Lucasian Primality Criterion for Specific Class of $k \cdot 2^n - 1$

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Abstract: Conjectured polynomial time primality test for specific class of numbers of the form $k \cdot 2^n - 1$ is introduced .
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1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $k \cdot 2^n - 1$ with k odd , $k < 2^n$ and $n > 2$, see Theorem 5 in [1] . In this note I present polynomial time primality test for numbers of the form $k \cdot 2^n - 1$ with $3 \mid k$ that is special case of Riesel test .

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$, where m and x are nonnegative integers .

Conjecture 2.1. *Let $N = k \cdot 2^n - 1$ such that $n > 2$, $k > 0$, $3 \mid k$, $k < 2^n$ and*

$$\begin{cases} k \equiv 1 \pmod{10} \text{ with } n \equiv 2,3 \pmod{4} \\ k \equiv 3 \pmod{10} \text{ with } n \equiv 0,3 \pmod{4} \\ k \equiv 7 \pmod{10} \text{ with } n \equiv 1,2 \pmod{4} \\ k \equiv 9 \pmod{10} \text{ with } n \equiv 0,1 \pmod{4} \end{cases}$$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(18)$, thus
N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

References

[1] Riesel, Hans (1969) , ”Lucasian Criteria for the Primality of $N = h \cdot 2^n - 1$” , *Mathematics of Computation* (AmericanMathematical Society), 23 (108): 869-875 .