Bat catches Fly in Schwarzschild spacetime

Dmitri Martila (eestidima@gmail.com)

Estonian Physical Society

Lääne 9-51, Tartu 50605, Estonia

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Abstract

According to a distant observer, a Fly escaping towards a black hole never reaches the event horizon. Let "Fly" be a synonym for a space ship with food while "Bat" stands for a space ship with starving astronauts. The question then arises, how long the Bat has to wait before reaching the Fly.

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I. INTRODUCTION

What unites such titans of thoughts like Hawking and Susskind? The merciless battle [6] around the event horizon, which gave birth to the *information loss paradox*. Where analogous paradoxes come from in the first place?

Since the discovery of the Schwarzschild solution in 1916, the coordinate singularity at the event horizon of a black hole [7–11] has called for a lot of speculations. Just like the Twin Paradox in Special Relativity, this question still trains the minds. In 2009, Gerard 't Hooft stated that "there is no physical singularity or curvature singularity at $r \to 2m$" [1]. Contrary to this statement, in 2008 it was claimed "that using nonlinear superdistributional geometry and super generalized functions it seems possible to show that the horizon singularity is not only a coordinate singularity" [12]. I have critically examined Ref. 7 and calculated by my own that this method artificially produced a singular layer of *exotic matter* stuck to the horizon area. However, exotic matter is ruled out by the *energy conditions* [9]. I found the same fatal error in another contemporary work on "Distributional Geometry" [13].

Scientists from Tartu University also published papers on the event horizon [14]. From the geodesic deviation (exceeding the first order) it was found that the passage through the event horizon is locally detectable. This distantly indicates that there could be something to find. The first paper in Ref. [14] was published in the same year as I completed my master thesis at the Tartu department with *cum laude*. Still, the area of black hole event horizon attracts stable and vivid interest. The literature brief review points out that the coordinate singularity at the event horizon might have a physical meaning. Therefore, please allow me to *explore strange new worlds ... to boldly go where no man has gone before* [21].

The question posed in the abstract is a variation of a question given by Richard T. Hammond in 1986 [2]. However, I give a different answer: because the proper time and the road to the horizon are finite, the Fly descending with fixed speed can be reached at any time. This result strongly supports the existence of a naked curvature singularity at the event horizon [3]. In the following sections this point will be strengthened. As the statement contradicts the dogmatic physical worldview, in the Discussion section I answer to possible criticism. I express my faith that the naked singularities have been found and can be observed. Even though it is shown that physically reasonable solutions can have the nakedness of singularities [4], they are far from the actual experimental detection. Contrary

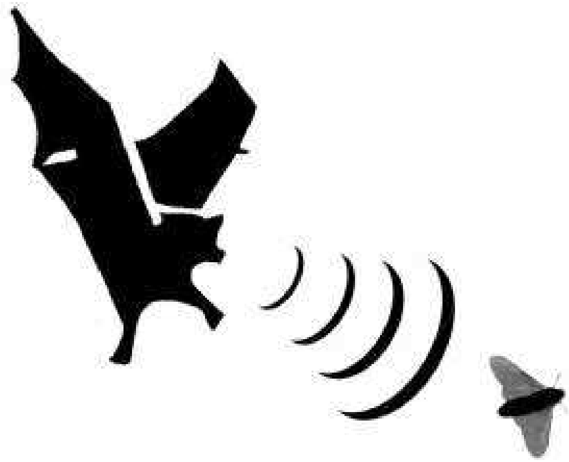

FIG. 1: “Bat” and “Fly”

to that, recognizing the physical-ness of coordinate singularities makes them automatically experimentally verifiable, because the event horizon is contained in the definition of experimentally verified black holes (e.g., in 2007 the MASTER Team discovered the Black Hole Ergosphere by surrounding the event horizon [5]).

II. THE FUNNY SITUATION OF BAT AND FLY

For large distances we are used to measure distances with locators. Let us use a more illustrative description and call the one locator space station as “Bat” and the another one as “Fly” (see Fig. 1). Our statement is that the Fly can never hide in the “black hole”. The radar pulse delays of the Bat are becoming shorter and shorter (see Fig. 2). And even if the Fly flew at nearly the speed of light, the Bat will overtake her at the surface of the “black hole”. After all, there is no signal delay $\Delta\tau \to 0$. The delay of pulse determines the distance.

III. CALCULATIONS

Let us consider the Scharzschild metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2. \qquad (1)$$

A stationary observer observes the same speed at all points on a trajectory [8],

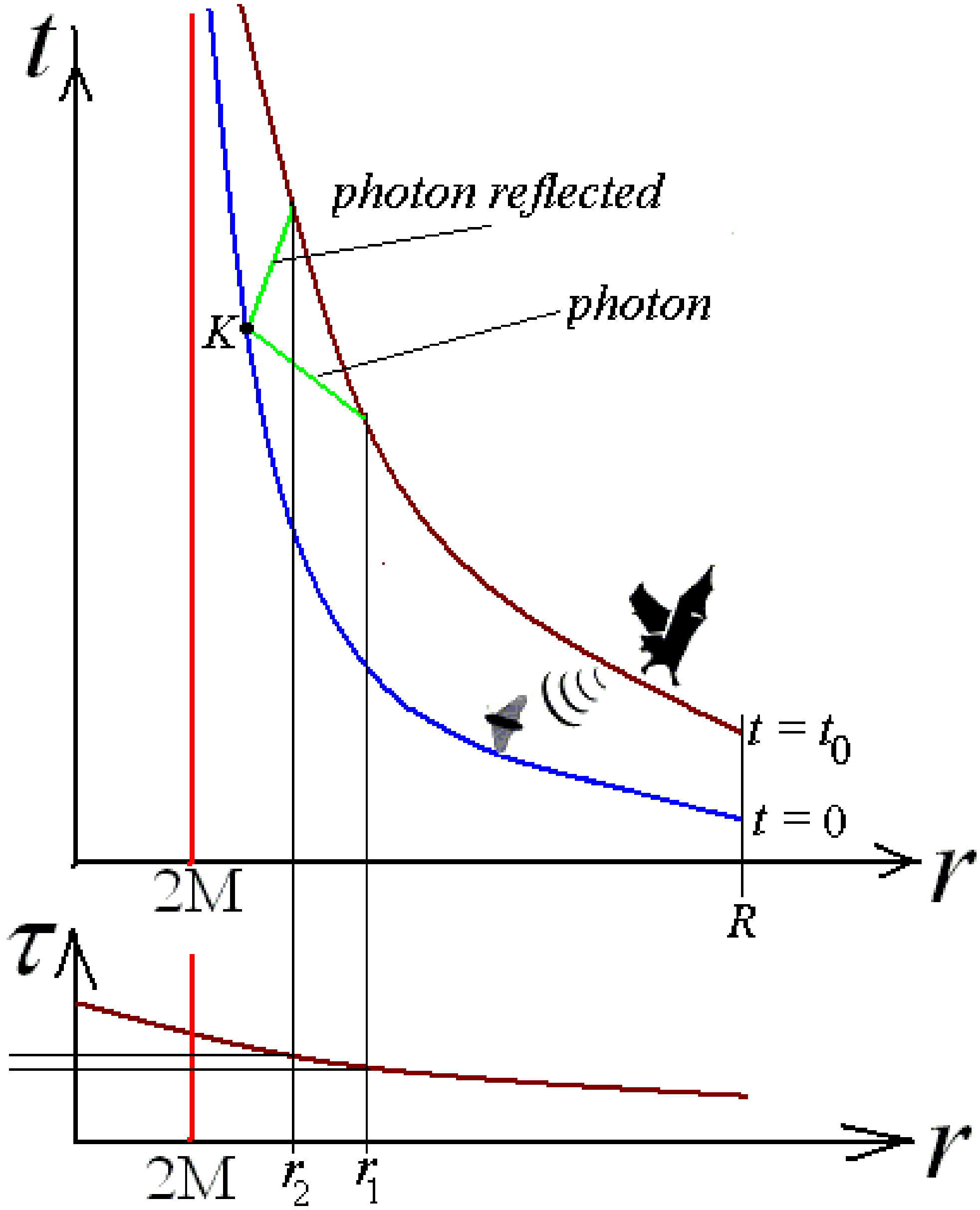

FIG. 2: The trajectory of the device "Bat", $t = t(r)$, increases indefinitely $t \to \infty$ while approaching the horizon $r = 2M$. The Bat releases a photon at $r = r_1$ and Bat catches it at $r = r_2$, hereby $2M < r_2 < r_1$. Then while $r_1 \to 2M$ the $r_2 \to 2M$. Therefore, the difference $r_1 - r_2$ tends to zero. Since the Bat's proper time is a regular function of r (ie, without the discontinuity at the horizon), the proper time interval $\Delta\tau = \tau_2 - \tau_1$ between the sending and receiving pulse (which can be labeled with verbal phrase if pulses is the whole sequence) tends to zero. And that means that the measured sonar distance $L = c\,\Delta\tau/2$ between Bat and Fly becomes zero.

$$v = \frac{d\hat{r}}{d\hat{t}} = \frac{\sqrt{g_{rr}}\, dr}{\sqrt{-g_{tt}}\, dt} \,. \tag{2}$$

So

$$dt = \frac{1}{v}\, S^2\, dr \,, \tag{3}$$

where

$$S(r) = \frac{1}{\sqrt{1 - 2M/r}} \tag{4}$$

has a singularity at $r = 2M$. The "geometrized" mass of spacetime is given by $M = G\, m/c^2$, where m is the mass of the black hole in kilogram, G is the gravitational constant, and $c = 1$ is the speed of light.

The Bat and Fly start motion from the same $r = R$ at coordinate time $t = t_0 > 0$ and $t = 0$ respectively. Bat holds local velocity v_B constant, and Fly holds v_F. Therefore the point $K(t_k, r_k)$ of reflection of pulse from "Fly" has:

$$\left(\frac{1}{v_F} - 1\right) \int_{r_k}^{r_1} S^2 dr + \left(\frac{1}{v_F} - \frac{1}{v_B}\right) \int_{r_1}^{R} S^2 dr = t_0 \,. \tag{5}$$

Here $1/v_F - 1$ is positive, because we do not consider superluminal *warp drive* (however we can). If $v_B < v_F$ the $1/v_F - 1/v_B$ is negative. Integrals are positive.

Let us check $v_F = v_B$ case. Let us take fixed $r_1 \neq 2M$, which is not infinitesimally close to $2M$. Then because first integral diverges as $r_k \to 2M$ and is zero if $r_k = r_1$, then solution is possible for any value of t_0 and speed v_F, including the photonic limit $v_F \to 1$.

Let us check $v_B < v_F$ case. Common sense tells us, that Bat never catches Fly. But it always catches! Let us take fixed $r_1 \neq 2M$, which is not infinitesimally close to $2M$. Then the second negative addition can be large, but fixed. Then the first integral, which is unlimited as $r_k \to 2M$, certainly give us solution within $2M < r_k < r1$, hereby $r_k \neq 2M$.

Notice, that always holds

$$2M < r_k < r_2 < r_1 \,. \tag{6}$$

Thus, as $r_1 \to 2M$, must be $r_2 - r_1 = 0$.

IV. PROPER TIME OF BAT

Stationary observer measures time-intervals [8]

$$d\tau_r = \frac{dt}{S} \,. \tag{7}$$

The time inside the object (flying by) relates to that observer's time via Special Relativity

$$d\tau = \sqrt{1-v^2}\; d\tau_r = \frac{\sqrt{1-v^2}}{S}\; \frac{dt}{dr}\; dr\,, \tag{8}$$

where from Eq(3) holds

$$\frac{dt}{dr} = \frac{S^2}{v}\,. \tag{9}$$

So, result

$$\tau = \sqrt{\frac{1}{v^2} - 1}\; \int_r^R S\, dr \tag{10}$$

is always regular, $\tau < \infty$. Note, that if $v = 1$ it's zero (proper time of the luminal travel); if $v = 0$ it's infinite (there is no movement).

Decision.

This "impressive" (JMP Editor) geometrical enigma should encourage you to look more carefully on the curvature of spacetime geometry.

V. SINGULAR CURVATURE TENSORS

I argue below, that removing coordinate singularity from components of the tensor, doesn't remove it from the tensor itself. Using the Einstein summation rule, where any pair of indexes means summation from 1 to 4. Traditionally [8, 9] the (fourth rank) tensor is being written as

$$\mathbf{R} = R_{\alpha\beta\eta\gamma}\, \mathbf{V}^{\alpha} \otimes \mathbf{V}^{\beta} \otimes \mathbf{V}^{\eta} \otimes \mathbf{V}^{\gamma}\,, \tag{11}$$

where functions $R_{\alpha\beta\eta\gamma}$ are called "components", but the followings are "tensor products" of the basis vectors $\{\mathbf{V}^1, \mathbf{V}^2, \mathbf{V}^3, \mathbf{V}^4\}$. Components of the basis vector number 2, calculated in their own basis will be $V^2_i = (0, 1, 0, 0)$.

Basis vectors itself make the metric $\mathbf{V}^{\gamma}\, \mathbf{V}^{\omega} = g^{\gamma\,\omega}$, where the scalar product is also connected with metric, thus $g^{i\,s}\, V^{\gamma}_i\, V^{\omega}_s = g^{\gamma\,\omega}$. This always holds, because basis vector components in their own basis are Kronecker deltas, i.e $V^{\gamma}_i = \delta^{\gamma}_i$.

Taking two basis vector fields $\mathbf{V}^1$ and $\mathbf{V}^3$. Multiplying basis vectors with curvature tensor we get function, which doesn't depend on the choice of metric (coordinates), i.e. it's true scalar

$$\mathbf{\Psi} \equiv \mathbf{R}\; \mathbf{V}^3\mathbf{V}^1\mathbf{V}^3\mathbf{V}^1 = \tag{12}$$

$$= R^{i\,k\,g\,n}\, V^3_i V^1_k V^3_g V^1_n = R^{i\,k\,g\,n}\, \delta^3_i \delta^1_k \delta^3_g \delta^1_n =$$

$$= R^{3131} = M\, S^2/r^5\,.$$

It's large near horizon, where $r \approx r_s$. Let's recheck. Perform the coordinate change $\{x^\beta\} \to \{w^\kappa\}$. First coordinate system "A" and second system "B". Vectors $\mathbf{V}^1$ and $\mathbf{V}^3$ are two basis vectors from "A" and their components are Kronecker deltas. But they are not the basis vectors of the "B". So their components are different, [9] i.e. $(dx^\beta/dw^\kappa)\delta^3_\beta = dx^3/dw^\kappa$ for vector $\mathbf{V}^3$. For transformation of covariant components $R^{i\,k\,g\,n}$ one uses the opposite matrixes, i.e. (dw^κ/dx^n).

Thus, the invariant $\mathbf{\Psi}$ shows singular behavior of the compounds of the curvature tensor near the black hole horizon. Of course some quantities are not singular there. But at famous hypothetical singularity in the center ($r \approx 0$) not all quantities are singular. For example the scalar curvature R^γ_γ with concrete physical meaning [17]. It is zero everywhere [8], also in the closest neighborhood of $r = 0$.

Any tensor (except zero rank) is determined not only by its components, but also by the basis vector fields. I argue, that if in certain coordinate system the components of curvature tensor are not singular, the basis takes the singularability over. One may be sure: the singular curvature could induce singular behavior of other locally detectable quantities. Some research group has published in Astronomische Nachrichten (highly cited, oldest astronomical journal), what the Kretschmann scalar can be singular near non-spherical horizon [18] with singular orbital acceleration even for spherical horizon [18].

Therefore the parallel transport of a vector A^β along a spacetime loop, which is not infinitesimally small, can result in a catastrophic change of $A^\beta A_\beta$ when a part of the loop approaches the event horizon.

VI. THE FORCE

The curviness of spacetime drives all effects in General Relativity. It's described by Riemann curvature tensor. The singularity of the curvature tensor COMPONENTS is called coordinate singularity, because can be removed by coordinate transformation [7–11]. But I'm pointing your attention, that there's emissaries from curvature, that are invariant under coordinate transformations and showing singular behavior according to astronaut-observer. Like the blueshift of in-falling light, the proper acceleration g and force F, which could break the bones of astronaut on the very dense planet. For him it doesn't matter, which coordinate system has described this deadly real force; the invariant scalar of Minkowski force for stationary body $K^\nu\,K_\nu = F^1\,F_1 + F^2\,F_2 + F^3\,F_3$.

The late 2011 Wikipedia elegantly wrote: "The surface gravity, g, of an astronomical or other object is the gravitational acceleration experienced at its surface. The acceleration of a test body at the event horizon of a black hole turns out to be infinite.. Because of this, a renormalized value is used." I disagree, that only because of very high values NEAR the horizon one can not use there conventional definition of surface gravity. Taking accelerometers the finite values would measure stationary observers hanging near the horizon. This corresponds to my singularity definition in Introduction.

Seeing the accelerated fall of ball (g) in our cabin, we can not say whether cabin stands on dense planet, or is being accelerated by the rockets with g. Would it be the latter case, the inertial force would have been simply $m_o\,g$. The m_o is the proper mass in Special Relativity, which is as known – invariant. *The equivalence principle guarantees that a gravity field (a central force) cannot be distinguished from forces due to uniform acceleration* [15], thus the same valued force $m_o\,g$ would experience astronaut on dense planet. Thus, the force is singular function of r, as you see in [16]; the book [8] on the star solution shows, that Archimedes force compensating the gravity acting on small section of star is of form $F \sim S/r^2$. Wald is solidaristic: *of course, the locally exerted force... becomes infinite on the horizon*, page 332 in Ref. [9]. I'm correcting him: *of course, the locally exerted force has singularity near the horizon, according to the definition from my Introduction.* More clear: *. . . near the critical state of the system, which would contain the event horizon*: a dense object has no event horizon [19], but the spacetime curvature is so strong, that situation is very similar to the area near the event horizon.

On the finite, *renormalized surface gravity* $\kappa \equiv M/r^2$. On pages 332, 158 in [9] the κ gained presentation as not local, measurable for infinite long weightless string (a test mass m_1 is attached to the end of string, hanging motionless near the Black Hole. The distant free end is without any mass attached, but is watched to be accelerated with κ by local observer. You can not attach a mechanism m_2 to this end, otherwise the κ changes its definition value. Thus, there is renormalized non-local acceleration κ, but not the renormalized force $F_2 = m_2\,\kappa$). So it is not local characteristic of localized thing – horizon! In the coordinate "map" of Universe ($r, t < \infty$) the famous central singularity (at $r \approx 0$) is absent [10]. So the $\kappa = 1/(4M)$ in [10] is the maximum strength of black hole gravity in this map.

It's superweak for supermassive $M \to \infty$ black hole, which sheds doubt on possibility to renormalize the surface gravity.

VII. RESULTS AND CONCLUSION

Although the surface gravity g has suffered procedure of Renormalization, nevertheless this operation left the gravitational attraction (force F) and curvature scalar $\mathbf{\Psi}$ to include singular function $S(r)$.

Here was argued, that I found the naked singularity of curvature [4], predicted by General Relativity. It's my personal joy and triumph, I would like to share. Because recently Pankaj S Joshi has published: *... one of the most important unsolved problems in astrophysics. Opening of naked singularities could change the search strategy unified theory of all physical interactions and not only because of the possibility of direct observational tests of this theory.* [4]

We like to remove unwanted strange things [7] but for some true researcher they are still like splinter in the brain. In General Relativity all effects are driven by the curvature of spacetime and too many things are singular near event horizon. Like the redshift of light [8], clocks slowing [8], near light-speeds of massive test particles [8].

VIII. DISCUSSION

An opponent might say: "in local coordinate frame (*tetrad* [9]) the Riemann tensor is not singular". I argue. Introducing coordinate frames, one changes the coordinate system. So this case belongs to Section V.

An opponent might say: "the geodesic deviation equation [8, 9] shows, that there is no catastrophic deformation of the in-falling body-astronaut passing the horizon. Where is the singularity, if it's real?". I argue. Such equation assumes, that proper time is absolute: if astronaut would compare the clocks attached to his head and legs, they always show the same time [8]. But as we know, the atomic clocks in building's basement are going slower, than at the roof. Then opponent might refer to the strong equivalence principle [15]. But this principle silently assumes, that the laboratory dimensions are negligible in respect to curvature. But this is not true in case, if the coordinate singularity is real. Just like we all were taught, that the central singularity ($r \approx 0$) rips apart the in-falling body by *tidal forces* [9]. Imagine, if you -*superhero*- standing on shore will throw a black hole into lake, you never see the waterline descending, because the forward front of water enters black hole on the bottom at infinite coordinate time [8, 10]. But your hand-clock measures just this time [8, 10]. But this is absurd: black holes "eat" the matter, even stars [9]. So the only logical solution is that super-strong gravitation of naked singularity compresses water near the horizon of black hole, becoming its assimilated part. Such innovative *tidal force is well defined* [9], see page 68.

An opponent might say: "the geodesic trajectory of in-falling test particle can be mathematically continued inside the horizon. If there would be singularity, the geodesic were be terminated at the horizon, just like it does at the central singularity". I argue. Who can guarantee, that in infinite distance coordinate future (the Earth time) the area, where the spacetime metrics changes the signature (so the future-directed worldline tangent vector becomes past-directed [10]), it still has physical meaning? My own calculations show, that after test particle crosses the horizon, it leaves our Universe (the *coordinate map* [9]). It's because the Jacobian determinant connecting *Schwarzschild coordinates* $\Leftrightarrow$ *Comoving coordinates of gravitational collapse* turns to un-allowed zero [16], after the star surface drops down the horizon sphere. In a deeper sense the event horizon in General Relativity is the un-crossable light-speed barrier in Special Relativity. Because the principles of equivalence are foundations of Einstein's Relativity.

An opponent might say: "the geometrical nature of the General Relativity does not allow any force-interpretation". I disagree. The Curvature of Geometry is producing the value on Dynamometer, which designed to measure the forces. Otherwise the Newton would not introduced his *Inverse Square Law* $F \sim 1/r^2$. So the General Relativity shall not contradict the Classical Theory. Wald is solidaristic: *we may meaningfully speak of the gravitational force field of the Earth*, page 68 in Ref. [9].

An opponent might say: "there are papers you don't know, where is proved that there are no stationary observers in Schwarzschild spacetime. Especially near the horizon". I argue. First of all, the spacetime asymptotically becomes flat. And in flat spacetime certainly can exist observers with stationary position. Secondly, if even in principle, even *thought experimentally* (Ref. [9]: page 158) the observer can not hold its position (even for a second), then it's the additional proof. That there is the real singularity. Where physical and philosophical concepts like time, energy, stationarity loose their common sense.

IX. ON EXISTENCE OF BLACK HOLES

I present objections against the statements of the popular press: "UNC professor says black holes can't exist" (WNCN News), "Big Bang was not, and black holes do not exist – proved mathematically", "Black holes dont exist - Physicist claims "proof" Black holes do not exist, Big Bang wrong " (YouTube). Sadly, but this was deduced from Houghton paper [19], which has different value. So let me note following on [19]:

1) the authors use exotic matter ("negative energy flux"). But sadly, there is no emphasizes on issue of violation of the energy conditions (weak, strong or dominant ones: good review is in Wikipedia). So, such kind of articles would justify: faster than light travel, wormholes, time machines and what is most crucial here – the exotic matter can give the anti-gravitation, which can stabilize the collapsing star. These hypothetical exotics are usually with their paradoxes. So it is not surprising the result authors got: there are no black holes, only a temporary collapse.

2) Sadly, while talking about Oppenheimer Snyder (OS) model the Houghton missed mentioning the Marshall's article [20]. But I quote: "At this point OS made a fatal error by choosing an..." (Marshall). So it is sad to quote Houghton: "We use the matching of metrics at the surface of the star, illustrated in Section 2.2 for the OS model".

3) Houghton: "More explicitly, the surface gravity of the black hole is defined in terms of the 4-acceleration of an external observer. If κ were increasing with time, so would the acceleration of inertial relative to freely falling observers." My warning is following. On the finite, renormalized surface gravity $\kappa \equiv M/r^2$: on pages 332, 158 in Ref. [9] the κ gained presentation as not local, measurable for infinite long weightless string. So it is not local characteristic of localized surface! Therefore, the subject of your first sentence is surface gravity g, but subject of the second sentence is κ. Moreover, free falling observer, which happen to be commoving with a free falling particle (Houghton's "inertial") do not observe any acceleration of this particle.

4) Houghton: "We can equivalently deduce the bounce of the star and show that it is reached before the horizon forms, from the Einstein equations and the total energy conservation". Me: the energy topic is not lucid enough in General Relativity.

5) Houghton: "We conclude that the star never enters the Schwarzchild surfaces, meaning the bounce occurs before the formation of an event horizon. The reason behind this result lies on the fact that the inclusion of negative energy radiation in the interior of the star, violates the energy condition of the PenroseHawking singularity theorem". Me: from the very beginning there were no problem of forming the event horizon. According to conventional physics this surface forms in infinite distant coordinate time. Who can guarantee, that in actual infinity the Universe and matter are there? I respectfully suggest to correct your word "Schwarzchild" into Schwarzschild.

Finally, there is always the mixture of ordinary and exotic matter in Houghton's contribution. I mean, even if in some point the total energy density is zero, it does not mean, that there are no matter present. But in real world there is some kind of annihilation of the opposites. So if the energy is zero, there is no ordinary and no exotic matter present.

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