Contracted-Tensor Covariance Constraints on Gravity Theory

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Abstract

A fundamental theorem underpinning Einstein's gravity theory is that the contraction of a tensor is itself a tensor of lower rank. However this theorem is not an identity; its demonstration cannot be extended beyond space-time points where the space-time transformation in question has a Jacobian matrix with exclusively finite components and that matrix' inverse also has exclusively finite components. Space-time transformations therefore cannot be regarded as physical except at such points; indeed in classical theoretical physics nonfinite entities don't even make sense. This, taken together with the Principle of Equivalence, implies that metric tensors can be physical only at space-time points where they and their inverses have finite components exclusively, and as well have signatures which are identical to the Minkowski metric tensor's signature. For metric-tensor solutions of the Einstein equation there can exist space-time points where these physical constraints on the solution are flouted, just as there exist well-known solutions of the Maxwell and Schrödinger equations which also defy physical constraints—and therefore are always discarded. Instances of unphysical solutions of the Maxwell or Schrödinger or Einstein field-theoretic equations can usually be traced to subtly unphysical initial inputs or assumptions.

Contracted-tensor covariance constraints on space-time transformations

A key building block of Einstein's gravity theory is the requirement that the contraction of an upper index with a lower index of any tensor *is always itself a tensor whose rank is two less than that of the antecedent tensor* [1], e.g., the contraction T^μ_μ of an arbitrary second-rank mixed-index tensor T^μ_ν *is always itself a scalar*. This universal contracted-tensor covariance requirement is readily seen to boil down to [2],

$$(\partial\bar{x}^\alpha/\partial x^\mu)(\partial x^\nu/\partial\bar{x}^\alpha) = \delta^\nu_\mu, \tag{1}$$

where $\bar{x}^\alpha(x^\mu)$ is an arbitrary *suitable* space-time transformation and $x^\nu(\bar{x}^\alpha)$ is that transformation's *inverse*.

For the space-time transformation $\bar{x}^\alpha(x^\mu)$ *to be locally suited to* Eq. (1) *at a space-time point* x^μ, its Jacobian matrix $\partial\bar{x}^\alpha/\partial x^\mu$ *must exist at* x^μ, with all the components of that matrix (i.e., all the partial derivatives $\partial\bar{x}^\alpha/\partial x^\mu$ at x^μ) well-defined in terms of the finite real numbers, and that well-defined local Jacobian matrix at x^μ must furthermore possess *a matrix inverse*, all of whose matrix components are *also* well-defined in terms of the finite real numbers. Under *those circumstances* Eq. (1) will actually *hold locally at the space-time point* x^μ because Eq. (1) is in fact *a mathematical theorem*, i.e., Eq. (1) is clearly *a corollary of the calculus chain rule*.

It is, however, crucially important to understand that although Eq. (1) is a mathematical theorem, *it does not hold unconditionally*, i.e., *it is not a mathematical identity*. For example, if any of the components of the above-discussed Jacobian matrix that arises from the space-time transformation $\bar{x}^\alpha(x^\mu)$ should locally touch an *infinite* value, or if any of the components of *the matrix inverse* of that Jacobian matrix should locally touch infinity, then at *that* space-time point the left-hand side of Eq. (1) *is ill-defined in terms of the finite real numbers*, while the right-hand side of Eq. (1) *remains well-defined in terms of the finite real numbers*. Thus Eq. (1), notwithstanding its status as a theorem, *is not even unconditionally self-consistent*, let alone unconditionally true.

Therefore since Einstein's gravity theory incorporates contracted-tensor covariance, which boils down to Eq. (1), it *cannot* also incorporate *mathematically unrestricted space-time transformations*. At the very least, a space-time transformation must be regarded as out-of-bounds to Einstein's gravity theory—i.e., as *unphysical*—at any space-time point where a component of its Jacobian matrix touches infinity or a component of the *inverse* of that matrix touches infinity.

Such exclusion of *infinite* mathematical entities is of course obviously *also required by classical theoretical physics precepts*, and therefore would not have occasioned Einstein the slightest qualm. Because of Einstein's Principle of Equivalence, however, local space-time transformations *occupy a central position in his gravity*

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theory, so the physical need to bar infinities from the components of the Jacobian matrices and the Jacobian matrix inverses of those transformations results in *clarification* of the *entirety* of that theory.

Contracted-tensor covariance constraints on metric tensors

In consequence of the Principle of Equivalence, every metric tensor is locally the congruence transformation of the Minkowski metric tensor with the Jacobian matrix of a space-time transformation [3]. In the preceding section we have learned that space-time transformations are physically acceptable only at space-time points where all the components of their Jacobian matrix are finite real numbers and all the components of the *inverse* of that Jacobian matrix *as well* are finite real numbers. In view of the strictly finite-component nature of the physically acceptable congruence transformation matrices and their inverses, and also the finite-component nature of the Minkowski metric tensor and its (identical) inverse, it is apparent that metric tensors are physically acceptable only at space-time points where all their components are finite real numbers and all the components of their inverses *as well* are finite real numbers. Moreover, because of the mathematical Sylvester law of eigenvalue signature inertia for such congruence transformations, metric tensors furthermore are physically acceptable only at space-time points where their eigenvalue *signature* is identical to the $(+,-,-,-)$ eigenvalue signature of the Minkowski metric tensor [4].

However *metric-tensor solutions of the Einstein equation* don't necessarily adhere at every space-time point to the conditions needed for physical acceptability which have just been pointed out; in fact it is *commonplace* for all kinds of *well-established field-theoretic equations* to yield *physically unacceptable solutions*.

To try to understand this phenomenon and its appropriate handling, we begin with a simple instance of it which afflicts source-free electromagnetism.

Unphysical static uniform-field solutions of source-free electromagnetism

The source-free electromagnetic field equations, namely,

$$\nabla \cdot \mathbf{E} = 0,\ \ \nabla \times \mathbf{E} = -(1/c)\dot{\mathbf{B}},\ \ \nabla \cdot \mathbf{B} = 0 \ \text{ and } \ \nabla \times \mathbf{B} = (1/c)\dot{\mathbf{E}}, \tag{2}$$

are clearly satisfied by *all* static uniform $\mathbf{E}$ and $\mathbf{B}$ fields. However *unless* those static uniform electromagnetic fields *completely vanish* the resulting electromagnetic field *energy*, which is given by $(1/2)\int d^3\mathbf{r}(|\mathbf{E}|^2 + |\mathbf{B}|^2)$, patently *diverges*, so all nontrivial static uniform *source-free* electromagnetic field-equation *solutions* are in fact *unphysical*, and indeed are *shunned* in electromagnetic theory.

The electromagnetic field *energy divergences* which occur for these unphysical static uniform field solutions of the source-free electromagnetic field equations are strikingly analogous to *the divergent wave-function normalization factors* which can occur in cases of *unphysical wave-function solutions of Schrödinger equations*.

Unphysical non-normalizable Schrödinger-equation solutions

The stationary-state Schrödinger equation for the simple harmonic oscillator in configuration representation is notable for possessing only solutions which manifest divergent wave-function normalization factors—except at a countable set of energies of measure zero.

At every energy value this stationary-state Schrödinger equation in configuration representation possesses two linearly independent parabolic cylinder function solutions; as $x \to +\infty$ *or* as $x \to -\infty$, the various linear combinations of the two solutions can be either strongly unbounded in magnitude or can strongly approach zero. The integral over the real line of the square of any linear combination of the two solutions *diverges* unless the energy has one of the discrete values $(n + (1/2))\hbar\omega$, $n = 0, 1, 2, \ldots$, where ω is the natural angular frequency of the oscillator.

The *discarding* of *all* of the unphysical non-normalizable solutions of this stationary-state Schrödinger equation is what produces the well-known *discrete* energy spectrum of the quantized simple harmonic oscillator, along with *its very, very particular* accompanying parabolic cylinder function Schrödinger-equation solutions that *happen* to strongly approach zero *both* as $x \to +\infty$ *and* as $x \to -\infty$; these very particular stationary-state Schrödinger-equation solutions comprise *the physical energy-eigenfunction set* for the simple harmonic oscillator.

Of course the simple harmonic oscillator is only *one example* of the way in which *the merciless discarding* of unphysical non-normalizable solutions of the Schrödinger equation can punch gaps in energy spectra. The discrete part of the hydrogen-atom energy spectrum is similarly linked to Schrödinger-equation solutions which are either strongly unbounded in magnitude or strongly approach zero as $r \to \infty$, with the latter behavior occurring only for a discrete set of energies. Of course there exist myriad *other* physical systems whose character is associated with the *discarding* of unphysical non-normalizable Schrödinger-equation solutions: such discarding needn't necessarily produce *discrete* energy spectra, in some circumstances the result is merely *gaps* in the energy spectrum.

Nor is divergence of the wave-function normalization factor the *only* way in which a Schrödinger-equation solution *can violate a physical constraint*. Many physical systems are inherently *rotationally periodic*. The *discarding* of Schrödinger-equation solutions *which unphysically don't conform to the rotational periodicity* of such a system can *also* punch gaps in energy spectra.

We next examine side by side the free-particle system in one dimension and the simplest free-rotator system, whose stationary-state Schrödinger-equation solutions *are precisely analogous* but whose energy spectra *are utterly dissimilar*, a striking example of a situation wherein *the existence of a physical constraint*, namely the free rotator's rotational periodicity, *compels massive discarding of solutions of the underlying fundamental field equation*.

Free-particle versus free-rotator Schrödinger equations and energy spectra

In one dimension the kinetic energy and Lagrangian of a free particle is $(1/2)M\dot{x}^2$; the kinetic energy and Lagrangian of the simplest free rotator is $(1/2)I\dot{\theta}^2$, where I is the free rotator's moment of inertia.

From its Lagrangian we obtain the free particle's canonical momentum $p = M\dot{x}$, which implies that $\dot{x} = p/M$. We use this last result to eliminate $\dot{x}$ in favor of p in the expression $(1/2)M\dot{x}^2$ of the free particle's kinetic energy, and thereby obtain the free particle's formal Hamiltonian, namely,

$$H_{\mathrm{par}}(x, p) = p^2/(2M). \tag{3a}$$

Following the same route for the free rotator, we note that its Lagrangian $(1/2)I\dot{\theta}^2$ yields its canonical momentum $L = I\dot{\theta}$, which we see *is its angular momentum*. Noting that $\dot{\theta} = L/I$, we eliminate $\dot{\theta}$ in favor of L in the expression for the free rotator's kinetic energy to produce it's formal Hamiltonian, namely,

$$H_{\mathrm{rot}}(\theta, L) = L^2/(2I). \tag{3b}$$

As is well known, the classical free-particle canonical momentum p is quantized by turning it into an operator $\widehat{p}$ that, in configuration representation (i.e., for free-particle wave functions whose arguments are x and time), is given by,

$$\widehat{p} = -i\hbar(\partial/\partial x). \tag{4a}$$

The free particle's configuration-representation quantized Hamiltonian operator $\widehat{H}_{\mathrm{par}}$ is then obtained by inserting Eq. (4a) into its classical Hamiltonian given by Eq. (3a),

$$\widehat{H}_{\mathrm{par}} = \widehat{p}^2/(2M) = -(\hbar^2/(2M))(\partial^2/\partial x^2). \tag{4b}$$

The configuration-representation quantization of the classical free rotator (i.e., for free-rotator wave functions whose arguments are θ and time) follows a formally completely parallel route to that taken in Eqs. (4a) and (4b) for the free particle. Configuration-representation quantization of the classical free rotator's canonical (i.e., angular) momentum L is of course given by,

$$\widehat{L} = -i\hbar(\partial/\partial\theta). \tag{5a}$$

The free rotator's configuration-representation quantized Hamiltonian operator $\widehat{H}_{\mathrm{rot}}$ is then obtained by inserting Eq. (5a) into its classical Hamiltonian given by Eq. (3b),

$$\widehat{H}_{\mathrm{rot}} = \widehat{L}^2/(2I) = -(\hbar^2/(2I))(\partial^2/\partial\theta^2). \tag{5b}$$

For the free particle it is apparent from Eqs. (4b) and (4a) that any eigenfunction of the momentum operator $\widehat{p}$ will also be an eigenfunction of the free-particle Hamiltonian operator $\widehat{H}_{\mathrm{par}}$. From Eq. (4a) it is easily verified that the eigenfunction of $\widehat{p}$ which has the arbitrary momentum eigenvalue p_0 is given by $N_{\mathrm{par}}\exp(ip_0x/\hbar)$, where N_{par} is its constant normalization factor. This eigenfunction of $\widehat{p}$ isn't square integrable over the real x-values, but *it does remain bounded* as $|x|\to\infty$ and *it as well oscillates strongly* as $|x|$ increases without bound, so it *is* subject to Dirac's delta-function normalization, which yields,

$$N_{\mathrm{par}} = (2\pi\hbar)^{-\frac{1}{2}}. \tag{6}$$

This free-particle momentum eigenfunction's corresponding *energy eigenvalue* for the free-particle Hamiltonian operator $\widehat{H}_{\mathrm{par}}$ of Eq. (4b) is clearly seen to be the nonnegative energy value $p_0^2/(2M)$. Therefore, since p_0 is an arbitrary real momentum eigenvalue, the energy spectrum of $\widehat{H}_{\mathrm{par}}$ *encompasses all nonnegative energies*.

For the free rotator one proceeds similarly. From Eq. (5a) the eigenfunction of $\widehat{L}$ which has the angular momentum eigenvalue L_0 is given by $N_{\mathrm{rot}}\exp(iL_0\theta/\hbar)$, where N_{rot} is its constant normalization factor. This free-rotator angular momentum eigenfunction's corresponding *energy eigenvalue* for the free-rotator Hamiltonian operator $\widehat{H}_{\mathrm{rot}}$ of Eq. (5b) is seen to be the nonnegative energy value $L_0^2/(2I)$.

The considerations in the foregoing paragraph however fail to address head-on a critical *difference* between the one-dimensional free-particle system of Eqs. (4a) and (4b) and the free rotator system of Eqs. (5a) and (5b), namely that the free rotator system (but *not* the one-dimensional free-particle system) *is rotationally periodic*. The crucial technical issue which arises from this rotational periodicity requirement is that the eigenfunction $N_{\mathrm{rot}}\exp(iL_0\theta/\hbar)$ of the foregoing paragraph is rotationally periodic *only when* $L_0 = 0, \pm\hbar, \pm 2\hbar, \pm 3\hbar, \ldots$. All the eigenfunctions $N_{\mathrm{rot}}\exp(iL_0\theta/\hbar)$ which have *other values of* L_0 are *unphysical* because they *aren't* rotationally periodic *and must be discarded*. On a less critical level, its rotational periodicity implies that an eigenfunction $N_{\mathrm{rot}}\exp(iL_0\theta/\hbar)$ which *does* have a physically permitted value of L_0 only needs to be normalized on the interval $0\le\theta<2\pi$, where it certainly *is* square integrable. Therefore,

$$N_{\mathrm{rot}} = (2\pi)^{-\frac{1}{2}}. \tag{7}$$

With the above massive *discarding of its unphysical eigenfunctions*, the *energy spectrum* of the free-rotator Hamiltonian operator $\widehat{H}_{\mathrm{rot}}$ of Eq. (5b) has been reduced to the *discrete* energy points $(n\hbar)^2/(2I)$, where $n = 0, 1, 2, 3, \ldots$, which certainly *contrasts starkly* with the energy spectrum of the free-particle Hamiltonian operator $\widehat{H}_{\mathrm{par}}$ of Eq. (4b) that *encompasses all of the nonnegative energies*.

These last three sections unmistakably drive home the point that *physically unacceptable solutions* of *well-established field-theoretic equations* absolutely *must be discarded*.

In the earlier sections we established that a *necessary* condition for a metric tensor to be physically acceptable at a given space-time point is that its eigenvalue signature at that point must be identical to the $(+,-,-,-)$ eigenvalue signature that is possessed by the Minkowski metric tensor, and that that metric tensor and its inverse must at that point *possess exclusively finite components*.

With the two foregoing paragraphs firmly in mind, we now scrutinize some metric-tensor solutions of the Einstein equation which are unphysical at certain space-time points, starting with the Schwarzschild solution.

Are Schwarzschild-solution unphysical points really located in empty space?

First (and, as it happens, foremost) it is to be noted that empty-space Schwarzschild metric-tensor solutions *exhibit no unphysical points at all* if the spatial extent d of the gravitational *source* of effective mass $M>0$ *is sufficiently large*, i.e., if $d \gtrsim (G/c^2)M$. That notwithstanding, empty-space Schwarzschild metric-tensor solutions *can at least be cogitated on* in the case that the gravitational source of effective mass $M>0$ *is shrunk to a point*, a mainstay *Newtonian* idealization. For such a *point-mass* source of effective mass $M>0$, empty-space Schwarzschild metric-tensor solutions *exhibit a shell of unphysical points whose radius is of order* $(G/c^2)M$.

But *can a point-mass source of effective mass* $M>0$ *be a self-consistent idealization in a relativistic theory* where mass and energy, in particular *negative gravitational energy*, intermingle?

Let's *check* the self-consistency of the idealized effective *positive* point-mass M by *pulling that object apart* into *two* identical such objects, separated by a distance d, with the mass of each denoted as $(M_>/2)$. Then,

$$Mc^2 = M_> c^2 - (G(M_>/2)^2/d). \tag{8a}$$

Of course as $d \to \infty$, $M \to M_>$. But to check the self-consistency of the *effective positive point-mass idealization* we must consider the *opposite* limit, namely *the point-mass reassembly limit* $d \to 0$, and explore whether in *that* limit the effective mass M given by Eq. (8a) *can in fact maintain a positive value*.

Clearly if we hold $M_>$ *fixed* while $d \to 0$ *that won't be the case*. So let's pump up $M_>$ *before* we start to shrink d. But increasing $M_>$ can't be done with abandon because the *negative* gravitational energy second term on the right-hand side of Eq. (8a) *dominates* if $M_>$ is made large enough. However for every fixed value of the separation d we can find that value of $M_>$ *which maximizes the value of* M. That value of $M_>$ turns out to be $2(c^2/G)d$, and the corresponding *maximum value of* M *for a given separation* d is,

$$M_{\max}(d) = (c^2/G)d. \tag{8b}$$

Eq. (8b) shows that *even when* $M_>$ *is thus optimally chosen at every value of* d, the limit $d \to 0$ *doesn't result in a positive effective point-mass*.

We therefore see that once an ostensible *positive* effective point-mass *is disassembled*, "all the king's horses and all the king's men cannot put it back together again". Thus the mainstay Newtonian *positive point-mass* idealization *is clearly not self-consistent in a relativistic theory*.

In addition, Eq. (8b) draws our attention to *an inherent self-gravitational limit on a system's effective mass that is proportional to its largest linear dimension, with the constant of proportionality of the order of* (c^2/G) [5].

The fact that a system of a given size is inherently self-gravitationally limited to an effective mass of *at most* of order (c^2/G) times that size *renders harmless* the shell of *unphysical points* of the empty-space Schwarzschild metric-tensor solution for a source of effective mass M: that unphysical shell, whose radius is of order $(G/c^2)M$, *isn't* in fact located in the empty-space region where the Schwarzschild solution is valid but is *instead* tucked into the confines of the gravitational *source* of effective mass M *whose own radius is inherently self-gravitationally obliged to be of order* $(G/c^2)M$ *or greater*.

The proposition that a source of positive effective mass M *never* has a radius as small as the radius of the shell of unphysical points of its Schwarzschild solution is supported by the behavior of any self-gravitationally shrinking Oppenheimer-Snyder dust cloud in the "standard" coordinate system that is customarily used to express the Schwarzschild solution. In those "standard" coordinates the radius of a shrinking Oppenheimer-Snyder dust cloud of positive effective mass M is at *all* finite times *larger* (if only barely) than the radius $2(G/c^2)M$ of the shell of unphysical points of the Schwarzschild solution which has a source of the *same* positive effective mass M and is expressed in the *same* "standard" coordinates [6].

Although the metric tensor associated with the shrinking Oppenheimer-Snyder dust cloud is physically exemplary throughout space-time *once it is mapped into "standard" coordinates*, its form in the "comoving" coordinate system—*whose utilization is what makes analytic solution of that dust cloud's Einstein equation feasible* [7]—exhibits periodically repeated *unphysical space-time points*.

Unphysical metric-tensor behavior at infinitely many space-time points of one coordinate system that leaves no apparent trace in another coordinate system is of course a bewildering puzzle which we now briefly discuss.

The tangled tale of two Oppenheimer-Snyder coordinate systems

Before we attempt to unravel the mystery of the apparent disparity of the Oppenheimer-Snyder metric tensor in "comoving" and "standard" coordinates, let's try to gain a little insight into the physical behavior of a self-gravitationally shrinking dust cloud. To establish a toehold on that physics, let's momentarily jettison relativity and revert to Newtonian gravity. To make this tractable "on the back of an envelope", let's even jettison the cloud in favor of merely two point-mass dust particles which start from relative rest and proceed to accelerate toward each other under the influence of their mutual gravitational attraction.

This is a highly degenerate gravitational orbit problem which plays out in one dimension and produces some wild behavior. As with all Newtonian gravitational orbit problems, the net energy is negative and is conserved. However, when these idealized point particles meet, their gravitational potential energy obviously attains $-\infty$, so their kinetic energy is forced to $+\infty$, which implies that their meeting coincides with their having *infinite speed*. The resulting degenerate one-dimensional orbit, which like all simple Newtonian two-particle orbits is *periodic*, isn't described by a run-of-the-mill elementary function, but by a *cycloid* as a function of *time*. Cycloids possess the periodic *cusps* which correspond to the periodic bouts of *infinite speed* that are the notable feature of these orbits when the particles meet; cycloids also possess the periodic broad "flat spots" which characterize the gradual slowing toward the transient halt that these particles periodically experience in the vicinity of their greatest mutual separation.

Neither infinite gravitational potential energy nor infinite speed can be expected to survive the imposition of relativity on these "crazy" Newtonian cycloidal orbits. Instead the extremely strong gravitational potential will have a profound effect which is completely alien to Newtonian gravity theory, namely *tremendous gravitational time dilation* (i.e., gravitational redshift). That will drastically slow down the progression of the Newtonian periodic cycloidal motion; indeed it turns out that this motion's very first infalling "crunch" stage has its time duration dilated to *an infinite time interval* in the "standard" coordinate system.

Astonishingly, however, the dominant feature of the metric-tensor solution of the Einstein equation for the Oppenheimer-Snyder dust cloud in the spherically-symmetric *"comoving" coordinate system* is the archetypal Newtonian *periodic cycloid in time*, with its cusps *completely intact* [8]; in those "comoving" coordinates *there is no discernible trace of the drastic gravitational time dilation which would be expected*. Moreover, the *cusps* of that cycloid cause this spherically-symmetric "comoving" metric tensor *to periodically violate the metric-tensor Minkowski signature requirement* and therefore the Principle of Equivalence.

So how does a metric-tensor solution of the *Einstein* equation end up actually staking out *Newtonian* territory and thumbing its nose at both gravitational time dilation and the Principle of Equivalence? Examination of the metric tensor *form* that is used for the spherically-symmetric "comoving" coordinate system reveals that it *forces* the metric tensor component g_{00} *to assume the value unity under all circumstances* [9].

Forcing the value unity on the metric tensor component g_{00} *does not occur* for the spherically-symmetric "standard", "isotropic" or "harmonic" coordinate systems. As a matter of fact $(g_{00})^{-\frac{1}{2}}$ is the *gravitational time dilation factor* in the static limit [10]. Therefore, it is in the nature of the spherically-symmetric "comoving" coordinate system *to completely suppress relativistic gravitational time dilation in the static limit*. Judging from the *results* of the solution of the Einstein equation for the Oppenheimer-Snyder dust cloud in spherically-symmetric "comoving" coordinates, complete suppression of relativistic gravitational time dilation would appear to extend far beyond the static limit in that coordinate system. Indeed those results *suggest that spherically-symmetric "comoving" coordinates subvert the Einstein equation into yielding unphysical relativistically-deficient quasi-Newtonian solutions*.

Then why is it that everything apparently comes right when the Einstein-equation metric tensor solution for the Oppenheimer-Snyder dust cloud in "comoving" coordinates is mapped into "standard" coordinates, *including complete absence of unphysical space-time points*, notwithstanding that these are a periodic fixture in the "comoving" coordinate system?

The technical answer to this question is that the *mapping* from the unphysical "comoving" metric tensor for the Oppenheimer-Snyder dust cloud to that metric tensor's "standard" counterpart *also comes out to be unphysical*, namely *divergent*, on a very substantial subset of space-time [11]. Inter alia, all the periodic unphysical space-time points of the "comoving" metric tensor for the Oppenheimer-Snyder dust cloud are mapped to *infinite time* in the "standard" coordinate system, and the very first infalling "crunch" stage of the Newtonian cycloid in time in the "comoving" coordinate system has its time duration dilated to *an infinite time interval* in the "standard" cooordinate system.

In other words, *both* the "comoving" metric tensor for the Oppenheimer-Snyder dust cloud *and* its space-time mapping to the "standard" coordinate system *are riddled with unphysical space-time points*, but the Oppenheimer-Snyder dust cloud metric tensor in the "standard" coordinate system *which is the result of combining the two* manifests *no* unphysical space-time points: the unphysical (i.e., *divergent*) points of the *mapping* send *all* the unphysical space-time points of the "comoving" metric tensor for the Oppenheimer-Snyder dust cloud to *infinite time*, along with *almost the entire periodic history of the cycloidal behavior in time* of that "comoving" metric tensor, *aside* from its very first infalling "crunch" stage, whose time duration is dilated to an infinite time interval.

The apparent moral of this story is that metric-tensor solutions of the Einstein equation in the spherically-symmetric "comoving" coordinate system are unphysical relativistically-deficient distortions of the field dynamics *which are unsuited to direct interpretation*. However, *if it is feasible to work out the unphysical mapping of a given unphysical Einstein-equation metric-tensor solution in spherically-symmetric "comoving" coordinates to a physically acceptable spherically-symmetric coordinate system*, then *the result* of applying that particular unphysical mapping to that given unphysical spherically-symmetric "comoving" Einstein-equation metric-tensor solution can confidently be expected to be *a physically acceptable* spherically-symmetric metric tensor. This general discussion encompasses in particular the bewildering path that Oppenheimer and Snyder followed to evident success, albeit *they apparently didn't actually discern the unphysical relativistically-deficient quasi-Newtonian character* of the cycloidal metric-tensor solution of the Einstein equation which is obtained for the dust cloud in spherically-symmetric "comoving" coordinates.

Since the three-momentum density of the Oppenheimer-Snyder dust fluid vanishes in the "comoving" coordinate system by definition, no shrinkage of the dust cloud can occur in that coordinate system, only periodically singular *variation in the dust cloud's energy density* as the cloud swaps energy with the gravitational field. Dust cloud shrinkage of course *does* occur in the "standard" coordinate system, and as was mentioned at the end of the previous section, the radius of a shrinking dust cloud of effective mass M is at all finite times larger (if only barely) than the radius $2(G/c^2)M$ of the shell of unphysical points of the Schwarzschild metric tensor whose source also has effective mass M and which is itself expressed in the "standard" coordinate system [6].

References

[1] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, New York, 1972), Section 4.3, p. 96.

[2] S. Weinberg, op. cit., Section 4.3, p. 97.

[3] S. Weinberg, op. cit., Eqs. (3.6.1)–(3.6.4), pp. 85–86.

[4] S. Weinberg, op. cit., Section 3.6, p. 86.

[5] S. K. Kauffmann, arXiv:1212.0426 [physics.gen-ph] (2012).

[6] S. Weinberg, op. cit., Section 11.9, p. 347.

[7] S. Weinberg, op. cit., Section 11.9, pp. 342–345.

[8] S. Weinberg, op. cit., Eqs. (11.9.24)–(11.9.26), pp. 344–345.

[9] S. Weinberg, op. cit., Eq. (11.8.8), p. 340 and Eq. (11.9.1), p. 342.

[10] S. Weinberg, op. cit., Eq. (3.5.2), p. 79.

[11] S. K. Kauffmann, viXra:1408.0240, Relativity and Cosmology (2014).