An Aspirations Model of Decisions in a Class of Ultimatum Games

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Abstract

We propose a novel model of aspiration levels for interactive games, termed *economic harmony*. The model posits that the individuals' levels of outcome satisfaction are proportional to their actual outcomes relative to their aspired outcomes. We define a state of harmony as a state at which the interacting players' levels of outcome satisfaction are equal, and underscore the necessary condition for the manifestation and sustenance of harmony situations on the behavioral level. We utilize the proposed model to predict the transfer decisions in a class of two-person ultimatum games, including the standard ultimatum game, an ultimatum game with asymmetric information about the size of the "pie", an ultimatum game with varying veto power. We also apply the model to predicting behavior in a three person ultimatum game with uncertainty regarding the identity of the recipient and in a sequential common pool resource game. For all the aforementioned games, we show that the model accounts well for large sets of experimental data, and with the predictions of game theory and other relevant models of interactive situations. Strikingly, for the standard ultimatum game, the model predicts that the allocator should transfer a proportion of the entire sum that equals 1- Φ, where Φ is the famous Golden Ratio, most known for its aesthetically pleasing properties.

Keywords: Aspiration Level, Fairness, Ultimatum Game, Dictator Game, Common Pool Resource Dilemma, , Golden Ratio.

1. Introduction

In the standard ultimatum game (Güth, Schmittberger & Schwartze, 1982; Camerer & Thaler, 1995), one player (the allocator) receives an amount of monetary units, and must decide how much to keep for herself and how much to transfer to another player (the recipient). The recipient replies either by accepting the proposed offer, in which case both players receive their shares, or by rejecting the offer, in which case the two players receive nothing. In the "strategy protocol", instead of the accept/reject options, before being informed about the allocators' offers, recipients are requested to register their minimum acceptable offer. The allocator in the game has complete entitlement to make any offer, between zero and the entire amount, while the recipient has complete entitlement to veto any offer.

Game theory predicts that a rational allocator, who believes that the recipient is also rational, should offer the smallest amount possible, since the recipient, being rational, will accept any positive offer. Experimental findings of numerous ultimatum studies refute this prediction. The modal offer in most experiments is the equal split, and the mean offer is ≈ 40% of the entire amount. In the first experimental study by Güth, et al. (1982), the modal offer of was 50% of the entire amount and the mean offer was 41.9%. Since 1982 numerous studies have repeatedly replicated these results. For examples, despite differences in culture, socio-economic background, and type of currency (Kahneman, Knetsch, & Thaler (1986), reported a mean offer of 42.1% (for commerce students in an American university), and Suleiman (1996) reported a mean of 41.8% for Israeli students. A more recent meta-analysis on ultimatum experiments conducted in twenty six countries with different cultural backgrounds (Oosterbeek, Sloof, & Van de Kuilen, 2004), reported a mean offer of 41.5%, and yet another large cross-cultural study conducted in 15 small-scale societies (Henrich et al., 2005), reported a mean offer of 40.5%. In stark difference with the prediction of game theory, in the above cited studies, and in many other uncited studies, division of ≈ 60-40 (%), for the allocator and recipient, respectively, seem to be robust across countries, cultures, socio-economic levels, monetary stakes, etc.

Other models, including Inequality Aversion (IA) theory (Fehr & Schmidt, 1999), and the theory of Equity, Reciprocity and Competition (ERC) theory (Bolton & Ockenfels, 2000), which were proposed to account for the cooperation and fairness observed in strategic interactions, yield better predictions, but nonetheless, they fall short of predicting the observed 60-40 split. The ERC model posits that, along with pecuniary gain, people are motivated by their own payoffs, relative to the payoff of others. Despite its relative success in predicting behavior in other games, with regard to the standard ultimatum game the prediction of ERC is uninformative, as it predicts that the proposer should offer any amount that is larger than zero and less or equal 50%. Inequality Aversion (IA) assumes that in addition to the motivation for maximizing own payoffs, individuals are motivated to reduce the difference in payoffs between themselves and others, although with greater distaste for having lower, rather than higher, earnings than. The prediction of IA with regard to the ultimatum game is also problematic, as it requires an estimation of the relative weight of the fairness component in the proposer's utility function.

In the present paper we propose a novel psychological model of decisions in interactive situation and tests its prediction, using a large set of data from standard ultimatum games and from several variants of the game, including an ultimatum game with varying veto power, an ultimatum game with asymmetric information about the size of the "pie", and a three person ultimatum game with uncertainty regarding the identity of the responder. For all the mentioned games, we derive predictions for the allocators' decisions, and compare them with experimental data, as well as with the predictions of game theory and of other relevant models. We show that the proposed models' predictions fit well with all the tested data.

The remainder of the paper is organized as follows: Section 2 gives a brief review of previous psychological models of level of aspiration. Section 3 presents the proposed model. Sections 4-7 detail applications of the model for the standard ultimatum game, and to three variants of the game, including a three-person ultimatum game, and test the derived prediction using experimental data. Section 8 discusses extensions of the model to other, n-person, strategic games, and section 9 concludes

2. **Level of aspiration models: A brief review**

Previous models of aspiration level have almost conclusively dealt with the effects of aspirations decisions in individuals' choice situations. These models differ from classical utility models in proposing the individual's aspiration level as a natural reference point in his or her scale of utility, instead of the arbitrary zero point. The view that individuals value their payoffs by comparing them to their levels of aspiration has a long tradition in psychology (e.g., Hilgard, Sait, & Margaret, 1940; Lewin, Dembo, Festinger, & Sears, 1944; Simon, 1959). It has been also well studied as in the domain of individual decision-making under risk, especially in relation to Security-Potential/Aspiration (CP/A) theory (Lopes, 1987, 1996; Lopes & Oden, 1999; Rieger, 2010). Several studies have also incorporated individual models of aspirations in predicting behaviors in interactive situations (e.g. Siegel, 1957; Crowne, 1966; Hamner & Donald, 1975; Rapoport & Kahan, 1983; Komorita & Ellis, 1988; Thompson, Mannix, & Bazerman, 1988; Tietz, 1997). As examples, Crowne (1966) investigated the role of aspiration level in players' choices in a repeated prisoner's dilemma game, and Komorita and Ellis (1988) studied its role in coalition formation. Common to CP/A theory and similar models is the use of Herbert Simon's conceptualization of satisficing vs. non-satisficing outcomes (Simon, 1959). According to Simon's conceptualization, an outcome greater than the aspiration levels is evaluated as satisfactory, while an outcome smaller than the aspiration level is evaluated as unsatisfactory.

3. The proposed model

Conceptually, the proposed level of aspirations resembles the aforementioned individual choice models, in its proposition that individuals make choices based on their levels of satisfaction from optional outcomes, and that in the decision process they evaluate actual outcomes through comparing them with aspired outcomes. Nonetheless, the new model differs from previous models in two significant aspects: 1. the model treats interactive situations rather than individual choice ones, 2. Levels of satisfaction are defined as functions of the *ratio* between the individuals' actual outcomes and their aspired outcomes, rather than the common definition as functions of the difference between the actual and aspired outcomes. We contend that the ratio scale is preferable to the commonly used difference scale for defining outcome satisfaction. First, the ratio scale is most common in physics, biology, evolution, and other exact sciences; Second, it is the standard definition practice in psychophysics, starting with Fechner’s law (Fechner, 1860; Gescheider, 1997) and Steven’s power law (Stevens, 1946, 1960), to more recent theories of audio and visual perception (Luce, Steingrimsson, & Narens, 2010; Palmer, 1999) and signal detection (Posch, 1999); Third, the ratio scale is dimensionless and does not depend on the measurement units of the divided goods; Fourth, all types of statistical measures are applicable to ratio scales, and only with these scales may we properly indulge in logarithmic transformations (Stevens, 1946).

Assuming self-interested players, the model posits that the players' outcome satisfaction levels are proportional to their *actual* payoffs, *relative* to the payoffs to which they *aspired*. In formal terms, the level of satisfaction (LS_i) of an individual i who is allocated x_i monetary units, when he or she had aspired to receive A_i monetary units, is assumed to be a function of x_i/A_i, or LS_i = F(x_i/A_i), where F(.) is an increasing function with its argument.

Now consider a dyadic interaction in which one player (an allocator) have complete entitlement on a monetary sum of money, S, and that he or she must decide how much to keep and how much to transfer to a second player (a recipient). Without loss of generality, assume that S = 1 Monetary unit (MU). Assuming that the allocator keeps a portion of x and transfers $x_r = 1 - x$ to the recipient, the levels of pay satisfaction of the two players, as prescribed by the model, will be $LS_a = F_a(x_a/A_a)$ and $LS_r = F_r(x_r/A_r) = F_r((1\text{-}x)/A_r)$, for the allocator and recipient, respectively, where A_a and A_r are the maximal payoffs to which the allocator and the recipient aspire, respectively. For simplicity, we assume linear relationships, such that $LS_a = x/A_a$ and $LS_r = (1\text{-}x)/A_j$. The *point(s) of harmony* at which two players will be equally satisfied with their payoffs should satisfy:

$$LS_a = LS_r, \quad \text{...... (1)}$$

or:

$$\frac{x}{A_a} = \frac{1-x}{A_r}, \quad \text{...... (2)}$$

which yields:

$$x = \frac{A_a}{A_a+A_r} \quad \text{...... (3)}$$

And:

$$x_r = 1 - \frac{A_a}{A_a + A_r} = \frac{A_r}{A_a + A_r} \quad \text{...... (4)}$$

Determining x which guarantees a fair allocation, in the sense of equal levels of satisfaction, requires the assessment, or measurement of the players' maximal aspirations. In the absence of any constraints on the allocator's decision, a self-interested allocator would aspire for the entire sum ($i.e., A_a$ = 1). Hypothesizing about the recipient's aspired payoff is trickier. We consider two plausible possibilities: (1) the recipient might aspire to receive *half* of the net profit; (2) he or she might aspire to receive a sum that equals the sum the allocator keeps for himself or herself. Although at first sight, the two conjectures seem identical, they are not.

Under the first assumption, we have A_a = 1 and $A_r = \frac{1}{2}$. Substitution in equations 3 and 4 yields:

$$x = \frac{A_a}{A_a + A_r} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3} \quad \text{...... (5)}$$

And: $x_r = 1 - \frac{2}{3} = \frac{1}{3}$ ...... (6)

On the other hand, under the second assumption, we have A_a = 1 and $A_r = x_a$. Substitution in equations 3 yields:

$$x = \frac{1}{1 + x} \quad \text{...... (7)}$$

Solving for x we get:

$$x^2 + x - 1 = 0 \quad \text{...... (8)}$$

Which solves for:

$$x = \frac{-1 \pm \sqrt[2]{1^2 + 4}}{2} = \left(\frac{-1 \pm \sqrt[2]{5}}{2}\right) \quad \text{...... (9)}$$

For positive x values, we get:

$x = \frac{\sqrt[2]{5}-1}{2} = \Phi \approx 0.62$ …… (10)

Where Φ is the famous Golden Ratio [(1)] (Livio, 2002; Posamentier & Lehmann, 2007). The corresponding portion for the recipient is: x_r = (1- φ) ≈ 0.38.[(1)]

In summary, the proposed model predicts that if the recipient aspires to receive 50% of the total amount, harmony between the two players' levels of satisfaction is achieved if the allocator transfers one *third* of the total amount to the recipient. On the other hand, if the recipient aspires to be treated equally (i.e., $A_r = x_a$), "harmony" is achieved if the allocator transfers a portion of *1- Φ ≈ 0.38* of the total amount to the recipient. Since self-interested allocators would not transfer to recipients more than they would keep to themselves, the difference between the predicted harmony points is less than 5%. In accounting for empirical reports of outcome satisfaction, and in the absence of additional information about the interacting individuals' aspiration levels, the model predicts a mean transfer to the recipient in the range between about 0.33 and 0.38 of the entire amount.

It is important to stress that while the predicted points of harmony prescribes that the interacting players would be equally satisfied, no expectations could be upheld to their emergence as *behavioral* outcomes. In the absence of binding rules (e.g., a minimum transfer, or sanctions for allocating unfairly), rational allocators will strive to maximize their personal payoffs, rather than equalize the two players satisfaction levels. In game theoretical terms, the points of harmony, predicted by the model, are not behavioral equilibrium points. For a point of harmony to emerge and stabilize on the behavioral level, it must be supported by an external social or institutional mechanism.

(1) The Golden ratio is commonly known as $\varphi = \frac{\sqrt[2]{5}+1}{2} \approx 1.618$. The two Golden Ratios Φ and φ are related by the relationships $\Phi = 1- \varphi = \frac{1}{\varphi}$.

An efficient mechanism which has proven to be effective in inhibiting the motivation to maximize own profits and to enhance cooperation is sanctions (see, e.g., Fehr & Fischbacher, 2004; Samid & Suleiman, 2008; O'Gorman et al. 2009). In the present context, the importance of efficient sanctions for achieving fairness harmony on the behavioral level will be demonstrated in the following sections, which the derived harmony solutions are applied for predicting allocators' behavior in experimental ultimatum games.

4. Predicting offers in a class of ultimatum games

We tested the model using a large set of data from the standard ultimatum game and other variants of the game, including a three person ultimatum game with uncertainty regarding the identity of the responder. We also tested the model's prediction for data from experiments on the sequential common pool resource (CPR) game.

4a. Application to the standard ultimatum game

As noted above, for a point of harmony to be stable, it must be supported by an efficient external mechanism, such as an institutional or social sanctioning mechanism. In experimental economics, there is ample evidence demonstrating the effectiveness of sanctions, whether by a second party, third party, or an institution, in enhancing cooperation and fairness in resource allocation games (e.g., Boyd & Richerson, 1992; Fehr, & Gächter, 2002; Fehr & Fischbacher, 2004; Henrich, et al. 2005; Samid & Suleiman, 2008; O'Gorman et al., 2009; Baldassarria & Grossman, 2011). The ultimatum game structure endows the recipients with efficient, although costly, punishment power, as they could punish allocators whose offers are perceived to be unsatisfactory. Game theory, being indifferent to the recipients' veto power, predicts that a rational allocator, who believes that the recipient is also a rational player, should offer the smallest amount possible, since the recipient, being rational, will accept any positive offer. In contrast the proposed model predicts an offer in the range 0.33-0.38 of the entire amount, with preference for the upper limit; i.e., the Golden Ratio division. I tested the model's predictions using two large data sets on the ultimatum game: (1) a meta-analysis on 75 ultimatum game experiments conducted in 26 countries with different cultural backgrounds (Oosterbeek, Sloof, & Van de Kuilen, 2004); (2) a large cross-cultural study conducted in 15 small-scale societies, including three groups of foragers, six groups of slash-and-burn horticulturalists, four groups of nomadic herders, and two groups of small-scale agriculturalists (Henrich et al., 2005).

For the two tested studies, the frequencies of offers are depicted in Figure 1. The figure shows that the two distributions are well-behaved and quite similar to each other. The reported mean offers are 0.395 and 0.405, for the Oosterbeek et al. and the Henrich et al. studies, respectively; both close to the Golden prediction of ≈0.382. A Two one-sided test of equivalence (TOST), see e.g., Jen-pei Liu & Shein-Chung (1997), validates this conjecture. For the Oosterbeek et al. study, the analysis yielded significant results for the upper and lower bounds of the equivalence range (upper bound=42.016, $p<0.0001$; lower bound=34.377, $p= 0.0425$; overall significance=0.0425). For the Henrich et al. study, the results were also significant (upper bound=42.016, $p= 0.012$; lower bound=34.377, $p=0.0255$; overall significance= 0.0255).

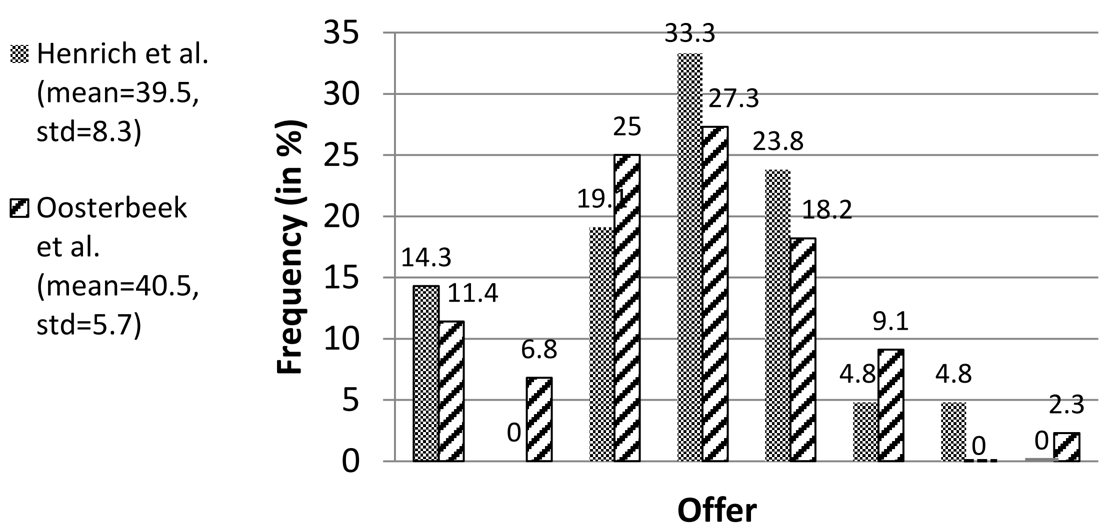

Figure 1: Distributions of offers in two large-scale ultimatum studies

4b. Ultimatum game with varying veto power

Although the data of the two multi-cultural studies, as well as numerous other ultimatum studies, strongly support the model's Golden Ratio prediction, a proper investigation of the effectiveness of punishment in ultimatum bargaining requires testing the allocators' offers under different levels of punishment efficacy. For this purpose I used data from several experiments on the δ-ultimatum game (Suleiman, 1996). In the δ-ultimatum game, acceptance of an offer of [x, S-x] entails its implementation, whereas its rejection results in an allocation of [δ x, δ (S-x)], where δ is a "shrinkage" factor known to both players ($0 \leq \delta \leq 1$). Varying the reduction factor, results in different recipients' punishment efficacy. For δ =0, the game reduces to the standard ultimatum game, in which the recipient has maximal punishment power, while for δ =1 the game reduces to the dictator game (Camerer & Thaler, 1995; Bardsley, 2008) in which the recipient is powerless. While game theory is indifferent to the value of δ, predicting that for all δ values ($0 \leq \delta \leq 1$) the allocator should keep almost all the amount, the proposed model predicts that proposer's confronting stronger responders (in higher δ conditions), should offer more than proposer's confronting weaker responders (in lower δ conditions).

Table 1 depicts the mean offers reported by fours experiments using the δ-ultimatum game: two conducted on Israeli students (Suleiman, 1996, 2014), and two conducted on Dutch students from (van Deijk, et al., 2004; Handgraaf et al., 2008).

Table 1

Mean offers (in %) in four experiments using varying punishment efficacy

Study	**Punishment Efficacy**					
	Strong			**Weak**		
	δ=0	δ=0.1	δ=0.2	δ=0.8	δ=0.9	δ=1
Suleiman (1996)	42	-	43	27	-	30
Suleiman (2014) (Repeated Game)	39	-	-	19	-	-
Mean offer (Israeli Participants)	**41.3**			**25.3**		
van Deijk, et al. (2004) (pro-selves)	46	-	-	-	32	-
Handgraaf et al. (2008) (exp.1)	48	43	-	-	36	44
Mean offer (Dutch Participants)	**45.7**			**37.3**		
Overall mean offer (In %)	**43.5**			**31.3**		

The top row indicates the values of the δ parameter implemented in each experiment, ranging from δ = 0 (a standard ultimatum game) to δ = 1 (dictator game. As seen in the table, for both Israeli and Dutch students, the mean offers under strong punishment efficacy (δ = 0, 0.1. 0.2) are significantly higher than the mean offers under weak, or no punishment efficacy (δ = 0.8, 0.9. 1). In all tested experiments, the differences in offers made under high and low punishment efficacy were found to be statistically significant. g Interestingly, the mean offers reported in the two experiments on Israeli participants are quite close to the Golden Ratio prediction, while for Dutch participants under the high and low punishment efficacy conditions, the mean offers were higher than the offers made by Israeli participants. Note also that the mean offers made in games in which responders has no power ($\delta = 1$), are larger than the mean offers made in games in which responders has little power ($\delta = 0.8$, 0.9). This phenomenon is particularly manifest Dutch students, for whom the difference was found to be statistically significant (see, Handgraaf et al, 2008).

4c. Ultimatum game with one-sided uncertainty about the "pie" size

Rapoport, Sundali, & Seale (1996) employed a one-sided uncertainty "demand game" (Mitzkewitz and Nagel, 1993), in which the proposer knows the value of the shared pie but the responder only knows its probability distribution. The pie size $\$k$ was distributed uniformly over the interval (a, b). The distribution of k was common knowledge, but only the proposer knew its realization **K**. The Responder only knew x, the amount which the proposer decided to keep. The study utilized a between-subject design with three uncertainty conditions: 0 - 30 (a = 0, b = 30), 5 - 25 (a = 5, b = 25) and 10 - 20 (a = 10, b = 20). The three uniform distributions have different support but the same expected value: $\frac{a+b}{2}$ = 15. The main result is that the proposer's proportional share of the pie increased as the uncertainty about the pie size increased. The mean proportional demands for the three experimental conditions were: 0.72 for condition 0-30, 0.66 for condition 5-25, and 0.65 for condition 10-20.

Derivation of harmony point prediction of the demand x (see Appendix A) yields:

$$x = \frac{-(b+a)+\sqrt{(b+a)^2 + 16\, b(b+a)}}{8} \qquad \text{........ (11)}$$

For condition 0-30, substituting a= 0, b= 30 in Eq. 11 gives:

$$x_1 = \frac{-30+\sqrt{30^2 + 16\ 30x30}}{8} \quad = 11.712$$

Which yields a predicted ratio of $r_1 = \frac{11.712}{15} = 0.781$

Similarly for conditions 5-25 and 10-20 we get: x_2 = 10.447 and x_3 = 9.059, with corresponding ratios of r_2= 0.697 and r_3= 0.601, respectively. Figure 2 depicts the mean observed demands for the three experimental conditions, alongside with the predicted harmony demands.

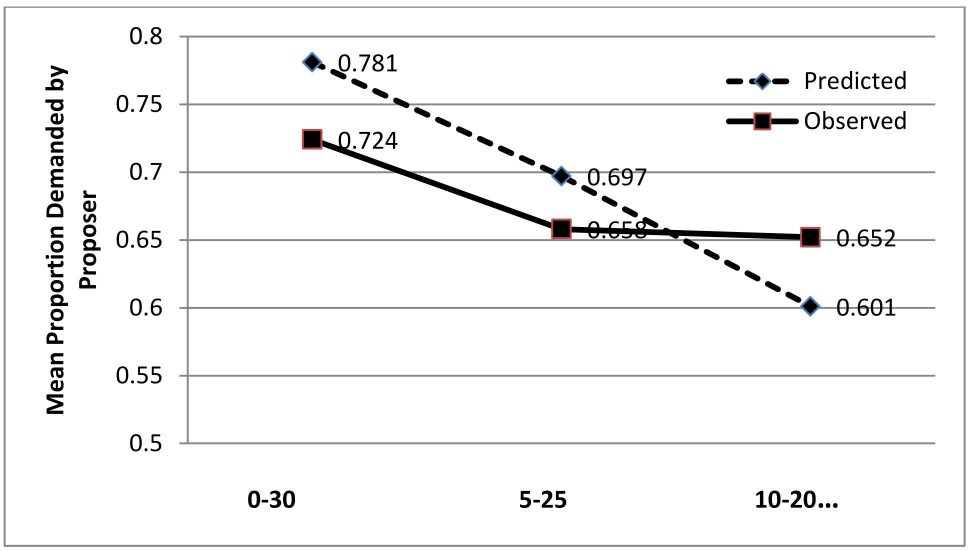

Figure 2: Observed demands in the Rapoport et al. (1996) study, together with the predicted harmony demands

As shown in the figure, the model's prediction captures the results' pattern quite nicely. For the three experimental conditions the demand values predicted by the theory are close to the observed ones (mean square error ≈ 0.002), and like the experimental results, they increase monotonically with the increase in the level of uncertainty about the pie size. The theoretical and observed results suggest that under conditions of higher uncertainty about the pie size, proposers can "hide" behind the "veil of ignorance", and demand larger portions of the pie.

4d. A three-person ultimatum game

I also tested the model's prediction of offers in a three-person ultimatum game (Kagel & Wolfe, 2001), which was specifically designed to test the predictions of ERC and IA. In this game, Player **X** offers to split a sum of money $*m*, allocating ($m$-($y$ + z), y, z) for herself, player **Y**, and player **Z**, respectively. One of the latter players is chosen at random (with probability p) to accept or reject the offer. If the responder accepts, then the proposed allocation is binding, as in the standard ultimatum game. However, if the responder rejects, then both she and **X** receive zero payoffs, and the non-responder receives a "consolation prize" of $*c*. The consolation prize to the non-responder was varied across four conditions with ($)$c$ = 0, 1, 3, 12. The probability of designating **Y** or **Z** as responder was $p = \frac{1}{2}$ and the amount to be allocated in all conditions was m = $15. The findings contradicted the predictions of both ERC and IA. Frequent rejections were detected, when both theories call for acceptance. In addition, the effect of the consolation prize for the non-responder on the probability of responders' acceptance rate was insignificant, and did not increase monotonically with the size of the consolation prize, as both theories suggest.

The modal offer of the proposer was the equal distribution of ($5, $5, $5), and the median was ($7, $4, $4). The proposers' mean demand was quite similar across all treatments, including the one with high "consolation" payoff of c=12. It decreased only slightly over the course of the repeated game. The reported rejection rates were lowest for the equal distribution ($5, $5, $5), and the ($6, $4.50, $4.50) distribution, equaling 1% and 6%, respectively, with corresponding expected demand of $4.96 and $5.67. The modal distribution was (7, 4, 4) and it yielded nearly maximum expected demand with relatively few rejections.

Derivation of the "harmony" point(s) for this game (see appendix B) yields:

$$\bar{x} = \frac{m}{4}(\Phi + 1) \approx 1.62 \ \frac{m}{4} \qquad \text{…….. (12)}$$

Where $\bar{x}$ is the predicted **X**'s demand, *m* is the pie size and Φ is the Golden Ratio (≈ 0.62). For *m* = $15 we get:

$$\bar{x} = 1.62 \frac{\$15}{4} \approx \$6.08$$

Thus, the predicted harmony distribution is (6.08, 4.46, 4.46). This result is almost identical to the second least rejected distribution of (6, 4.5, 4.5) and to the median distribution of at which the share of **X** relative the shares of **Y** and **Z** is $\frac{7}{(7+4)} = \frac{7}{11}$, ≈ 0.64, a value very close the Golden Ratio (≈ 0.62). Moreover, contrary to the predictions of both ERC and IA, and in accordance with the experimental findings, the "harmony" model predicts that the amount *x*, kept by **X** is independent on the consolation prize *c* (see Appendix B, and Eq. 12).

4e. Sequential common pool resource dilemma game

Another n-person game, which has the structure of an ultimatum game, is the sequential common pool resource dilemma (CPR) game. CPR games model situations in which a group of people consume a limited shared resource. Under the *sequential protocol of play* (Harrison & Hirshleifer, 1989), with a step-level rule, individual requests are made in an exogenously determined order, which is common knowledge, such that each player knows his position in the sequence and the total requests of the players who have preceded him in the sequence. If the total request does not exceed the pool size, all players receive their requests. However, if the total request exceeds the pool size, all players receive nothing (Rapoport, Budesu, & Suleiman, 1993; Budescu, Au, & Chen, 1997; Budescu & Au, 2002).

The n-person sequential CPR with a step-level resource has the structure of an n-person ultimatum game played by the strategy method, in which any player can "veto" the request decisions of the players precedes him in the sequence, simply by making a sufficiently large request. The game theoretic sub-game perfect equilibrium of the sequential CPR game prescribes that the first mover should demand almost all the amount available in the common pool, leaving an infinitesimally small portion to others. This prediction is strongly refuted by experimental results, showing that first movers do exploit their positions but that they leave much of the resource for others' consumption. Moreover, studies reveal a robust *position effect*: individual requests are inversely related to the players' positions in the sequence with the first mover requesting most, and the last mover requesting the least (Suleiman & Budescu, 1999; Budescu & Au, 2002, Larrick & Blount, 1995).

To derive the harmony solution for the sequential CPR game, consider a game with n players. Denote the requests of players occupying positions *i* and *i*+1 in the sequence by r_i and r_{i+1} (i = 1, 2, … n-1). The CPR game is similar in its structure to the UG. For two successive players r_i and r_{i+1}, the game is reduced to a two-person ultimatum game, in which harmony is achieved when $r_i = \Phi$ (≈ 0.62), and $r_{i+1} =$ 1- Φ (≈ 0.38), or

$$\frac{r_{i+1}}{r_i} = \frac{1-\Phi}{\Phi} = \Phi \ (i = 1, 2, \dots n\text{-}1) \qquad \dots\dots (13)$$

Where $\Phi \approx 0.62$ is the Golden Ratio.

I tested the above prediction using data reported in one study using groups of three players (first row) and and three studies using groups of five players (Rapoport et al., 1993; Budescu, Rapoport, & Suleiman, 1995; Budescu, Suleiman, & Rapoport, 1995). In all studies the pool size was 500 points. The resulting predictions (using Eq. 12 and a pool size of 500), together with the experimental results are depicted Figure 3. As shown in the figure the match between the theoretical predictions and the experimental results is impressive, although the requests of players appearing later in the sequence are underestimated by the theory. For the three players game, a Kolmogorov-Smirnov test revealed that the differences between the theoretical and the observed requests are non-significant.The maximum difference between the cumulative distributions is 0.091 (p=1). A similar conclusion holds for the five players game. The maximum difference between the cumulative distributions is 0.20 with a corresponding (p = 0.975).

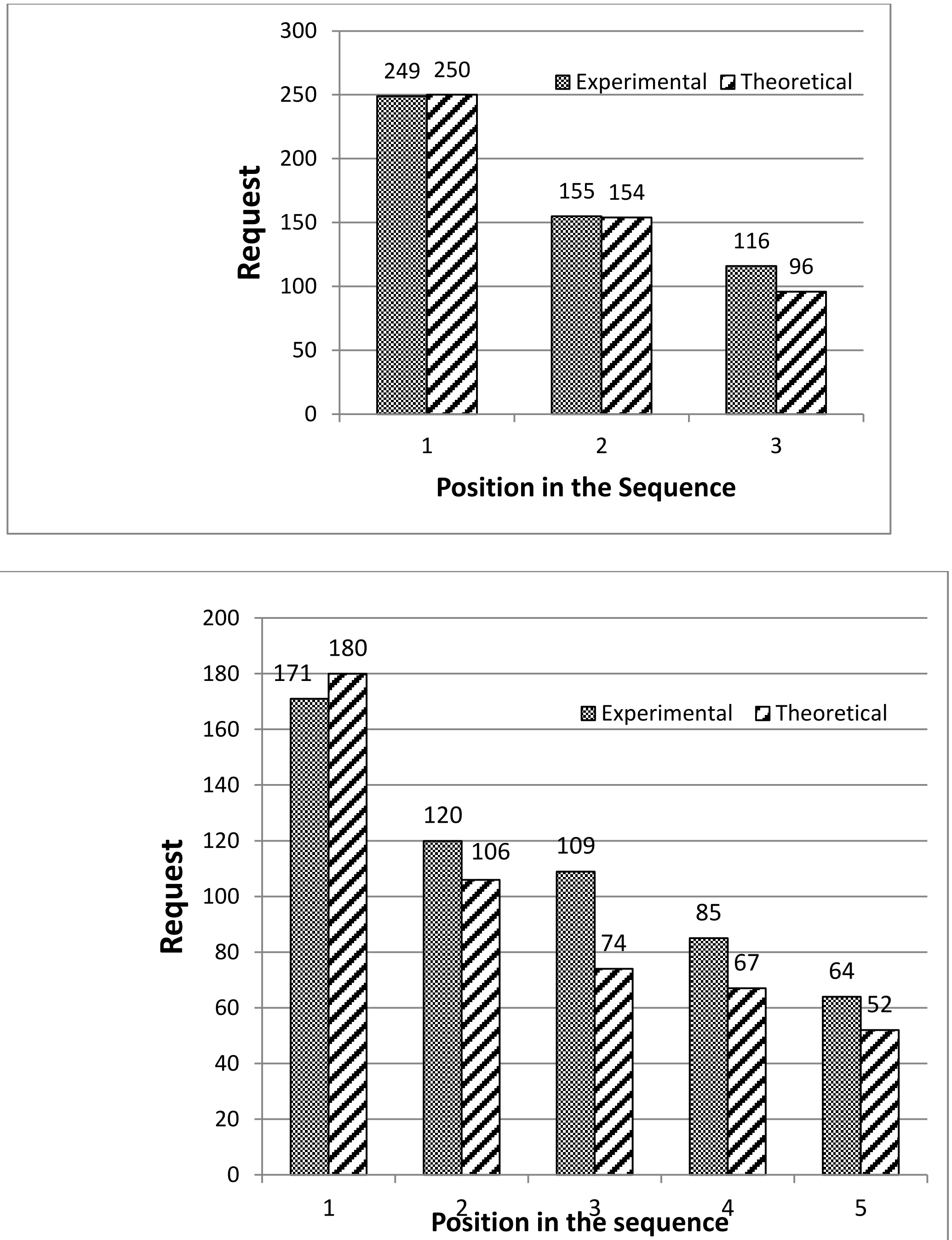

Figure 3. Empirical and predicted mean of requests in a sequential CPR Dilemma with resource size = 500 for n=3 players (upper panel) and n=5 players (lower panel).

5. Relaxing the rationality assumption

In deriving the "harmony" points in section 3, we assumed that a rational allocator would aspire for the entire amount. We relax the model by assuming that allocators might aspire to receive any amount between the entire amount, S, and (1- α) S, α is a *security* parameter (Lopes, 1987), $0 \le \alpha \le 0.5$. Under the assumption that the recipient aspires to receive $\frac{1}{2}S$, Eq. 2 becomes:

$$\frac{x}{1-\alpha} = \frac{1-x_{\alpha}}{0.5} \qquad \text{….. (14)}$$

Solving for x we get:

$$x = \frac{1-\alpha}{(1.5-\alpha)} \qquad \text{….. (15)}$$

And:

$$x_{r} = 1 - \frac{1-\alpha}{(1.5-\alpha)} = \frac{0.5}{(1.5-\alpha)} \qquad \text{….. (16)}$$

Similarly, assuming that the recipient aspires to be treated equally ($A_{r} = x$), we have:

$$\frac{x}{1-\alpha} = \frac{1-x}{x} \qquad \text{….. (17)}$$

Solving for x yields:

$$x^2 + (1-\alpha)\, x - (1-\alpha) = 0 \qquad \text{…. (18)}$$

Which solves for:

$$x = \frac{\sqrt[2]{(1-\alpha)^2 + 4\,(1-\alpha)} - (1-\alpha)}{2} \qquad \text{….. (19a)}$$

And

$$x_r = 1 - \frac{\sqrt[2]{(1-\alpha)^2 + 4\,(1-\alpha)} - (1-\alpha)}{2} = \frac{3 + \alpha + \sqrt[2]{(1-\alpha)^2 + 4\,(1-\alpha)}}{2} \qquad \text{…… (19b)}$$

For the solutions in equations 15 and 18b, the offers x_r as functions of α in the range α = 0 - 0.5 are depicted in Figure 4.

As expected, the figure shows that two functions increase monotonically with α.

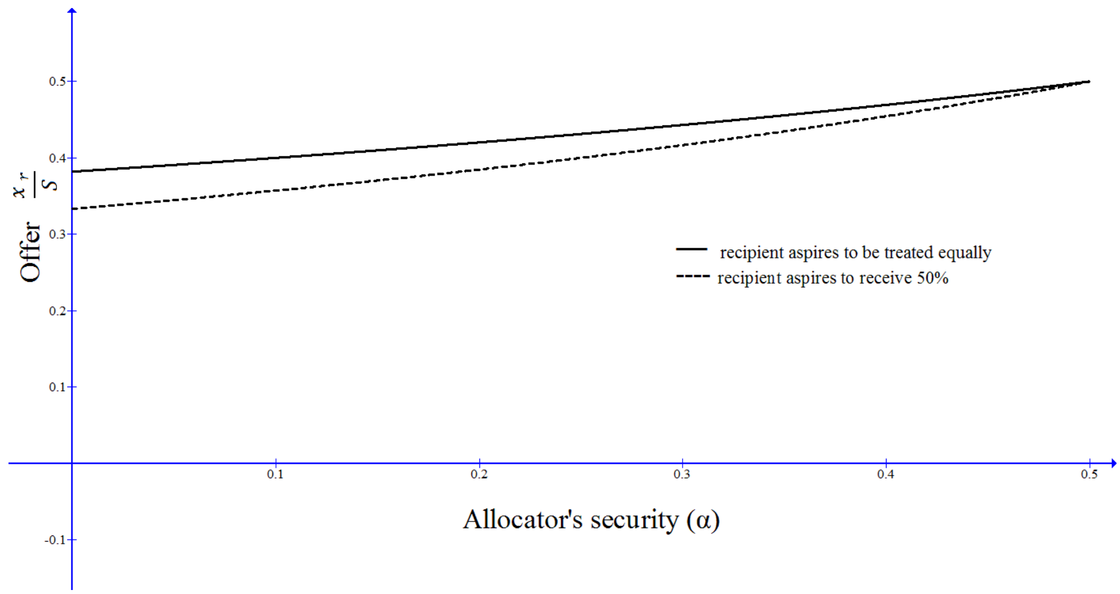

Figure 4: Predicted offers as function of the proposer's security level

Assuming that allocators would aspire for any amount in the range (1, $\frac{3}{4}$), the predicted mean offers could be calculated by integrating the functions in Equations 16 and 19b over the specified range. Under the assumption $A_r = \frac{1}{2}$, from Eq. 16 we have:

$$x_r = \frac{1}{0.25} \int_0^{0.25} \frac{0.5}{(1.5-\alpha)}\, d\alpha = -\,2\ln(3\text{-}2\alpha)\,]_0^{0.25} \approx 0.37 \qquad \text{….. (20)}$$

While under the assumption $A_r = x_a$, using Eq. 19b we get:

$x_r = \frac{1}{0.25} \int_0^{0.25} \left(\frac{3+\alpha+\sqrt[2]{(1-\alpha)^2+4(1-\alpha)}}{2}\right) d\alpha$

$=2\ [3\alpha - \frac{\alpha^2}{2} - \frac{1}{2}(\frac{\alpha}{2} - \frac{3}{2})\ \sqrt[2]{\alpha^2 - 6\alpha + 5}\ + 2\ln(3\text{-}\alpha\text{-}\ \sqrt[2]{\alpha^2 - 6\alpha + 5}\ )]_0^{0.25} \approx 0.40$ …. (21)

As could be verified, these predictions are only slightly higher (less than 2-3%) than the comparable predictions obtained under the complete rationality assumption.

6. Relaxing the linearity assumption

For proposer and responder with power level of satisfaction functions of the form $LS_a = (\frac{x}{A_p})^a$ and

$LS_r(x) = (\frac{1-x}{A_r})^b$, applying the harmony principle in Eq. 2 yields:

$(\frac{x}{A_a})^a = (\frac{1-x}{A_r})^b$ …….. (22)

Setting $A_a = 1 - \alpha_a$ and $A_r = x - \alpha_r$ we obtain:

$(\frac{x}{1-\alpha_a})^a = (\frac{1-x}{x-\alpha_r})^b$ ……. (23)

For the case of $\alpha_p = \alpha_r = 0$ we obtain:

$\frac{x^{a+b}}{(1-x)^b} = 1$ …… (24a)

Which could be written as:

$x\ (x^{\beta} + 1\ ) = 1$ …… (24b)

Where $\beta = \frac{a}{b}$

Figure 5 depicts the predicted offer for a wide range of β values (on a log scale). For practical cases it is plausible to assume that players are generally risk averse, normally preferring a sure thing to a gamble of equal expected value, and a gamble of low variance over a riskier prospect (Kahneman & Lovallo, 1993), and that allocators, who face the risk of losing a larger amount (in case or rejecting their offers), will display more risk aversion than recipient. In terms of the model prediction, we thus may assume that, in general, a, b, and β are smaller than 1. For $0.5 \leq a, b \leq 1$, we have $0.5 \leq \beta \leq 2$. Substitution in Eq. 24b and solving for x we get $0.570 \leq x \leq 0.682$, implying that the offer should range between 0. 430 (for $\beta = \frac{a}{b} = 0.5$) and 0.318 (for $\beta = \frac{a}{b} = 2$). Numerical integration over the range β = 0.5- 2 yield an average offer of about 0.369; which is only about 1% less than the 0.382 predictions obtained under the linearity assumption.

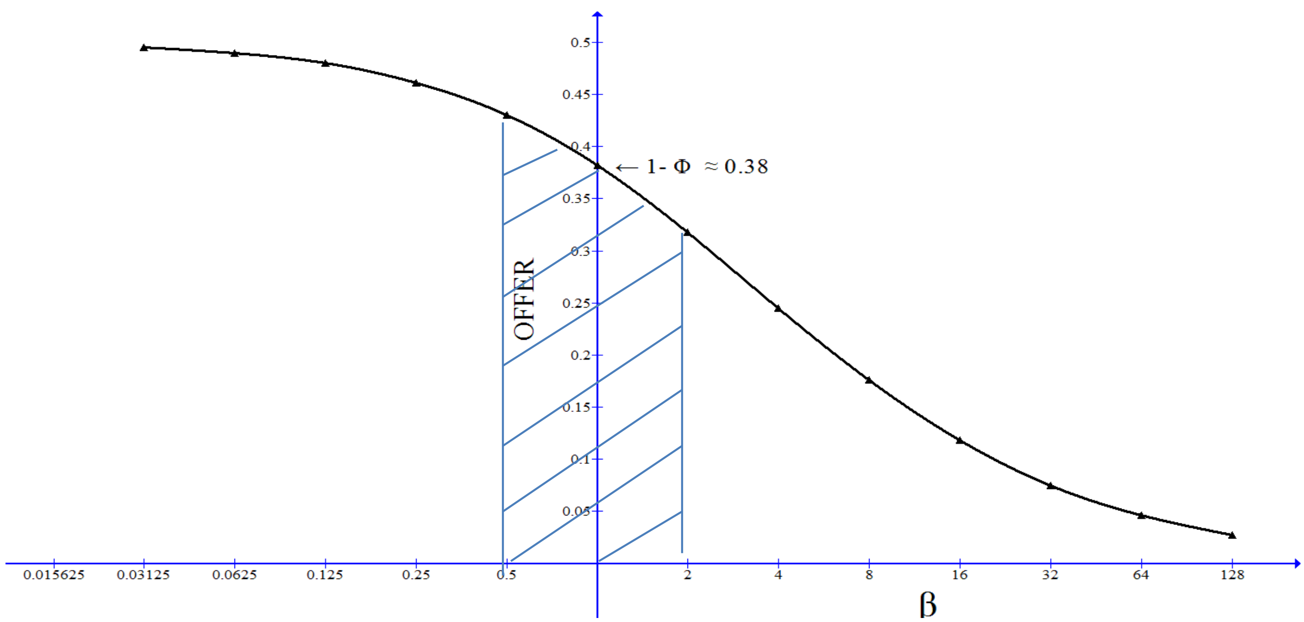

Figure 5: The predicted offer as a function of the ratio between the proposer and the responder risk indexes

7. Summary and concluding remarks

The present paper proposed a formal model, based on aspiration levels considerations and utilized it to predict the allocators' offers in a class of ultimatum games. For the standard ultimatum game, the model yields two “harmony” points, at which the levels of outcome satisfaction of the two interacting players are equal. Under the assumption that recipients aspires to receive 50% of the total net profit, the predicted “harmony” offer is *one third* of the total amount. On the other hand, assuming recipients aspires to receive an amount that equals the amount that the allocators keep for themselves; the predicted harmony offer is $1-\varphi \approx 0.38$ of the total amount, where $\varphi \approx 0.62$ is the Golden Ratio. Contrasting the model's predictions for the standard ultimatum game, and for variants of the game, including a three-person ultimatum game, revealed that the model yields impressive point predictions of observed mean offers. Also, Generalization of the model to predicting behavior in n-person person games, including public goods and common pool resource dilemma games seems quite tractable.

In section 3 we emphasized that a point of harmony will not emerge as a behavioral outcome, unless it is supported by an effective punishment. This conjecture is supported by the results of the experiment (reported in section 50), which clearly shows that convergence to the predicted Golden Ratio solution occurred only under high, but not under low punishment efficacy. Interestingly, while some learning seems to affect the behavior of allocators under low punishment, under high punishment and no-prime, the effect of learning seems marginal or nonexistent (see Fig.4). This raises the intriguing possibility, supported by the finding of one-shot games, that individuals are endowed with initial propensities to divide according to the Golden Ratio.

Several theories, including Inequality Aversion (IA) (Fehr and Schmidt, 1999), ERC (Bolton & Ockenfels, 2000), have been proposed to explain cooperation and fairness in strategic games. As demonstrated in section 4, the proposed "harmony" model outperforms the predictions of the above mentioned theories. This is particularly significant, given the fact that does not include any parameter to account for the possibility of cooperation and fairness, as done in previous models. It follows that under plausible assumption about the players' aspirations, *fairness emerges as a possibility among completely self-regarding individuals*.

The emergence of the Golden Ratio as a "fair" division adds to its numerous appearances in the arts and human's aesthetic taste (See, e.g., Livio,2002; Pittard, Ewing, & Jevons, 2007; Green, 1995), and in physics and life sciences (Klar, 2002; Shechtman et al., 1984; Coldea et al., 2010; Suleiman, 2013), including the human brain's functioning (Weiss & Weiss, 2003; Roopun et al., 2008; Conte et al., 2009) suggesting the intriguing possibility that our aesthetical and ethical preferences are to a large extent, "hard wired", with deep evolutionary roots. At the cognitive level, one might hypothesize that at the psychophysiological and neurological levels, humans' responses to receiving fair offers are similar to their responses to aesthetically pleasing visual or auditory stimuli. In a recent fMRI study, in a recent fMRI study (Wang et al., 2014) reported that participants who performed aesthetic judgments on both faces and scenes containing moral acts, exhibited common involvement of the orbitofrontal cortex (OFC), inferior temporal gyrus and medial superior frontal gyrus, suggesting that both types of judgments are based on the orchestration of perceptual, emotional and cognitive components. Another study by Chapman et al. (2009), published in *Science*, the authors demonstrated that photographs of disgusting contaminants, and receiving unfair offers in the ultimatum game, evoked similar activation of the muscle region of the face characteristic of an oral-evoke physiological responses similar to responses evoked by favorable stimuli.

References

Baldassarria, D., & Grossman, G. Centralized sanctioning and legitimate authority promote cooperation in humans. *Proceedings of the National Academy of Sciences*. U.S.A., 108 (27), 11023–11027, 2011.

Bardsley, N. Dictator game giving: altruism or artefact? *Experimental Economics*, 11, 122 -133, 2008.

Bolton, G. E., & Ockenfels, A. ERC: A theory of equity, reciprocity, and competition. *American Economic Review*, 90(1), 166-193, 2000.

Boyd, R., & Richerson, P. J. Punishment allows the evolution of cooperation (or anything else) in sizable groups. *Ethology and Sociobiology*, 3, 171-19, 1992.

Budescu, D.V., Au, W. A model of sequential effects in common pool resource dilemmas. *Journal of Behavioral Decision Making*, 15 (1), 37–63, 2002

Budescu, D.V., Au, W., & Chen, X. Effects of protocol of play and social orientation in resource dilemmas. *Organizational Behavior and Human Decision Processes*, 69 (3), 179 -193, 1997.

Budescu, D. V., Suleiman, R., & Rapoport, A. Positional order and group size effects in resource dilemmas with uncertain resources. *Journal of Organizational Behavior and Human Decision Processes*, *61* (3), 225-238, 1995.

Budescu, D. V., Rapoport, A., & Suleiman, R. (1995). Common pool resource dilemmas under uncertainty: qualitative tests of equilibrium solutions. *Games and Economic Behavior*, *10*, 171-201.

Camerer, C. & Thaler, R. H. Ultimatums, dictators and manners. *Journal of Economic perspectives*, 9 (*2*), 209-219, 1995.

Camerer, C. *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton: Princeton University Press, 2003.

Chapman, H. A., Kim, D. A. Kim, Susskind, J. M., & Anderso, A. K. In Bad taste: evidence for the oral origins of moral disgust. *Science*, *323*, 1222-1226, 2009.

Coldea, R., Tennant, D. A., Wheeler, E. M., Wawrzynska, E., Prabhkaran, D., Telling, M., Habicht, K., Smeibid, P., & Kiefer, K. Quantum criticality in an Ising chain: Experimental evidence for emergent E8 symmetry. *Science*, 327(5962), 177-180, 2010.

Conte, E., Khrennikov, A., Federici, A., & Zbilut, J. P. Variability of brain waves: A new method based on a fractal variance function and Random Matrix Theory. *Chaos, Solitons & Fractals*, 41(5), 2790-2800, 2009.

Crowne, D. P. Family orientation, level of aspiration, and interpersonal bargaining, *Journal of Personality and Social Psychology*, 3 (6), 641-664, 1966.

Fechner, G. T. *Elemente der Psychophysik* (Elements of Psychophysics), 1860.

Fehr, E., & Fischbacher, U. Social norms and human cooperation. *Trends in Cognitive Sciences*, *8*, 185-190, 2004.

Fehr, E., & Gächter, S. Altruistic punishment in humans. *Nature*, *415*, 137-140, 2002.

Fehr, E., & Schmidt, K. M. A theory of fairness, competition, and cooperation. *Quarterly Journal of Economics*, 114, 817-868, 1999.

Green, C. D. All that glitters: a review of psychological research on the aesthetics of the golden section. *Perception*, 24 (8) 937 - 968, 1995.

Güth, W., Schmittberger, R., & Schwartze, B. An experimental analysis of ultimatum games. *Journal of Economic Behavior and Organization*, 3, 367-388, 1982.

Harrison, G., & Hirshleifer, J. An experimental evaluation of weakest-link/best-shot models of public goods. Journal of Public Economics, 97, 201-225, 1989.

Hamner, W. C., & Donald L. H.). The effects of information and aspiration level on bargaining behavior. *Journal of Experimental Social Psychology*, 11 (4), 329–342, 1975.

Henrich, J. et al. "Economic man” in cross-cultural perspective: Behavioral experiments in 15 small-scale societies. *Behavioral and Brain Sciences*, 28, 795–855, 2005.

Hilgard, E. R., Sait, E. M., & Margaret, G. A. Level of aspiration as affected by relative standing in an experimental social group. *Journal of Experimental Psychology*, 27 (4), 411-421, 1940.

Kagel, J. H., & Wolfe, K. W. Test of fairness models based on equity consideration in a three-person ultimatum game. *Experimental Economics*, *4*, 203-219, 2001.

Kahneman, D., & Lovallo, D. Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking. *Management Science*, 39 (1), 17-31, 1993.

Kahneman, D. Knetsch, J. L., & Thaler, R. H. Fairness and the assumptions of economics. *Journal of Business*, 59, S285-S300, 1986.

Klar, A. J. S. Fibonacci’s flowers. *Nature*, 417, 595, 2002.

Komorita S. S., & Ellis, A., L. Level of aspiration in coalition bargaining. *Journal of Personality and Social Psychology*, 54 (3), 421-431, 1988.

Larrick, R. P., & Blount, S. Social context in tacit bargaining games. In R. M. Kramer, & D. M. Messick (Eds.). *Negotiations as a Social Context* (pp. 268-284). Sage Publications, 1995.

Lewin, K., Dembo, T., Festinger, L., & Sears, P. S. Level of aspiration. In, J. Hunt (Ed.). *Personality and the behavior disorders* (pp. 333-378). Oxford, England: Ronald Press, 1944.

Liua, J-P, & Chow, S-C. A two one-sided tests procedure for assessment of individual bioequivalence. *Journal of Biopharmaceutical Statistics*, 7 (1), 49-61, 1997

Livio, M. *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. New- York: Broadway Books, 2002.

Lopes, L. L. Between hope and fear: The psychology of risk. *Advances in Experimental Social Psychology*, 20, 255-295, 1987.

Lopes, L. L. Algebra and process in the modeling of risky choice. In J. Busemeyer, R. Hastie, & D. L. Medin (Eds.), *Decision making from a cognitive perspective* (vol. 32, pp. 177- 220). San Diego: Academic Press, 1995.

Lopes, L. L., & Oden, G. C. The role of aspiration level in risky choice: A comparison of cumulative prospect theory and SP/A theory. *Journal of Mathematical Psychology*, *43*, 86-313, 1999.

Luce, R. D., Steingrimsson, R., and Narens, L. Are psychophysical scales of intensities the same or different when stimuli vary on other dimensions? Theory with experiments varying loudness and pitch. *Psychological Review*, 117 (4), 1247-1258, 2010.

Mitzkewitz, M., & Nagel, R. (1993). Experimental results on ultimatum games with incomplete information. *International Journal of Game Theory*, *22*, 171-198.

Nikolic, S.T., Cosic, L., Pecujlija, M., & Miletic, A. The effect of the Golden Ratio on consumer behavior. *African Journal of Business Management*, *5*, 8347-8360, 2011.

O'Gorman, R., Henrich, J., & Van Vugt, M. Constraining free riding in public goods games: designated solitary punishers can sustain human cooperation. *Proceedings of the Royal Society B,* 276, 323-329, 2009.

Oosterbeek, H., Sloof, R., & Van de Kuilen, G. Cultural differences in ultimatum game experiments: Evidence from a meta-analysis. *Journal of Experimental Economics*, 7 (2), 171-188, 2004.

Palmer, S. E. Vision Science - Photons to Phenomenology, MIT Press, Cambridge, MA, 1999.

Pittard, N., Ewing, M., & Jevons, C. Aesthetic theory and logo design: examining consumer response to proportion across cultures. *International Marketing Review*, 24(4), 457-473, 2007.

Posamentier, A. S., & Lehmann, I. *The fabulous Fibonacci Numbers*. Prometheus Books, 2007.

Posch, M. Win–stay, lose–shift strategies for repeated games - memory length, aspiration levels and noise, *Journal of Theoretical Biology*, 198, 183–195, 1999.

Rapoport, Am., & Kahan, J. P. Standards of fairness in 3-quota 4-person games. In R. Tietz (Ed.), *Aspiration levels in bargaining and economic decision-making* (pp. 337-351). New York: Springer-Verlag, 1983.

Rapoport, A., Budescu, D.V. & Suleiman, R. Sequential requests from randomly distributed shared resources. *Journal of Mathematical Psychology*, 37, 241-265, 1993.

Rapoport, A., Sundali, J. A., & Seale, D. A. Ultimatums in two-person bargaining with one-Sided uncertainty: Demand games. *Journal of Economic Behavior & Organisation*, 30, 173-196, 1996.

Rieger, M. O. (2010). SP/A and CPT: A reconciliation of two behavioral decision theories. *Economics Letters*, 108 (3), 327–329, 2010.

Roopun, A. K., Kramer, M. A., Carracedo, L. M., Kaiser, M., Davies, C. H., Traub, R. D., Kopell, N. J., & Whittington, M. A. Temporal interactions between cortical rhythms. *Frontiers in Neuroscience, 2(2)*, 145-154, 2008.

Samid, Y, & Suleiman, R. Effectiveness of coercive and voluntary institutional solutions to social dilemmas. In A. Biel, D. Eek, T. Garling, & M. Gustafsson (Eds.) .*New Issues and Paradigms in Research on Social Dilemmas* (pp. 124-141). Springer, 2008.

Shechtman D., Blech I., Gratias D., & Cahn J. W. Metallic phase with long-range orientational order and no translational symmetry. *Physical Review Letters*, 53, 1951, 1984.

Siegel, S. Level of aspiration in decision making. *Psychological Review,* 64, 253-262, 1957.

Simon, H. A. Theories of decision-making in economics and behavioral science. *The American Economic Review*, 49 (3), 253-283, 1959.

Stevens, S. S. On the theory of scales of measurement, *Science*, 103 (2684), 677-680, 1946.

Stevens, S.S. The psychophysics of sensory function. *American Scientist*, 48 (2), 226-253, 1960.

Suleiman, R. Expectations and fairness in a modified ultimatum game. *Journal of Economic Psychology, 17,* 531-554, 1996.

Suleiman, R., & Budescu, D. V. Common pool resource (CPR) dilemmas with incomplete information. In D. V. Budescu, I. Erev, & R. Zwig (Eds.), *Games and human behavior* (pp. 387-410). Lawrence Erlbaum Associates, Inc. Publishers, 1999.

Suleiman, R. The dark side revealed: A complete relativity theory predicts the content of the universe. *Progress in Physics, 4,* 34-40, 2013.

Thompson, L. L., Mannix, E. A., & Bazerman, M. H. (1988). Group negotiation: Effects of decision rule, agenda, and aspiration. *Journal of Personality and Social Psychology*, 54 (1), 86-95.

Tietz, R. (1997) Adaptation of aspiration levels - theory and experiment. In W. Albers, W. Güth, P. Hammerstein, B. Moldovanu & E. van Damme (Eds.). *Understanding strategic interactions. Essays in honor of Reinhard Selten* (pp. 345-364). Berlin and Heidelberg: Springer.

Wang, T., Mo, L., Mo, C., Tan, L. H., Cant, J. S., Zhong, L., & Cupchik, G. Is moral beauty different from facial beauty? Evidence from an fMRI study. *Journal of Social Cognitive and Affective Neuroscience*. doi: 10.1093/scan/nsu123, 2014.

Weiss, H., & Weiss, V. The golden mean as clock cycle of brain waves. *Chaos, Solitons, & Fractals,* 18 (4), 643-652, 2003.

Appendices

Appendix A- Derivation of the harmony demand in an ultimatum game with one-sided uncertainty

For prediction of the harmony demand, *x*, in an ultimatum game with one-sided uncertainty (Rapoport et al. 1996), let **K** denote the realization of the random variable *k***.** Like in the standard UG the aspiration levels for the proposer and responder are **K** and *x*, respectively. From Eq. 2 we can write:

$$\frac{x}{\mathbf{K}} = \frac{k-x}{x} = \frac{k}{x} - 1 \qquad \text{……..(1a)}$$

Calculating the expected value of both sides of the equation we get:

$$\frac{x}{\mathbf{K}} = \frac{1}{x}\,\mathrm{E}(k) - 1 \qquad \text{……..(2a)}$$

Since the recipient is informed that the allocator has kept *x*, she concludes that the conditional distribution of *k*, P(*k/x*) is distributed uniformly in (x, b). The expected value E(k) is given by:

$$\mathrm{E}(k) = \frac{1}{b-x}\int_x^b k\,\mathrm{P}(k/x)dk = \frac{1}{(b-x)} \cdot \frac{1}{2}(b^2 - x^2) = \frac{(b+x)}{2} \qquad \text{..…..(3a)}$$

Substitution the value of E(k) in Eq. 2a yields:

$$\frac{x}{\mathbf{K}} = \frac{(b+x)}{2x} - 1 \qquad \text{........ (4a)}$$

The expected value of the realization of *k* across all trails is equal to: **K**= $\frac{b+a}{2}$. Substitution in Eq. 4a gives:

$$\frac{x}{\frac{b+a}{2}} = \frac{(b+x)}{2x} - 1 \qquad \text{........ (5a)}$$

After simplification, equation 5a could be written as:

$$4x^2 + (b + a)\,x - b\,(b + a) = 0 \qquad \text{........ (6a)}$$

Thus the positive solution for *x* equals:

$$x = \frac{-(b+a)+\sqrt{(b+a)^2 + 16\,b(b+a)}}{8} \qquad \text{........ (7a)}$$

Appendix B – Derivation of EH prediction for a three-person ultimatum game

To derive the harmony solution for the three-person game with uncertainty regarding the identity of the responder, I decompose the three-person game into two identical two-person games, one between **X** and **Y** and another between **X** and **Z**. In each game the proposer **X** must split a cake of size $\frac{m}{2}$ in a two-person compound game, in which with probability $p = \frac{1}{2}$ the counterpart is responder in a standard ultimatum game, and with probability 1- $p = \frac{1}{2}$ she is recipient in a dictator game. For the ultimatum game, the harmony model yields a Golden Ration split of $[\frac{m}{2}\Phi, \frac{m}{2}(1-\Phi)]$. Since in the dictator game no sanctions could be applied for punishing an unfair proposer, the model predicts a split of $(\frac{m}{2}, 0)$.

The expected payoffs for **X** in her play with **Y** and with **Z** are equal $(\bar{x}_{XY} = \bar{x}_{XZ})$, and are given by:

$$\bar{x}_{XY} = \bar{x}_{XZ} = \frac{m}{2}\Phi\, p + \frac{m}{2}(1\text{-}p) = \frac{m}{2}(\Phi\, p + 1 - p) = \frac{m}{2}[1\text{- p }(1\text{- }\Phi)] \qquad \text{….. (1b)}$$

Substituting $p = \frac{1}{2}$ we get:

$$\bar{x}_{XY} = \bar{x}_{XZ} = \frac{m}{2}(\Phi + 1) \qquad \text{…….. (2b)}$$

And the expected payoff $\bar{x}$ is:

$$\bar{x} = \frac{1}{2}(\bar{x}_{XY} + \bar{x}_{XZ}) = \frac{m}{4}(\Phi + 1) \approx 1.62\ \frac{m}{4} \qquad \text{…….. (3b)}$$

For m = \$15 we get:

$$\bar{x} = 1.62\ \frac{\$15}{4} \approx \$6.08$$