Single valued neutrosophic numbers and their applications to multicriteria decision making problem

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Abstract

In this paper, we firstly introduced single valued neutrosophic numbers which is a generalization of fuzzy numbers and intuitionistic fuzzy numbers. A single valued neutrosophic number is simply an ordinary number whose precise value is somewhat uncertain from a philosophical point of view. Then, we will discuss two special forms of single valued neutrosophic numbers such as single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers. Also, we give some operations on single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers. Finally, we give a single valued trapezoidal neutrosophic weighted aggregation operator(SVTNWAO) and applied to multicriteria decision making problem.

Keyword 0.1 *Neutrosophic set, single valued neutrosophic numbers, trapezoidal neutrosophic numbers, triangular neutrosophic numbers, aggregation operators, decision making.*

1 Introduction

In 1965, fuzzy set theory has long been presented to handle vagueness, inexact and imprecise data by Zadeh [41]. After Zadeh, fuzzy sets, especially fuzzy numbers have been widely studied and applied to various fields, such as decision-making, pattern recognition, game theory and so on. In fuzzy sets, the degree of memberships of the elements in a universe is a single value but, those single values cannot provide any additional information because, in practice, information regarding elements corresponding to a fuzzy concept may be incomplete. The fuzzy set theory is not capable of dealing with the lack of knowledge with respect to degrees of memberships, Atanassov[3] proposed the theory of intuitionistic fuzzy sets the extension of Zadehs fuzzy sets by using a non-membership degree cope with the presence of vagueness and hesitancy originating from imprecise knowledge or information. Because of the restriction that "$0 \leq$membership degree + non-membership degree ≤ 1", intuitionistic fuzzy sets were extended by Smarandache[23], in 1995, to neutrosophic sets which can only handle incomplete information not the indeterminate information and inconsistent information from philosophical point of view. This theory is very important in many applications since indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent and also the theory generalizes the concept of the classical sets, fuzzy sets, intuitionistic fuzzy sets and so on. Recently, some fuzzy models with fuzzy sets have been researched by many authors (e.g. [8, 9]) and some neutrosophic models with neutrosophic sets have been researched by many authors (e.g. [1, 2, 21, 22, 24, 26]).

Yue [40] developed a multiple attribute group decision making model based on aggregating crisp values into intuitionistic fuzzy numbers. Ye [34] modeled an extended technique for order preference by similarity to ideal solution (TOPSIS) method for group decision making with interval-valued intuitionistic fuzzy numbers to solve the partner selection problem under incomplete and uncertain information environment. Also he introduced a multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Chen et al. [5] developed a approach to tackle multiple criteria group decision-making problems in the context of interval-valued intuitionistic fuzzy sets. Ban [4] investigated some parameters for intuitionistic fuzzy numbers such as value, ambiguity, width and weighted expected value to use in construct a approximation operators. Zhang and Liu [42] presented a weighted arithmetic averaging operator and a weighted geometric average operator and used to the decision making area after defined the score function and the accuracy function. Xu [25] proposed a method for the comparison between two intuitionistic fuzzy values based on score function and accuracy function and developed some aggregation operators.

Li [13], Nehi [18], Wei [31] and Jianqiang and Zhong [15] introduced the trapezoidal intuitionistic fuzzy numbers and gave some operations for them. Then, they proposed some averaging operators. Palanivelrajan and Kaliraju [19] investigated the algebraic properties of intuitionistic fuzzy numbers and developed a new concept called trapezoidal intuitionistic fuzzy number group. Yu [39] researched the aggregation methods of the intuitionistic trapezoidal fuzzy information. He introduced a Generalized Intuitionistic Trapezoidal Fuzzy Weighted Averaging operator and showed the evaluation of the teaching quality based on the proposed operator under intuitionistic trapezoidal fuzzy environment. Gani et al. [14] proposed the group decision making problems in which all the information provided by the decision makers is expressed as decision matrices where each of the elements are characterized by intuitionistic trapezoidal fuzzy numbers and the information about attribute weights are known.

Li [16] defined the concept of triangular intuitionistic fuzzy numbers and develop a new methodology for ranking triangular intuitionistic fuzzy numbers. Mahapatra and Roy [17] gave intuitionistic fuzzy number and its arithmetic operations based on extension principle of intuitionistic fuzzy sets. Also they presented a system to automobile system by intuitionistic fuzzy system. Wang et al. [27] introduced some new arithmetic operations and logic operators for triangular intuitionistic fuzzy numbers and applies them to fault analysis of printed circuit board assembly system and compared to the existing ones. Farhadinia and Ban [12] developed a novel method to extend a similarity measure of generalized trapezoidal fuzzy numbers to similarity measures of generalized trapezoidal intuitionistic fuzzy numbers. Esmailzadeh and Esmailzadeh [11] constructed a new method to calculate the distance between intiuitionistic fuzzy sets, in especial triangular intuitionistic fuzzy number, on the basis of α-cuts. Das and Guha [10] presented a new method has been proposed for ranking of triangular intuitionistic fuzzy number by determining centroid point of triangular intuitionistic fuzzys number and compare the presented method with the existing ranking results. Chen and Li [6] presented Dynamic Multi-Attribute Decision Making(DMADM) model on the basis of triangular intuitionistic fuzzy numbers, to solve the DMADM problem, where all the decision information takes the form of triangular intuitionistic fuzzy numbers. Wan [29] and Wan et al. [30] studied on multi-attribute group decision making problems by developing a new decision method based on power average operators of triangular intuitionistic fuzzy numbers. Recently, some intuitionistic models with intuitionistic sets have been researched by many authors such as: [6, 20, 32, 33, 35, 37, 38].

Single valued neutrosophic numbers are a special case of single valued neutrosophic sets and are of importance for neutrosophic multiattribute decision making problems. As a generalization of intuitionistic numbers, a single valued neutrosophic number seems to suitably describe an ill-known quantity. The paper is organized as follows. In section 2, the concepts of fuzzy sets, intuitionistic fuzzy sets, intuitionistic numbers and neutrosophic sets are introduced. In section 3, we propose the concept of single valued neutrosophic numbers, single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers with operations. In section 4, single valued neutrosophic trapezoidal weighted aggregation operator of single valued trapezoidal neutrosophic numbers is given. Then, score function and accuracy function of the single valued trapezoidal neutrosophic numbers are introduced. In section 5, a method for multi-criteria group decision making based on the proposed operator is presented. Also, we applied it to the evaluation of teaching quality. Finally, the paper is concluded in section 7.

2 Preliminary

In this section, we recall some basic notions of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy numbers and neutrosophic sets. For more details, the reader could refer to [3, 13, 22, 23, 24, 29, 30, 28, 41].

Definition 2.1 *[41] Let E be a universe. Then a fuzzy set X over E is a function defined as follows:*

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where $\mu_X : E \to [0.1]$.

Here, μ_X called membership function of X, and the value $\mu_X(x)$ is called the grade of membership of $x \in E$. The value represents the degree of x belonging to the fuzzy set X.

Definition 2.2 *[3] Let E be a universe. An intuitionistic fuzzy set K on E can be defined as follows:*

$$K = \{< x, \mu_K(x), \gamma_K(x) >: \ x \in E\}$$

where, $\mu_K : E \to [0,1]$ *and* $\gamma_K : E \to [0,1]$ *such that* $0 \le \mu_K(x) + \gamma_K(x) \le 1$ *for any* $x \in E$.

Here, $\mu_K(x)$ *and* $\gamma_K(x)$ *is the degree of membership and degree of non-membership of the element* x*, respectively.*

Definition 2.3 *[23] Let* E *be a universe. A neutrosophic sets(NS)* A *in* E *is characterized by a truth-membership function* T_A*, a indeterminacy-membership function* I_A *and a falsity-membership function* F_A*.* $T_A(x)$*;* $I_A(x)$ *and* $F_A(x)$ *are real standard elements of* $[0,1]$*. It can be written as*

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E,\, T_A(x), I_A(x), F_A(x) \in]^-0, 1[^+\}.$$

There is no restriction on the sum of $T_A(x)$*;* $I_A(x)$ *and* $F_A(x)$*, so* $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.4 *[28] Let* E *be a universe. A single valued neutrosophic sets(SVNS)* A *,which can be used in real scientific and engineering applications, in* E *is characterized by a truth-membership function* T_A*, a indeterminacy-membership function* I_A *and a falsity-membership function* F_A*.* $T_A(x)$*;* $I_A(x)$ *and* $F_A(x)$ *are real standard elements of* $[0,1]$*. It can be written as*

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E,\, T_A(x), I_A(x), F_A(x) \in [0,1]\}.$$

There is no restriction on the sum of $T_A(x)$*;* $I_A(x)$ *and* $F_A(x)$*, so* $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.5 *[13, 29] An intuitionistic fuzzy number* $\tilde{a}$ *denoted by* $\tilde{a} = \langle(\underline{a}_1, a_{1l}, a_{1r}, \overline{a}_1); w_{\tilde{a}}), (\underline{a}_2, a_{2l}, a_{2r}, \overline{a}_2; u_{\tilde{a}})\rangle$*, whose membership function for* $\mu_{\tilde{a}} : R \to [0, w_{\tilde{a}}]$ *and nonmembership function* $\nu_{\tilde{a}} : R \to [u_{\tilde{a}}, 1]$ *are defined as follows:*

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\mu l}(x) & (\underline{a}_1 \le x < a_{1l}) \\ w_{\tilde{a}} & (a_{1l} \le x < a_{1r}) \\ f_{\mu r}(x) & (a_{1r} \le x < \overline{a}_1) \\ 0 & otherwise \end{cases} \quad and \quad \nu_{\tilde{a}}(x) = \begin{cases} f_{\nu l}(x) & (\underline{a}_2 \le x < a_{2l}) \\ u_{\tilde{a}} & (a_{2l} \le x < a_{2r}) \\ f_{\nu r}(x) & (a_{2r} \le x < \overline{a}_2) \\ 1 & otherwise \end{cases}$$

respectively, where $\underline{a}_1, a_{1l}, a_{1r}, \overline{a}_1, \underline{a}_2, a_{2l}, a_{2r}, \overline{a}_2, w_{\tilde{a}}$ *and* $u_{\tilde{a}}$ *are real numbers. The values* $w_{\tilde{a}}$ *and* $u_{\tilde{a}}$ *represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that* $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ *and* $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$.

For some specific values of the parameters $\underline{a}_1, a_{1l}, a_{1r}, \overline{a}_1, \underline{a}_2, a_{2l}, a_{2r}, \overline{a}_2, w_{\tilde{a}}$*, and* $u_{\tilde{a}}$*, we can further construct some particular forms of intuitionistic fuzzy numbers such as trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers as follows;*

1. *If* $(\underline{a}_1, a_{1l}, a_{1r}, \overline{a}_1) = (\underline{a}_2, a_{2l}, a_{2r}, \overline{a}_2)$*, then intuitionistic fuzzy number is reduced to trapezoidal intuitionistic fuzzy numbers* $\tilde{a} = \langle(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}\rangle$.

2. *If* $a = a_1 = a_2$*, then the trapezoidal intuitionistic fuzzy number is reduced to the triangular intuitionistic fuzzy number* $\tilde{a} = \langle(\underline{a}, a, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}\rangle$.

3 Single Valued Neutrosophic Numbers

In this section, we will propose the concept of single valued neutrosophic numbers, single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers with operations. In the following, some definitions and operatios on intuitionistic sets defined in [13, 29, 39], we extend these definitions and operatios to single valued neutrosophic sets [28].

Definition 3.1 *Let* $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0,1]$ *be any real numbers. A single valued neutrosophic number* $\tilde{a} = \langle([a_1, b_1, c_1, d_1)], w_{\tilde{a}}),\ ([a_2, b_2, c_2, d_2], u_{\tilde{a}},)\ ([a_3, b_3, c_3, d_3], y_{\tilde{a}})\rangle$*, is a special single valued neutrosophic set on the set of real numbers* R*, whose truth-membership function* $\mu_{\tilde{a}} : R \to [0, w_{\tilde{a}}]$*, a indeterminacy-membership function* $\nu_{\tilde{a}} : R \to [u_{\tilde{a}}, 1]$ *and a falsity-membership function* $\lambda_{\tilde{a}} : R \to [y_{\tilde{a}}, 1]$ *as given by;*

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\mu l}(x) & (a_1 \le x < b_1) \\ w_{\tilde{a}} & (b_1 \le x < c_1) \\ f_{\mu r}(x) & (c_1 \le x \le d_1) \\ 0 & otherwise \end{cases} \qquad \nu_{\tilde{a}}(x) = \begin{cases} f_{\nu l}(x) & (a_2 \le x < b_2) \\ u_{\tilde{a}} & (b_2 \le x < c_2) \\ f_{\nu r}(x) & (c_2 \le x \le d_2) \\ 1 & otherwise \end{cases}$$

and

$$\lambda_{\tilde{a}}(x)=\begin{cases} f_{\lambda l}(x) & (a_3\le x<b_3)\\ y_{\tilde{a}} & (b_3\le x<c_3)\\ f_{\lambda r}(x) & (c_3\le x\le d_3)\\ 1 & otherwise\end{cases}$$

respectively, where the functions $f_{\mu l}:[a_1,b_1]\to[0,w_{\tilde{a}}]$, $f_{\nu r}:[c_2,d_2]\to[u_{\tilde{a}},1]$ $f_{\lambda r}:[c_3,d_3]\to[y_{\tilde{a}},1]$ *are continuous and nondecreasing, and satisfy the conditions:* $f_{\mu l}(a_1)=0, f_{\mu l}(b_1)=w_{\tilde{a}}, f_{\nu r}(c_2)=u_{\tilde{a}}$, $f_{\nu r}(d_2)=1$, $f_{\lambda r}(c_3)=y_{\tilde{a}}$, *and* $f_{\lambda r}(d_3)=1$*; the functions* $f_{\mu r}:[c_1,d_1]\to[0,w_{\tilde{a}}]$, $f_{\nu l}:[a_2,b_2]\to[u_{\tilde{a}},1]$ *and* $f_{\lambda l}:[a_3,b_3]\to[y_{\tilde{a}},1]$ *are continuous and nonincreasing, and satisfy the conditions:* $f_{\mu r}(c_1)=w_{\tilde{a}}, f_{\mu r}(d_1)=0$, $f_{\nu l}(a_2)=1$, $f_{\nu l}(b_2)=u_{\tilde{a}}$, $f_{\lambda l}(a_3)=1$ *and* $f_{\lambda l}(b_3)=y_{\tilde{a}}$. $[b_1,c_1], a_1$ *and* d_1 *are called the mean interval and the lower and upper limits of the general neutrosophic number* $\tilde{a}$ *for the truth-membership function, respectively.* $[b_2,c_2], a_2$ *and* d_2 *are called the mean interval and the lower and upper limits of the general neutrosophic number* $\tilde{a}$ *for the indeterminacy-membership function, respectively.* $[b_3,c_3], a_3$ *and* d_3 *are called the mean interval and the lower and upper limits of the general neutrosophic number* $\tilde{a}$ *for the falsity-membership function, respectively.* $w_{\tilde{a}}$, $u_{\tilde{a}}$ *and* $y_{\tilde{a}}$ *are called the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree, respectively.*

Clearly; the single valued neutrosophic numbers are a generalization of the intuitionistic fuzzy numbers [13]. Thus, the neutrosophic number may express more uncertainty information than the intuitionistic fuzzy number.

For some specific values of the parameters $a_1,b_1,c_1,d_1,\ a_2,b_2,c_2,d_2,\ a_3,b_3,\ c_3,d_3,w_{\tilde{a}},\ u_{\tilde{a}}$ and $y_{\tilde{a}}$ we can further construct some particular forms of single valued neutrosophic numbers such as; single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers.

Definition 3.2 *A single valued trapezoidal neutrosophic number* $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{a}},u_{\tilde{a}},\ y_{\tilde{a}}\rangle$ *is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy-membership, and a falsity-membership are given as follows:*

$$\mu_{\tilde{a}}(x)=\begin{cases} (x-a_1)w_{\tilde{a}}/(b_1-a_1) & (a_1\le x<b_1)\\ w_{\tilde{a}} & (b_1\le x\le c_1)\\ (d_1-x)w_{\tilde{a}}/(d_1-c_1) & (c_1<x\le d_1)\\ 0 & otherwise,\end{cases}$$

$$\nu_{\tilde{a}}(x)=\begin{cases} (b_1-x+u_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1\le x<b_1)\\ u_{\tilde{a}} & (b_1\le x\le c_1)\\ (x-c_1+u_{\tilde{a}}(d_1-x))/(d_1-c_1) & (c_1<x\le d_1)\\ 1 & otherwise\end{cases}$$

and

$$\lambda_{\tilde{a}}(x)=\begin{cases} (b_1-x+y_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1\le x<b_1)\\ y_{\tilde{a}} & (b_1\le x\le c_1)\\ (x-c_1+y_{\tilde{a}}(d_1-x))/(d_1-c_1) & (c_1<x\le d_1)\\ 1 & otherwise\end{cases}$$

respectively.

If $a_1\ge 0$ and at least $d_1>0$, then $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is called a positive single valued trapezoidal neutrosophic number, denoted by $\tilde{a}>0$. Likewise, if $d_1\le 0$ and at least $a_1<0$, then $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is called a negative single valued trapezoidal neutrosophic number, denoted by $\tilde{a}<0$. A single valued trapezoidal neutrosophic number $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ may represent an ill-known quantity of the range, which is approximately equal to the interval $[b_1,c_1]$.

The single valued trapezoidal neutrosophic numbers are a generalization of the intuitionistic trapezoidal fuzzy numbers [13]. Thus, the neutrosophic number may express more uncertainty information than the intuitionistic fuzzy number.

Example 3.3 $\tilde{a}=\langle(1,2,5,6);0.8,0.6,0.4\rangle$ *is a single valued trapezoidal neutrosophic number, whose membership and nonmembership functions are given as follows:*

$$\mu_{\tilde{a}}(x)=\begin{cases} 0.8(x-1) & (1\le x<2)\\ 0.8 & (2\le x\le 5)\\ 0.8(6-x) & (5<x\le 6)\\ 0 & otherwise\end{cases}\qquad \nu_{\tilde{a}}(x)=\begin{cases} 1.4-0.4x & (1\le x<2)\\ 0.6 & (2\le x\le 5)\\ 0.4x-1.4 & (5<x\le 6)\\ 1 & otherwise,\end{cases}$$

and

$$\lambda_{\tilde{a}}(x) = \begin{cases} 1.6 - 0.6x & (1 \leq x < 2) \\ 0.4 & (2 \leq x \leq 5) \\ 0.6x - 2.6 & (5 < x \leq 6) \\ 1 & otherwise, \end{cases}$$

respectively.

Definition 3.4 *Let* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}},\ u_{\tilde{a}}, y_{\tilde{a}} \rangle$ *and* $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{b}},\ u_{\tilde{b}}, y_{\tilde{b}} \rangle$ *be two single valued trapezoidal neutrosophic numbers and* $\gamma \neq 0$, *then*

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $\tilde{a} - \tilde{b} = \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

3. $\tilde{a}\tilde{b} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$

4. $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$

5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$

6. $\tilde{a}^{-1} = \langle (1/d_1, 1/c_1, 1/b_1, 1/a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \ (\tilde{a} \neq 0)$.

Example 3.5 *Let* $\tilde{a} = \langle (2, 4, 5, 8); 0.2, 0.3, 0, 5 \rangle$ *and* $\tilde{b} = \langle (1, 3, 6, 7); 0.4, 0.5, 0, 6 \rangle$ *be two single valued trapezoidal neutrosophic numbers, then*

1. $\tilde{a} + \tilde{b} = \langle (3, 7, 11, 15); 0.2, 0.5, 0.6 \rangle$

2. $\tilde{a} - \tilde{b} = \langle (-5, -2, 2, 7); 0.2, 0.5, 0.6 \rangle$

3. $\tilde{a}\tilde{b} = \langle (2, 12, 30, 56); 0.2, 0.5, 0.6 \rangle$

4. $\tilde{a}/\tilde{b} = \langle (\frac{2}{7}, \frac{2}{3}, \frac{5}{3}, 8); 0.2, 0.5, 0.6 \rangle$

5. $\tilde{a}^{-1} = \langle (\frac{1}{8}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}); 0.2, 0.5, 0.6 \rangle$

6. $2\tilde{a} = \langle (4, 8, 10, 16); 0.2, 0.3, 0, 5 \rangle$

Note 3.6 *It is easily shown that the results obtained by multiplication and division are not always single valued trapezoidal neutrosophic numbers. But, for the sake of convenience, we still use single valued trapezoidal neutrosophic numbers to express these computational results approximately.*

Remark 3.7 *If* $0 \leq w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \leq 1$, $0 \leq w_{\tilde{a}} + u_{\tilde{a}} + y_{\tilde{a}} \leq 1$, $y_{\tilde{a}} = 0$ *and* $0 \leq w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \leq 1$, $0 \leq w_{\tilde{b}} + u_{\tilde{b}} + y_{\tilde{b}} \leq 1$, $y_{\tilde{b}} = 0$, *then the single valued trapezoidal neutrosophic numbers* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ *and* $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ *degenerate to the intuitionistic trapezoidal fuzzy numbers [13]* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, 0 \rangle$ *and* $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{b}}, u_{\tilde{b}}, 0 \rangle$, *respectively. As a result, Definition 3.4 is reduced to*

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle$

2. $\tilde{a} - \tilde{b} = \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle$

3. $\tilde{a}\tilde{b} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$

4. $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$

5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); w_{\tilde{a}}, u_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$

6. $\tilde{a}^{-1} = \langle (1/d_1, 1/c_1, 1/b_1, 1/a_1); w_{\tilde{a}}, u_{\tilde{a}} \rangle \;\; (\tilde{a} \neq 0)$.

Hence, the algebraic operations over the single valued trapezoidal neutrosophic numbers are a generalization of those over the intuitionistic trapezoidal fuzzy numbers [13]. Thus, the neutrosophic number may express more uncertainty information than the intuitionistic fuzzy number.

Definition 3.8 *A triangular neutrosophic number* $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ *is a special neutrosophic set on the real number set* R*, whose truth-membership indeterminacy-membership and falsity-membership functions are defined as follows:*

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)w_{\tilde{a}}/(b_1 - a_1) & (a_1 \leq x < b_1) \\ w_{\tilde{a}} & (x = b_1) \\ (c_1 - x)w_{\tilde{a}}/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1 - x + u_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ u_{\tilde{a}} & (x = b_1) \\ (x - b_1 + u_{\tilde{a}}(c_1 - x))/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 1 & otherwise, \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1 - x + y_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ y_{\tilde{a}} & (x = b_1) \\ (x - b_1 + y_{\tilde{a}}(c_1 - x))/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 1 & otherwise, \end{cases}$$

respectively.

If $a_1 \geq 0$ and at least $c_1 > 0$ then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a positive triangular neutrosophic number, denoted by $\tilde{a} > 0$. Likewise, if $c_1 \leq 0$ and at least $a_1 < 0$, then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a negative triangular neutrosophic number, denoted by $\tilde{a} < 0$.

A triangular neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ may express an ill-known quantity about a, which is approximately equal to a.

Definition 3.9 *Let* $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$*, and* $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$*, be two single valued triangular neutrosophic numbers and* $\gamma \neq 0$ *be any real number. Then,*

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $\tilde{a} - \tilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{b}} \vee y_{\tilde{b}} \rangle$

3. $\tilde{a}\tilde{b} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2; w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (a_1c_2, b_1b_2, c_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1c_2, b_1b_2, a_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$

4. $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/c_2, b_1/b_2, c_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1/a_2, b_1/b_2, a_1/c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$

5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$

6. $\tilde{a}^{-1} = \langle (1/c_1, 1/b_1, 1/a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \;\; (\tilde{a} \neq 0)$.

Likewise, it is easily proven that the results obtained by multiplication and division of two single valued triangular neutrosophic numbers are not always single valued triangular neutrosophic numbers. However, we often use single valued triangular neutrosophic numbers to express these computational results approximately.

Example 3.10 *Let* $\tilde{a} = \langle (2, 4, 5); 0.3, 0.4, 0.5 \rangle$ *and* $\tilde{b} = \langle (1, 2, 4); 0.7, 0.8, 0.9 \rangle$ *be two single valued triangular neutrosophic numbers then,*

1. $\tilde{a}+\tilde{b}=\langle(3,6,9);0.3,0.8,0.9\rangle$
2. $\tilde{a}-\tilde{b}=\langle(-2,2,4);0.3,0.8,0.9\rangle$
3. $\tilde{a}\tilde{b}=\langle(2,8,20);0.3,0.8,0.9\rangle$
4. $\tilde{a}/\tilde{b}=\langle(\frac{1}{2},2,5);0.3,0.8,0.9\rangle$
5. $5\tilde{a}=\langle(10,20,25);0.3,0.4,0.5\rangle$
6. $2\tilde{b}=\langle(2,4,8);0.7,0.8,0.9\rangle$
7. $\tilde{a}^{-1}=\langle(\frac{1}{5},\frac{1}{4},\frac{1}{2});0.3,0.4,0.5\rangle$
8. $\tilde{b}^{-1}=\langle(\frac{1}{4},\frac{1}{2},1);0.7,0.8,0.9\rangle$

Remark 3.11 *If* $0\le w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\le 1$, $0\le w_{\tilde{a}}+u_{\tilde{a}}+y_{\tilde{a}}\le 1$, $y_{\tilde{a}}=0$ *and* $0\le w_{\tilde{b}},u_{\tilde{b}},y_{\tilde{b}}\le 1$, $0\le w_{\tilde{b}}+u_{\tilde{b}}+y_{\tilde{b}}\le 1$, $y_{\tilde{b}}=0$, *then the single valued triangular neutrosophic numbers* $\tilde{a}=\langle(a_1,b_1,c_1);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ *and* $\tilde{b}=\langle(a_2,b_2,c_2);w_{\tilde{b}},u_{\tilde{b}},y_{\tilde{b}}\rangle$ *degenerate to the intuitionistic triangular fuzzy numbers [13]* $\tilde{a}=\langle(a_1,b_1,c_1);w_{\tilde{a}},u_{\tilde{a}},0\rangle$ *and* $\tilde{b}=\langle(a_2,b_2,c_2);w_{\tilde{b}},u_{\tilde{b}},0\rangle$, *respectively. As a result, Definition 3.9 is reduced to*

1. $\tilde{a}+\tilde{b}=\langle(a_1+a_2,b_1+b_2,c_1+c_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle$
2. $\tilde{a}-\tilde{b}=\langle(a_1-c_2,b_1-b_2,c_1-a_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle$
3. $\tilde{a}\tilde{b}=\begin{cases}\langle(a_1a_2,b_1b_2,c_1c_2;w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1>0,c_2>0)\\ \langle(a_1c_2,b_1b_2,c_1a_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1<0,c_2>0)\\ \langle(c_1c_2,b_1b_2,a_1a_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1<0,c_2<0)\end{cases}$
4. $\tilde{a}/\tilde{b}=\begin{cases}\langle(a_1/c_2,b_1/b_2,c_1/a_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1>0,c_2>0)\\ \langle(c_1/c_2,b_1/b_2,a_1/a_2);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1<0,c_2>0)\\ \langle(c_1/a_2,b_1/b_2,a_1/c_3);w_{\tilde{a}}\wedge w_{\tilde{b}},u_{\tilde{a}}\vee u_{\tilde{b}}\rangle & (c_1<0,c_2<0)\end{cases}$
5. $\gamma\tilde{a}=\begin{cases}\langle(\gamma a_1,\gamma b_1,\gamma c_1);w_{\tilde{a}},u_{\tilde{a}}\rangle & (\gamma>0)\\ \langle(\gamma c_1,\gamma b_1,\gamma a_1);w_{\tilde{a}},u_{\tilde{a}}\rangle & (\gamma<0)\end{cases}$
6. $\tilde{a}^{-1}=\langle(1/c_1,1/b_1,1/a_1);w_{\tilde{a}},u_{\tilde{a}}\rangle$ $(\tilde{a}\neq 0)$.

Hence, the algebraic operations over the single valued triangular neutrosophic numbers are a generalization of those over the intuitionistic triangular fuzzy numbers. Thus, the neutrosophic number may express more uncertainty information than the intuitionistic fuzzy number.

4 Aggregation operators of the single valued trapezoidal neutrosophic number and its score function and accuracy function

In this section, a neutrosophic trapezoidal weighted aggregation operator of neutrosophic trapezoidal numbers is given. Then, score function and accuracy function of the single valued trapezoidal neutrosophic numbers are introduced. Some of it is quoted from application in [39].

Definition 4.1 *Let* $\tilde{a}_j=((a_j,b_j,c_j,d_j);w_{\tilde{a}_j},u_{\tilde{a}_j},y_{\tilde{a}_j})$ $(j=1,2,...,n)$ *be a collection of single valued trapezoidal neutrosophic numbers. Then neutrosophic trapezoidal weighted aggregation operator(SVTNWAO)is defined as;*

$$SVTNWAO(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=(w_1\tilde{a}_1^{\gamma}+w_2\tilde{a}_2^{\gamma}+\cdots+w_n\tilde{a}_n^{\gamma})^{\frac{1}{\gamma}}$$

where $\gamma>0$, $w=(w_1,w_2,...,w_n)^T$ *is a weight vector associated with the SVTNWAO operator, with* $w_j\ge 0$, $j=1,2,3,...,n$ *and* $\sum_{j=1}^{n}w_j=1$.

Definition 4.2 *We defined a method to compare any two single valued trapezoidal neutrosophic numbers which is based on the score function and the accuracy function. Let* $\tilde{a}_1 = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$ *be a single valued trapezoidal neutrosophic number, then*

$$S(\tilde{a}) = \frac{1}{16}[a + b + c + d] \times (2 + \mu_{\tilde{a}} - \nu_{\tilde{a}} - \gamma_{\tilde{a}})$$

and

$$A(\tilde{a}) = \frac{1}{16}[a + b + c + d] \times (2 + \mu_{\tilde{a}} - \nu_{\tilde{a}} + \gamma_{\tilde{a}})$$

is called the score and accuracy degrees of $\tilde{a}_1$, *respectively,*

Example 4.3 *Let* $\tilde{a} = \langle (1, 4, 5, 6); 0.9, 0.5, 0.1 \rangle$ *be a single valued trapezoidal neutrosophic number then,*

$$S(\tilde{a}) = \frac{1}{16}[1 + 4 + 5 + 6] \times (2 + 0.9 - 0.5 - 0.1) = 2.3$$

$$A(\tilde{a}) = \frac{1}{16}[1 + 4 + 5 + 6] \times (2 + 0.9 - 0.5 + 0.1) = 2.5$$

Definition 4.4 *Let* $\tilde{a}_1$ *and* $\tilde{a}_2$ *be two single valued trapezoidal neutrosophic numbers;*

1. *If* $S(\tilde{a}_1) < S(\tilde{a}_2)$, *then* $\tilde{a}_1$ *is smaller than* $\tilde{a}_2$, *denoted by* $\tilde{a}_1 < \tilde{a}_2$
2. *If* $S(\tilde{a}_1) = S(\tilde{a}_2)$;
   (a) *If* $A(\tilde{a}_1) < A(\tilde{a}_2)$, *then* $\tilde{a}_1$ *is smaller than* $\tilde{a}_2$, *denoted by* $\tilde{a}_1 < \tilde{a}_2$
   (b) *If* $A(\tilde{a}_1) = A(\tilde{a}_2)$, *then* $\tilde{a}_1$ *and* $\tilde{a}_2$ *are the same, denoted by* $\tilde{a}_1 = \tilde{a}_2$

Theorem 4.5 *Let* $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j})$ $(j = 1, 2, ..., n)$ *be a collection of single valued trapezoidal neutrosophic numbers,* $w = (w_1, w_2, ..., w_n)^T$ *be a weight vector of* $\tilde{a}_j$ *with* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$. *Then their aggregated value by using SVTNWAO operator is also a neutrosophic trapezoidal number and*

$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \Big(\Big[\textstyle\sum_{j=1}^{n} w_j a_j^{\gamma}, \sum_{j=1}^{n} w_j b_j^{\gamma}, \sum_{j=1}^{n} w_j c_j^{\gamma}, \sum_{j=1}^{n} w_j d_j^{\gamma} \Big]; \bigwedge_{j=1}^{n} w_{\tilde{a}_j}, \bigvee_{j=1}^{n} u_{\tilde{a}_j}, \bigvee_{j=1}^{n} y_{\tilde{a}_j} \Big)$$

Proof: The proof can be made by using mathematical induction on n as;
Assume that,

$$\tilde{a}_1 = \langle [a_1^{\gamma}, b_1^{\gamma}, c_1^{\gamma}, d_1^{\gamma}]; w_{\tilde{a}_1}^{\gamma}, u_{\tilde{a}_1}^{\gamma}, y_{\tilde{a}_1}^{\gamma} \rangle$$

and

$$\tilde{a}_2 = \langle [a_2^{\gamma}, b_2^{\gamma}, c_2^{\gamma}, d_2^{\gamma}]; w_{\tilde{a}_2}^{\gamma}, u_{\tilde{a}_2}^{\gamma}, y_{\tilde{a}_2}^{\gamma} \rangle$$

be two single valued trapezoidal neutrosophic numbers then,
for $n = 2$, we have

$$w_1 \tilde{a}_1^{\gamma} + w_2 \tilde{a}_2^{\gamma} = \left\langle \left(\sum_{j=1}^{2} w_j a_j^{\gamma}, \sum_{j=1}^{2} w_j b_j^{\gamma}, \sum_{j=1}^{2} w_j c_j^{\gamma}, \sum_{j=1}^{2} w_j d_j^{\gamma} \right); \wedge_{j=1}^{2} w_{\tilde{a}_j}, \vee_{j=1}^{2} u_{\tilde{a}_j}, \vee_{j=1}^{2} y_{\tilde{a}_j} \right\rangle$$

If holds for $n = k$, that is

$$w_1 \tilde{a}_1^{\gamma} + w_2 \tilde{a}_2^{\gamma} + \cdots + w_k \tilde{a}_k^{\gamma} = \left\langle \left(\sum_{j=1}^{k} w_j a_j^{\gamma}, \sum_{j=1}^{k} w_j b_j^{\gamma}, \sum_{j=1}^{k} w_j c_j^{\gamma}, \sum_{j=1}^{k} w_j d_j^{\gamma} \right); \wedge_{j=1}^{k} w_{\tilde{a}_j}, \vee_{j=1}^{k} u_{\tilde{a}_j}, \vee_{j=1}^{k} y_{\tilde{a}_j} \right\rangle$$

then, when $n = k + 1$, by the operational laws in Definition3.9, I have

$$w_1 \tilde{a}_1^{\gamma} + w_2 \tilde{a}_2^{\gamma} + \cdots + w_{k+1} \tilde{a}_{k+1}^{\gamma} = \left\langle \left(\sum_{j=1}^{k} w_j a_j^{\gamma}, \sum_{j=1}^{k} w_j b_j^{\gamma}, \sum_{j=1}^{k} w_j c_j^{\gamma}, \sum_{j=1}^{k} w_j d_j^{\gamma} \right); \wedge_{j=1}^{k} w_{a_j}, \vee_{j=1}^{k} u_{\tilde{a}_j}, \vee_{j=1}^{k} y_{\tilde{a}_j} \right\rangle$$

$$+\left\langle\left(w_{k+1}\, a_{k+1}^{\gamma}, w_{k+1}\, b_{k+1}^{\gamma}, w_{k+1} c_{k+1}^{\gamma}, w_{k+1} d_{k+1}^{\gamma}\right); w_{\tilde{a}_{k+1}}, u_{\tilde{a}_{k+1}}, y_{\tilde{a}_{k+1}}\right\rangle$$

$$=\left\langle\left(\sum_{j=1}^{k+1} w_j a_j^{\gamma}, \sum_{j=1}^{k+1} w_j b_j^{\gamma}, \sum_{j=1}^{k+1} w_j c_j^{\gamma}, \sum_{j=1}^{k+1} w_j d_j^{\gamma}\right); \wedge_{j=1}^{k+1} w_{\tilde{a}_j}, \vee_{j=1}^{k+1} u_{\tilde{a}_j}, \vee_{j=1}^{k+1} y_{\tilde{a}_j}\right\rangle$$

therefore proof is valid.

Example 4.6 *Let*

$$\tilde{a}_1 = \big([0.214, 0.321, 0.545, 0.781]; 0.5, 0.4, 0.9\big),$$
$$\tilde{a}_2 = \big([0.241, 0.462, 0.656, 0.963]; 0.8, 0.2, 0.4\big),$$
$$\tilde{a}_3 = \big([0.311, 0.414, 0.531, 0.722]; 0.6, 0.3, 0.7\big),$$
$$\tilde{a}_4 = \big([0.268, 0.321, 0.581, 0.745]; 0.7, 0.2, 0.5\big),$$

be four single valued trapezoidal neutrosophic numbers, and $w = (0.4, 0.1, 0.2, 0.3)^T$ be the weight vector of $\tilde{a}_j (j = 1, 2, 3, 4)$. When the parameter γ takes different values, different aggregated values of the single valued trapezoidal neutrosophic numbers can be obtained.

When $\gamma = 1$,

$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \big([0.2523, 0.3537, 0.5641, 0.7766)]; 0.5, 0.4, 0.9\big)$$

and their score is 0.1460.

When $\gamma = 3$,

$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \big([0.0171, 0.0472, 0.1818, 0.4792]; 0.5, 0.4, 0.9\big)$$

and their score is 0.0543.

When $\gamma = 5$,

$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \big([0.0013, 0.0069, 0.0597, 0.3071]; 0.5, 0.4, 0, 9\big)$$

and their score is 0.0281.

The SVTNWAO operator has be following desirable properties.

Property 4.7 *Let (for $j = 1, 2, \ldots, n$) $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j})$ and $\tilde{b}_j = ((e_j, f_j, g_j, h_j); w_{\tilde{b}_j}, u_{\tilde{b}_j}, y_{\tilde{b}_j})$ be two collection of single valued trapezoidal neutrosophic numbers, $\tilde{a} = ((a, b, c, d); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}})$ be a single valued trapezoidal neutrosophic numbers and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector related to the SVT-NWAO operator, such that $w_j \geq 0$, $j = 1, 2, \ldots, n$, and $\sum_{j=1}^{n} w_j = 1$, and $\gamma > 0$ then, for $j = 1, 2, \ldots, n$ we have*

1. *If $\forall j$, $a_j = a$, $b_j = b$, $c_j = c$, $d_j = d$, $w_{\tilde{a}_j} = w_{\tilde{a}}$, $u_{\tilde{a}_j} = u_{\tilde{a}}$ and $y_{\tilde{a}_j} = y_{\tilde{a}}$ then*
$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$$

2. *If*
$$a_j^- = \Big(\big(min\{a_j\}, min\{b_j\}, min\{c_j\}, min\{d_j\}\big); min\{w_{\tilde{a}_j}\}, max\{u_{\tilde{a}_j}\}, max\{y_{\tilde{a}_j}\}\Big)$$
$$a_j^+ = \Big(\big(max\{a_j\}, max\{b_j\}, max\{c_j\}, max\{d_j\}\big); max\{w_{\tilde{a}_j}\}, min\{u_{\tilde{a}_j}\}, min\{y_{\tilde{a}_j}\}\Big)$$
then,
$$a^- \leq SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq a^+$$

3. *if $a_j \leq e_j$, $b_j \leq f_j$, $c_j \leq g_j$, $d_j \leq h_j$, $w_{\tilde{a}_j} \leq w_{\tilde{b}_j}$, $u_{\tilde{a}_j} \geq u_{\tilde{b}_j}$,and $y_{\tilde{a}_j} \geq y_{\tilde{b}_j}$, for all j then,*
$$SVTNWAO(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq SVTNWAO(\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_n)$$

5 Multicriteria decision making based on single valued trapezoidal neutrosophic numbers

In this section, we will apply the SVTNWAO to multicriteria decision making problem and illustrate the effectiveness of the aggregation operators by the example of evaluation of the teaching quality. Some of it is quoted from application in [39].

Assume that there is a multi-criteria decision making problem under neutrosophic trapezoidal environment. Let $X = \{x_1, x_2, ..., x_m\}$ be the set of alternatives and $U = \{u_1, u_2, ..., u_n\}$ the set of criteria. The decision makers evaluate the objects (the criterion u_j for the alternative x_i) and expressed by single valued trapezoidal neutrosophic numbers $\tilde{a}_{ij}$. Based on the SVTNWAO, we construct a decision making method by the following algorithm;

Algorithm:

Step 1. The experts evaluate the objects (the criterion u_j for the alternative x_i) and express by single valued trapezoidal neutrosophic numbers $\tilde{a}_{ij}$ as a Table.

Step 2. Find single valued trapezoidal neutrosophic numbers by the SVTNWAO operator when the parameter γ takes different values,

Step 3. Calculate score values found by the SVTNWAO operator and the ranking of alternatives

Example 5.1 *Assume that a government wants to fill a position. There are five candidates* (x_i ($i = 1, 2, 3, 4, 5$)) *and they will be evaluated by experts panel of the Management Science and Engineering Institute from the following three aspects, their teaching attitude* (u_1), *ability* (u_2) *and content* (u_3). *These 5 teacher who have good teaching attitude could tend to generate a multiplier effect while teaching ability will be evaluated by their professional knowledge and social practice. The last factor is the teachers teaching content, to check whether the content is closely around teaching Guidance. The weight vector of the three criteria is supposed as* $(0.4, 0.3, 0.3)^T$. *Then,*

Step 1. *The experts evaluate the teachers and showed their evaluation results in Table 1.*

	u_1	u_2	u_3
x_1	$\langle(0.3,0.4,0.5,0.7);0.5,0.4,0.3\rangle$	$\langle(0.2,0.3,0.5,0.6);0.5,0.3,0.7\rangle$	$\langle(0.1,0.2,0.7,0.8);0.9,0.1,0.5\rangle$
x_2	$\langle(0.2,0.5,0.6,0.9);0.8,0.2,0.4\rangle$	$\langle(0.2,0.4,0.6,0.8);0.1,0.2,0.3\rangle$	$\langle(0.2,0.3,0.6,0.7);0.5,0.3,0.8\rangle$
x_3	$\langle(0.3,0.4,0.7,0.8);0.6,0.3,0.2\rangle$	$\langle(0.3,0.5,0.8,0.9);0.2,0.5,0.8\rangle$	$\langle(0.3,0.4,0.5,0.6);0.8,0.2,0.6\rangle$
x_4	$\langle(0.3,0.5,0.8,0.8);0.7,0.2,0.5\rangle$	$\langle(0.2,0.3,0.7,0.8);0.9,0.8,0.7\rangle$	$\langle(0.4,0.6,0.7,0.8);0.5,0.4,0.2\rangle$
x_5	$\langle(0.4,0.6,0.7,0.8);0.3,0.5,0.6\rangle$	$\langle(0.1,0.3,0.5,0.7);0.9,0.7,0.5\rangle$	$\langle(0.4,0.5,0.6,0.7);0.9,0.3,0.6\rangle$

Table 1*: single valued trapezoidal neutrosophic decision table*

Step 2. *For* $\gamma = 1, 2, 4$ *and* 6, *the aggregated single valued trapezoidal neutrosophic numbers found by the SVTNWAO was shown in Table 2.*

	$\gamma = 1$	$\gamma = 2$
x_1	$\langle(0.2100,0.3100,0.5600,0.7000);0.5,0.4,0.7\rangle$	$\langle(0.0510,0.1300,0.3220,0.4960);0.5,0.4,0.7\rangle$
x_2	$\langle(0.2000,0.4100,0.6000,0.8100);0.1,0.3,0.8\rangle$	$\langle(0.0400,0.1750,0.3600,0.6630);0.1,0.3,0.8\rangle$
x_3	$\langle(0.3000,0.4300,0.6700,0.7700);0.2,0.5,0.8\rangle$	$\langle(0.0900,0.1870,0.4630,0.6070);0.2,0.5,0.8\rangle$
x_4	$\langle(0.3000,0.4700,0.7400,0.8400);0.5,0.8,0.7\rangle$	$\langle(0.0960,0.2350,0.5500,0.7080);0.5,0.8,0.7\rangle$
x_5	$\langle(0.3100,0.4800,0.6100,0.7400);0.3,0.7,0.6\rangle$	$\langle(0.1150,0.2460,0.3790,0.5500);0.3,0.7,0.6\rangle$
	$\gamma = 4$	$\gamma = 6$
x_1	$\langle(0.0038,0.0132,0.1158,0.2578);0.5,0.4,0.7\rangle$	$\langle(0.0003,0.0019,0.0462,0.1397);0.5,0.4,0.7\rangle$
x_2	$\langle(0.0016,0.0351,0.1296,0.4574);0.1,0.3,0.8\rangle$	$\langle(0.0001,0.0077,0.0467,0.3265);0.1,0.3,0.8\rangle$
x_3	$\langle(0.0081,0.0367,0.2377,0.3996);0.2,0.5,0.8\rangle$	$\langle(0.0007,0.0076,0.1304,0.2783);0.2,0.5,0.8\rangle$
x_4	$\langle(0.0114,0.0663,0.3079,0.5082);0.5,0.8,0.7\rangle$	$\langle(0.0015,0.0205,0.1754,0.3699);0.5,0.8,0.7\rangle$
x_5	$\langle(0.0180,0.0730,0.1357,0.3079);0.3,0.7,0.6\rangle$	$\langle(0.0029,0.0236,0.0657,0.1754);0.3,0.7,0.6\rangle$

Table 2*: The single valued trapezoidal neutrosophic numbers obtained by the SVTNWAO*

Step 3. *The corresponding score values and the ranking of alternatives was shown in Table 3.*

	Y_1	Y_2	Y_3	Y_4	Y_5	*ranking*
$\gamma = 1$	*0.1558*	*0.1263*	*0.1221*	*0.1469*	*0.1338*	$x_1 > x_4 > x_5 > x_2 > x_3$
$\gamma = 2$	*0.0851*	*0.0774*	*0.0758*	*0.0993*	*0.0806*	$x_4 > x_1 > x_5 > x_2 > x_3$
$\gamma = 4$	*0.0342*	*0.0390*	*0.0384*	*0.0559*	*0.0345*	$x_4 > x_2 > x_3 > x_5 > x_1$
$\gamma = 6$	*0.0165*	*0.0238*	*0.0235*	*0.0355*	*0.0167*	$x_4 > x_2 > x_3 > x_5 > x_1$

Table 3: *The score values and the ranking of alternatives*

6 Conclusion

In this work, we have defined single valued neutrosophic numbers which is a generalization of fuzzy numbers and intuitionistic fuzzy numbers and we have presented two special forms of single valued neutrosophic numbers such as single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic numbers with operations. Finally, we have developed a single valued trapezoidal neutrosophic weighted aggregation operator(SVTNWAO) and applied to multicriteria decision making problem. In future work, we will applied this concept to game theory, algebraic structure, optimization and so on.

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