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UNCERTAINTY EVALUATION FOR AN ULTRASONIC DATA FUSION BASED TARGET DIFFERENTIATION PROBLEM USING GENERALIZED AGGREGATED UNCERTAINTY MEASURE 2

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Abstract
The purpose of this paper is uncertainty evaluation in a target differentiation problem. In the problem ultrasonic data fusion is applied using Dezert-Smarandache theory (DSmT). Besides of presenting a scheme to target differentiation using ultrasonic sensors, the paper evaluates DSmT-based fused results in uncertainty point of view. The study obtains pattern of data for targets by a set of two ultrasonic sensors and applies a neural network as target classifier to these data to categorize the data of each sensor. Then the results are fused by DSmT to make final decision. The Generalized Aggregated Uncertainty measure named GAU2, as an extension to the Aggregated Uncertainty (AU), is applied to evaluate DSmT-based fused results. GAU2, rather than AU, is applicable to measure uncertainty in DSmT frameworks and can deal with continuous problems. Therefore GAU2 is an efficient measure to help decision maker to evaluate more accurate results and smoother decisions are made in final decisions by DSmT in comparison to DST.

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1. Introduction

Ultrasonic sensors are widely used in robotics applications such as localization, target differentiations and mapping. A measurement scheme is proposed which uses two sets of ultrasonic sensors to determine location and type of target surface (see Ref. 1). In this study, concentration is on target differentiation based on pattern of data which are obtained by a set of two ultrasonic sensors. The target classification is performed by employing time of flight (TOF) of the sensors. Classification of different targets by using neural networks is achieved for outcomes of each sensor. Afterwards the results are fused together to make final decision.
There has been much research on sensor fusion methods in recent years. The evidence theory, also known as Dempster–Shafer theory, is one of the most popular frameworks to deal with uncertain information. This theory is often presented as a generalization of probability theory,

where the additivity axiom is excluded. The Dezert–Smarandache Theory (DSmT) is a theory of plausible and paradoxical reasoning proposed by Dezert and Smarandache (see Refs. 2-5). It can be considered as an extension of the classical Dempster–Shafer theory (DST)[6] but with fundamental differences. DSmT is able to solve complex static or dynamic fusion problems beyond the limits of the DST framework, especially when conflicts between sources become large and when the refinement of the frame of the problem under consideration becomes inaccessible because of the vague, relative and imprecise nature of elements[3]. There are some successful applications of DSmT in target type tracking[7] and robot map building[8-10]. Efficiency of DSmT in comparison to DST is confirmed in sonar grid map building[9].

On the other hand, uncertain information often exists on all levels of fusion process which are usually related to physical constraints, detection algorithms, and the transmitting channel of the sensors[11]. Therefore, it is important to have an uncertainty evaluation after sensor fusion for better decision making. Hartley[12] and Shannon[13] respectively established the field of information theory. Hartley measure and Shannon entropy have been used in the possibilities and probabilities frameworks, respectively. Based on these approaches, information or preferably uncertainty-based information can be quantified by different general measures commonly called measures of uncertainty[14].

Several theories have been developed to deal with uncertainty such as probability theory, fuzzy sets theory, possibility theory, evidence theory and rough sets theory. Instead of opponents, they should rather be seen as complementary, each of them being designed for dealing with different types of uncertainty. Three main types of uncertainty have been identified[15]: fuzziness, conflict, and non-specificity, the latter two being unified under the term ambiguity. Different measures of ambiguity, often called measures of total uncertainty have been proposed. Among them, a measure of aggregated uncertainty named AU is proposed[16,17]. This measure is defined in the framework of evidential theory that aggregates the non-specificity and conflict. It has been proved that this measure satisfies the five Klir & Wierman's requirements[14,18]. It have been formalized, within a broad range of theories of imprecise probabilities, the notion of a total aggregated measure of uncertainty and various disaggregations into measures of non-specificity and conflict[19]. As another uncertainty measure, a new measure of aggregated uncertainty is introduced, named AM for Ambiguity Measure that aims at eliminating the shortcomings of AU such as computing complexity[20]. By AM, an alternative to measure ambiguity in Dempster–Shafer theory is offered. But actually, their proposed measure is not, in a general sense, sub-additive. It is showed this by a specific counterexample which clearly demonstrates that their assumption in the last step of the proof is incorrect and that AM indeed violates sub-additivity[21].

In spite of efficiency of AU measure, this uncertainty measure and its associated algorithm for computing[18], has devoted for DST framework and cannot be applied to DSmT directly. As mentioned before, DSmT is a generalization of DST. Two generalized AU measures, which are named GAU1 and GAU2, have been introduced by the authors (see Ref. 22). It is proved that the new measures have enough efficiency to evaluate the DSmT based results. In this paper these new measures are used. An experimental setup which is based on ultrasonic sensors is configured. Neural networks are used in the first level of the fusion process. Neural networks are trained by acquired data of the set of ultrasonic sensors and then outputs are used to perform the differentiation task. Finally, the obtained results of the DSmT-based decision maker are evaluated by the proposed uncertainty measures.

This paper is organized as following: A short review on DSmT that are considered in uncertainty analysis is mentioned in Sec. 2. Sec. 3 is devoted to represent GAU2 as the new measure and a short discussion on the AU and GAU1 measures. In Sec. 4, experimental studies are carried out on uncertainty measurement for a target classification problem. Finally, some concluding remarks are presented in Sec. 5.

2. Dezert–Smarandache Theory

Dezert–Smarandache Theory is a theory of plausible and paradoxical reasoning[3-5]. The development of DSmT arises from the necessity to overcome the inherent limitations of Dempster–Shafer Theory (DST)[6] which are closely related with the acceptance of Shafer's model for the fusion problem under consideration. This means the frame of discernment $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ is implicitly defined as a finite set of exhaustiveand exclusivehypotheses. The Dedekind's lattice, also called in the DSmT framework hyper-power set D^Θ is defined as the set of all composite propositions built from elements of Θ with ∪ and ∩ operators such that:

(1) $\emptyset, \theta_1, \theta_2, \dots, \theta_n \in D^\Theta$

(2) $if\ \ A, B \in D^\Theta\ \ then\ \ A \cup B \in D^\Theta\ ,\ A \cap B \in D^\Theta$

(3) No other elements belong to D^Θ, except those obtained by using rule (1) or rule (2).

DSmT starts with the notion of free DSm model, denoted $M^f(\Theta)$, and considers Θ only as a frame of exhaustive elements $\{\theta_1, \theta_2, \dots, \theta_n\}$ which can potentially overlap. When the free DSm model holds, the classic commutative and associative DSm rule of combination is performed. From a general frameΘ, a map is defined as $m(.): D^\Theta \longrightarrow [0,1]$ associated to a given body of evidence as:

$$m(\emptyset) = 0\ \ ,\ \ \ \sum_{A \in D^\Theta} m(A) = 1\ \ \ ,\ \ \ 0 \le m(A) \le 1 \qquad (2.1)$$

The quantity $m(A)$ is called the generalized basic belief assignment/mass (gbba) of *A*. The generalized belief and plausibility functions are defined in almost the same manner as within the DST, *i.e.*

$$Bel(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B) \qquad (2.2)$$

$$Pl(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B) \qquad (2.3)$$

When the free DSm model $M^f(\Theta)$ holds for the fusion problem under consideration, the classic DSm rule of combination $m_{M^f(\Theta)} = m(.) = [m_1 \oplus m_2](.)$ of two independent sources of evidences over the same frame with belief functions $Bel_1(.), Bel_2(.)$ associated with gbba $m_1(.), m_2(.)$ corresponds to the conjunctive consensus of the sources. It is given by:

$$\forall C \in D^\Theta\ \ ,\ \ m_{M^f(\Theta)}(C) = m(C) = \sum_{\substack{A,B \in D^\Theta \\ A \cap B = C}} m_1(A) m_2(B) \qquad (2.4)$$

Since D^Θ is closed under ∪ and ∩ set operators, this new rule of combination guarantees that $m(.)$is a proper generalized belief assignment, *i.e.*$m(.): D^\Theta \longrightarrow [0,1]$.

3. Uncertainty Measurement

Measuring uncertainty or information means assigning a number or a value from some ordinal scale to a given model of an epistemic state. Two types of classical evidential based uncertainties, non-specificity and conflict are often measured as part of the fusion techniques such as DST fusion (18). All of the uncertainty measures attempt to measure uncertainty in bits. One bit of uncertainty is the amount of total uncertainty regarding the truth or falsity of one proposition. One of the most appropriate uncertainty measures which are developed in DST

frameworks is the Aggregate Uncertainty (AU) measure. While the goal of information fusion is to reduce the global uncertainties, (18) explored the concept of comprehensive uncertainty measurement in the DST framework.

Definition3.1. The measure of the Aggregated Uncertainty contained inBel, denoted as$AU(Bel)$, is defined by:

$$AU(Bel) = max\{-\sum_{\theta\in\Theta} p_\theta \, log_2 \, p_\theta\} \quad (3.1)$$

where the maximum is taken over all $\{p_\theta\}_{\theta\in\Theta}$such that $p_\theta \in [0,1]$ for$\theta \in \Theta$, $\sum_{\theta\in\Theta} p_\theta = 1$ and for all $A \subseteq \Theta$, $Bel(A) \leq \sum_{x\in A} p_x$.

It is proved that the measure satisfies all the properties for a reasonable uncertainty measurement, specifically the sub-additivity and additivity which are defined[14]. Algorithm of computing AU was originated[18]. The algorithm is applied for DST framework while it cannot be used for DSmT directly. The reason is hidden in the algorithm of computing AU and especially in the main difference of DST and DSmT. In the DST, the frame of discernment of the fusion problem under consideration assumed to have exhaustive and exclusive elementary hypotheses but in DSmT these conditions are violated. The algorithm of computing AU measure states that at least one part of information which is determined by$A \cap B$ will be missed if anyone wants to apply this algorithm to DSmT. Accordingly, uncertainty measurement would not be accurate. Two Generalized Aggregate Uncertainty measures, which are named GAU1 and GAU2, have been developed[22]. The idea of generalizing Aggregated Uncertainty measure in GAU1 to evaluate DSmT, is disjointing the free DSm model to separated sets (Fig. 1). In this manner, the main problem of fusion still have two events whereas there are three separated events such as a Shafer's model with 3 events. Therefore, the same algorithm of computing for AU measure can be used as the algorithm of computing the measure GAU1 after the mentioned extension in order to evaluate the DSmT-based fusion results.

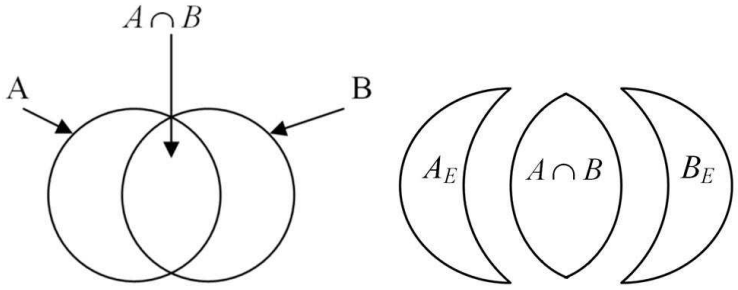

Fig. 1. Disjointing of framework of free DSm model with two jointed events to three excluded sets

Although GAU1 is applicable to evaluate uncertainty in DSmT framework, the extension that is used in GAU1 is true for the problems where the refinement is possible. There are some cases that the refinement is not possible, for example when the frontiers of the sets in the frame of discernment are not clear. So this refinement may not work for any frame of discernment. The new uncertainty measure which has been called, Generalized Uncertainty measure 2 or GAU2 has been introduced to overcome this limitation[22]. In GAU2, despite of GAU1, clarity of the frontiers of the sets in the frame of discernment is not necessary. Therefore GAU2 is a suitable uncertainty measure for continuous frameworks. Consider the set of non-exclusive events $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ or$\Theta = \{\theta | \theta = \theta_i, i = 1,2, \dots, n\}$. GAU2 is defined based on a class of

probability distribution of events of a set such as $\Theta_P$whereas the entropy of Shannon is maximized. Θ_P is equal to:

$$\Theta_P = \left\{\theta_P | \theta_P = \theta_{P_j}, j = 1,2,\dots,n_{\Theta_P}\right\} = \left\{\theta_P | \theta_P = \theta_{P_j} = \bigcap_{\substack{i\in I \\ \emptyset\neq I\subseteq\{1,2,\dots,n\}}} \theta_i\right\}$$

or in a simple form: $\Theta_P = \left\{\{\theta_i\}_{i=1,2,\dots,n}, \{\theta_i \cap \theta_j\}_{\substack{i,j=1,2,\dots,n \\ i\neq j}}, \{\theta_i \cap \theta_j \cap \theta_k\}_{\substack{i,j,k=1,2,\dots,n \\ i\neq j\neq k}}, \dots\right\}$

And cardinality of Θ_P is: $|\Theta_P| = n_{\Theta_P} = \sum_{i=1}^{n}\binom{n}{i}$.

Definition 3.2. The measure of the Generalized Aggregated Uncertainty 2 contained inBel,$GAU2(Bel)$, which is defined by:

$$GAU2(Bel) = max\left\{-\sum_{\theta_P\in\Theta_P} p_{\theta_P}\, log_2\, p_{\theta_P}\right\} \qquad (3.2)$$

$p_{\theta_P}$is the associated probability distribution assignment of each event of Θ_P and the maximum is taken over all$\{p_{\theta_P}\}_{\theta_P\in\Theta_P}$such thatfor all $\theta_P \in \Theta_P$, $0 \le p_{\theta_P} \le 1$,

$\sum_{\theta_P\in\Theta_P}(-1)^{\alpha(\theta_P)+1}p_{\theta_P} = 1$, $\alpha(\theta_P = \bigcap_{i\in I}\theta_i) = |I| where\ (\emptyset \neq I \subseteq \{1,2,\dots,n\})$,

$$Bel(A) \le \sum_{\theta_P\in A} p_{\theta_P}\ forall\ \emptyset \neq A \subseteq \Theta_P$$

The generalized algorithm for computing the GAU2 measure is:

Input: a frame of discernmentΘ (with n non-exclusive events), a generalized belief function Bel onΘ

Output: $GAU2(Bel), \{p_{\theta_P}\}_{\theta_P\in\Theta_P}$ such that:

$$GAU2(Bel) = -\sum_{\theta_P\in\Theta_P} p_{\theta_P}\, log_2\, p_{\theta_P}$$

$$\Theta_P = \left\{\theta_P | \theta_P = \theta_{P_j}, j = 1,2,\dots,n_{\Theta_P}\right\} = \left\{\theta_{P_1}, \theta_{P_2}, \dots, \theta_{P_{n_{\Theta_P}}}\right\}$$

$$p_{\theta_P} = p_{\theta_{P_j}}|_{\theta_P=\theta_{P_j}, j=1,2,\dots,n_{\Theta_P}} \ , \sum_{\theta_P\in\Theta_P}(-1)^{\alpha(\theta_P)+1}.p_{\theta_P} = 1 \quad , \quad 0 \le p_{\theta_P} \le 1$$

$$\alpha\left(\theta_P = \bigcap_{i\in I}\theta_i\right) = |I| where \quad (\emptyset \neq I \subseteq \{1,2,\dots,n\})$$

$$Bel(A) \le \sum_{\theta_P\in A} p_{\theta_P}\, forall \emptyset \neq A \subseteq \Theta_P$$

Line 1) begin

Line 2) make

$$\Theta_P = \left\{\theta_P | \theta_P = \theta_{P_j}, j = 1,2,\dots,n_{\Theta_P}\right\} = \left\{\theta_P | \theta_P = \theta_{P_j} = \bigcap_{\substack{i\in I \\ \emptyset\neq I\subseteq\{1,2,\dots,n\}}} \theta_i\right\}$$

Line 3) $Y = \Theta_P$, $Bel' = Bel$

Line 4) while Y $\neq \emptyset$ and $Bel'(Y) > 0$ do

Line 5) find a non-empty set $A \subseteq \Theta_P$ such that $Bel(A) \,/\, |A|$ is maximal if there are more such sets A than one, take the one with maximal cardinality endif
Line 6) for each $\theta_P \in A$ do$p_{\theta_P} = Bel'(A)/|A|$ endfor
Line 7) for each $B \subseteq (Y - A) \cup (Y \cap A)$ do

$$Bel'(B) = Bel'(B \cup A) - Bel'(A) + Bel'(B \cap A)$$endfor

Line 8) $Y = (Y - A) \cup (Y \cap A)$
Line 9) endwhile
Line 10) if $Bel'(Y) = 0$ and $Y \neq \emptyset$ then
Line 11) for all $\theta_P \in Y$ do $p_{\theta_P} = 0$ endfor
Line 12) endif
Line 13) $GAU2(Bel) = -\sum_{\theta_P \in \Theta_P} p_{\theta_P} \, log_2 \, p_{\theta_P}$
Line 14) end
Clearly, one may see the differences between the above algorithm and the algorithm of AU measure. The differences are:

- Replacing the set of non-exclusive events Θ by the new set Θ_P
- The condition imposed to θ_P in the Definition 3.2
- Lines 7 and 8 of the GAU2 measure algorithm to continue the computations of the next probability assignments

4. Experimental Study: Ultrasonic Sensors for Target Classification

4.1. *Experiment Setup*

In the experimental setup, two identical acoustic transmitter/receiver pairs with center-to-center separation *d=35 cm* are employed. The common targets that there exist in real environment of mobile robot applications such as *Plane*, *Cylinder* with diameter 20 cm and *Corner* with 90° angle are considered. TOF data are collected at 20 sensor locations which are located at 5 different angles from $\varphi = -30°$ to $\varphi = +30°$ in *15°* increments, and from *r = 0.5 m* to *r = 2 m* in 0.5 *m* increments separately (Fig. 2). To disaffect the distances of targets in target classification, the data are normalized regarding distance. Consequently, the targets can be classified regardless to the mentioned positions. Fig. 3 indicates normalized data of ultrasonic sensors for these 20 positions for the targets.

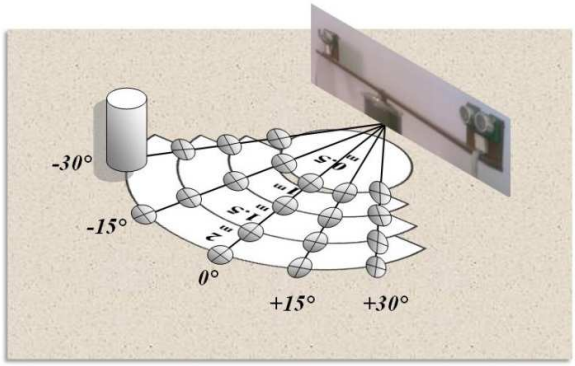

Fig. 2. Experiment setup and 20 different positions of targets

TOF signals of each sensor pair are used as input signals of a neural network. The hidden layer comprises 50 neurons and hyperbolic tangent as nonlinear functions and linear functions at the output layer with 3 neurons. For each sensor, One set of data are collected for each target location for each target primitive, resulting in 60 (=4 ranges×5 angles×3 target types) sets of data. The network is a multi-layer perceptron (MLP) network with a learning constant equal to 0.9, momentum constant equal to 0.5, and a sigmoid-type nonlinearity. The neural network estimates the target type using these data.

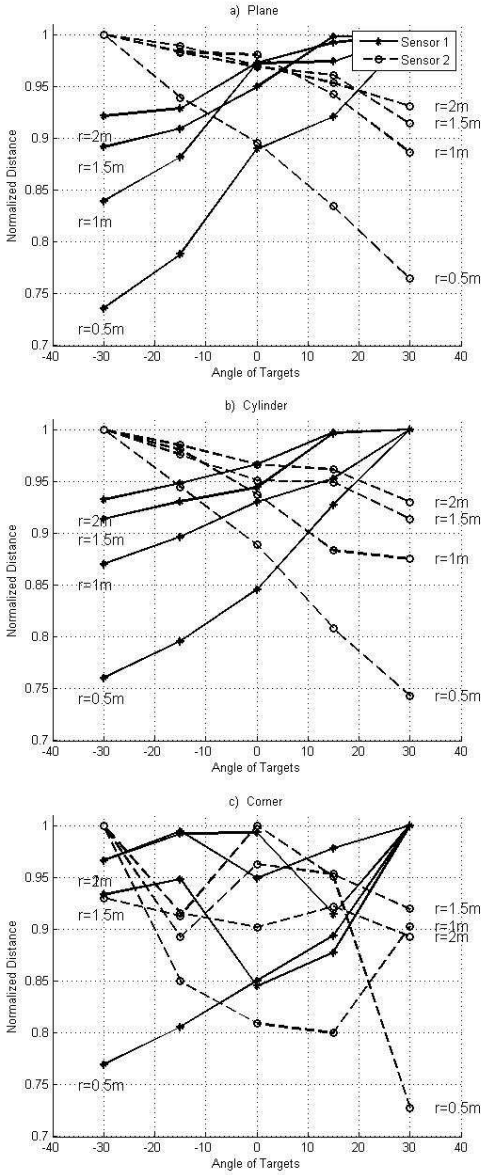

Fig. 3. Normalized data of ultrasonic sensors in 20 positions (4 ranges and 5 angles) for the targets *"Plane"*, *"Cylinder"* and *"Corner"*

4.2. *Sensor Fusion with Uncertainty Measurement*

In this section, DST and DSmT are applied to the results that are obtained by neural networks in order to differentiate the target types. After the results of two sensors are fused, uncertainty measurement has been carried out according to AU measure for DST and the GAU1 and GAU2 measures for DSmT.

Table 1 gives the results of correct target type classification that are considered as basic belief assignment of each sensor of the three targets for the case that the target object is *"Plane"*. Accordingly, each sensor by using a trained neural network presents a quantity to differentiate the targets. In this table, *"P"* is used to represent *"Plane"*, *"Cy"* is for *"Cylinder"* and *"Co"* is for *"Corner"* and "Θ" is devoted to represent total ignorance *i.e.*$\Theta = P \cup Cy \cup Co$.

Table 1. Outputs of neural network based classifier as basic belief assignment and DST based fusion results; Target type: *"Plane"*

2^{Θ}	*Sensor1*	*Sensor2*	$m_{DST}(A)$
P	*0.7333*	*0.5333*	*0.8698*
Cy	*0*	*0.1333*	*0.0178*
Co	*0*	*0.0667*	*0.0592*
$P \cup Cy$	*0.1333*	*0.1333*	*0.0355*
$P \cup Co$	*0.0667*	*0*	*0.0059*
$Cy \cup Co$	*0.0667*	*0.0667*	*0.0118*
Θ	*0*	*0.0667*	*0*

4.2.1. *Results of DSmT-based Fusion and Uncertainty evaluation by GAU2 Measure*

Table 2 illustrates the results of DSmT-based fusion results. When DSmT is used, the number of events to be decided is more than number of events in DST because of using hyper-power set and free DSm model. DSmT fusion showed its capabilities in continuous problems as well as problems with non-exclusive events. Basically, DST has not enough efficiency to deal with problems with such models. On the other hand, advantages of using DSmT fusion should be studied in uncertainty point of view as well. Therefore, DSmT-based fusion results of the target differentiation are evaluated by GAU2 measure. According to Definition 3.2, members of the set $\Theta_P$are defined as: $\Theta_P = \{\theta_1, \theta_2, \dots, \theta_7\}$ where: $\theta_1 = P$,$\theta_2 = Cy$,$\theta_3 = Co$,$\theta_4 = P \cap Cy$,$\theta_5 = P \cap Co$,$\theta_6 = Cy \cap Co$, $\theta_7 = P \cap Cy \cap Co$.

The first step of the algorithm of computing is illustrated in Table 2. The maximum value of $Bel(A)/|A|$ is obtained for the event$P$, therefore$p_P = 0.8755$. According to the algorithm, by discarding the event *P* and consequently its union the results the GAU2 measure continues. It is concluded that $p_{Cy} = 0.1957$,$p_{Co} = 0.1022$, $p_{P \cap Cy} = 0.0978$,$p_{P \cap Co} = 0.0489$, $p_{Cy \cap Co} = 0.0267$, $p_{P \cap Cy \cap Co} = 0$. Therefore uncertainty in the DSmT-based results by GAU2 (Eq. (6)) is equal to 1.6453.

To investigate the uncertainty improvement in the results of DSmT fusion, uncertainties in the results of each sensor have to be considered. Similarly uncertainty in each sensor can be computed by the measure GAU2. The value of uncertainties in Sensor1 and Sensor2 by the GAU2 are computed by the algorithm and are equal to 1.1034 and 1.4036, respectively. Uncertainty evaluation of DSmT fusion by GAU2 shows that the DSmT reduces the amount of uncertainty in final decisions. Uncertainty in DSmT fusion results are less than the sum of

uncertainties in Sensors1&2. So it can be concluded that DSmT has improved the results in uncertainty point of view.

Table 2. The 1st step of the algorithm of computing GAU2 measure for the DSmT results; Target type: *"Plane"*

D^Θ	$m_{DSmT}(A)$	$Bel(A)$	$Bel(A)/\|A\|$
P	*0.6444*	*0.8755*	*0.8755*
Cy	*0.0267*	*0.1334*	*0.1334*
Co	*0.0089*	*0.0667*	*0.0667*
$P \cup Cy$	*0.0267*	*0.9734*	*0.4867*
$P \cup Co$	*0.0044*	*0.9199*	*0.4600*
$Cy \cup Co$	*0.0089*	*0.3156*	*0.1578*
$P \cap Cy$	*0.0978*	*0.0978*	*0.0978*
$P \cap Co$	*0.0489*	*0.0489*	*0.0489*
$Cy \cap Co$	*0*	*0*	*0*
$P \cap (Cy \cup Co)$	*0.0844*	*0.1467*	*0.0734*
$Cy \cap (P \cup Co)$	*0.0089*	*0.0978*	*0.0489*
$Co \cap (P \cup Cy)$	*0.0089*	*0.0489*	*0.0245*
$P \cup (Cy \cap Co)$	*0.0089*	*0.9022*	*0.4511*
$Cy \cup (P \cap Co)$	*0.0178*	*0.2934*	*0.1467*
$Co \cup (P \cap Cy)$	*0.0044*	*0.2622*	*0.1311*
$P \cap Cy \cap Co$	*0*	*0*	*0*
$(P \cap Cy) \cup (P \cap Co) \cup (Cy \cap Co)$	*0*	*0*	*0*
Θ	*0*	*1*	*0.3333*

Table 3 summarizes the results of uncertainty measurement in the experiment. In the case of conflict measurements, DSmT must be used instead of DST. Also these experiments demonstrate that DSmT presents smoother decisions, especially in continuous models. Since AU is presented for DST and cannot be applied to the DSmT results, GAU1 and GAU2 are applicable as uncertainty measure for DSmT fusion results. Moreover this study shows the efficiency of DSmT to improve the final results in uncertainty point of view. Additionally, application of the GAU2 measure has not the limitation of the GAU1 measure to deal with events with non-distinguishable borders.

Table 3. Uncertainty measurement for sensor 1&2, DST and DSmT fusion results

Results	*Sensor 1*	*Sensor 2*	*Sensor1+ Sensor2*	*DST*	*DSmT*
Uncertainty measurement by AU	*1.1035*	*1.2730*	*2.3765*	*0.6680*	-
Uncertainty measurement by GAU1	*2.7259*	*2.8078*	*5.5337*	-	*2.4866*
Uncertainty measurement by GAU2	*1.1034*	*1.4036*	*2.5070*	-	*1.6453*

5. Conclusions

In this paper, uncertainty evaluation problem in a decision making system is considered. An experimental setup of ultrasonic sensors is established to study target differentiation problem and uncertainty measurement in decision making. A common neural network is used as classifier for each sensor path to get the classification results of the sensors. DSmT overcomes the limitations of DST. On the other hand, AU cannot be applied to DSmT because of the involved assumption in the algorithm of computing AU which states the events of frame of discernment must be without community. Generalized AU measures, *i.e.*GAU1 and GAU2 have been developed to

overcome this limitation DSmT and the associated uncertainty measures are applied to the results of sensors and the results are discussed in details. The final decision in the presented configuration has uncertainty less than each sensor's measurement. On the other hand, efficiency of Generalized Uncertainty measures to measure uncertainty and more accurate results and smoother decisions are made in final decisions by DSmT in comparison to DST are validated. The following suggestions might be considered as further studies;

- employing other classification methods instead of neural networks
- utilizing ultrasonic echo signal amplitudes as acquired data in addition to TOF data
- looking for an uncertainty measure with less complexity than AU, GAU1 and GAU2 in computation, which satisfies the requirements of uncertainty measures

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