Revival of the "Original" Yukawa Charge Exchange Interaction and the Liquid Drop Character of the Nucleus

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Abstract

The reason, as to wherefrom arises the liquid drop character of the nucleus, has been a source of puzzlement since the birth of nuclear physics. We provide an explanation of the same by reviving the very first and the "original" charge exchange interaction model suggested by Yukawa in 1935.

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The liquid drop character of the nucleus has been a source of puzzlement since the birth of nuclear physics. At present the Independent Particle character of the nucleus, built around the concept of Charge Independence and the Generalized Pauli Principle, is known to provide a very successful picture of the nucleus. This of course, is in complete variance with respect to the Nuclear Drop model of the nucleus. We know that the Independent Particle Model character basically arises and is consistent with quantum mechanics. And this of course ensures its success. But we have no idea as to wherefrom the liquid drop character of the nucleus arises. We look into this open-ended issue.

The very first and the "original" exchange interaction as suggested by Yukawa in 1935, in standard notation, was [1];

$$V = -g^2(\tau_+(1)\tau_-(2) + \tau_-(1)\tau_+(2))\frac{e^{-\mu r}}{r} \tag{1}$$

This interaction acts between a neutron-proton pair. It is zero when acting between a pair of protons or a pair of neutrons. This interaction conserves electric charge. It was exactly this that was needded to bind a deuteron and to explain the scattering data in the neutron-proton case. It works for the mirror pair as well.

It was only later in 1936, that on realizing that the interaction may also be mediated by neutral pions, that Yukawa and Kemmer independently, incorporated the exchange of neutral pion as well. This led to the full SU(2) isospin symmetry playing its role in nuclear physics, This leads to the concept of Charge Independence in nuclear physics - which turned out to be a very fruiful idea. The subsequent discovery of the mesons, π^+, π^-, π^0 with masses around 140 Mev, ensured the success of the full isospin group in nuclear physics.

In the early days of nuclear physics Rarita and Schwinger had employed exchange forces of two different kind [2]. One was the so called "symmetrical type" in which the isotopic dependence of the potential was

$$\vec{\tau_1}.\vec{\tau_2} \tag{2}$$

This is the standard isospin contribution which one uses along with the Generalized Pauli Exclusion Principle. The other one was called the "charged theory" in which the isosin dependence was as

$$\vec{\tau_1}.\vec{\tau_2} - \tau_{3,1}\tau_{3,2} \tag{3}$$

Note that this charged theory was modelled after the "original" Yukawa interaction term above. Because of the existence of all the charge states of the pion, soon the original idea of Yukawa got subsumed in the bigger SU(2) group. Everyone convinced himself/herself that the original idea of Yukawa interaction mediated by π^+, π^- was nothing but a subset of the larger set π^+, π^-, π^0 and which was anyway existing with a mass of about 140 MeV. And thus all the putative successes of the "original" Yukawa model gets attributed to the pion of the full SU(2) isospin group. And this is what is believed to be true as of now!

However, let us re-visit the whole idea once again. Note that for pion $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, these cartesian components do not correspond to the physical pion. Only when these are written as suitable complex linear combinations in terms of the polar coordiantes as $\frac{\pi_1 + i\pi_2}{\sqrt{2}}$, $\frac{\pi_1 - i\pi_2}{\sqrt{2}}$, and $\pi_3 = \pi^0$ is that these can be associated with the charged pion states as π^-, π^+, π^0 with mass about 140 MeV. Clearly the original charged states π^-, π^+ of Yukawa's 1935 idea, do not correspond to anything known physically except as subset of the charged pion set at 140 MeV.

But note that the mathematical method adopted above to go from the cartesian components to the spherical polar components is the same as adopted for the polarization states of the photon [3]. There the polarization vector $\vec{\epsilon}$, the cartesian components (as in the case of pions above), do not correspond to anything physical, and only in terms of the spherical polar coordinates $(\epsilon_x + i\epsilon_y), (\epsilon_x - i\epsilon_y), \epsilon_z$ do these correspond to the physical polarization states of the massive photon as (+1, -1, 0). So there is a parrallelism between the massive and charged pion states with the massive photon's polarization states. However, in the case of the photon, we have the case of real photons which are massless and for which we know that there are only two tansverse states of polarisation (+1 , -1). The longitudinal polarization state is missisng. One may ask as to how come, the same massless states of pion with charge zero, does not exist as a mediator of the strong interaction? One problem that one faces right away is that, for photon, the real photon is massless with only two trasverse polarization states, and it is the same photon, which under specific situations, becomes massive and gets all the three polarization states. However for the pions, we know of but one massive pion with the three charge states. Physically this pion does not change mass in any particular situation and there is no physically detected pion of mass zero. However, if the parallelism between the photon polarization states and the pion charge states makes sence, there should be a massless pion with only two charge states +1 and -1 and which should be physically relevant. Let us take this point of view seriously. This we do and assume that this massless "pion" is a different entity with respect to the massive pions and must be somehow "hidden". But now it should have specific physical effects which would manifest its presence as a physical reality. What possible physical effects such a massless "pion" may manifest?

As seen above, Yukawa did propose the charged current interaction but without invoking masslesness for it. In fact, he demanded that these be massive to explain the finite range of the strong interaction. And later the discovery of the three charged pions at about 140 MeV mass, confirmned this logic. But the point is that it is possible to have another independent charge current interaction without a neutral pion, to explain the proton-neutron interaction. Does this interaction exist independently of the charge independent isospin interaction as manifestd in the SU(2) framework? Today, the dominant Independent Particle Model of nuclear physics is modelled after the SU(2) isopsin group with Charge Independence and Generalized Pauli Principle as its base.

However if this were the whole story, then there would be no need of the massless "pion" model. But the reality is that the nuclear physics phenomenon can be equally well described by treating the nucleus as made up of two independent Fermi seas of protons and neutron separately. In that picture the neutron and protons are treated as distinguishable. This point is considered in detail in several places. In fact it has been shown convincingly at several places that these two pictures of the nucleus, as consisting of independent and distinguishable proton and neutron seas, and the other one where the Generalized Pauli Principle treating p-n as identical, both give equivalent descriptions of the nucleus. This is well recorded, for exapmle in Blatt and Weisskopf [4, p. 153-156], Brink [5. p. 16-18] , Lawson [6, p. 107-122].

If we treat these two as independent pictures, as these indeed clearly are (as shown above [4,5,6]), then it would be logical to assume that these should be based on their own "independent" exchange pictures. We of course have the three pion charge states and their exchanges as the base for the Generalized Pauli Principle model leading to the Independent Particle Model. And next, the other picture of charged current interaction mediated by "massless" pions comes to our rescue to be used as the charged current interaction between neutrons and protons in the other independent picture of the nuclear reality. Let us see in what manner we can undertand this independent neurton and proton Fermi sea model in terms of this "original" exchange picture of Yukawa.

To understand this we go to Sacks derivation of how the above "Original" interaction of Yukawa may manifest itself. We revive here an old model of Sachs [7] of a phenomenological exchange potential. We know that the Majorana potential is the space exchange potential. Let us define it as [7, p. 60]

$$V = \frac{1}{2} \sum_{j,k\, j \neq k} J(\vec{x}_j - \vec{x}_k) P_{jk} \tag{4}$$

where J is a simple attractive function and P is the space exchange operator. Note that the full attractive potential produced by J is felt by nucleons in the same state only. **The problem is that it is known that there is no classical analogue of the Majorana potential**. To undesrtand this and to get a better feel of the nature of the potential, let us consider the electric current and the charge density. Define charge density as

$$\rho(\vec{r}) = \sum_{k=1,A} e_k \int {\psi^2}_{r_k=r} d^{3(A-1)} \vec{r} \tag{5}$$

Here integration is over all 3(A-1) nucleon configuration space of all the particles other than the k-th. e_k is the charge of the particle-k. A sum over all spins is also implied.

The equation of continuity is

$$div\vec{S} + \frac{\partial \rho}{\partial t} = 0 \tag{6}$$

The current density $\vec{S}$ is defined such that the above continuity equation holds good. For ordinary forces used in nuclear physics, both in the isopin Generalized Pauli Exclusion Principle formalism or the ordinary neutron-proton formalism, this holds true. This current given by

$$\vec{S}_0(\vec{r}) = \frac{\hbar}{2Mi}\sum_k e_k \int \{\psi^* grad_k \psi - \psi grad_k \psi^*\}_{r_k=r} d^{3(A-1)}\vec{r} \tag{7}$$

satisfies the continuity equation by virtue of the time dependent Schroedinger equation.

Problem arises, however if the potential in the Schroendinger equation involves a space exchange operator. $\vec{S}_0$ does not satisfy the above continuity equation anymore [7,8]. A satisfactory current $\vec{S}$ is obtained [7,8,9] only by adding an appropriate quantity $\vec{S}_x$ to $\vec{S}_0$ as

$$\vec{S} = \vec{S}_0 + \vec{S}_x \tag{8}$$

where $\vec{S}_x$ is the space exchange current. They suggest a simple framework to extend this structure as [7,8,9]

$$\vec{S}_x(\vec{r}) = -\frac{ie}{\hbar}\langle \sum_{\pi\nu} \vec{r}_{\pi\nu}[\int_0^1 d\alpha \delta(\vec{r} - \vec{r}_\pi - \alpha \vec{r}_{\pi\nu})] J(\vec{r}_{\pi\nu}) P_{\pi\nu}\rangle \tag{9}$$

The indices π, ν are labels for proton and neutron. Note that this cotribution comes only from points lying on a line connecting a proton and a neutron. This current flows on a filament connecting unlike particles. They interpret this as the electric charge switching back and forth between a neutron-proton pair, thereby producing a current along the line joining them [7, p. 62, 8,9].

In the above, they assumed a putative massive pion. But as we saw, we have to actually treat this "pion" as distinct from the physical pion of mass 140 Mev. Our "pion" here is massless. and as such it should mediate the above charge exchange of Sachs model while travelling with the velocity of light. It should take only about 10^{-22} seconds to go through a nucleus of size $\sim$ 10 fm or so. How will this manifest itself?

We know that due to the 140 MeV massive pions the range of the strong interaction would be of the order 1 fm or so. Now this new massless pion mediated charged current will carry the charge right through from one end of the nuclear surface to the other end of the surface. So for the N=Z nucleus this interaction will be sending the charges across to and fro from one end of the boundary (or nuclear surface) to the other. Thus this should be a "surface" effect as in the liquid drop model. Thus we should not interpret this as some charged current, but actually as oscillation of a charged liquid drop with a period of of about 10^{-22} seconds. Question, does this make sence for a nucleus? Below as shown in the Appendix, indeed this model prediction holds good with respect to the physical reality of the nuclear liquid drop model. **Hence the massless charged pion exchange should be the basis as to why the nucleus behaves as a liquid drop**. This is the main result of this paper.

So, it is a fact, that there are two equally successful and independent models of the nucleus. The first one, using full SU(2) isospin symmetry, with proton and neutron treated as indistingishable particles and with 140 MeV massive pions π^+, π^-, π^0 as mediators, and thus having Charge Independence as a base, leads to today's successful Independent Particle Shell Model. The other one, treats protons and neutrons as distinguishable particles and provides an equally successful description of the nucleus as the first one above (see e.g. Blatt and Weisskopf [4, p. 153-156], Brink [5. p. 16-18] , Lawson [6, p. 107-122]). But then what are the mediators of this independent model ? We have shown here that these were actually provided by the very first and the "original" charge current exchange mechanism of Yukawa in 1935 ; but with an additional factor of our new demand, that these charged "pions" π^+, π^-, being diiferent entities from the 140 MeV massive pions, be actually massless. Exchange of these massless "pions" is shown here to be naturally explained by the oscillating charge model of Sachs [7,8,9]. We have thus shown how this leads to providing the liquid drop character to the nucleus.

Appendix : Oscillations of nucleus as charged liquid drop

The presure of rupture of a nuclear drop of charge Q and radius R due to the Coulomb force, given below on the left-hand-side, for stabilty should be balanced by the pressure created by surface tension α as

$$\frac{q^2}{4\pi R^4} \approx \frac{2\alpha}{R} \tag{10}$$

For Fermium isotope, with Z= 100 and A=256, of size $\sim$ 10 fm, this gives a surface tension $\alpha \sim 10^{20}$ dyne/cm.

Let us use dimensional analysis to obtain the period of nuclear oscillations. We assume that the nuclear vibrations of an excited nucleus is caused only by the surface tension. For the sake of simplicity we are neglcting the charge of the nucleus. (However, it turns out that a more exact calculation including the charge as well, does not affect the order of magnitude of the the period of oscillations as found here). Hence the period of oscillation T will depend upon the surface tension α, the density ρ, and the size R. Thus

$$T \sim \rho^x R^y \alpha^z \tag{11}$$

where x,y and z are unknown parameters. Finally we get

$$T \sim (\frac{\rho}{\alpha})^{\frac{1}{2}} R^{\frac{3}{2}} \tag{12}$$

With $\rho \sim 10^{14} \frac{gm}{cm^3}$ this gives the value of the period of nuclear oscillations as $T \sim 10^{-21} sec$. The frequency of oscillations is thus $\omega \sim 10^{21} sec^{-1}$.

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