-system (relative to  $\mathbf{R}$ ) and  $\mathbf{L}$  is the angular momentum of the free-system relative to the reference frame S (the free-system of N particles must be three-dimensional, and the relative distances between the N particles must be constant)

## The Equation of Motion

From the third transformation it follows that the acceleration  $\mathbf{a}_a$  of a particle A of mass  $m_a$  relative to a reference frame S, is given by:

$$\mathbf{a}_a = \mathbf{F}_a/m_a + 2\boldsymbol{\omega} \times (\mathbf{v}_a - \mathbf{V}) - \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_a - \mathbf{R})] + \boldsymbol{\alpha} \times (\mathbf{r}_a - \mathbf{R}) + \mathbf{A}$$

where  $\mathbf{F}_a$  is the resultant force acting on particle A ( $\hat{\mathbf{a}}_a = \mathbf{F}_a/m_a$ )

## Observations

The alternative classical mechanics of particles presented in this paper is invariant under transformations between reference frames and can be applied in any reference frame without the necessity of introducing fictitious forces.

This paper considers, on one hand, that not all forces obey Newton's third law (in its strong form or in its weak form) and, on the other hand, that all forces are invariant under transformations between reference frames ( $\mathbf{F}' = \mathbf{F}$ )

Additionally, from the equation of motion it follows that a reference frame S is inertial when ( $\boldsymbol{\omega} = 0$  and  $\mathbf{A} = 0$ ) and that it is non-inertial when ( $\boldsymbol{\omega} \neq 0$  or  $\mathbf{A} \neq 0$ )

## Bibliography

- D. Lynden-Bell and J. Katz**, Classical Mechanics without Absolute Space (1995)
- J. Barbour**, Scale-Invariant Gravity: Particle Dynamics (2002)
- R. Ferraro**, Relational Mechanics as a Gauge Theory (2014)
- A. Torassa**, General Equation of Motion (2013)
- A. Torassa**, Alternative Classical Mechanics (2013)
- A. Torassa**, A Reformulation of Classical Mechanics (2014)
- A. Torassa**, A New Principle of Conservation of Energy (2014)

## Appendix

For a system of N particles, the following definitions are also applicable:

$$\text{Angular Momentum} \quad \hat{\mathbf{L}}' \doteq \sum_i m_i (\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) \times (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm})$$

$$\text{Work} \quad \hat{W}' \doteq \sum_i \int_1^2 \mathbf{F}_i \cdot d(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) = \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm})^2$$

$$\text{Kinetic Energy} \quad \Delta \hat{K}' \doteq \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm})^2$$

$$\text{Potential Energy} \quad \Delta \hat{U}' \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm})$$

$$\text{Lagrangian} \quad \hat{L}' \doteq \hat{K}' - \hat{U}'$$

where  $\hat{\mathbf{r}}_{cm}$  and  $\hat{\mathbf{v}}_{cm}$  are the inertial position and the inertial velocity of the center of mass of the system of particles.  $\sum_i \int_1^2 m_i \hat{\mathbf{a}}_i \cdot d(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) = \sum_i \int_1^2 m_i (\hat{\mathbf{a}}_i - \hat{\mathbf{a}}_{cm}) \cdot d(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) = \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm})^2$

The angular momentum  $\hat{\mathbf{L}}'$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its strong form.

$$\hat{\mathbf{L}}' = \text{constant}$$

$$d(\hat{\mathbf{L}}')/dt = \sum_i m_i (\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) \times (\hat{\mathbf{a}}_i - \hat{\mathbf{a}}_{cm}) = \sum_i m_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times \hat{\mathbf{a}}_i = \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0$$

$$\hat{\mathbf{L}}' \doteq \sum_i m_i (\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) \times (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm}) = \sum_i m_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times [\mathbf{v}_i - \boldsymbol{\omega} \times (\mathbf{r}_i - \mathbf{r}_{cm}) - \mathbf{v}_{cm}]$$

The mechanical energy  $\hat{E}'$  of a system of N particles remains constant if the system is only subject to conservative forces.

$$\hat{E}' \doteq \hat{K}' + \hat{U}' = \text{constant}$$

$$\Delta \hat{E}' = \Delta \hat{K}' + \Delta \hat{U}' = 0$$

$$\Delta \hat{K}' \doteq \sum_i \Delta^{1/2} m_i (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_{cm})^2 = \sum_i \Delta^{1/2} m_i [\mathbf{v}_i - \boldsymbol{\omega} \times (\mathbf{r}_i - \mathbf{r}_{cm}) - \mathbf{v}_{cm}]^2$$

$$\Delta \hat{U}' \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_{cm}) = \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\mathbf{r}_i - \mathbf{r}_{cm})$$

where  $\mathbf{r}_{cm}$  and  $\mathbf{v}_{cm}$  are the position and the velocity of the center of mass of the system of particles relative to a reference frame S, and  $\boldsymbol{\omega}$  is the angular velocity of the free-system relative to the reference frame S. If the system of particles is isolated and if the internal forces obey Newton's third law in its weak form then:  $\sum_i - \int_1^2 \mathbf{F}_i \cdot d(\mathbf{r}_i - \mathbf{r}_{cm}) = \sum_i - \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i$

# The New Principle of Conservation of Energy

- versions 1 & 2 -

For a system of N particles, the following definitions are also applicable:

$$\text{Work} \quad W \doteq \sum_i \left( \int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta \frac{1}{2} \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right) = \Delta K$$

$$\text{Kinetic Energy} \quad \Delta K \doteq \sum_i \Delta \frac{1}{2} m_i (\bar{\mathbf{v}}_i \cdot \bar{\mathbf{v}}_i + \bar{\mathbf{a}}_i \cdot \bar{\mathbf{r}}_i)$$

$$\text{Potential Energy} \quad \Delta U \doteq - \sum_i \left( \int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta \frac{1}{2} \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right)$$

where  $\bar{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_{cm}$ ,  $\bar{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{v}_{cm}$ ,  $\bar{\mathbf{a}}_i = \mathbf{a}_i - \mathbf{a}_{cm}$ ,  $\mathbf{r}_i$ ,  $\mathbf{v}_i$  and  $\mathbf{a}_i$  are the position, the velocity and the acceleration of the  $i$ -th particle,  $\mathbf{r}_{cm}$ ,  $\mathbf{v}_{cm}$  and  $\mathbf{a}_{cm}$  are the position, the velocity and the acceleration of the center of mass of the system of particles,  $m_i$  is the mass of the  $i$ -th particle, and  $\mathbf{F}_i$  is the resultant force acting on the  $i$ -th particle. If the system of particles is isolated and if the internal forces obey Newton's third law in its weak form then:  $\sum_i \left( \int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta \frac{1}{2} \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right) = \sum_i \left( \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i + \Delta \frac{1}{2} \mathbf{F}_i \cdot \mathbf{r}_i \right)$

The new principle of conservation of energy establishes that if a system of N particles is only subject to conservative forces then the mechanical energy  $E$  of the system of particles remains constant.

$$E \doteq K + U = \text{constant} \quad [ \Delta E = \Delta K + \Delta U = 0 ]$$

The new principle of conservation of energy is invariant under transformations between reference frames since the kinetic energy  $K$ , the potential energy  $U$  and the mechanical energy  $E$  of a system of N particles are invariant under transformations between reference frames ( $\mathbf{F}' = \mathbf{F} \mid m' = m \mid \bar{\mathbf{r}}' = \bar{\mathbf{r}} \mid \bar{\mathbf{v}}' \cdot \bar{\mathbf{v}}' + \bar{\mathbf{a}}' \cdot \bar{\mathbf{r}}' = \bar{\mathbf{v}} \cdot \bar{\mathbf{v}} + \bar{\mathbf{a}} \cdot \bar{\mathbf{r}}$ )

The new principle of conservation of energy can be applied in any reference frame without the necessity of introducing fictitious forces and without the necessity of introducing additional external variables (such as  $\boldsymbol{\omega}$ ,  $\mathbf{R}$ ,  $\mathbf{V}$ ,  $\dot{\omega}_S$ , etc.)

# Alternative Classical Mechanics IV

Alejandro A. Torassa

Creative Commons Attribution 3.0 License  
(2014) Buenos Aires, Argentina  
atorassa@gmail.com

- version 2 -

This paper presents an alternative classical mechanics which is invariant under transformations between reference frames and which can be applied in any reference frame without the necessity of introducing fictitious forces. Additionally, a new principle of conservation of energy is also presented.

## The Dynamic Reference Frame

The dynamic reference frame  $\check{S}$  is basically a reference frame that can be used to obtain kinematic quantities (such as dynamic position, dynamic velocity, etc.) starting primarily from dynamic quantities (such as force, mass, etc.)

The dynamic position  $\check{\mathbf{r}}_a$ , the dynamic velocity  $\check{\mathbf{v}}_a$  and the dynamic acceleration  $\check{\mathbf{a}}_a$  of a particle A of mass  $m_a$  relative to the dynamic reference frame  $\check{S}$ , are as follows:

$$\check{\mathbf{r}}_a \doteq \int \int (\mathbf{F}_a/m_a) dt dt$$

$$\check{\mathbf{v}}_a \doteq \int (\mathbf{F}_a/m_a) dt$$

$$\check{\mathbf{a}}_a \doteq (\mathbf{F}_a/m_a)$$

where  $\mathbf{F}_a$  is the resultant force acting on particle A.

## The New Dynamics

[1] A force is always caused by the interaction between two particles.

[2] The resultant force  $\mathbf{F}_a$  acting on a particle A of mass  $m_a$  produces a dynamic acceleration  $\check{\mathbf{a}}_a$  according to the following equation:  $\check{\mathbf{a}}_a \doteq \mathbf{F}_a/m_a$

[3] This paper considers that not all forces obey Newton's third law (in its strong form or in its weak form)

## The Definitions

For a system of N particles, the following definitions are applicable:

$$\text{Mass} \quad M \doteq \sum_i m_i$$

$$\text{Linear Momentum} \quad \check{\mathbf{P}} \doteq \sum_i m_i \check{\mathbf{v}}_i$$

$$\text{Angular Momentum} \quad \check{\mathbf{L}} \doteq \sum_i m_i \check{\mathbf{r}}_i \times \check{\mathbf{v}}_i$$

$$\text{Work} \quad \check{W} \doteq \sum_i \int_1^2 \mathbf{F}_i \cdot d\check{\mathbf{r}}_i = \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i)^2$$

$$\text{Kinetic Energy} \quad \Delta \check{K} \doteq \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i)^2$$

$$\text{Potential Energy} \quad \Delta \check{U} \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d\check{\mathbf{r}}_i$$

$$\text{Lagrangian} \quad \check{L} \doteq \check{K} - \check{U}$$

## The Principles of Conservation

The linear momentum  $\check{\mathbf{P}}$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its weak form.

$$\check{\mathbf{P}} = \text{constant} \quad [d(\check{\mathbf{P}})/dt = \sum_i m_i \check{\mathbf{a}}_i = \sum_i \mathbf{F}_i = 0]$$

The angular momentum  $\check{\mathbf{L}}$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its strong form.

$$\check{\mathbf{L}} = \text{constant} \quad [d(\check{\mathbf{L}})/dt = \sum_i m_i \check{\mathbf{r}}_i \times \check{\mathbf{a}}_i = \sum_i \check{\mathbf{r}}_i \times \mathbf{F}_i = 0]$$

The mechanical energy  $\check{E}$  of a system of N particles remains constant if the system is only subject to conservative forces.

$$\check{E} \doteq \check{K} + \check{U} = \text{constant} \quad [\Delta \check{E} = \Delta \check{K} + \Delta \check{U} = 0]$$



## The Transformations

The dynamic position  $\check{\mathbf{r}}_a$ , the dynamic velocity  $\check{\mathbf{v}}_a$  and the dynamic acceleration  $\check{\mathbf{a}}_a$  of a particle A relative to a reference frame S, are given by:

$$\check{\mathbf{r}}_a = \mathbf{r}_a + \check{\mathbf{r}}_S$$

$$\check{\mathbf{v}}_a = \mathbf{v}_a + \check{\boldsymbol{\omega}}_S \times \mathbf{r}_a + \check{\mathbf{v}}_S$$

$$\check{\mathbf{a}}_a = \mathbf{a}_a + 2\check{\boldsymbol{\omega}}_S \times \mathbf{v}_a + \check{\boldsymbol{\omega}}_S \times (\check{\boldsymbol{\omega}}_S \times \mathbf{r}_a) + \check{\boldsymbol{\alpha}}_S \times \mathbf{r}_a + \check{\mathbf{a}}_S$$

where  $\mathbf{r}_a$ ,  $\mathbf{v}_a$  and  $\mathbf{a}_a$  are the position, the velocity and the acceleration of particle A relative to the reference frame S.  $\check{\mathbf{r}}_S$ ,  $\check{\mathbf{v}}_S$ ,  $\check{\mathbf{a}}_S$ ,  $\check{\boldsymbol{\omega}}_S$  and  $\check{\boldsymbol{\alpha}}_S$  are the dynamic position, the dynamic velocity, the dynamic acceleration, the dynamic angular velocity and the dynamic angular acceleration of the reference frame S relative to the dynamic reference frame  $\check{S}$ .

The dynamic position  $\check{\mathbf{r}}_S$ , the dynamic velocity  $\check{\mathbf{v}}_S$ , the dynamic acceleration  $\check{\mathbf{a}}_S$ , the dynamic angular velocity  $\check{\boldsymbol{\omega}}_S$  and the dynamic angular acceleration  $\check{\boldsymbol{\alpha}}_S$  of a reference frame S fixed to a particle S relative to the dynamic reference frame  $\check{S}$ , are as follows:

$$\check{\mathbf{r}}_S \doteq \int \int (\mathbf{F}_0/m_s) dt dt$$

$$\check{\mathbf{v}}_S \doteq \int (\mathbf{F}_0/m_s) dt$$

$$\check{\mathbf{a}}_S \doteq (\mathbf{F}_0/m_s)$$

$$\check{\boldsymbol{\omega}}_S \doteq \pm |(\mathbf{F}_1/m_s - \mathbf{F}_0/m_s) \cdot (\mathbf{r}_1 - \mathbf{r}_0) / (\mathbf{r}_1 - \mathbf{r}_0)^2|^{1/2}$$

$$\check{\boldsymbol{\alpha}}_S \doteq d(\check{\boldsymbol{\omega}}_S)/dt$$

where  $\mathbf{F}_0$  and  $\mathbf{F}_1$  are the resultant forces acting on the reference frame S in the points 0 and 1,  $\mathbf{r}_0$  and  $\mathbf{r}_1$  are the positions of the points 0 and 1 relative to the reference frame S and  $m_s$  is the mass of particle S (the point 0 is the origin of the reference frame S and the center of mass of particle S) (the point 0 belongs to the axis of dynamic rotation, and the segment 01 is perpendicular to the axis of dynamic rotation) (the vector  $\check{\boldsymbol{\omega}}_S$  is along the axis of dynamic rotation)

## The Equation of Motion

From the third transformation it follows that the acceleration  $\mathbf{a}_a$  of a particle A of mass  $m_a$  relative to a reference frame S, is given by:

$$\mathbf{a}_a = \mathbf{F}_a/m_a - 2\check{\omega}_S \times \mathbf{v}_a - \check{\omega}_S \times (\check{\omega}_S \times \mathbf{r}_a) - \check{\alpha}_S \times \mathbf{r}_a - \check{\mathbf{a}}_S$$

where  $\mathbf{F}_a$  is the resultant force acting on particle A ( $\check{\mathbf{a}}_a \doteq \mathbf{F}_a/m_a$ )

## Observations

The alternative classical mechanics of particles presented in this paper is invariant under transformations between reference frames and can be applied in any reference frame without the necessity of introducing fictitious forces.

This paper considers, on one hand, that not all forces obey Newton's third law (in its strong form or in its weak form) and, on the other hand, that all forces are invariant under transformations between reference frames ( $\mathbf{F}' = \mathbf{F}$ )

Additionally, from the equation of motion it follows that a reference frame S is inertial when ( $\check{\omega}_S = 0$  and  $\check{\mathbf{a}}_S = 0$ ) and that it is non-inertial when ( $\check{\omega}_S \neq 0$  or  $\check{\mathbf{a}}_S \neq 0$ )

## Bibliography

- D. Lynden-Bell and J. Katz**, Classical Mechanics without Absolute Space (1995)
- J. Barbour**, Scale-Invariant Gravity: Particle Dynamics (2002)
- R. Ferraro**, Relational Mechanics as a Gauge Theory (2014)
- A. Torassa**, General Equation of Motion (2013)
- A. Torassa**, Alternative Classical Mechanics (2013)
- A. Torassa**, A Reformulation of Classical Mechanics (2014)
- A. Torassa**, A New Principle of Conservation of Energy (2014)

## Appendix

For a system of N particles, the following definitions are also applicable:

$$\text{Angular Momentum} \quad \check{\mathbf{L}}' \doteq \sum_i m_i (\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) \times (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm})$$

$$\text{Work} \quad \check{W}' \doteq \sum_i \int_1^2 \mathbf{F}_i \cdot d(\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) = \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm})^2$$

$$\text{Kinetic Energy} \quad \Delta \check{K}' \doteq \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm})^2$$

$$\text{Potential Energy} \quad \Delta \check{U}' \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm})$$

$$\text{Lagrangian} \quad \check{L}' \doteq \check{K}' - \check{U}'$$

where  $\check{\mathbf{r}}_{cm}$  and  $\check{\mathbf{v}}_{cm}$  are the dynamic position and the dynamic velocity of the center of mass of the system of particles.  $\sum_i \int_1^2 m_i \check{\mathbf{a}}_i \cdot d(\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) = \sum_i \int_1^2 m_i (\check{\mathbf{a}}_i - \check{\mathbf{a}}_{cm}) \cdot d(\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) = \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm})^2$

The angular momentum  $\check{\mathbf{L}}'$  of an isolated system of N particles remains constant if the internal forces obey Newton's third law in its strong form.

$$\check{\mathbf{L}}' = \text{constant}$$

$$d(\check{\mathbf{L}}')/dt = \sum_i m_i (\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) \times (\check{\mathbf{a}}_i - \check{\mathbf{a}}_{cm}) = \sum_i m_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times \check{\mathbf{a}}_i = \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0$$

$$\check{\mathbf{L}}' \doteq \sum_i m_i (\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) \times (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm}) = \sum_i m_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times [\mathbf{v}_i + \check{\boldsymbol{\omega}}_S \times (\mathbf{r}_i - \mathbf{r}_{cm}) - \mathbf{v}_{cm}]$$

The mechanical energy  $\check{E}'$  of a system of N particles remains constant if the system is only subject to conservative forces.

$$\check{E}' \doteq \check{K}' + \check{U}' = \text{constant}$$

$$\Delta \check{E}' = \Delta \check{K}' + \Delta \check{U}' = 0$$

$$\Delta \check{K}' \doteq \sum_i \Delta^{1/2} m_i (\check{\mathbf{v}}_i - \check{\mathbf{v}}_{cm})^2 = \sum_i \Delta^{1/2} m_i [\mathbf{v}_i + \check{\boldsymbol{\omega}}_S \times (\mathbf{r}_i - \mathbf{r}_{cm}) - \mathbf{v}_{cm}]^2$$

$$\Delta \check{U}' \doteq \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\check{\mathbf{r}}_i - \check{\mathbf{r}}_{cm}) = \sum_i - \int_1^2 \mathbf{F}_i \cdot d(\mathbf{r}_i - \mathbf{r}_{cm})$$

where  $\mathbf{r}_{cm}$  and  $\mathbf{v}_{cm}$  are the position and the velocity of the center of mass of the system of particles relative to a reference frame S, and  $\check{\boldsymbol{\omega}}_S$  is the dynamic angular velocity of the reference frame S. If the system of particles is isolated and if the internal forces obey Newton's third law in its weak form then:  $\sum_i - \int_1^2 \mathbf{F}_i \cdot d(\mathbf{r}_i - \mathbf{r}_{cm}) = \sum_i - \int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i$