Recurrent formulas which conduct to probably infinite sequences of primes and a generalization of a Cunningham chain

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I define few formulas which conduct from any odd prime respectively from any pair of distinct odd primes to an infinity of probably infinite sequences of primes, also to such sequences of a certain kind of semiprimes, and I also make a generalization of a Cunningham chain of primes of the first kind, respectively of the second kind.

Conjecture 1:

Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Examples:

First such prime for [p, n] = [104729, 2]

: 104729*2 + 1 = 209459 prime.

First such prime for [p, n] = [104729, 3]

: 104729*3 + 2 = 314189 prime.

First such prime for [p, n] = [104729, 4]

: 104729*4 + 3 = 314189 prime.

First such prime for [p, n] = [104729, 5]

: 104729*5 + 4 = 523649;
: 523649*5 + 4 = 2618249 prime.

Conjecture 2:

Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Examples:

First such prime for [p, q, n] = [104729, 3, 2]

: 104729*2 + 3 = 209461;
: 209461*2 + 3 = 418925;
: 418925*2 + 3 = 837853 prime.

First such prime for [p, q, n] = [104729, 7, 3]

: 104729*3 + 7*2 = 314201;
: 314201*3 + 7*2 = 942617;
: 942617*3 + 7*2 = 2827865;
: 2827865*3 + 7*2 = 8483609 prime.

First such prime for [p, q, n] = [104729, 13, 4]

: 104729*4 + 13*3 = 418955;
: 418955*4 + 13*3 = 1675859 prime.

First such prime for [p, q, n] = [104729, 17, 7]

: 104729*5 + 17*4 = 523713;
: 523713*5 + 17*4 = 2618633 prime.

Definition:
(generalization of a Cunningham chain of the first kind)

We name a Cunningham-Coman chain of primes of the first kind the primes p1, p2, ..., pk obtained through the recurrent formula: p2 = 2*p1 + q, p3 = 4*p1 + 3*q, p4 = 8*p1 + 7*q, ...,pi = 2^(i – 1)*p1 + 2^(i – 1) – 1, where q is an odd prime.

Example:

For q = 5, we have the following Cunningham-Coman chain of length 6 starting with p1 = 13: 13, 31 (= 2*13 + 5), 67 (= 4*13 + 3*5), 139 (= 8*13 + 7*5), 283 (= 16*13 + 15*5), 571 (= 32*13 + 31*5).

Conjecture 3:

Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Note: In a previous paper, "Two exciting classes of odd composites defined by a relation between their prime factors" I defined Coman semiprimes of the first kind the semiprimes p*q with the property that q1 – p1 + 1 =

p2*q2, where the semiprime p2*q2 has also the property that q2 – p2 + 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk – qk + 1 is a prime. I also defined Coman semiprimes of the second kind the semiprimes p*q with the property that q1 + p1 - 1 = p2*q2, where the semiprime p2*q2 has also the property that q2 + p2 - 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk + qk - 1 is a prime.

Examples:

First such semiprime for [p, n] = [104729, 2]

: 104729*2 + 1 = 209459;
: 209459*2 + 1 = 418919;
: 418919*2 + 1 = 837839;
: 837839*2 + 1 = 1675679;
: 1675679*2 + 1 = 3351359;
: 3351359*2 + 1 = 6702719 = 139*48221, which is a Coman semiprime because 48221 – 139 + 1 = 7*6869 and 6869 – 7 + 1 = 6863 which is prime.

Conjecture 4:

Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Examples:

First such semiprime for [p, q, n] = [104729, 7, 3]

: 104729*3 + 7*2 = 43*7307, which is a Coman semiprime of the second kind because 7307 + 43 – 1 = 7349, which is prime.

First such semiprime for [p, q, n] = [104729, 11, 5]

: 104729*5 + 11*4 = 523689;
: 523689*5 + 11*4 = 2618489 = 547*4787, which is at the same time a Coman semiprime of the first kind because 4787 – 547 + 1 = 4241, which is prime, and a Coman semiprime of the second kind, because 4787 + 547 – 1 = 5333, which is also prime.

Definition:

We name *an absolute Coman semiprime* a semiprime which is at the same time Coman semiprime of the first kind and Coman semiprime of the second kind.

Conjecture 5:

Let p be an odd prime; then the formula ((((n*p - n + 1)*p - n + 1))*p - n + 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 6:

Let p and q be distinct odd primes; then the formula ((((n*p - (n + 1)*q)*p - (n + 1)*q)*p - (n + 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Definition:
(generalization of a Cunningham chain of the second kind)

We name a Cunningham-Coman chain of primes of the first kind the primes p1, p2, ..., pk obtained through the recurrent formula: p2 = 2*p1 - q, p3 = 4*p1 - 3*q, p4 = 8*p1 - 7*q, ...,pi = 2^(i – 1)*p1 - 2^(i – 1) + 1, where q is an odd prime.

Conjecture 7:

Let p be an odd prime; then the formula ((((n*p - n + 1)*p - n + 1))*p - n + 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 8:

Let p and q be distinct odd primes; then the formula ((((n*p - (n + 1)*q)*p - (n + 1)*q)*p - (n + 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.