a dirge, from some human compulsion ,... to again, join to that which isn't broken ,... as-in place-recordings of <u>an</u> ongoing <u>attempt</u> ,... and because time is a great thickener of things -and yet, always still - short $..^{\downarrow}$ $..^{\downarrow}$

and then:

(*wrapping things-up*,... *and folding*) into, *or* (*sometimes* with a permission, along-side of or then from an outside taking-in) an already ancient philosophy-and-conversation : on the toolset of-or-into 'purifying' (symbolic and graphic)-techniques.

and as_... this journey, of a few *cardinality* howlings into the wind : ... is culled, with a simple and naïve-observation that it seems completely-reasonable "to <u>speak</u> of <u>a faithful</u>-projection of color *onto* -a- visually *subjecti vised* -point · ", but yet or still, somewhat more difficult to speak of <u>a similar</u> pointed*-representation*, for 'locally-emergent' and/or 'distributed-geometric' objects, such as : dimensionally-*simple* piped-triangles \triangle , or, in some other *complexity* fractalings * , _ present (after one more cycle-of-completion), an ..un-ordered,..*already* existent,.. further generalization to the infinitesimals ,...-and/or an adjoining of: (standard -And- non-standard analysis). By an

interior color-theory of abstract terminal...(s)

– a Construction of \mathbb{R}^+

•

- (an) internal Spectral-carrying-Structure for numbers
- logarithmically contained laminar flows (\mathbb{R}_{CLF})
- real-analytic(domain bounded) color algebras, knitted into geometries
- predictive -Opaqueness and an upto type-structuring of frames
- an application to inertial-maps and localized-time(like *representations*)
- a tools section

tagged for now ... with a wild or then whimsical warning paradox, and where as we <u>must</u> .. also touch on this: mostly as it seems, it *still* remains *remarkably* degenerate, let us <u>just briefly</u> risk , the intensely difficult, well-spring of platonic *externalities* ..., out of the nut-shell: and more, coarsely-or-physically speaking

> "as ,some, first-and/or-final system...:) -nothing exists:- (... preserved as- or in bare-potential* then is or may-be associated with any obvious- representative model which presents-inconsistency. of course quite a bit, in a larger-sense, frees up from that "! and more so, any sufficiently unleashed operational in-coherence - returns then, also- a climb-into such) fully-degenerate neighborhoods (")

Anyway, various paradoxical-classes, as often-is *method-usual* (stratified, domain <u>restricted</u> -or-ignored), start with, a *little* again - <u>anthropormorhic</u> - view of territory (as it is: commensurate) - and thus – we take a further re-fining of ... or a new

- reconnoitering: with-in ,a wave-and-saturation of in-the-air-algebraic-approaches and discrete-instantiation, there is a simply fascinating, 'sometimes'-forgotten < multi-threaded > history on, the science and way to assign a meaning-and-role to "symbolic-numbers". (in <u>practical aspect</u>) numbers have become -or- remain "namings"; which then if-sensed, as abstracted-Domain "Objectification-counting-localization-or-quotient.ing-,...,-descriptors ", embody
- * <u>by</u>-this '<u>registration</u>': [also some-tending*-to inducement -of- distinguishable- <u>codings</u> -forms](p_adic -or- not) which then <u>is</u>* or are implicit-of, or at the least coding consistent-with, an <u>aliasing</u> for ...such (<u>frame</u>)-adjoined ("possibilities- or- new recordings" - or- renormalizations, ...,) to or by 'those' apparent "<u>system-and-constraints</u>".

and so *this as poetry*, in a label *'immersive*-context', of trace-*capturing*-[*separate*] -*framings of* discourse:: partly inorder to retain, update ,or, merely fill-in *those* internal-type-capabilities *which* may *remain* or be , *a* vehicle at given *interfaces*, then <u>is</u>, a concrete-<u>return</u> and *expansion down*-into a *language-passed*-point.(*ing*)--dimensional(critical or)*then* componet-structure philosophies, as specifically-and-representationally related to:

[<u>name.(ing)-instantiation-potential(s)</u>], and/or, the *information-history-inherintly* carried-forward with-in an [abstract'mark.(ings)'-back-ground] by, a sub-class, underpinning, and, in a sense to be made precise from *with-in*, then *"descriptive positions"-given by-*or-*over*, *number-constructed alias-spaces and maps*.

further this provides , as such , from some opposite tenet \star ,

<u>a</u> "*method* and *example*" of dressing-or-*writing* external-constraint ,into, point*ed*->*geometric*-context(s) ; and, by weakening (or making porous) the convention that *isomorphic*-objects are *in-a-way* identical : brings *classic*-definition to [*color-(evil*)]-systems or *modulation*-shells as seen by a *partial(ization)*-of-number(s), where in the end, of course these examples may then again be embedded in classes of identifiers, for other generalized spaces.

and so, and somewhat as a "review to the literature", the workings begin by carrying-into "convergent foundations";

- a. in the first section: [a <u>completion now</u> of the "interior"] of <u>a</u> *naming*-conveyer for the *irrational*-real-numbers with a <u>constant-representative</u>¹ (and/or a distributed-overlapping identity-element). *these* representatives then act as a reference for sub-typing the [latent-<u>sequence</u>-(ie.an available-alternate '*labeling*)-*space*'] which
- * forms, the underlying *monogenerative* {*"infinite" dimensional-unit*}-and/or-*universe* of-each *separating- 'Object_*or-element-or-<u>point</u>': in then {*a* "re-emergent constructive"-contrast}, of the-*1-dimensional* system \mathbb{R}_{\dagger} , and also, of the (*externally-algebraically-closed*) hypercomplex systems, which immediately or conjointly follow ...
- with this simple addition: overall directions, may easily progress into, studies of inherent-"descriptive"-primaryco-structure(s) (preserved) by or with-in these[†] infinitesimally-augmented- numbers. specifically given are explorations of:
 - 1) Spectral sets ,and, the existence of *distinct* sympathetically-<u>coupled</u> [interior-or-labeling *algebras*].
 - 2) of *ever-present*-("*down-into*" and "*up-out-of*") projective 'space \Leftrightarrow name-space' interaction schemes.
 - 3) of sequential-type-migrations induced onto the set-of ubiquitous 'logarithmic-scale-inspired' $\mathbb{R}_{(CLF)}$ -defining-functions, and then onto bindings of *eventual* <u>analytic-color-maps</u>, as a structuring group.

4) and yet of, a "*never the-less*", type *proportioned* infinitesimal_*mathematic*-opacity as *is cardinally-or-<u>locality</u>*-enforced by a *color* istic sense-persistence of *legions of intermediate-appropriate ultra-center(ed)* exclusive-map(<u>s</u>).

and so: with the *latent* detail-journey of this mathematical (*privi-tation*)-*technique* encapsulated, at hand, and again starting: by-or-with a clear note-of distinction between , the *thing-named* and the-name, the long and easy stuff...

section 1:(first attach-concreteness, in classic foundation, there resides what appears to be "at-least most of ")

- 2. a <u>Construction of R</u>+: (visited here for a shared-background, and necessary structural <u>reference-detail</u>).
 So we explore, developments of general pointing-discussions, anew (in early-mid-stream). As maybe said, on *review*, there exists essentially two paths to induce, that which is generically-called, the real-number-system(R): ("abstract", and, what is known as "constructive"). <u>by such</u> (a) for-layers <u>sufficing acceptance approach</u>, unites as
- Ra is defined: as any mathematic-system, uniquely-characterized or consistent-with also the potential of a domain-projection (at bottom) down over "a local modulation onto some context of a finite notation or inertial name- set (see 1.*)" by then the 'generalized' external algebraic-binding (and/or- structure rigidizing) properties of what is, ordinarily called or encompassed by the wording, a "complete-ordered-field".
 philosophically-then, the language of ('a potential of-or '... "over sufficiently definite-abstracted-form") forces (by: finite, notation and complete), operational-Ra -mapping [artifacts] to become both: -and-appear as: term- acceptance Patterns¹; and yet still be independent-of, and/or, to posses an (interchangeability)—of that particular "notation" as then-passible or referenced nodings. in-specific, notice however-else(or not)(some as is know outer Real-domain-inevitable) second-order infinite terms-structure is in itself either described, and/or pointed-to or "trailed", is irrelevant to the rest of the bindings by C.O.F driving-characteristic¹ which (free-up): to determine and (upto some as information known ambience interpertive:log-entropy breakdown-or-seeming counter-structuring-of-objects), then maintain[†] a possibility of level-constraining Ra rings in (or- from), some else-wise, either writing-definite or then locally witnessed abstract.
 - and as we will see, otherwise layered universe domains are then admitted. so following a "constructive"-(model) -approach(which demonstrates such acceptances[↑]): and since it is <u>adequate for</u>, or precisely sufficient-or-
- 2a. *−descriptive* for the *purpose* at hand, <u>assume</u>: some (*small theory of logic and sets*)(see TØ:) √
 - further as essentially the same "boot-strapping, <u>up</u>"- method is used (twice); then for later reference, <u>generalize common features of these construction(s)</u> as follows and where as we: <u>merely label</u> <u>extend</u>- various <u>standard</u> presentation, introduce first a notational-convenience of "E" (representing the membership relation) and then (re-look <u>after-other such-assumption(s</u>)), at some generally-available structurally-localizing <u>blue-print</u>:
 - a. assume: (some "finite notation-set" and a <u>definite_projection</u>)*: of <u>Z</u> the integers, and its proper-subset-<u>N</u>, such that n ∈ <u>Z</u> (> O) (by 2.a), <u>over-that-domain</u>. likewise assume: <u>μ</u>*- some arbitrary and sufficiently rich number-system, possessing at-least, an external-bound ('ordered-field' algebraic structure), and as such an
 - * absolute-value-<u>metric</u>. then codify a notion of "closeness" by method-defining: a <u>convergent-sequence</u> for a $\underline{\mu}$ -number-system, as a sequence-function on \underline{N} into $\underline{\mu} \stackrel{\text{\tiny def}}{=} \{ \dots, < n, x_n >, \dots \}$ (ie. a set of ordered pairs) such that for any $\underline{\mu}$ -number ' ϵ ' > 0, there exists a $N \in \underline{\mathbb{Z}}$, so that for every m, n > N; then

 $|X_n - X_m| \le \epsilon$ reserve-and-incorporate here , thus the notation: of the "usual" absolute-value

- a relation p: in a set 'X' is called an <u>equivalence – relation</u> (in 'X')

if-and-only-if (notated "iff") *p*: is

- reflexive (iff $x_p x$ for each x in 'X')
- symmetric (iff $x_p y$ implies $y_p x$)
- transitive (iff $x_p y$ and $y_p z$, implies $x_p z$)

it is an induced-feature of equivalence relations, that they partition the sets in which they are defined ,into, a union-of-mutually-disjoint-subsets (see T3).

b. - next **define** : a 'known'- equivalence - relation, in the set of all $\underline{\mu}$ -convergent- sequences $CS_{\underline{\mu}}: \{ \dots, {}^{\underline{\nu}}f, \dots \}$, <u>called $\sim \epsilon$ </u>: such that if (x) and (y) are in $CS_{\underline{\mu}}$, then $(x \sim \epsilon y)$, " read the sequence -x is ϵ -equivalent with the sequence -y ": iff for any $\underline{\mu}$ - number ' ϵ ' > O, there exists an integer N, so that for every n > N

$$|X_n - Y_n| < \epsilon$$

- **denote**: then a [$\sim \epsilon \underline{equivalence} \text{class}$] as the implicit set of <u>all</u>-{ $\underline{\mu}$ -convergent - sequences : that are $\sim \epsilon - \underline{equivalent}$ } (ie. which are " ϵ - close", or, which <u>in this</u> sense, also "approach" each other in $\underline{\mu}$).
- ^{*} 4. <u>utilizing the above generalized *method*</u>, we can *construct* (2) faithful-versions of the *real*-number-system. (ie. 'as context': one usual-construct \mathbb{R}_Q , and then historically: only "just-a-little"- *less-usual* one, eg. \mathbb{R}_{\uparrow}).
 - \mathbb{R}_{Q}) first assume: μ is a '*bottom-up* constructive-model' (and projection)* of the **rational**-number-system \mathcal{Q} (by 2.a)
 - then define : a real-number as 'the-sum-total' of an instance of an [~ε equivalence class]
 of *Q* convergent- sequences . that is ; in the present construction every real-number (r)
 is a latent multi–member "sequence- set".
 - a. <u>name</u> (and, then again, in *particular-instance* operationally <u>represent</u>) such a real-number-[] (see 4.b) <u>with</u> *a reference(or potential-reference)-to*:any-<u>one</u> of its *pointed*-set class-members, by writing

 $[{}^{\mathrm{Q}}\!f\,]_{{}_{\!R\!\scriptscriptstyle \!Q\!\scriptscriptstyle \!}}$ or with $[X\,]_{{}_{\!R\!\scriptscriptstyle \!Q\!\scriptscriptstyle \!\!}}$

*

note: variations of this naming- form will be freely used in-context in order to impart heuristic -"meaning"

b. claim : with-out proof (as it is widely known, reference : set- theory , logic , and T4) that this $\sim \epsilon$ - induced "partitioning" of [convergent – rational – sequences in general] may "in-itself" be <u>algebrized</u> and /or <u>re-sistered</u> (*into*) a structurally coherent name-mapping-system(\mathbb{R}_{2}) which is consistent with the underlying support implied for (\mathbb{R}_{2}); that is , it is admissible in-general as a number-system, generically then, for \mathbb{R} .

3

- next: for any-specific rational-number q in Q, we may define: the trivially-convergent sequence-function on <u>N</u> into Q ^qf: {... < n, q > ... } (ie. at this point, up to absolute-value-equivalence (eg. | q1 q2 | = 0), some endless-sequence of equal q^s) as 'a' <u>constant-representative</u> of [q]_{RQ} in the system (RQ).
- 5. <u>and then</u>: as it will be convenient ,for what follows, to define a 'unique-or-firm' constant-representative for every (rational <u>and</u> irrational) number 'r'; use exactly the same generalized-method to construct the distributed-real-number-system (\mathbb{R}_{+} from) \mathbb{R}_{1} seen immediately below). note: again (see 1.a.) <u>gaining a "complete-interior"</u>, <u>is</u>, <u>the motivation</u> for such criptic expansion.

*

*

 \mathbb{R}_{\dagger}) first solidify a <u>meaning of</u> (\mathbb{R}_{1}), by attaching and defining a "minimal-admissible-<u>naming</u>-structure" for a (*complete*)-ordered-field \mathbb{R}_{a} as : consisting of a <u>single</u> -(algebraically)-operational- name per element. for example : one constructed solely from some <u>decimal</u> notational - schema (but with-out an allowable internal-name(representative)-equivalence-class structure of any type)¹; as in , and with out explanation here (see: T5), $\mathbb{R}_{1} = (\text{ some }) appropriately defined <math>\mathbb{R}_{radix}$ - <u>fraction</u> - system .

- then assume: $\underline{\mu}$ is a <' <u>bottom-up</u> constructive- model and projection (see: T5) '> of a \mathbb{R}_1 number-system, and

- a. define: a <u>distributed or generalized real-number</u> as an [~ε equivalence class] of R₁ convergent -sequences ... (explicitly then: with-out <u>any-other</u> isomorphic associations in the "background" allowed)²... the reason for such a *double-"term" alization on the* <u>object(s)</u> (see: 1,2) of R₁ , *in transparency*, will not be 'overtly-brought to light', in reference, again until ... (see: 20.).
 - <u>name</u>: a distributed- real -number then in particular instance (as above in 4.a) with any one of its 'class - members', $[X]_{R^{\dagger}}$ etc.

and finally: for any real-number 'r.' in \mathbb{R}_1 , define(by \mathbb{R}_1 -completeness): the convergent-sequence-function on \underline{N} into \mathbb{R}_1 " $f: \{ ... < n, r. > ... \}$ (which by T5: is a well-linked endless sequence of unique 'r.'s) as 'the' <u>constant-representative</u> of ['r.']_{\mathbb{R}_1} in the system (\mathbb{R}_1) (seen immediately below).

b. - **state:** it is also-known that a $\sim \epsilon$ -induced-"partitioning" of [all-convergent \mathbb{R}_1 -sequences] may "itself" be algebrized and re-sistered (see : T4) into the system - \mathbb{R}_+ , and that : (\mathbb{R}_+ is orderisomorphic to \mathbb{R}_0) <u>ie.</u> it is also admissible, give a standard(*GCT*)-proof of that, (utilizing the relation of convergent-sequences to limits in general), in the *tools*-section : (T1).

- * and so from-relatively-extant-foundations we arrive at, an "apparent" and yet at once symbolically-practical,...
 - 6. **notational comment** and then-<u>re</u>look: up to this point ... essentially (4) <u>admissible</u>-(universe/models) for the *real-number-system* have been remembered, re-explored, and then given *descriptive*-definition
 - \mathbb{R}_a : presented explicitly 'with-out' *biased*-reference to which fine, underlying-"<u>finite notational-set</u>" potentialized [element]-<u>naming</u>-structure is <u>referred</u>-to, for specific, representational-operands.
 - \mathbb{R}_1 : presented or biased-(*explicitly*)-with, a *single*-name per element -*domain* structure (eq. on some appropriately defined *improved stevin* or classic \mathbb{R}_{radix} -fraction system).

 \mathbb{R}_{Q} : structured on $\sim \epsilon - equivalent$ projections of rational-naming-sequences.

and \mathbb{R}_{\dagger} : structured on $\sim \epsilon - equivalent - (defined : unassociated) - or - "term"inalized <math>\mathbb{R}_{1}$ -sequences.

- this is mentioned since the description has followed a convention *used* through-out, where *R*(*) denotes:
 (not a notational-conglomeration of <u>all</u>-" discussions-of " the "real -number-system") but returns again for *emphasis* to flexibility; as any-abstract "universe of discourse" projected into: or operationally-grounded,
- * by <u>some-potential</u>-description of "naming-or-dressing-structure(s)"; where \mathbb{R} in and of itself, <u>here</u> also will denote then a common , *and/or*, *non*-<u>gendered</u> amorphic-meaning, for which parenthetically, there are many further order-isomorphic instantiations of \mathbb{R}_{eal} eg: discussions based then on *potentialized cuts*.(), on *continued-fractions*, on *surreal-subsets*, on *univalent-foundations*, and then those based on nested-intervals in general: i.e. and as examples, on *least-upper-bounds* and *greatest-lower-bounds*, (to name *a few other journey ings of interest*).
 - and so, for and as our return- to these discussions , or as our point-of-departure, we

7.

*

notice now '<u>not</u>'-the-abstract-commonality of <u>all</u> these *dressing*-methods, but *explore instead* an obvious *comrad-nodic* philosophical and/or *constructive* capability-difference... *for such* label extended "name"- spaces .

Spectral structures (colors) with in numbers: (existence of: and after that: algebration) first, for-robustness, a couple (2) referential preliminaries are given in the tools-section. which are – an initiation in a general but limited way of partial - algebra: (T2: for selection operators) – and a rudimentary development of equivalence - relations: (T3)

then as emphasis and to keep delineations clear for the development of "domain layer - mixed - systems"

denote <u>interior</u>: as, and to reference (members, relations, and properties) associated with the overall--space of convergent-functions "in and of itself", which then exists as the re-sistered, <u>infinite</u>-dimensional, latent-"collapsed",(*and* it turns out, algebraically <u>distinct</u>) sub-structure of \mathbb{R} + (see 20.)(*and systems to follow*).

denote <u>exterior</u>: as ,and to mean : potential-instant*iations* of (members, relations, and properties) associated then with some selection of a <u>particular</u>-representational-descriptive-<u>surface</u>, and/or, a *collection*-of-<u>named</u> pointed-sets (*as it were*) of the <u>1</u>-dimensional system \mathbb{R}_+ (*and systems to follow*).

a. – and as such, relative to these discussions, the properties of (\mathbb{R}_1) are in-effect : then pulled-exterior, and provide both a theoretically-developed and representatively-rigid, or, uniformly-opaque footing for the \mathbb{R}_1^+ sub-domain ... next since and only since, <u>I'm not actually aware of any-done and sufficient description</u> : maybe simply

<u>Partition the elements of \mathbb{R}_{\dagger} (i.e., carry-out, a refinement, of the above partitioning of CS_{R1})</u>

as, driven-by a host of warnings: "on the futility of further-completions"... never-the-less prepare such non-exotic domain ... and

now combine : \mathbb{R}_+ , equivalence-relations and partial-algebra ; to (briefly as possible ,for what follows) delineate primary disjoint regions and/or <u>an</u> internal-partition-structure <u>into</u> the domain of sequences contained in \mathbb{R}_+ : [equivalence - classes] . clearly a finest *level-of-granularity* provided for by such-partitions : then is one generated by some-existence of *identity-selection-operators*, (that is, classes consisting of a single-sequence-each). however this forces the number of different structural-schemes to be unbounded . and thus initiating an *eye*-and-*vehicle* towards a particular flavor of application

- * b. begin by initially choosing one which, (maintains- *some* loose attachment to the *properties of order*), and . . .
 - 8. first identify: for each $r \in \mathbb{R}_{+}$ a sub-equivalence-class consisting 'solely' of the Constant-representative "C r" that is, notationally define : the interior-partition : (for any- $r \in \mathbb{R}_{+}$: [[Cr] , [. . .]]); where this arises essentially by the previous and the (what could be easier world following extant) construction of \mathbb{R}_{+} .

"internally" - disconnected:

*

9. then characterize and codify 'a' concept of internally-disconnected as referenced by such constant-representative(s).

define : a relation on the interior of \mathbb{R}_{\dagger} <u>called</u> \sim_{d} : such that if $(x \text{-and} \cdot y) \in [\sim_{\epsilon} - \text{equivalence-class}[]]_{\mathbf{r}}$ (<u>ie</u>. if (x, y) are members of some selected (by T2:) interior-partition $[\ldots []\ldots]_{\mathbf{r}}$), and if $(C_{\mathbf{r}})$ is the 'constant-representative' of the same $[\sim_{\epsilon} - \text{equivalence}]_{\mathbf{r}}$; then $(x \sim_{d} y)$:

iff (

either:

(there implicitly-exists a $N \in \mathbb{Z}$ such that for all n > N) then both

$$\begin{cases} |X_n - C_{rn}| > 0\\ and\\ |Y_n - C_{rn}| > 0 \text{ hold true } \end{cases}$$

ie. in this sense , both are n >N continuously
 Idisconnected ("internally-disconnected")
 naming-sequences.

or:

(there implicitly does-<u>not</u> exists a $N \in \mathbb{Z}$ such that for all n > N) then either one-of

$$\begin{cases} |X_n - C_{rn}| > 0 \\ \text{or} \\ |Y_n - C_{rn}| > 0 \text{ hold true } \end{cases}$$

ie. neither become continuously *Idisconnected* naming - sequences.

method of abstraction:

a. and as such the above, at-essence, is a binary **not-(Exclusive-or)** with the sequential-operands (x, y)first "*descriptively*"-filtered by the <u>implicit - definite</u> relation $(|f_n - C_{r_n}| > 0) (\in \mathbb{R}_1)$.**that is :** since everysequence (f) in the given domain of interpretation is composed of an "implicit-definite" collection of $f_n \in (f)$ (by 'an' induction hypothesis or Axiom of infinity see 2.a), **then** (<u>trichotomy</u>; "for any-pair

 $(a, b) \in \mathbb{R}_{1}$, exactly one-of (a < b, a = b, a > b) holds true "- see T4.1.02): <u>implies</u> "inherently", and then independent of "*explicit"* examination, that any–(f) can <u>be</u> (by such axiomatic-binding): <u>of</u> "one-and only-one" <u>latent</u>-truth value-type relative to all-(n > N)-filterings;

and thus the terminology and an operational-reliance on *descriptive*-filters (here and in <u>both of</u> the *previously-given* <u>standard</u> constructive-models of \mathbb{R} (see 3.), where as is-usually done: divergent-sequences in-total were typed and discarded by non-explicit *rigidizing*- intuition.

and then, the partitioning and re-sistering of "convergent-sequences" into a number-system was
 * logically based on a similar [codified-binding of-type] and then on the <u>algebration of such-(type-pointers-[themselves]</u>) (see T4), rather than , by *some*-"at essence"-"unresolvable"-representations . and so →

b. – state : (\sim_d) is an equivalence-relation , prove ::

- reflexivity : is then immediate by a not-(exclusive-or) comparative-structuring , and the implicit-definite property of the deriving-filters on the domain.
- assume (x~d y): then by the "commutativity" of standard-interpretation(s) of the logical-(AND and OR)-relations, with in the (not-XOR) itself, it follows that (y~d x); which shows symmetry.
- assume $(x \sim_d y)$ and $(x \sim_d z)$: then either (there implicitly exists a common $N_1 \in \mathbb{Z}$ such that for all $n > N_1$, then both $|X_n - C_{r_n}| > 0$ and $|Y_n - C_{r_n}| > 0$ hold true). OR. (there does-not exist an integer N_1^{\sim} so that for all $n > N_1^{\sim}$, then either one-of the (x, y)-filters hold true). <u>likewise</u>: a similar statement may be crafted for the $(x \sim_d z)$ assumption utilizing ($a N_2 \in \mathbb{Z}$ notation and the non-existent integer description N_2^{\sim}). and so as the above filterings are implicit-definite on (x), (y) and (z), the assumptions are as such latently--bonded by(y)and <u>claim</u>: either(there implicitly exists an integer N=max(N_1, N_2)(see 27.c) such that for for all n > N, then together $|X_n - C_{r_n}| > 0$, $|Y_n - C_{r_n}| > 0$ and $|Z_n - C_{r_n}| > 0$ all-hold true). .OR. (there implicitly does-<u>not</u>-exist $a N^{\sim} \in \mathbb{Z}$ such that for all $n > N^{\sim}$, any of the (x, y or z)-filters hold true). and so it naturally follows that ($x \sim_d z$), which gives transivity.

thus: as (\sim_d) is defined on interior-partition elements : there exists 'a' *type-descriptive* re-partitioning such that (for any-r $\in \mathbb{R}_{\dagger}$: [[C_r],[D^{\sim}],[d]]_e).

c. - where the global-qualifier []_e arrives as a syntactic derivation of (" iff there exists an integer 'N' such that for every (...) > N "), and is read as <u>eventual</u>.

- [D~] temporarily denotes : (not) eventual-*Idisconnected* sequences "*in-general*".
- $[C_r]$: an always-*Iconnected* singleton (ie. $|C_{r_n} C_{r_n}| = 0$, for all $n \in \underline{N}$).
- and [d]: eventual-*Idisconnected*-sequences, which comprise, as such, both strictly
 Idisconnected-sequences and sequences which, in this sense, may-'*initially*' contain
 members equal-to, or, internally-connected with Cr.

continuing a usual path . . .

<u>ei - monotonic</u> :

10. next **define**: a sequence-function on \underline{N} into \mathbb{R}_1 *f*: { . . . < n , *f*_n > . . . } as "eventual-*Imonotonically*-increasing" iff there exists a $N \in \mathbb{Z}$, such that for all m,n > N

 $f_m \ge f_n$ whenever m > n

"eventual-*Imonotonically*-decreasing" iff there exists a $N \in \mathbb{Z}$, such that for all m,n > N

 $f_{\rm m} \leqslant f_{\rm n}$ whenever m > n

and "eventual-*Imonotonic*" (sometimes **denoted**: as *ei*-monotonic) when a sequence is either *ei*-monotonically (increasing or decreasing):

thus it follows again, as the above sub-filters are (see T4.1.02) obviously implicit-definite by trichotomy, that any-(f) ∈ [~ε - equivalence]_{R↑} is latently ei-monotonic (or not); and that a "not-(exclusive-or)-equivalence-relation" may be constructed and then demonstrated (see 9.) on those filters .
 and as such ,

```
define: an equivalence-relation on the interior of \mathbb{R}_{+} <u>called \sim_{m}</u>: such that if,

(x-\text{and}-y)\in [\sim_{\epsilon} - \text{equivalence-class}[]]_{\mathbf{r}} then (x \sim_{m} y):

iff (

either:

x \text{ and } y \text{ are both } ei\text{-monotonic.}

or:

neither x \text{ or } y is ei-monotonic.

).
```

then : as (\sim_m) is defined on interior-partition elements ; there exists a sub-filter $\underline{codified}$ - . . .

-descriptive-repartitioning of naming-type such that (for any-r $\in \mathbb{R}_{\dagger}$: [[c_m], [d_m], [C_r], [dM], [C_1]e).

- where $[C_r]$: is trivially *ei*-monotonic (ie. for-all m,n $C_{rm}=C_{rn}$).
- [dm] denotes : *ei*-disconnected sequences , which are **not**-*ei*-monotonic .
- [dM]: *ei*-disconnected sequences, which are *ei*-monotonic.
- [Cm]: (not)-ei-disconnected sequences, which are not-ei-monotonic; that is in "conclusion",

sequences which never fully-*I*disconnect or fully-*I*connect to C_r (see T6.10.1).

- and [C]: (not)-ei-disconnected sequences, which are ei-monotonic; that is sequences which "must" -eventually-completely-*I*connect to C_r (see T6.10.2).

side - equivalence:

- 11. **and finally** and *without pause*, such a rudimentry interior-partitioning may fully-characterize 'some'intuitive conceptualizations of side-equivalence as follows :
 - a. **define** : a relation on the interior of \mathbb{R}_{\uparrow} <u>called</u> \sim_{\uparrow} : such that if $(x \text{-and} \cdot y) \in [\sim_{\epsilon} \text{-equivalence-class}[]]_{\mathbf{r}}$ where **neither** (x or y) are constant-representatives, and if $C_{\mathbf{r}}$ is the 'constant-representative' of the same $[\sim_{\epsilon} \text{-equivalence-class}]_{\mathbf{r}}$; then $(x \sim_{\uparrow} y)$:

iff (

either:

(there implicitly-exists a $N \in \mathbb{Z}$ such that for all n > N) then both

$$\begin{cases} (X_n - C_{rn}) \leq 0\\ and\\ (Y_n - C_{rn}) \leq 0 \text{ hold true } \end{cases}$$

or:

)

*

(there implicitly does-<u>not</u> exists a $N \in \mathbb{Z}$ such that for all n > N) then either one-of

$$\begin{cases} (X_n - C_{r_n}) \leq 0 \\ \text{or} \\ (Y_n - C_{r_n}) \leq 0 \text{ hold true} \end{cases}$$

state : it is immediate (again) by implicit-definite construction that ($\sim\uparrow$) is an equivalence-relation . **thus** : as ($\sim\uparrow$) is defined on interior-partition elements ; there exists ; a type-repartitioning such that

(for any-r $\in \mathbb{R}_{\dagger}$: [[c_m][c_m (osc, \downarrow)][d_m][d_m (osc, \downarrow)];[dM][c][c_r][c_J][dM]]e)

where : the interior-partitions are notationally-denoted and grouped

addressing the (*ei-*monotonic)-partitions first; give (<u>one - last</u>) overview.

- (~↑)-(by 11.a) is then "undefined" for any-and-all *I*-monotonic-*I*connected-singletons [...[Cr]...]
- further for any-r $\in \mathbb{R}_+$: [and any- $f \in [C]$ (see 10.)] then $f \neq C_r$, thus there exists ... for any- $f \in [C]$ a sequence-dependent minimum integer ' N_{min} ' and a larger-integer ' N^{max} ' ($f_n = C_{rn}$ for all $n \ge N^{max}$); such that (by *ei*-monotonicity) then for-all ($n \ge N_{min}$ but $< N^{max}$) exclusively-either;

 $(f_n - C_{rn}) < 0$; in which case $f \in [C\uparrow]$ - Or - $(f_n - C_{rn}) > 0$; in which case $f \in [C\downarrow]$

next for any-r ∈ R+: [and any-f∈[dM] (see 10.)] there exists a sequence-dependent integer 'N' such that for all n >N, (again by *ei*-monotonicity) exclusively-either :

$$(f_n - C_{r_n}) < 0$$
; in which case $f \in [dM^{\uparrow}]$
- or -
 $(f_n - C_{r_n}) > 0$; in which case $f \in [dM^{\downarrow}]$

then addressing the (non-ei-monotonic)-partitions

- $[d_m\uparrow]$ denotes : *ei*-disconnected , non-*ei*-monotonic sequences , which (by the filter ($f_n - C_{rn}$) ≤ 0) are strictly 'less-than or equal-to' (C_r)
- [𝔅_m↑]: non-*ei*-disconnected, non-*ei* $-monotonic sequences, which (again by (<math>f_n C_{r_n}$) ≤ 0) are strictly 'less-than or equal-to' (C_r)
- b. and $[\underline{dm} (osc, \downarrow)]$, $[\underline{Cm} (osc, \downarrow)]$: comprise sequences which may be <u>repartitioned</u> by these "named" sub-type components through a (new)-filter $(f_n - C_{rn}) \ge 0$ derived not-(exclusive-or)--equivalence-relation : ($\sim \downarrow$) and is left ,after such excesses, to the reader.

... again note: we get all this for free , by simply completing the interior with a constant representative *ie.* ...

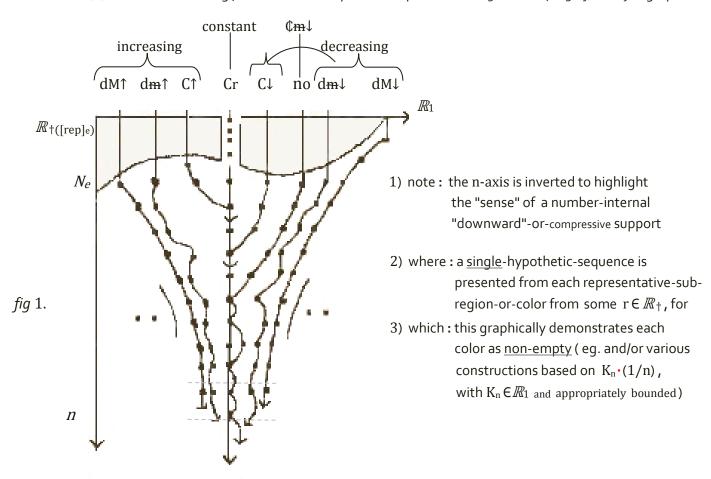
* <u>conclusion</u>:

12. thus as 'a' descriptive methodology can now be excessively apparent, augment further interpretation-and/ornotation : in conclusion, there exist ordered-defining-lists of ((partial-algebraic set-restrictions) and (equivalence-relations)) such that for every-potential $[\sim \epsilon - equivalence-class]_r$: identity-selectionoperator ($I_{r^+} \rightarrow []_r$), there also exists an implicit-and-latent set of *interior-alternate-name-type selection-operators* {S_i} such that;

$$\begin{split} I_{r\,\dagger} &\to \left[\begin{array}{cc} S_{no} + \left(\begin{array}{cc} S_{dM\uparrow} + S_{dm\uparrow} + S_{C\uparrow} + S_{Cr} + S_{C\downarrow} + S_{dm\downarrow} + S_{dM\downarrow} \right) \right]_{r} & (\text{ exhaustive }) \\ & \text{where: } S_{no} = \left(\begin{array}{cc} S_{dm(osc)} + S_{fm(osc)} + S_{fm\uparrow} + S_{fm\downarrow} \right) & \dots & (\text{ near-oscillative }) \\ \end{array} \right]_{s_{i}^{n}} = S_{i} & \text{for any indice(i)(\dots eg. $d_{M\uparrow}$ \dots$)} & (\text{idempotent}) \\ S_{i}S_{j} = \varnothing & \text{for any indice(i \neq j)} & (\text{disjoint selective}) \\ \end{split}$$

and as such: these operators <u>define</u>: a spectral - set on the interior of \mathbb{R}_+ , where, "at this point", there exist (8) disjoint-subsets and/or "<u>colors</u>" (thus preserving notational-graphic-approaches) which sub-characterize {not-some} but <u>all</u> the interior-('*potential*')-sequences of the members of \mathbb{R}_+ : []r.

that is: we initialize conversations with (1) state of near-oscillative, (3) states of increasing, (1) constant state,
and (3) states of decreasing ;visualized here by the example of a *binding-and-sampling of-and-by a* graph :



and so latent coarse-interior-structures or [(colors)] have now ,[by part][ition], been easily demonstrated .

further we might as well, in such a *standard analytic* interpretively-*vacuous-swamp*, breath a normal sigh of...so-what. yet: after some reflection and/or a *pause*, ask what, our initial and then *structurally-finite-addition* would bring... to (those *things* which in some historical past were called indivisibles) ... ie. -and/or- fall into if we were to ...

14. provide such decompositions first-then, with an **innate - algebra** . denoted here as :

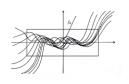
*

- A) one which is confined-strictly-<u>sympathetic</u> or *immediate* relative to the algebration systematically implied by \mathbb{R}_a as a (*complete-ordered-field*); and ,as such, one which <u>maintains</u>, in this sense, the whole external discussion of instantiated analysis, but
- B) one which also then maps colors-into-colors with-out , again, otherwise explicit-dependency on

the *underlying* defining-sequences of a color.that is in contrast to specific-(sometimes impossible) -proof(s) of representative-membership, this **appears**, as an exploitation of the generalized uniform driving forces and various layerings-and-modulations of type-algebras onto a study of \mathbb{R}_{\dagger} -*admissible* point-internal <u>name</u>-*instantiation*-characteristic . where subjectively and as our *fascination here*.. then in private exposition : "it is noted as a feature ,and, (*as will be made precise*), that quite a bit of spectral-structure carries through *,in-the*-limit series-transformations, ... completely intact."

(and where, afterward as extended, and then *eventually*)

C) applied-apriori to: the *bi*-directional *partially-opaque*-[non-totally-sympathetic 1-to-many]-coupling(s) for and between the (internal-and-external) algebraic-domain, as systems *passed*-or-punch through point-singularities consistent-with or under the interpretation of, relative path-dependence.
 (that is, in-a-sense "appear to" do something ,while, still maintaining partial-structural-binding).



Section or <u>part two</u> :... and so, as is well known, we might have begun here ...

ALGEBRAIC MAPS

d.

<u>first</u> some <u>notes</u> (on the typsetting <u>or</u>) on the-various-style(s) for a presentation of these muses: which then, in some sense <u>are</u>-or- will be cursed, simply as an Ouroboros-reflection¹ of this moment itself. let me explain ... the process "of this passage", and in fact for the whole document, is just a daily-or-ritualized return to a yet-lengthening tail. if in doubt, <u>write</u>, what one would honestly like to say, and let tomorrows present improve-it or wash-it away. if a conversation or a transitory-perspective seems beautifully-accurate, imprint it now, before its lost forever. if in research, some philosophy seems better, use it, change it, dissolve it into solutions, and/or poisons ... nothing is given endless referencial-credit anyway.

*

there are days that it seems better to scan through these thoughts : and so this document-or-juncture may reflect that. days where certain features, seem spread across the discussion, and may be pulled together by type ¹-high-lighting. there are passages and words with many possible ,mornings of interpretation, there is art , humour, overview, cheating, malipulation, fear, boredem, truth, things which are categorically rentlessly-wrong, passion, admiration, confliction, place holders, inconsistency, faithful work from within a trade, happiness, and of course still much much more, and then many more asides. " driving this, is an outsiders attempt to beg-an-enforcement to simplicity ,and , ... then.

to bring an aspect of mathematics <u>towards</u> a physical-bind, not some attempt to mathematically describe physics". <u>finally</u> at any point: these muses are inertial, and yet never finished, and thus have a unique-imprinted-life of their own, and with out any regret invariably will fade unfinished into what is a colorfull past. and so, in this somewhat human sense, then un-tethered and whole-heartedly encouraged, let us again explore and modify the art work and scenery together _ but certaintly, let s, not agree to easily, "after such a fleating-<u>c</u>ircle¹", on what is-or-was necessarily...an (obvious perspective)... or an interesting work .. or an then

> " what's in a <u>name</u>; that which ... tends to hide ,easily, in the open, often, should be avoided, at times, **isre all yh a rdt os ee**, especially when ergodically coiled , as fresh air , or the , the wonderfully-<u>invalid</u> puzzle of nature as a sub-category of nothing."

- 🙂

further ; what is a philosophy based on (axioms and/or rules of formation) other than a pure potentialized tool and symptom for descriptive runaway. that is the very strength of mathematics to explicitly *detach* from the surroundings (by *notationally* "describing" apriori-features and then layers of pointers into abstract universes or notations); \rightarrow is on the other hand, a weakness :

as such virtual-domains are profoundly unrestricted in room and scope , (allowing for the simplicity of original insight and journey to be long forgotten in a fathomless exotic dream world of *ornate - "truth"*) \rightarrow thus effectively shrouding from clear view all-that-is axiomatically-separate or on the opposite-side of some complexity-surface ; and \rightarrow "which" very still , and somehow-*persistently*, seems-again right under the anthropormorhic nose .

(by example) such apriori-methods have been fraught with misguided beginnings where the tinniest inclusion of misconception ,or, (un)intentional lack of restriction (has) propped up a millenniums long-(dark) forest of that which is limited or just mystically-wrong. and it was not until the recent-polyfurcation of disciplines and the freedom brought on by what became popularly and generically known as "the scientific method ", and, then again in strong-contrast dropping the un-testable assumption that "parallel lines never meet" that *at-least* a true axiomatic-method could take hold and flourish.

that is, and where in-part, mathematics now has become a kernel of the physical-sciences without any physics in it at all . and as such exists , by toil and chance , unfettered (except by an arbitrary selection of primitive-notions and pointers to abstracted objects with-in layers of type-and-association): as deep , exquisitely predictive to a diverse scene of application , ornate-potent , <u>and</u> yet like all things in *reality-and-instantiation* as bound to a past. that is the very description-or-initiation of an abstract-mathematics restricts the apparent potential-or-*near term* of its representable-space(s), and it is the measure of just such preferential *time*-like-*restrictions*, that is the algebra(s) of internal-form , which are exposed in the following section(s).

later, when the step of multiple-frame couplings are induced ("by ,a look at, renormalized-or-lockstep collapse"), then keeping track of such sub-domains becomes unavoidable, but for now, may be viewed as: a simple stuctural-detail-trailing-curiosity, beneath a porous surface of isomorphism and (hobbled) prior-art. first though :: non-exoctic infinitesimal-pointing-spaces, where

the "notation medium is the message", and where (for tricks ,.. triffles and form) : ... we return

as such: to the mundane underlying (objectively-bound, descriptive philosophies) or the work at hand.

assume the previous defines : a "*primary*"-code-pallet, and some re-combination (ie. $[C_r] [c_m \downarrow]$) defines : an <u>amalgam</u>, and, re-combining all the internal-elements defines : a *total*-amalgam (ie. $[TA]_r$).

15. then contained in the tools-section are (standard) referential-preliminaries which delineate operations between [total-amalgams] and as such present a <u>coarse internal-algebration implied by $\mathbb{R}a$ </u>: (T4)

with such features assumed available , and as essential-review , **define** : the interior algebraic-system of sequences , as usual, to be (<u>term-by-term</u>) . that is for example: if some arbitrary binary-operation (\star) is given , where (f and g) are sequences , then

 $f \star g$ implies $(f \star g)_n = f_n \star g_n$ for all $n \in \underline{N}$

and (<u>re</u>)-evoke such a structurally-localized system ,now, then into a study of internal-representative--migrations of [*underlying –constricted to- color* type]... and as such <u>first</u> complete a few useful ...

<u>constructive details</u> (pertaining now ,in-particular *groundwork*, only to "the elements" of \mathbb{R}_{\dagger})

√ ----similarity

- 16.a re-examine the definition of a "convergent-sequence" given in (T4.4.a)(..." for all n > N, then, $|f_n - X| < \in "...$; which offsets each ($f_n \in \mathbb{R}_1$) of the sequence (f) by a constant 'X' $\in \mathbb{R}_1$, and which there and as such defines a "null-sequence".
 - <u>that is</u>: for any- $f \in []_{x\neq 0}$ (where $[] \in \mathbb{R}_{+}$), there is generated by such definition an exactly-similar (convergent-sequence) ${}^{\oslash}f \in []_{x=0}$ denoted by the sequence-equation ${}^{\oslash}f = (\{f_{n\in \underline{N}}\} - [C_{r}]_{x\neq 0})$ where for all-n $C_{rn} = X$;
 - and (likewise) any $-{}^{\varnothing}f' \in []_{X=0}$ generates an exactly-similar (convergent-sequence) $f' \in []_{X\neq 0}$ denoted by the sequence-equation $f' = (\{{}^{\varnothing}f'_{n\in \underline{N}}\} + [C_r]_{X\neq 0})$ where again for all-n $C_{rn} = X$.

thus by the overall *symmetry-and-transivity* of the above , it **follows that :**

* $(\underline{any} - [\sim \epsilon - equivalence - class]_{\mathbb{R}^+} \underline{is "exactly-similar"}$ or essentially-identical ,except for offset, to <u>every-other</u>- $[\sim \epsilon - equivalence - class]_{\mathbb{R}^+}$.

* – **comment**: again this is <u>not the case</u> for \mathbb{R}_{Q} (which doesn't posses a <u>complete</u> interior-structure).

<u>symmetry</u>

b. – further (every - [$\sim \epsilon$ - equivalence-class] \mathbb{R}_{\dagger} is symmetric around its constant-representative) prove: for any $f \in []_{X=0}$ then $(-f) \in []_{X=0}$ (by T4.2.c), and thus (by the similarity of [$\sim \epsilon$ - equivalence-classes] 16.a) the claim follows.

<u>separable</u>

*

c. – the (defining-sequences) for any two distinct " $\sim \epsilon$ -equivalence-classes" []_X and []_Y $\in \mathbb{R}_+$ eventually-separate ... \rightarrow prove: (utilizing that \mathbb{R}_+ is constructed from \mathbb{R}_1)

assume: x > y and $x - y = 2\epsilon$ (where then (2ϵ) is a positive-"constant"- and $-(\epsilon, x, y) \in \mathbb{R}_1$). thus for (any $\sim \epsilon$ -sequences f_x , and $C_r) \in []_x$ where then $(C_{rn} = X \text{ for all } n)$ there exists (by 3.b) an integer N such that for all n > N then $|f_{Xn} - x| < \epsilon$; that is (for all $-f_x \in []_x$ "eventually" $x - \epsilon < (f_x) < x + \epsilon$ holds). likewise for $[]_y$, then (for all $-f_y \in []_y$ "eventually" $y - \epsilon < (f_y) < y + \epsilon$ holds). **however:** by assumption $(x = 2\epsilon + y)$ so $(x - \epsilon = y + \epsilon)$, thus it follows implicitly that every $-f_y$ is *separated-by the eventual-relation*(s) " $(f_y) < y + \epsilon < (f_x)$ " from all $-f_x$. and so by a similar argument for y > x; $[]_x$ and $[]_y$ separate in \mathbb{R}_1 (as well as being disjoint (see: T3) in \mathbb{R}_+)

- **note**: relative to an exterior-context of the-global-structuring of \mathbb{R}_{+} [its points] as such necessarily appear topologically "**closed**". regardless , it then follows that :

sundries (and at times useful &- uniformly-bounded-representations)

d. - for any- $\epsilon > 0$ ($\epsilon \mathbb{R}_1$) and any- $(f) \epsilon [color-(or amalgam)]_r$ ($\epsilon \mathbb{R}_{\dagger}$), there latently exists (by 3.a) a ("minimum-unique"-sequence-dependent integer N_{ϵ}) $\epsilon \mathbb{Z}$: such that the $f_n \epsilon(f)$ may be then considered implicitly re-indexed (by $\breve{n} = n - N_{\epsilon}$): such that for all -(\breve{m} , \breve{n}) >0 then $|f_{(\breve{n})} - f_{(\breve{m})}| < \epsilon$.

that is the collection of (all)-such-identically and latently re-indexed (f) sequences: (f $_\epsilon$) \in [some color]r ($\in \mathbb{R}_+$),

if all those: (\check{n}) are-then *considered* restricted from \underline{Z} to \underline{N} : implicitly defines a recasting of that color to a $\underline{\epsilon}$ -uniformly-bounded-representation (..." []]"...) which by definition "*captures*" all - (f) \in [the color(amalgam)]_r ($\in \mathbb{R}_{\dagger}$).

e. - and as such, a []_X $\in \mathbb{R}_{\dagger}$ is called <u>positive</u>: iff a $\epsilon > 0$ ($\in \mathbb{R}_{1}$) may be shown (see T6.16.e) to exist such that there exists ϵ - uniformly-bounded-representation(s) of []_X and []_{Cr=0} such that strictly []]_{Cr=0} < []]_X; that is for (any-and-all ${}^{\circ}f_{(\tilde{n})}\epsilon$ (the ${}^{\circ}f_{\epsilon})\epsilon$ []]_{Cr=0}) <u>and</u> (any-and-all $g_{(\tilde{m})}\epsilon$ (the $g_{\epsilon})\epsilon$ []]_X) then ${}^{\circ}f_{(\tilde{n})} < g_{(\tilde{m})}$

(where of course in-application such proofs relie again at essence on $\,16.c\,)$.

- f. likewise, a []_X $\in \mathbb{R}_+$ is called <u>negative</u>: iff there exists some $\epsilon > 0(\epsilon \mathbb{R}_1)$ such that there exist (see T6.16.f) ϵ - uniformly-bounded-representation(s) such that strictly []]_X < []]]_{Cr} = 0 (again by 16.c).
- Έ_____

17. - <u>then focusing</u>, again, in on the thread of discussion, that beneath the "closed"-surface of <u>any</u>-point lies some and yet benign potential (coiled up in a virtual partition-<u>name</u>-space) : establish and *begin* an exploration of the coupled algebraic-maps with in 1-dimensional *mixed-layer-space*(s) (see 20.) as initiated by $(\mathbb{R}_{\dagger})^{(1)}$. along the way, both *side-algebras* and a (*)-*relative-internal-metric* will emerge, allowing for an (*)-*interior-calculus*, and confirming the appearance of ornateness ; which (of course), as is usual, and as it: is-compulsive, will also then be built upon (see: 14.d).

an internal-naming scheme (the-first iteration) that is, and

a. – where, the $f_n \in f(\in [C_r]_{\mathbb{R}^+})$ (ie. the various " f_n " of an equivalence-class for some C_r), are rewritten with-respect or in-reference to " C_r " itself; as

 $f_n = Cr_n \mp *f_n$ sometimes written: +/-

- where : all (* $f_n \in \mathbb{R}_1$) are understood ≥ 0 ;
- b. the($\mp *f_n$ thus associated) with any- $f \in [C_r]_{\mathbb{R}^+}$ (by 16.a) necessarily exist and form a null-sequence.
- c. the interpretation of "∓" is context-dependent, but at times for conciseness is (written as (∓) or as (+/-)) and: implies (2)-distinct sequence-*formulas*, which, (by T4.1.M3) distribute over parenthesis);
 ie.
- d. (+/-)(* f_{1n} + * f_{2n}) denotes for example some(C'r_n) offsets (* f_{1n} + * f_{2n}) and (- * f_{1n} * f_{2n}), which when taken over $n \in \underline{N}$ (by 17.b and T4.3 "the addition of null-sequences") define in and of themselves, null-sequence(s), and as such define-sequences which remain "in" [C'_r]_{R_{\uparrow}} (by T4.5).

multiplication: (by any "positive"- constant-representative) preserves color

18.a – and then, in order to tend towards making specific an exploration of a color-algebra, state that : multiplying each element of the 'external-real-line' by a common-positive-factor (> 0), causes a uniform relative change or "lensing" of the absolute-value-metric equal to the common factor

(eg. |ca - cb| = |c(a - b)| = |c||a - b| by T1.*theorem.*1.1. II).

thus for all-interior supportive ($f_n \in f \in [C_r]_{\mathbb{R}_+}$) rewritten as $(f_n = Cr_n \mp *f_n) \in \mathbb{R}_1$) (see 7.a),

it follows that $|c|f_n = (|c|Cr_n) \mp (|c|*f_n)$ (by T4.1.M3)

and as such : term-by-term multiplying a representative $f \in [C_r]_{\mathbb{R}^+}$ "in some particular-instance" by then some positive-constant- $|C|_r$ -representative , causes in effect a uniform-lensing of the (* f_n relative-to) the constant-representative then of []|c|c_r.

b. - where (by T4.2.c): the \mp (|c|* f_n) define a null-sequence, and thus (by T4.5) define a []|c|Cr "contained" - lensing.

- c. and where $\pm |c|^* f_n$ preserves color, briefly substantiate this claim.
 - first, since : each of the equivalence-relations involved in the above partitioning of *R*⁺ are based on descriptive e-filters on sequence(s) with respect to some member-element relative-ordering.
 eg: the *f* ∈ []_{*R*⁺} are partitioned (at essence) by the eventual-innate-validity (or-not) of the "forms" :
 - 1. $-|f_n C_{rn}| > 0$ with in the *ei*-disconnected-filter; which here then may rewritten as: $|(Cr_n \mp *f_n) - Cr_n| > 0$, (ie. $|\mp *f_n| > 0$).
 - 2. $(f_m \ge f_n \text{ .or. } f_m \le f_n)$ with-in the *ei*-monotonic-filter; which then may be rewritten as: $(\mp *f_m \ge \mp *f_n)$ or $(\mp *f_m \le \mp *f_n)$.
 - 3. $(f_n Cr_n) \leq 0$ with-in the "1"- side-filter ; rewritten as : $(\mp *f_n) \leq 0$
 - 4. $(f_n Cr_n) \ge 0$ with-in the " \downarrow "- side-filter ; rewritten as : $(\mp *f_n) \ge 0$
 - 5. and $[\ldots, [Cr], \ldots]$ may be rewritten such that the associated $*f_n = 0$, for all $n \in \underline{N}$ (ie. in one sense, all are identity - ordered).

then : each "color"-is-forced , and is 'some' [relative order-type of $(*f_n)$]_e implicitly . that is , either as a relation-type to zero , or, between $(*f_m, *f_n)$.

- d. thus since : (T4.1.M(1-5) and T4.1.E(1-3) derive various-similar *multiplicative-theorem*, such as : " $x \le y$ implies for 0 < c, that $cx \le cy$ ", and, "x > 0 implies for c > 0, that cx > 0"...), then for any $(f_m, f_n) \in f(\in \mathbb{R}_+)$ it follows that all the (f)-internal-member-order-relationships are preserved in and through the binary-operation(s) of $|C|_r f$.
 - and then or again, since the |c| f_n ∈ (|C|rf) may be rewritten as |c|f_n = (|c|Cr_n) ∓ (|c|*f_n), and {∓*f_n} defines an associated "null-sequence" (see 17.b), then {∓*f_n} and {∓|c|*f_n} (by 18.b, 18.C.(1-5), and the immediately above 18.d): are members, in and of themselves, of the same [relative-order-type] -or- [color (amalgam)]_{X=0}; and as such, the claim essentially (by 16.a) follows.. (ie. stated contextually here as
- $|C_r|[...[color]...]_{r.} = [...[color]...]_{|C_r|r.}$

*

* e. – <u>note</u>: therefore it also follows that, the collection of <u>all</u> - <u>positive</u>-constant-representatives, may be claimed as (contained-in or *at least* equal-to) the [(multiplicative)-spectral-identity-class] (notated: " $e_{i\otimes}$ ") associated with this over-all *re-sistered* <u>distributed</u> internal-domain of \mathbb{R}_{\uparrow} .

19. - addition: (by any constant-representative) preserves color

recast the $f_n \in f(\in [C_r]_{\mathbb{R}^+})$ by or with $Cr_n \mp *f_n$, and 'any-term' of some otherwise general constant--representative by or with $Crg_n \mp 0$ (both $\in \mathbb{R}_1$). thus it is immediate : that

 $(\operatorname{Cr}_n \mp *f_n) + (\operatorname{Cr}_n \mp 0) = (\operatorname{Cr}_n + \operatorname{Cr}_n) \mp *f_n$

and besides-offset when taken over $n \in \underline{N}$, that addition by constant-representatives (by 18.c.(1-5)) preserves color, which, then in a fashion similar to the above derives the

* a. – <u>statement</u>: that the collection of <u>all</u>-constant-representatives, is (contained-in or equal-to) the [(additive)-spectral-identity-class] (notated : " $e_{i\oplus}$ ") associated with this

over-all distributed internal-domain of \mathbb{R}_{\dagger} .

and that : ($e_{i\otimes}$ and $e_{i\oplus}$) as such at least $\underline{\mathsf{partially}}\text{-}\mathsf{overlap}$.

* 20. – some overviews on ($e_{i\otimes} \bigcap e_{i\oplus}$) overlap ... "or the crux of the matter".

examine [zero] the additive-identity-element for the 1-dimensional exterior ring of $(\mathbb{R}_{\dagger})^{(1)}$ (ie. "0" + [any-number] = [any-number]); its "interior" however, (by: the existence of 1-1 maps between sequences-and-series) is a **very general** infinite-dimensional re-sistered space with-out any representative restriction impressed on it other-than the ϵ -(0)-convergent-equivalence of its member sequences(see 5.b)

further (for emphasis, by 16.c-e) \in -uniformly-bounded-representations of []]0 and []]1 may be shown to exist; such that they are "completely-disjoint or separate" in \mathbb{R}_1 ; and thus : as "one" is the exterior multiplicative-identity-element (1·r = r) for the real-number-system ; it is <u>apparently-immediate</u>: that the current spectral-decomposition of the interior of \mathbb{R}_+ (by : overlap) then shall represent the existence of an algebraically-<u>distinct</u>, sympathetically (see 14.A, 15.)-coupled-system, which is latent (<u>by</u>: the order-isomorphism of \mathbb{R}_+ to \mathbb{R}) or effectively-ubiquitous or in 'a' <u>genetic-background</u> of (all- \mathbb{R}), and as such intrinsic to its *hypercomplex*-closures (*eg.* the *complex*-numbers \mathbb{C}_+ ...and *etal.*), and thus and finally then as, coupled to all representations (or point-associated representations) which *by nature* are functionally and/or concretely-constructed on or over(\mathbb{R}, \mathbb{C}_- .)..(unless again <u>'term'ally</u>-restricted ,as in \mathbb{R}_1 , now formally see:T.5)

- a. and it is ,at essence, this "coupling" and non-trivial distinctness of (algebra and dimension) which inspires and forces the layer-description "mixed-domain", ie. as: an infinite-dimension exists in a continuous-and-invariant mix or coupling with-and-to any-such finite-dimension(s).
 - and, then again, it is the <u>partial</u>-overlapping(see: 21.a) of (e_{i⊗} and e_{i⊕}) which will-in-hindsight give interior-color-algebra (among its' many-other properties) also a < 'meta-stable, A-symmetric' > time-like feel, regardless . . . then examine,

(-1)c rotation:

21. - state multiplication by (-1) re-orders the external-real-line reflectively around the origin: thus for any- $f \in []_{R^{\dagger}}$; it follows naturally for (-1)(f); when "(-1)" is considered as the constant- (-1)c -representative and where the $f_n \in (f)$ may be rewritten " $f_n = Cr_n + - f_n$ "; that such multiplication internally generates (-1) $f_n = (-Cr_n) - + f_n$ relative to (f) (ie. at essence: $+ - \rightarrow -/+$). and thus, as the "side"-filters of (18.c.3 & 4) may be thought of as (-1)-"symmetric-duals" of each other : it also follows that (-1)c('any' - $f \in [some-color]c_r$) (see T6.21) maps into 'a' symmetric-color-dual. and as such (and with-out "much adieu" here) a claim may be easily generalized to an external-perspective represented for notational-emphasis by:

 $(-1)c_r[\ldots [color]\ldots]_{r.} = [\ldots [roloc]\ldots]_{-r.})$.

* a. – and where : this as such establishes the "partial"-overlap claim made above, since there exist $(f \notin \{Cr\})$.

side - addition :

- 22. with these (three) initiating features of color-algebra assumed in-hand, begin a discussion of side-addition (note: and then an extension to side-multiplication), starting with the following definition(s).
 - (2) sequences $(x), (y) \in \mathbb{R}^+$ are called <u>"left"-side-equivalent</u>: iff each is selected by 'an' amalgam-operator
 - a. $S_{LS} = S_{dM\uparrow} + S_{dm\uparrow} + S_{fm\uparrow} + S_{Cr} + (S_{Cr})$ (note: the inclusion of $[S_{Cr}] \subseteq e_{i\oplus}$)

that is or alternatively stated : there exist N_X , $N_Y \in \underline{Z}$ such that for all $n > (N_X \text{ or } N_Y)$, then (x) or (y) may be completely-represented from there on by some form : $\underline{C}(\underline{rn}) - \underline{*f}(\underline{n})$.

- **likewise** (2) sequences $(x), (y) \in \mathbb{R}^+$ are called <u>"right"-side-equivalent</u>: iff each is selected by 'an' amalgam-operator
 - b. $S_{RS} = S_{dM\downarrow} + S_{dm\downarrow} + S_{c\downarrow} + (S_{Cr})$ (again note: $[S_{Cr}] \subseteq e_{i\oplus}$)

that is or alternatively stated : there exist N_X , $N_Y \in \underline{Z}$ such that for all $n > (N_X \text{ or } N_Y)$, then (x) or (y) may be completely-represented from there on by some form : $\underline{C(rn) + *f(n)}$.

c. <u>addition</u> of (LS/RS)-same-side-equivalent(coloration) sequences , derives the easily-apparent-referencial rule: for all (r_1 , r_2) $\in \mathbb{R}_+$

 $[[LS]...]r_1 + [[LS]...]r_2 \Rightarrow [[LS]...]r_1 + r_2$ $[...[RS]]r_1 + [...[RS]]r_2 \Rightarrow [...[RS]]r_1 + r_2$

prove: assume (f_1, f_2) are (LS/RS-same-side-equivalent naming sequences) where the $(f_{1n} \in f_1 \in [C_r]_{r_1}, \text{ and } f_{2n} \in f_2 \in [C_r]_{r_2}) \in \mathbb{R}_1$ are rewritten as $(f_{1n} = C_{r_{1n}} \mp *f_{1n}, \text{ and}, f_{2n} = C_{r_{2n}} \mp *f_{2n})$ respectively. then "both" sequences (in unison) may be represented (note: the inclusion of S_{Cr}) by at least one common side form $(C(r_n) - *f_{(n)}) \underbrace{\text{or}} (C(r_n) + f_{(n)})$: exclusively, and so $(f_1 + f_2)$ may eventually be represented by

d.

 $(Cr_{1n}(\mp) * f_{1n}) + (Cr_{2n}(\mp) * f_{2n})$ $where here: (by 17.c) "(\mp)" represents a notational$ $(Cr_{1n} + Cr_{2n}) (\mp) (* f_{1n} + * f_{2n})$ combination of the (2) forms (ie. read high <u>or</u> Low)

and , where as (17.a defines all $f_{(n)} \ge 0$), and, (T4.1.a4 implies that for $(f_{1n}, f_{2n}) \ge 0$), that $0 \le f_{1n} \le (f_{1n} + f_{2n}) = f'_n$), and, (by 17.d "the addition of null-sequences" implies that f'_n , when taken over $n \in \mathbb{N}$, defines after algebraic-substitution of (Cr₁n + Cr₂n), strict-([]r₁+r₂)-internal-offsets): <u>then</u> $(f_1 + f_2)$ may eventually be represented by the "same"-one sufficient-form (C(r₁+r₂)_n - f'_n) <u>or</u> (C(r₁+r₂)_n + f'_n) as (f_1, f_2) themselves, and the claim.

- * more generally, since $\{ [S_{Cr}]_r \} \subseteq e_{i\oplus}$ (by 19.a), and each C_r then is such an identity-element sufficient; it follows also by a (C_r) -anti-symmetric (see 16.b) lack of additive-inverses [ie. *e*-not ($-(*f_n)$: for $*f_n \neq 0$)], that side-equivalent-sequences (in and of themselves) form an internal-spectral semi-group (or stable-descriptive-shell structure) which "algebraically"-projects-and-(<u>preserves</u>) information (via "semi-group"-stability) across the background of ,or, the distributed sub-domain of $(\mathbb{R})_{\dagger}$.
 - <u>note</u>: the (" \Rightarrow ") is a notational contraction, and simply represents *here* a mixed-domain map which is externally-equivalent (ie. =), and, internally into_*in a loose but then sufficiently appropriate sense* (ie. \rightarrow).

<u>color – dominance</u>

- 23. continuing with ground-work : exam color-dominance associated with a re-finement of the addition of (LS/RS-same-side-equivalent sequences). consider the partial LS/RS-operators descriptively selecting either 'ei-*disconnected* or (not)- ei-*disconnected* ' naming-sequences.
 - $1. \hspace{0.1in} S_{LS(d)} = S_{dM\uparrow} + S_{dm\uparrow} \hspace{0.1in} ; \hspace{0.1in} S_{LS(C)} = S_{\text{C}m\uparrow} + S_{C\uparrow} + S_{Cr}$
 - $2. \quad S_{RS(d)} = S_{dM\downarrow} + S_{dm\downarrow} \quad \text{;} \quad S_{RS(C)} = S_{\text{C}m\downarrow} + S_{C\downarrow} + S_{Cr}$

that is (by: 9. and 18.c.1 "ei-*disconnected* "), for any $f \in [...[]_{SLS/RS(d)...}]_r$ there exists a $N_d \in \underline{\mathbb{Z}}$, such that for all- $n > N_d$, the (f: associated): $*f(d)_n > 0$. and similarly (at least then, simply by 17.a), for any $f \in [...[]_{SLS/RS(C)...}]_r$ the (f: associated): $*f(c)_n \ge 0$. and as such, it is immediate that \neg

a. - <u>SLS/RS(d)</u> dominates <u>SLS/RS(C)</u> that is, since the addition of such LS/RS-same-side-equivalent sequences, may be eventually-completely-represented by 'one' of the exclusive-form(s): $(C(C)r_n + C(d)r_n)(\mp)(*f(C)n + *f(d)n)$ (by 22.d) then $0 \leq *f(C)n$ implies eventually (by T4.1.a4) that $0 \leq *f(d)n \leq (*f(C)n + *f(d)n)$ holds, and thus the final-form is also *Idisconnected*.

similarly:

- b. the addition of (2) SLS/RS(d)-same-side-equivalent sequences $(f_{1(d)}, f_{2(d)}) \in \mathbb{R}_+$ also necessarily map into $[\ldots []_{SLS/RS(d)} \ldots]r_1[d] + r_2[d]$; where a proof is almost identical to the immediately above (23.a) (except for with a replacement of : " $0 < *f_1(d)$ implies after (addition) that $\underline{0 < *f_2(d)n < (*f_1(d)n + *f_2(d)n)}$ eventually-holds ").
- * <u>more abstractly</u> the above (ie. 23.b) demonstrates the existence of an additive side-dominant (stable-shell structure):
 - 1. which interestingly doesn't contain members selected-by $\,S_{Cr}$. and
 - 2. which as such, <u>necessarily</u> exhibits [convergent-"<u>lensing</u>"]; that is for any $(*f_n, *g_n) > 0$, then, $(*f_n + *g_n)$ is greater-than either-of: $(*f_n \text{ or } *g_n)$ (note: <u>frame-opaqueness</u> will be tied to this later).
 - and which, at least simply by features brought into view by the "otherwise-general" descriptivebinding of non-monotonic naming-sequences to:[S_{LS/RS}(d)], while *still* forcing such side-lensings, "preserves-the-possibility" of a diverse-class of the other type-migrations (seen below). and as such, does not lay-hold or make completely-<u>rigid</u> future-instantiation (then) with-in this presently more coarsely-confined yet-stable (S_{LS/RS}(d): into)-domain.
 - (ie. and as a note: thus making more explicit the wording "shell").

monotonic - side - addition

24. – and next claim: adding (two) same-side (LS/RS)-*ei*-monotonic-naming-sequences produces <u>another</u> (of the) same-side (LS/RS)-*ei*-monotonic-sequence(s).

п

prove: first in a fashion similar to (17.c), utilize a notational combination "(\gtrless)" to denote (2) distinct exclusive sequence-formulas (ie. for the "LS..<u>or</u>..RS" case). and, **then assume**: *f*, *g* are same-side (LS/RS)-*ei*-monotonic sequence(s), thus (by 10. and "eventual") there exist integers $N (=\max(N_f, N_g))$ such that for all <u>fixed</u> m,n > N then

- 1. $f_{\mathrm{m}} \ (\gtrless) \ f_{\mathrm{n}}$ whenever m>n ;
- 2. $g_{\mathrm{m}}(\gtrless) g_{\mathrm{n}}$

```
however since ( by T4.1.a(1-4) and T4.1.E(1-3) ) :" x \ge y implies x + z \ge y + z"
and: " x \le y implies x + z \le y + z"
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it follows then that :

1. $\underline{f_m + g_n}$ (\gtrless) ($f_n + g_n$) where here (choose the fixed) : $g_n = z$ 2. $(g_m + f_m)$ (\gtrless) $\underline{g_n + f_m}$ and here (choose the fixed) : $f_m = z$

and therefore (by T4.1.a2) it is immediate that $(f_m + g_m)(\gtrless)(f_n + g_n)$ in general whenever (m>n) > N, and as such the claim.

therefore it also follows : (by 23.b)

a. - that $[\ldots []_{SLS/RS(d)M} \ldots]r_1 + [\ldots []_{SLS/RS(d)M} \ldots]r_2 \Rightarrow [\ldots []_{SLS/RS(d)M} \ldots]r_1+r_2$ and (by 23.a)

b. - that $[\ldots []_{SLS/RS(C)M\ldots}]r_1 + [\ldots []_{SLS/RS(d)M\ldots}]r_2 \Rightarrow [\ldots []_{SLS/RS(d)M\ldots}]r_{1+r_2}$

- where as would be expected, the (C)-ei-monotonic LS/RS-selection operators are :

$$\begin{split} S_{LS(C)} &= S_{C\uparrow} + S_{Cr} \\ S_{RS(C)} &= S_{C\downarrow} + S_{Cr} \end{split}$$

in their-notationally uncombined form(s).

* 25. <u>comment</u>: (as, an *injective* pre-amble to the color-properties of *power series* and *sequencial* expansions)

again restate , whether or not information other than color for the immediately above (24.b) (ie. $r_{[CM]}$, $r_{[dM]}$) \in [] $_{R^+}$ is apparent (or in-application may be recovered), then is irrelevant to "this" mapping; which still latently may, carry-"forward" ,regardless, as simply into an otherwise opaque [...[] $SLS/RS(d)M...]r_{[CM]+r_{[dM]}}$: and there in, by this *externally* (R_a)-imposed dormancy or rigidity , lies for example, potential (R_1 , ...)-point-*samplings* of the various-sets of <u>possible</u>-functions which may be embedded in or characterize particular geometries (or associated-geometries); constructing [elements] which are reflective-of, or, residual-to: (type-classings) -of- (path approaches "now lets say") under "apparent"-forcing , and then, mix-embed or <u>bind</u> such [objects] into environments of differentcharacteristic,producing <u>completely-local-intertial-regions</u> whose interiors(in this sense) representationally mimic "potential -past(s)-of" the sampled-space , while preserving the exterior-structural-binding or a fabric of the cross-embedded <mixed-into> domain . (that is, develop and maintain a possibility of point-binding--or-site projecting mathematic-structure by and through some initial *internal* driven-"<u>restriction</u>" of-and-to and-upto- some interpretively robust –and then- fabric separable-and/or-discrete-*ized* history-or-coloration(s) and -or- then by ... representative-supportive colored (name or n-tupled name)-spaces).

continuing with such eventual-discussions, and laying further necessary referential-groundwork ...

(non)-monotonic-side-addition :

26. – now examine (in general) and (in more-extended constructive-detail, as it is here that (*)relative internal-metric(s) first appear), the algebraically-defocusing family of partially-unstable-mappings:

 $[\ldots []SLS/RS(d)m\ldots]r_1 + [\ldots []SLS/RS(d)m\ldots]r_2$

first prove: (for any-f1 ∈ [... []SLS/RS(d)m...]r1 that such an interior-shell allows-for:
 (and so, there latently-exists) a same-side companion

 $f_2 \in [\dots [] SLS/RS(d) \oplus \dots]r_2 \quad \text{such that}$ $(f_1+f_2) \in [\dots [] SLS/RS(d) \dots]r_1+r_2 \quad (\in \mathbb{R}_+: \text{ and which is also of-the same-side})$ $). \qquad \uparrow$

to facilitate this observation introduce the utility and concept of an implicitly-constructed . . .

- a. <u>"step-function sheath"</u> as follows; begin by defining an implicit-set on a relationship between (2) integer- pointers and any-epsilon: \rightarrow and thus if *presented*-with some (other-wise unconfined) (LS/RS)-side-selectable naming-sequence $(f) \in \mathbb{R}^+$,
 - there is brought into adjoined-existence (by definition 22.a,b) a minimal eventual-integer (\underline{N}_{S}) such that (f) could-or-may be represented from there on , by one of the exclusive-side-form(s) "C(rn) (\mp) *f(n)", (ie. and again , either as (C(rn) + *f(n)) .or. as (C(rn) *f(n)))

(and) next for the $\underline{same}(f)$:

- there is also brought into latent-existence then (by 17.a and 3.b "the convergence of [f, Cr] ", for any (epsilon) $\epsilon > 0$ ($\epsilon \mathbb{R}_1$)): a ($\underline{N}\epsilon$) $\epsilon \mathbb{Z}$ so that for-all $n > N\epsilon$, the *f*-associated : (* $f_n < \epsilon$).

and as such identify a (outer-sheath generating) f-distinguished and (implicit) sub-set $\{*f_n\}_{(os)} \subseteq \{*f_n\}$ by the criteria that :

```
- for ( n> Ns ) <u>if</u> (then) :
each (*fn > 0)(\in \mathbb{R}_1), is in and of itself considered as a '\epsilon',
<u>and</u>
if for some *f("n"), (ie. the n) which indexes *f, also innately :
("n" = N\epsilon for that '\epsilon'=*f"n")
```

– then such (* f_n are) \in {* f_n }(os)

- <u>note</u>: the description of {*f_n}(os) then in-general relies on an implicit-definite philosophy(see 9.a), which (far from being an annoyance), *later*-<u>also</u> drives an underlying mechanism of pathologic-opaqueness, and allows-and-provides-for[relative virtual "local-ness"] with in partially-sympathetic multi-frame-couplings.
- 1. <u>returning</u>: { }(os) then is confined strict-monotonic, since by very definition : for any $\underline{m > n}$, if $(*f_m, *f_n) \in \{$ }(os), then (by the above epsilon-confinement) : {* $f_m < *f_n$ }(os).
- b. further and under tighter-constraints (for any- $f \in [\ldots [] S_{LS/RS(d)m} \ldots]r$), {* f_n }(os) then is :
 - 1. <u>non-empty</u>, prove : much as before, but with a few additional focusing details,

there inherently are associated with such <u>side-bound</u> <u>disconnected-(f),</u>

some $\underline{N_s}, \underline{N_d} \in \underline{\mathbb{Z}}$ such that for all $n > N_{sd}(=\max(N_s, N \text{ (see: 27.c)})$, then the rest of the *f*:associated **f*_n(are both <u>non</u>- Cr-oscillatory, and are (>0));

<u>also</u>

*

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again (by "convergence") for any \epsilon > 0 (\epsilon \mathbb{R}_1) there is a N \epsilon \epsilon \mathbb{Z}_2
```

such that for all $n > N \in , *f_n < \in ;$

and

then as well (by T1.theorem(1.7)) and thus essentially (see 7.a) by the "density"-properties of the \mathbb{R}_1 -system), for (any $f'_n > 0$) " in general ") there can be described : some- ϵ' ($\epsilon \mathbb{R}_1$) so that $\frac{f'_n > \epsilon' > \dots > 0}{2}$.

("now descriptively-generate the existence-of- a particular-'pattern-of' finite-packet(s) with-in these (f)").

 and so, for any-'such' *ei*-disconnected-(non)-monotonic-side-bound set {*fn } and then again for each (*f"n" ∈ {*fn}) such that :

 $n = any (N_{min} > N_{sd})$

there obviously, and, implicitly-(eventually)-exists some equal-or-larger ($N_{\epsilon'}$) max satisfying <u>both</u>: that,

there "is-or-maybe chosen" <u>some</u> (ϵ') < * $f_{N^{\min}}$ such that for all

 $\mathbf{n} > (N_{\epsilon'})^{\max}$, then ${}^*\!f_{\mathbf{n}} < \epsilon'$;

and

 $(N_{\epsilon'})^{max}$ - generates as such , a finite-set (or "packet") of $(*f_n) > \epsilon'$, indexed by and between $N_{min} \leq \{*f_{n}^{*}, s_{\epsilon'} \leq (N_{\epsilon'})^{max}\}$

- from a discussion of any of these *incrementally*-large finite-{sets} ϵ' the non-emptiness of {* f_n }(os) may be demonstrated: first with-in any potential-instantiation of 'a' {* f_n } ϵ' there necessarily-resides 'a': disjoint (ref: the points of \mathbb{R}_1 are separable) sub-set (\subseteq "{* f_n } ϵ' "), defined by the max1("{ f_n } ϵ' "), for a precise description of a binary-"max"-function (see: 27.c), which may then be generalized to-here. (note: this set may not be a singleton), and thus from that set; consider the unique-member represented or *pointed-to* simply and intuitively by the ordered-filterings:

 $f_{\bar{n}} = (\max_1 (" \{ f_{max_2(n)} \}_{\epsilon'}))$

clearly such a member meets the criteria : since , first of all, its the last-member of { equal (* $f_{(n>N_{sd})}$) in general } which are still: greater-than or equal-to (that is notationally)($\geq f_{\bar{n}} > \epsilon' > (all - f_{(n>N_{\epsilon'})})$; and thus, by being a maximum of some chosen " {* f_n } ϵ' ", then for all $n > \bar{n}$: * $f_n < f_{\bar{n}}$), and the assertion of non-emptiness.

- 3. further the (implicit)-existent {set}⊂ <u>N</u> which then indexes any-such {*f_n}(os) is necessarily continuous-and-"gappy": this follows first by the otherwise arbitrary definition of N_{min} above (see 26.b.2 ie. any); and then again as otherwise {*f_n} itself, would be eventually-(strict)-monotonic (see 26.a-b) which (by assumption) its not.
- 4. and at last bind the above to a notation : that is : (by the well-ordered properties of \underline{N}) (ref: order-theory) any potential-instantiation of such an indexing {set} $\subseteq \underline{N}$ contains a unique-minimum member (m_1). and thus {itself} may be tacitly considered indexed by the sequence-function on \underline{N} into \underline{N} ${}^{m}f$: { . . . m_p > . . . } such that (m_p = (the pth:(n)) indexing {* f_n }(os)) and

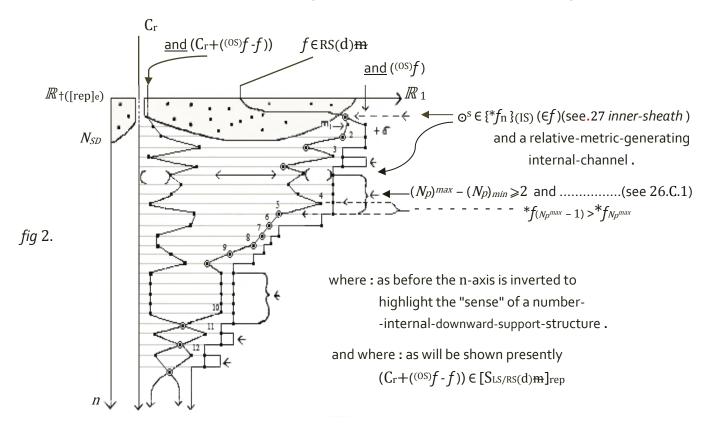
so $\{m_p\}$ arises then simply as a contiguous ordering for : latently-naming the $n \in \{*f_{n''}\}_{(os)}$

c. - as such it is now possible to **define** a "*non-tight-fitting*" : (outer)*f*-associated step-function-sheath(s) for $(any-f) \in [\ldots [] SLS/RS(d)m \ldots]r$ by the sequence-function(s) ($\in \mathbb{R}_1$)

" $^{(OS)}f_n = C(r_n) (\mp) * \underline{O}_n$ " such that :

 $*\underline{O}_{n} = \begin{cases} \text{for } n < m_{1} \text{ then} \\ *\underline{O}_{n} = *f_{n} & \dots \text{ ie. } \{*\underline{O}_{n}\} \text{ is identical or intersecting with } \{*f_{n}\} \text{ below } (m_{1}) \\ \text{for } m_{1} = n \leqslant (m_{2}+1) \\ *\underline{O}_{n} = (*f_{m_{1}}) + \delta & \dots \text{ ie. } *\underline{O}_{n} > *f_{n} \text{ where } \delta > 0(\in \mathbb{R}_{1}) \\ \text{for } p > 2 \text{ and for } (N_{p})_{min} < n \leqslant (N_{p})^{max} \\ \text{such that } ((N_{p})_{min} = m_{(p-1)} + 1) < n \leqslant ((N_{p})^{max} = m_{p} + 1) \text{ then} \\ *\underline{O}_{n} = *f_{m_{(p-1)}} \end{cases}$

- ... ie. inconculsion (see 26.d . .below) an appropriate sympathetic latent construction exists, giving a $n \ge m_1$ (in general) : {* f_n }(os) generated and confined step-monotonic, (non)-f-intersecting, sheathing-sequence of (f), such that from <u>there-on</u> all * $\mathcal{O}_n > *f_n$.
- reverse-engineered and generalized from, and thus inspired-and-demonstrated here by some .. and <u>notice now</u> the (to-be exploited) -localized-region, (as such), then of a visually-sufficient graph:



d. – first for completeness affirm the convergence-of at that ${}^{(OS)}f \in S_{LS/RS(d)}M$

*

- <u>convergence</u> follows: since as each member of the associated sequence ($\{* \underline{O}_n\}$ for $n > (m_2+1)$) is a member of $\{*f_n\}_{(os)}$ ($\in \{*f_n\}$): then a convergence-relation may be constructed for ($\{* \underline{O}_n\}$ or any re-arrangement of $\{* \underline{O}_n\}$) from the $\{*f_n\}_{(\in)}$ -relationship itself (see T4.appendix.1,2).
- <u>€ SLS/RS(d)M</u> follows : since (for n≥ $m_1 > N_{Sd}$), ^(OS)f is from there on (<u>by</u> order-preserving sympathetic-construction from {* f_n }(os)) "<u>monotonic</u>" : which then implicitly negates (Cr-oscillations) and forces ^(OS)f as "(f)"-same-side(<u>LS/RS</u>).
 - further: first since for p>2 and for all n> $m_{(p-1)}$; then all $*f_n < *f_{m_{(p-1)}}$ (by the definition of { }(os)) and : next since eventually every region of identical {* $\underline{O}_n = *f_{m_{(p-1)}}$ } (see 26.c.1 def : of $(N_p)_{min}$) are defined such that all- n >($m_{(p-1)}$ +1);

then : it follows that eventually all $*\underline{O}_n > f_n > 0$, and so ${}^{(OS)}f$ is necessarily <u>ei-disconnected</u>.

- and thus the assertion(s).
- e. now claim : the convergence of and that $(C_{(rn)}(\mp) \{ * \underline{O}_n *f_n \}) \in [SLS/RS(d)_{\mathbb{H}}]$ rep : however where as obviously, both $\{ * \underline{O}_n \}$ and $\{ *f_n \}$ are (by definition) *null-sequences*, and where as both the convergence of and then that $\{ (* \underline{O}_n *f_n) \} \in SLS/RS(d)$ are (see above 26.d) similarly immediate,

simply show { (* $\underline{O}_n - f_n$) } as (ei-<u>non</u>-monotonic) : thus

- for any of the $((N_p)_{min}, (N_p)^{max})$ -bounded regions for $(f, (^{OS})f)$ determined above (see 26.c.1) by p > 2 and $((N_p)^{max} (N_p)_{min} \ge 2)$ then: (by the visually-sufficient (fig 2.) and its generalization(s)(26.a-c)) $(\underbrace{(*f_{(N_p^{max} 1)} \in \underbrace{\{*f_n\}}_{(OS)} > \underbrace{*f_{N_p^{max}}}_{(N_p^{max})}))$ that is $(-*f_{N_p^{max}}) > (-(*f_{(N_p^{max} 1)}))$ (by $\in \mathbb{R}_1$), and as such, as (by 26.c.1) $*\underline{O}_{N_p^{max}} = *\underline{O}_{(N_p^{max} 1)}$ then after addition $(\underline{*O}_{N_p^{max}} \underbrace{*f_{N_p^{max}}}_{(N_p^{max} 1)}) > (\underline{*O}_{(N_p^{max} 1)} \underbrace{*f_{(N_p^{max} 1)}}_{(N_p^{max} 1)})$
- however since the $\{*f_n\}_{(os)}$ for all- $f \in [SLS/RS(d)_m]$ are 'gappy'- continuous (by 26.b.3), there continuously-exist such $((N_p)_{min}, (N_p)^{max})$ -bounded local-disturbances in-the-stream of these sequence(s);

and thus there does-(not)-exist a $N^{\sim} \in \underline{Z}$ such that for all-(m>n) > N^{\sim} then $(*\underline{O}_m - *f_m) \leq (*\underline{O}_n - *f_n)$. that is, it follows and expands more generally that $(\underline{(0S)}f - f)$ is (e-non-monotonic)

1. – and (by 16.a) that an exactly-similar sequence exists in every member of r $\in \mathbb{R}_+$

* f. - and therefore it also arises that : for any $f_1 \in [...[]_{SLS/RS(d)m...}]_{r_1}$ there implicitly exists an associated {* $\underline{O}_{2n} - f_{2n}$ } $\in [...[]_{SLS/RS(d)m...}]_{r_2}$ such that { $f_{1n} + (C_{2r_n} + ((^{OS})f_{2n} - f_{2n}))$ } $\in [...[]_{SLS/RS(d)M...}]_{r_1+r_2}$

- which of course (remember-now) is what was originally meant to be shown (see. 26).

* 27. <u>an aside</u> : and then remarshalling.

before completing the addition of opaquely-chosen SLS/RS(d)^m, another *cursory-and-brief* diversion : (where as since) essentially all the as-demonstrated-book-work and philosophy is done, it would be an absolute shame, at this juncture, not to (*at-least*) introduce (then for later, utility, of discussion) ... \rightarrow the notion of a "channel", and take by-example an initial foray into the idea of an internal-metric.

* behind the scenes of this presentation there has *always been* a guiding precept to do <u>nothing</u> except for to gain a simple-description-and-tools [*from with-in <u>extant</u>-capabilities*], for carrying-and-imprinting (in a most general way) the craft-of (notation and geometry) "onto" the *available naming*-structure(s)--and/or-*bound*aries(s) of "the conceptualization" of a point, (thus-shedding a 'still-worn' open/closed noose) however considering, that the over-all (state of the art), as it were, is intimately and (so-and-forever) tied to, and, a subtle carry-forward : of a descriptive premising (and maintenance) of abstracted construction(s) of the rope (in *our-and-a* globally-historic sense) and then its proper-sub-classification(s) (the compass and then (the ruler)), and, that "much" of the notationalization of those construction(s) was and is rooted in dimensionally- "purifying "-relation(s) of size as *initially*-notated by the rational-form, then :

attempting such a task (on first go-around) would be unnecessarily risky-and-obtuse with out some conceptualization (first) of a loosed-internal-measure . (and where , as such, (an appropriate-and-practical rooting) for the (now presently more)-generalized-descriptive-and/or-categoric method(s) will have been.. set forth): introduce the notion of a

a. <u>inner-sheath</u>:

briefly, as the methods and philosophies are almost-identicle to the immediately above (see: 26.), then (for any *f*-associated $\{*f_n\}$) $\in [\dots []_{SLS/RS(d)m} \dots]r$, an implicit generating sub-set $\{*f_n\}_{(IS)} \subset \{*f_n\}$ may be identified by a ("blue-sky")criteria for $n > N_{Sd}$ (that is we may look up to the:26.b.1 eventual-boundary)

first pre-define: $f_{(N_{sd+1})}$ (as a member of) $\{f_n\}_{(IS)}$ (note again see: fig 2.)

next: <u>for</u> $n > (N_{Sd} + 1)$

if (some $f_{n} \in \{f_n\}$ is) < $\{all \ f_n \text{ such that } (N_{sd} + 1) \leq n < n'' \}$,

<u>then</u> $f_{n} \in \{f_n\}_{(IS)}$ (ie. all such-members exist as "temporary" $f_{n>Nsd}$ -minima)

- <u>further and again</u>, the members of $\{*f_n\}_{(IS)}$ may be contiguously ordered by the descriptive notational convenience of $*f_{m_p}$ such that $(m_p = (\text{the } p^{\text{th}}:(n))$ indexing $\{*f_n\}_{(IS)}$ (see 26.b.4, ie.<u>here</u> $m_1=(N_{\text{Sd}}+1), \ldots$); and in particular the characteristics of (non-empty, strict-monotonic, continuous-gappy, and ei-disconnected) may be demonstrated (see T6.22.a) for any *f*-associated $(\{*f_n\}_{(IS)} \subset \{*f_n\}) \in [\ldots, [\ldots]_{\text{SLS/RS(d)}}, \ldots]_r$
- <u>that is</u> and for example then look at the $\odot^{s} \in f$ in (see fig 2.)
- b. and as such, much as before <u>define</u>: a (non-tight-fitting) (inner) *f*-associated step-function-sheath(s) for any- $f \in [\ldots []_{SLS/RS(d)m} \ldots]r$ by the sequence-function(s) ($\in \mathbb{R}_1$)

" (IS) $f_n = C(r_n) (\mp) * \underline{I}_n$ " such that :

$$*I_n = \begin{cases} \text{for } n \leqslant N_{\text{sd}} & \dots \text{ ie.} \{*\underline{I}_n\} \text{ is identical or intersecting with } \{*f_n\} \text{ upto-and-until} \\ & *\underline{I}_n = *f_n & \text{ the } N_{\text{sd}} \text{ - boundary. where after for } (p \in \underline{N}) \text{ and } n \geqslant m_{p=1} \text{ , then} \\ & \text{for } m_p \leqslant n < m_{(p+1)} & \dots \text{ that is: any } f \in \text{S}_{\text{LS}/\text{RS}(d)\text{m}} \text{ generates an inner-sheathing function,} \\ & *\underline{I}_n = *f_{\text{m}(p+1)} & \text{by a } latent\text{-construction based-on: a localized } \underline{\text{backwards-offset}} \\ & \text{and appropriate-finite-repetition of the members of } \{*f_n\}_{(\text{IS})}. \end{cases}$$

→ where (f) then separates its' inner-and-outer sheaths, since eventually by implicit-construction for <u>all</u> sufficiently-large $n \in \underline{N}$: ($0 < ^*\underline{I}_n < ^*f_n < ^*\underline{O}_n$), and, so the (inner-and-outer) sheaths form a disconnected channel around (f) which further, in and of itself, is ei-*disconnected* as a whole from C_r . thus demonstrating-<u>again</u> the somewhat *as-usual* counter-intuitive "roominess" with-in for example, *this* representative-model or (dressing -and/or- <u>naming</u>)-structure : of *abstract-geometric* "*point*(s)".

Begin a process of making *such a* 'concept of *bounded*-color flexibility' more precise. examine a (max representive)relative relation, which is defined

'at least' for any "(a,b) ≥ 0 <u>but</u> not-*both* (a,b)=0" by:

further, for-all (a,b,c) ≥ 0 , { if 'at most' one-member of the triad=0 };

<u>then</u>: $d^*(a,b)$ becomes a standard-metric relation under

the axiomization or restraint-criteria,

$$\begin{array}{lll} \mbox{A1.} & d^*(a,b) \geqslant 0 \ ; \\ \mbox{A2.} & d^*(a,b) = \ 0 & \mbox{iff } a = b \ ; \\ \mbox{A3.} & d^*(a,b) = \ d^*(b,a) \ ; \\ \mbox{A4.} & d^*(a,c) \leqslant \ d^*(a,b) + \ d^*(b,c) \ . \end{array}$$

where all must hold -true.

that the first three properties (ie. A.1-3) hold: is obvious,

there are (6)cases ,based on the orderings of (a,b,c) , for the fourth (A4.) ; similar 'proof-grammars' may be paired as below. for which (A4.) is first simplified by an instantiation relative to the particular axiomization-and-ordering ;and then *only-one* of the 'similar-grammars' is presented (for conciseness):

$$\begin{array}{c|c} a\leqslant b\leqslant c & \dots & \frac{c-a}{c}\leqslant \frac{b-a}{b}+\frac{c-b}{c} \ becomes \ \frac{b-a}{c}\leqslant \frac{b-a}{b} \ (but \ c\geqslant b \ and \ (b-a)\geqslant 0) \\ \hline \\ a\leqslant c\leqslant b \\ b\leqslant a\leqslant c & \dots & \frac{c-a}{c}\leqslant \frac{a-b}{a}+\frac{c-b}{c} \ becomes \ \frac{b-a}{c}\leqslant \frac{a-b}{a} \ (but \ left \ is\leqslant 0 \ ; while \ right \ is\geqslant 0.) \\ b\leqslant c\leqslant a \\ c\leqslant a\leqslant b & \dots & \left[\begin{array}{c} \frac{a-c}{a}\leqslant \frac{b-a}{b}+\frac{b-c}{b}=2-\frac{(a+c)}{b} \ (then \ mult. \ both \ sides \ by \ "ba") \ giving \dots \\ ba-bc \ \leqslant 2ba-ca-a^2 \ (but \ b\geqslant a), \ and \ (a+c) \ may \ be \ canceled). \end{array} \right]$$

let-s now take care of the: intended interpretation and motivation underlying the notation *max_{rep}(a,b)*, augmented here: as a restriction to stability | (for addition by zero with-in the absolute-value braces) |.
 ... by-example notice: if we simply-*initialize* (a=5) and (b=3); then under *one-possible* interpretation

$$d^{*}(a,b) = \frac{|a-b|}{max_{rep}(a,b)} \implies \frac{|5-3|}{5} \text{ or then} \implies \frac{|5-3+(1-1)|}{max_{rep}(a,b)} \dots \implies \frac{|4-2|}{4} \dots \implies \dots |1| \dots$$

which ,at-once *fascinating*, is restricted <u>here</u> by and< *with-in the-scope* of " max_{rep} " > to fix (a, and, b) as constants. that is: view max_{rep} then as an injective sample-and-<u>hold</u> in the denominator, *such-that* d*(a,b) becomes: notationally-concise(d).. and sufficient to that (*intuitive*)- description,... ..the "equals-sign" makes sense,.. and: d*(a,b) is as such in-the-short term stabilized . and then

- d. depictions of channel-width(s) [internally] defined by : <u>commonly</u>-indexed- members of the above inner/outer-sheaths, and, d*(a,b); <u>persist</u> strictly larger than zero, and, less than one: where (less than one) arrives, from inner-sheathing existing as *ei-disconnected*, and, (greater than zero) from local separation provided by (f_n) (again see. 26). _this, in effect produces a visual-interpretation which may, easily be *extended into*, a *collapsing- punctured- series* (of occupied $\frac{1}{2}$ disks), with referencing shifted to, or of-essence
 - * <u>folded</u>-in and compared-with an outer-converging sheath-horizon, instead of some ultimate-convergence as gifted by a constant-*representative*, (thus giving one perspective, example and/or visual domain for a non-degenerate (*)internal-metric).
- * 28. recall (again see 26.) that we are in a process of examining and/or then showing

 $[\ldots []SLS/RS(d)m\ldots]r_1 + [\ldots []SLS/RS(d)m\ldots]r_2$

as-and-to-be: "an algebraically-defocusing family of partially-unstable-internal-type-mappings", and

as such <u>prove</u>: (for any- $f_1 \in [...[]$ SLS/RS(d) $m \dots$]r1 that such a [coarsely bound] descriptive interior-shell provides-for: the choice of (and so, there latently-exists) a same-side *color-preserving* <u>companion</u>

 $f_2 \in [\dots [] SLS/RS(d)_{\mathfrak{m}} \dots]r_2$ such that $(f_1+f_2) \in [\dots [] SLS/RS(d)_{\mathfrak{m}} \dots]r_1+r_2$ ($\in \mathbb{R}_+$: and which is also of the same-side)).

the proof is obvious:

<u>first</u> <u>since</u>: we have assumed $f_1 \in [...[]_{SLS/RS(d)m...}]_{r_1}$, rewritten here (by 17.a) as $(f_1 = Cr_1 \mp *f_1)$; we may <u>then</u>: describe(by 16.a) an exactly-similar sequence $(f_2 = Cr_2 \mp *f_1) \in [...[]_{SLS/RS(d)m...}]_{r_2}$ such that , in effect, besides *constant-representative offsetting*, *f_1 is added to itself; that is then,

since { for-all 'r.' $\in \mathbb{R}_1$ | (r. + r. = (2)r.) }, essentially from the properties of a (complete, ordered, field see 2.), and so

<u>by</u> (term-by-term substitution see: 15., and the above), then $(f_1+f_2) = (Cr_1+Cr_2) \mp C_{r(2.)}*f_1$ where

 $C_{r(2.)}$ is, as such, a sequencially derived and then forced *positive*-constant-representive.

next by(18.e), since

 $C_{r(2.)} \in e_{i \otimes} <ie. is of the (multiplicitive)-spectral-identity-class for the "over-all" distributed-internal- domain >: and as (Cr_1+Cr_2) is simply the root-and/or-trunking (as it were) to the offset-foliation provided by (<math>\mp C_{r(2.)}*f_1$) the result ..follows.

- * a. then notice: this event, is *completely*-independent of underlying type. that is: <u>under the current spectralizaton</u> *every-*f* ∈ e_{i⊕(*f)} (ie... is-of the (additive)-spectral-identity-class associated-with that "particular" *f)
 restate this again: in effect "addings of [(*exactly-similar*)-name-element(s)] to themselves preserves color..."
 - b. finally: as sequencial addition is algebraically-localized by defining it to be term-by-term(again see: 15.);
 <u>then</u> the commutative-(and other)-properties of such binary-additions pass through uneffected into the internal-domain. <u>and as such both</u>, here and in the previous (see:26. et pre alibi.), ordering(s) of operands is irrelevant then to the-discussions for such families of mappings,
 - and as : in both the cases:(26. and 28.)

(LS/RS same-side ness) follows simply-then and-again from (22.c) same-side-addition,

.and.

(d) and/or ei-disconnected iness from (23.) color-dominance,

c. then: by the immediately above (28.) ,and/or, by a demonstration of an existence of both-cases...it follows that ,with-in the courseness of our descriptive-binding , the <u>partially</u>-unstable claim
 (ie. a potential loss of (any specific)- <u>monotic</u>- spectralization) may be robustly represented -and- notationalized here then as;

 $\left[\hdots \left[\hdots \right] SLS/RS(d) \mathbf{m} \dots \right] r_1 + \left[\hdots \left[\hdots \right] SLS/RS(d) \mathbf{m} \dots \right] r_2 \hdots \left[\hdots \right] SLS/RS(d) \dots \right] r_1 + r_2$