Gravitons, the Speed of Gravity, and the Generalized Newton Gravitational Law

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ABSTRACT: In many publications and web forum discussions the claims are constantly being made that in order for the orbits of planets around the Sun to be stable the gravity must propagate at much higher speeds than the speed of light *c*. In this paper it is shown on a simple and extreme example of two stars orbiting around each other in a circular orbit that this is not the case and that the assumption about the necessity for the large speed of gravity is unfounded. The explanation is based on the recognition that the Newton gravitational force has two components that are not necessarily collinear. This new fundamental finding is supported by modeling the field by gravitons that mediate the force of the field. This model finally leads to the generalization of Newton gravitational law that correctly accounts for the finite speed of gravity. From this result it is also found that the gravitational aberration angle is identical with the aberration angle of light, but is aiming in the opposite direction, lagging behind the source of the gravitational attraction.

Keywords[1]**:** speed of gravity, aether, dark matter, stability of planetary orbits, Newton third law of action and reaction, gravitons, generalized Newton gravitational law, aberration of gravity, barycenter

1. Introduction

There seems to persist, in the published literature, a strange controversy about the speed of gravity. Several well-known scientists have claimed that the speed of gravity is very high, much higher than the speed of light *c*. This claim is purportedly necessary and is in an agreement with the fact that the orbits of planets around the Sun are stable. The well-known proponent of this claim was, for example, Van Flandern (1998) who published a paper in the prestigious refereed journal, Physics Letters, where he discussed this issue in a great detail. This claim, however, was successfully criticized, according to this author's opinion, by Carlip (1999). Despite of this well based criticism the claims are still appearing in the literature again and again in support of the Van Flandern's position. A review of this topic can be found in Wikipedia: (http://en.wikipedia.org/wiki/Speed_of_gravity), with several interesting contributions from Kopeikin (2004) and the Kopeikin's critic (Samuel, 2004).

In this article a simple case of the two orbiting stars with identical masses moving in a circular orbit around each other is analyzed and a conclusion reached, which supports the Carlip's criticism. The circular orbit and the identical masses of the stars were selected for the purpose of simplifying the analysis and to show only the fundamental principle underlying the fact that the trajectories are stable. Extension to arbitrary masses and elliptical orbits can then be made, but this will not be pursued any further and in any more detail in this article.

2. Assumptions

In addition of these two simplifying assumptions stated above it will also be considered that the standard Newton gravitational physics operating in a flat space-time can be used, which means that the space-time curvature caused by the gravitational field of the stars can be neglected. However, the stars' gravitational and inertial masses will be considered dependent on velocity as derived in the Special Relativity Theory (SRT) or by the author elsewhere (Hynecek, 2011). Nevertheless, it is still necessary to keep in mind that the gravitational field is the distortion of the space-time, the distortion of the dark matter, or the distortion of the older aether, but it will be considered here to be so small that it will not significantly affect the distances that have to be input into the Newton gravitational law. It will also be considered that the gravity follows the wave equation thus propagating with the speed of light *c*, and is therefore subject to the rules of SRT. The gravitational field potential will be considered a scalar quantity and no other forces, in particular the gravitomagnetic force, will be considered. The finite speed of gravity propagation will manifest itself in the retarded positions of stars that will have to be used in the gravitational law.

3. Mathematical Description of the Problem

The instant positions of the star **A** and the star **B** on the circular orbit and along the x axis are shown in Figure 1. The figure also indicates the position of the star **B** when its gravitational field is sensed by the star **A**. This position is indicated by the letter **B'**. The position of the star **B** when it senses the field from the star **A** is then indicated by the letter **B''**.

The important point here is to realize that the centripetal force consists of the two vector components that are not collinear. This may seem odd on the surface and contradicting the Newton's third law, which claims that the action and reaction should always be balanced and pointed in opposite directions. What is balanced here is the sum of the two non collinear centripetal force vectors with the inertial centrifugal force of rotation. No other force, similar to the magnetic force of the Maxwell's EM field theory resulting from the motion of the source is assumed. It was shown elsewhere (Hynecek, 2013) that there is no such gravitomagnetic force.

The first component of the centripetal force vector is from the field of the star **B** when it was in the position **B'**. The time for the field to reach the star **A** is equal to:

$$t = a / c \qquad (1)$$

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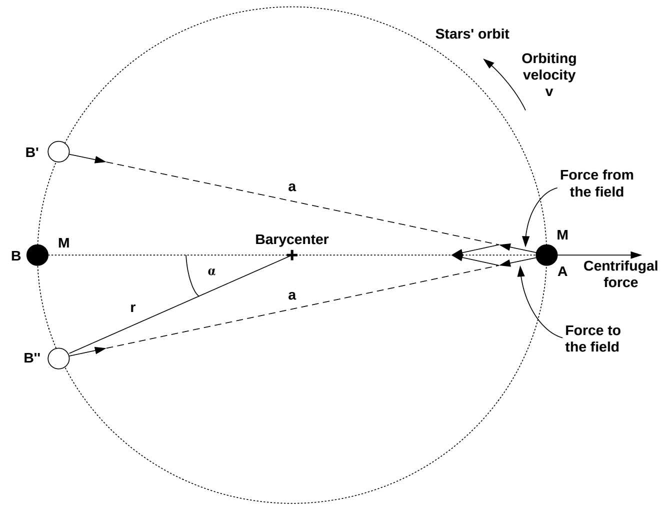

Fig.1. The two stars each having the mass ***M*** are orbiting each other on the same circular orbit. The centrifugal force balances the two centripetal force components, one received from the field of the star **B** when it was in the position **B'** and the other, the reaction force to the force delivered to the field by the star **A**. This force vector is aiming at the position **B''** where the star **B** will be in the future when this field reaches it.

The second component is the reaction force of the star **A** to the force that is exerted from its own field in order to drag it along as the star moves. It is important to realize that the space that has the field in it is a real physical entity that is distorted by the gravity of the stars' masses and thus has energy stored in it and can transmit force. The traditional concept of vacuum with nothing in it thus cannot work here. This can also be considered as an evidence for the existence of aether. The aether must be there to save the causality principle. The star **A** receives the force from the field and immediately adds to it the reaction force resulting from the force delivered to its own field. The result is then balanced by the inertial centrifugal force of rotation. It is shown in the next section of the paper that the angles these two centripetal force vectors span with the radial direction to the barycenter are identical and equal to $\alpha/2$. This is interesting since on the first look it seems as if the star **A** is anticipating where the star **B** will be in the future and is aiming the field force vector in that direction. How does it know? It will be shown later that this is the unavoidable consequence of motion of the star **A** along the same orbit with the same orbital velocity as the star **B**.

For the angle α in Figure 1 we have:

$$\alpha = \frac{\omega\, a}{c} \tag{2}$$

where ω is the angular velocity of the stars' rotation around the common barycenter. In the next step it is necessary to find the distance a over which the gravity is acting in order to use it in the Newton gravitational law. It is not difficult to show that for the small angles α it holds that:

$$a = 2r\cos(\alpha/2) \cong \frac{2r}{\sqrt{1+(\omega\ r/c)^2}} \tag{3}$$

where r is the radius of the orbit. The balance equation between the centrifugal and centripetal forces can then be written as:

$$\begin{gathered}\frac{\kappa M_0}{a^2}\left(1-\omega^2 r^2/c^2\right)\cos(\alpha/2) = \\ \frac{\kappa M_0}{a^2}\left(1-\omega^2 r^2/c^2\right)\frac{a}{2r} = \frac{\omega^2 r}{\sqrt{1-(\omega\ r/c)^2}}\end{gathered} \tag{4}$$

In this equation it was considered that the gravitational and the inertial masses depend on the velocity differently as follows:

$$M_g = M_0\sqrt{1-(\omega\ r/c)^2} \tag{5}$$

$$M_i = \frac{M_0}{\sqrt{1-(\omega\ r/c)^2}} \tag{6}$$

where for the velocity it was substituted that: $v = \omega\ r$. After rearrangement of Equation 4 and substitution for the distance a, still considering only small orbital velocities in comparison to c, the result becomes:

$$\omega^2 = \frac{\kappa M_0}{2r^2 a}\left(1-\omega^2 r^2/c^2\right)^{\frac{3}{2}} = \frac{\kappa M_0}{4r^3}\frac{1}{1+(1/8)R_s/r} \tag{7}$$

Here the symbol R_s is the orbiting star Schwarzschild radius: $R_s = 2\kappa M_0/c^2$ and κ is the gravitational constant. The term $\kappa M_0/4r^3$ follows also from the Kepler's third law for circular orbits. It is thus clear that the finite speed of propagation of the gravitational field does not cause any problems; it just slightly reduces the stars' orbital angular velocity ω. For the classical SRT mass dependence on velocity the resulting formula is similar with only a sign change in the denominator of the fraction in Equation 7:

$$\omega_{SRT}^2 = \frac{\kappa M_0}{4r^3}\frac{1}{1-(1/8)R_s/r} \tag{8}$$

The key point of this derivation and thus the proof that there is no net angular momentum loss or loss of energy up to and including the second order of v/c is due to the fact that the stars' acceleration vector is always perpendicular to the stars' velocity vector. This proof rests on the claim that the reaction force from the field of the star **A** is aiming exactly at the future position of the star **B**, which may seem strange and seemingly contradicts the causality principle. The details of the derivation of this fact are given in the next section.

4. Gravitons as the Exchange Force Particles

In order to demonstrate that there are two non-collinear components of the centripetal force derived from the gravitational field it is best to represent this field as an exchange force of graviton particles. However, these particles must have an unusual property with their linear momentums being negative, acting in the opposite direction to their velocity vector. It will also be assumed that the gravitons are moving similarly as photons with the speed of light c thus having only the inertial mass and no gravitational mass.

$$\vec{p}_g = -m_{ig}\vec{c} \tag{9}$$

One possible way to visualize the action of such particles is to consider them as holes or vacancies moving in the uniform crystalline-like aether-forming background. The vector diagram representing the interaction of these particles with the moving massive body **A** is shown in Figure 2.

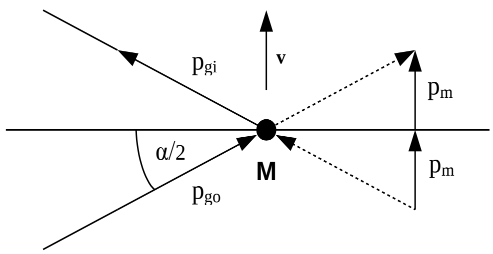

Fig.2. The vector diagram showing the impact of gravitons on the moving mass ***M***. Because the gravitons have a negative linear momentum the fictitious particles with a positive linear momentum were added to the diagram and are indicated there by the dotted lines. The mass ***M*** is moving with the velocity v in the y direction.

In order to better understand this diagram and the action of the negative linear momentum particles the gravitons can be replaced by fictitious particles with positive linear momentum acting on the body **A** from the opposite side. This is shown in the diagram by the dotted line vectors. The absolute value of the aggregate momentum of many incoming gravitons can then be found from the Newton second law as follows:

$$F_g = \frac{dp_g}{dt} = \frac{1}{2}\frac{\kappa M^2}{a^2} \tag{10}$$

Integrating the formula in Equation 10 over the time it takes for the gravitons to travel from the body **B** to the body **A** the aggregate linear momentum for the incoming gravitons is found and is equal to:

$$p_{gi} = \frac{1}{2}\frac{\kappa M^2}{ac} \tag{11}$$

The aggregate inertial mass of all the impinging gravitons is thus equal to:

$$m_{ig} = \frac{1}{2}\frac{\kappa M^2}{ac^2} \tag{12}$$

When these gravitons interact with the moving body **A**, they pick up the linear momentum from this body during the collisions, which is equal to:

$$\vec{p}_m = m_{ig}\bar{v} \tag{13}$$

This is also indicated in the drawing in Figure 2. Since the linear momentum conservation must be satisfied, the following equation for the outgoing graviton impact angle α must hold:

$$\left(2m_{ig}v\right)^2 = p_{gi}^2 + p_{go}^2 - 2p_{gi}p_{go}\cos\alpha \tag{14}$$

The solution of this equation is simple to find assuming that the absolute values of the graviton linear momentums are identical and that the velocities of the stars are small in comparison to c:

$$\sin(\alpha/2) = v/c \tag{15a}$$

$$\cos(\alpha/2) = \sqrt{1 - v^2/c^2} \cong \frac{1}{\sqrt{1 + v^2/c^2}} \tag{15b}$$

This is again the same formula as derived in Equation 3.

It is thus clear that the moving body must be exerting a force on the field and that the Newton gravitational force has always two components either collinear for stationary bodies or spanning a certain angle between them for the moving bodies.

This is typically not recognized in the main stream literature, which leads to problems such as the claims for necessity of the large propagation speed of gravity.

It would also be worthwhile to revisit the Lorentz force equation of Maxwell EM field theory for moving charged bodies to make sure that this problem is not repeated there. This work will be left for the future publications.

Finally it is necessary to mention that the Newton force component introduced in this article may be different than the self-force of charged particles described in many publications, for example in Wald (2009), and it is also different from the force of radiation reaction as discussed in a paper by Johnson and Hu (2001) when charged or only massive particles are accelerated.

5. Gravitational field vector direction of a moving mass

Another way to find the direction of force when the mass is moving with a velocity v along the x axis and is passing the observer located at the distance y from the x axis is with the help of the drawing in Figure 3. The observer senses the gravitational field intensity E_g described by the formula:

$$E_g = \frac{\kappa M}{a^2} \tag{16}$$

The distance a, however, must include the retarded position of the mass M that depends on the propagation speed of gravity. The amount of the retardation distance Δ is equal to $\Delta = a\,v/c$. The distance a is then calculated from the equation:

$$a^2 = y^2 + (x - av/c)^2 \tag{17}$$

and is equal to:

$$a = \frac{1}{(1 - v^2/c^2)}\left(-x\frac{v}{c} + \sqrt{x^2 + y^2\left(1 - v^2/c^2\right)}\right) \tag{18}$$

Knowing now the distance a the gravitational field intensity $E_g(x,y)$ can be calculated for any value of x and y as is shown in Figure 4. This equation represents a 2D surface and the direction at which the field intensity E_g is aiming at is found by the well-known principle of the steepest descent. By differentiating Equation 18 with respect to x, the result is:

$$x_{\max} = y\frac{v}{c} \tag{19}$$

From this result then follows that for the angle $\alpha/2$ it is:

$$\cos(\alpha/2) = \frac{1}{\sqrt{1+v^2/c^2}} \tag{20}$$

This is exactly the same result as in Equation 3. The motion of the mass along the x axis is thus responsible for the deviation of the maximum field intensity vector E_g from the perpendicular direction. As a result the field intensity vector of the star **A** is automatically aimed at the future position of the star **B**, since both stars are rotating at the same angular velocity ω and are following the same orbit.

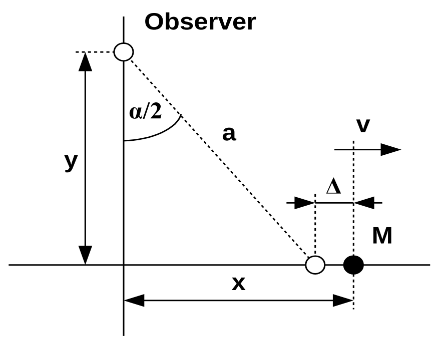

Fig.3. The gravitational field intensity E_g at the position of the observer from the moving mass M, which moves along the x axis. The retarded position of the mass M is indicated by the distance Δ.

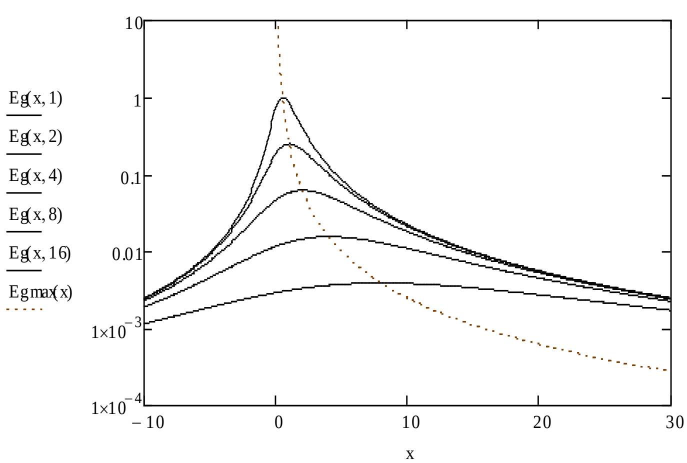

Fig.4. The normalized field intensity observed at the distance y from the x axis as a function of the position of the mass along the x axis. The locations of the maxima of the field intensity are indicated by the dotted line.

6. Generalized Newton Gravitational Law

Following the derivation of the field vector direction and the concept of gravitons as being the exchange particles of gravitational force, the Newton gravitational law for the force acting on the moving body **A** by the body **B,** which will be for the simplicity considered stationary, can thus be generalized to account for the finite speed of gravity as follows:

$$\vec{F}^{\perp}_{gAB}(r,t) = -\frac{\kappa M_A M_B \cos(\alpha/2)}{r^2(t)} \frac{(\vec{r}(t) - \vec{v}_{\perp} r(t)/c)}{r(t)\cos(\alpha/2)} \tag{21}$$

The derivation of this formula follows from the similar reasoning as the derivation of formula in Equation 4. In addition it was also considered that: $v << c$, and that $\vec{v}_{\perp}$ is the velocity vector component perpendicular to the resulting force vector direction, not to the vector r. By adding the force contribution due to the parallel velocity component and after somewhat lengthy but simple straight forward calculations as shown in the Appendix, Equation 21 can finally be rearranged to read:

$$\vec{F}_{gAB}(r,t) = -\frac{\kappa M_{A0} M_{B0} \sqrt{1-v^2/c^2}}{r^3(t)} \left(\vec{r}(t) - \vec{v}\frac{r(t)}{c} \right) \tag{22}$$

This is the generalized Newton gravitational law. In this equation the vector r represents the instantaneous physical distance from **B** to **A** and v is the velocity vector of the body **A**. The mass $\boldsymbol{M_A}$, depends on velocity and the dependence that was used in Equation 22 was according to Equation 5. The interesting feature of this formula is that the force acting on the body **A** is not aiming exactly in the direction of the body **B**. This is not expected from the traditional considerations of the Newton's law. The direction deviation, however, is very small and this may be the main reason why it is usually neglected. The formula must revert to the standard Newton gravitational law formula for $v = 0$, which is easily confirmed. For the standard SRT mass dependence on velocity of body **A** the formula in Equation 22 becomes:

$$\vec{F}_{gAB(SRT)}(r,t) = -\frac{\kappa M_{A0} M_{B0}}{r^3(t)\sqrt{1-v^2/c^2}} \left(\vec{r}(t) - \vec{v}\frac{r(t)}{c} \right) \tag{23}$$

Perhaps it might be possible to determine, from the very precise drop tower testing (Bremen, 2011), or from astronomical observations of stars orbiting the galaxy center in the Milky Way galaxy, which of these formulas actually corresponds to reality.

In the strong gravitational fields the natural distances between the bodies and the natural time are being distorted by the fields. This also applies to the speed of light that is reduced in the strong gravitational field as is well known. The exact calculation of force and consequently the motion of such very massive bodies thus becomes a complicated problem that can be handled only by numerical calculations.

7. Aberration of Gravity

An interesting observation that should also be commented on in Equation 22 or in Equation 23 is that the formulas are similar to formulas for the aberration of light that would follow from the observation of light propagation between these bodies. This is easily seen when the velocity vector v is perpendicular to r. Ofcourse, this relates only to the force vector direction and not to the absolute value. However, even though the magnitude of the angle for the gravitational aberration is the same as for the aberration of light: $\sin(\alpha_a) = v/c$, the direction is negative, aiming behind the source of gravity. This difference will, for example, cause the delay between the observation time of high noon and the high tide component that is caused by the Sun (Neap tides), not by the Moon. The delay, however, is very small:

$$t_d = \frac{2}{\pi}\frac{v_S}{c} 4.32 \cdot 10^4 s = 2.732 \ s \tag{24}$$

where v_S = 29.78 *km/sec* is the average orbital speed of Earth on its trajectory around the Sun. Such a small delay most likely cannot be observed.

8. Barycenter Coordinates and the Angular Momentum

Before the summary and conclusions of this paper it is also necessary to make a comment on the barycenter coordinates for the two orbiting bodies and calculate the total angular momentum.

It will now be assumed that the body **B** is also moving and that the total linear momentum of the system is zero. It must therefore hold that:

$$M_A \vec{v}_A = M_B \vec{v}_B \tag{25}$$

The formula in Equation 25 can be further modified using the relation: $\vec{v}_A = \vec{v}_{A\perp r} + \vec{r}_A(\vec{v}_A \cdot \vec{r}_A / r_A^2)$ to read:

$$M_A(\vec{v}_{A\perp r} + \vec{r}_A(\vec{v}_A \cdot \vec{r}_A / r_A^2)) = M_B(\vec{v}_{B\perp r} + \vec{r}_B(\vec{v}_B \cdot \vec{r}_B / r_B^2)) \tag{26}$$

By defining the distance between the mutually orbiting bodies as: $r = r_A + r_B$, using the relations: $v_{A\perp r} = r_A\omega$ and $v_{B\perp r} = r_B\omega$, and considering that the velocity vector is essentially independent of the radial vector, the distance from the barycenter to the body **A** is then found form Equation 26 as follows:

$$r_A = \frac{r}{1 + M_A / M_B} \tag{27}$$

From Equation 25 and Equation 26 also follows that: $\vec{r}_B \cdot \vec{v}_A = \vec{r}_A \cdot \vec{v}_B$. The SRT relativistic corrections can be added to the masses as well as already discussed previously. Equation 27 is the traditional formula for a barycenter as is widely popularized by NASA (2005) in their educational website.

In addition of the barycenter coordinates it is interesting to investigate how the angular momentum is affected by the generalized Newton gravitational law. For the body **A** the angular momentum is defined as follows:

$$\vec{\Omega}_A = M_A \vec{v}_A \times (\vec{r}_A - \vec{v}_A r_A / c) = M_A \vec{v}_A \times \vec{r}_A \tag{28}$$

Because the vector (cross) product of identical vectors is zero, the angular momentum is not affected by the new generalized Newton gravitational law.

Equation 28 can be modified using the relation: $\vec{v}_A = \vec{v}_{A\perp r} + \vec{r}_A(\vec{v}_A \cdot \vec{r}_A / r_A^2)$ resulting in the formula:

$$\Omega_A = M_A r_A v_{A\perp r} \tag{29}$$

which can be further modified to:

$$\Omega_A = M_A r_A^2 \omega \tag{30}$$

Using this formula and the formula in Equation 27 it is possible to derive the following relation between the angular momentums of mutually orbiting bodies.

$$\Omega_A M_A = \Omega_B M_B = \left(\frac{M_A M_B}{M_A + M_B}\right)^2 r^2 \omega \tag{31}$$

and finally find the expression for the total angular momentum of the system:

$$\Omega_A + \Omega_B = \frac{M_A M_B}{M_A + M_B} r^2 \omega \tag{32}$$

An interesting and the well-known fact to note here is that most of the total angular momentum of the system is carried by the star with the lighter mass.

9. Conclusions

In this article it was clearly shown that the standard Newtonian physics supplemented by the finite propagation speed c of the gravitational field does not lead to unstable trajectories of bodies in orbit. This derivation can be generalized to any masses and trajectories and also include the deformation of the natural space-time by the gravity. This was all possible by considering that there is a medium, invisible aether, or the transparent dark matter, in the space between the bodies that can be deformed and thus can propagate forces and store energy. The traditional vacuum whose existence is typically postulated between the gravitating bodies with nothing in it is thus not a reasonable concept and cannot explain the observed stability of planetary orbits. This stability can thus be considered as a proof of the aether existence or the existence of any other deformable medium irrespective of the name that may be chosen for it. The model that was used for the mathematical description of action of this medium was based on the graviton particles that have a negative linear momentum, the finite inertial mass, the zero gravitational mass, and propagate with the same velocity c as light.

It was also shown that the standard Newton gravitational force always consists of the two components: the force component of the field acting on the massive body and the force component of the massive body acting on the field. These two force components may not always be collinear. This fundamental finding thus allows the Newton gravitational law to be properly generalized and to correctly account for the motion of gravitating bodies.

Finally, it was also concluded that the aberration of gravity is identical to the aberration of light as for the magnitude of the angle, but it is aiming in the opposite direction, lagging behind the source.

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Appendix

The details of the derivation of generalized Newton gravitation law can be easily understood from the following steps:

When the velocity vector is perpendicular to the resulting force vector, the unit vector in the direction of the force can be written as follows:

$$\vec{f}_0^{\perp} = -\frac{\vec{r} - (\vec{v}_\perp / v_\perp) r \sin(\alpha/2)}{r(t)\cos(\alpha/2)} \tag{A1}$$

This can be clearly seen from the Figure 2. Substituting for the sin(a/2) the expression from Equation 15:

$$\sin(\alpha/2) = v_\perp / c \tag{A2}$$

the unit vector equation for the force direction can then be written as:

$$\vec{f}_0^{\perp} = -\frac{\vec{r} - \vec{v}_\perp r(t)/c}{r(t)\cos(\alpha/2)} \tag{A3}$$

For the gravitational force acting on the body **A** it is then possible to write:

$$\vec{F}_{gAB}^{\perp} = \frac{\kappa M_A M_B}{r^2(t)} \cos(\alpha/2)\, \vec{f}_0^{\perp} \tag{A4}$$

and simplify the result after the substitution from Equation A3 as follows:

$$\vec{F}_{gAB}^{\perp} = -\frac{\kappa M_A M_B}{r^3(t)} \left(\vec{r}(t) - \vec{v}_\perp \frac{r(t)}{c} \right) \tag{A5}$$

This is the simplified formula introduced in Equation 21 with cos(a/2) cancelled out.

In the next step the perpendicular velocity component can be expressed as follows:

$$\vec{v}_\perp = \vec{v} - \vec{f}_0(\vec{f}_0 \cdot \vec{v}) = \vec{v} - \vec{f}_0\left(\vec{f}_0 \cdot (\vec{v}_\perp + \vec{v}_\parallel)\right) = \vec{v} - \vec{f}_0 v_\parallel \tag{A6}$$

Substituting this expression into Equation A5 then results in:

$$\vec{F}_{gAB}^{\perp} = -\frac{\kappa M_A M_B}{r^3(t)} \left(\vec{r}(t) - \vec{v}\frac{r(t)}{c} + \vec{f}_0 v_\parallel \frac{r(t)}{c} \right) \tag{A7}$$

In the last step of the derivation it is only necessary to add to the expression the force contribution resulting from the parallel velocity component:

$$\Delta F_{gAB}^{\parallel} = \frac{\kappa M_A M_B}{r(t) c^2} v_\parallel \frac{c}{r(t)} \tag{A8}$$

which, interestingly enough, just cancels the last term in the parentheses in Equation A7. This results in the final expression for the generalized Newton gravitational law as follows:

$$\vec{F}_{gAB} = -\frac{\kappa M_A M_B}{r^3(t)} \left(\vec{r}(t) - \vec{v}\frac{r(t)}{c} \right) \tag{A9}$$

This formula was used in Equation 22 and Equation 23 after the substitution of respective dependencies of gravitational masses on velocity.

The formula in Equation A9 was derived in the reference frame of the body **B** that was not moving. When the body **A** is moving only in the direction of the vector r this formula can be simplified as follows:

$$F_{gAB}^{\parallel} = \frac{\kappa M_{A0} M_{B0}(1 - v^2/c^2)}{r^3(t)} \frac{r \pm v t_A}{\sqrt{1 - v^2/c^2}} \tag{A10}$$

where t_A was substituted back into the equation for the graviton time of flight from the body **B** to the body **A**. It is now easy to transform this formula, using the Lorentz coordinate transformation, so that the force on the body **A** is referenced to the coordinates moving with that body. The result is as follows:

$$F'_{gBA} = \frac{\kappa M_{A0} M_{B0}}{r'^2(t')} \frac{1}{\sqrt{1 - v^2/c^2}} \tag{A11}$$

This result thus reverts back to the standard Newton gravitational law as expected with the length contraction of the distance r included. This distance as it appears in the denominator is, of course, considered here as an instantaneous distance difference, the absolute value of the vector r. The apostrophe indicates the moving reference frame values. The correctness of Equation A11 can be easily confirmed directly as follows:

$$F'_{gBA} = \frac{\kappa M_{A0} M_{B0} \sqrt{1 - v^2/c^2}}{r'^2(t')(1 - v^2/c^2)} = \frac{\kappa M_{A0} M_{B0}}{r'^2(t')} \frac{1}{\sqrt{1 - v^2/c^2}} \tag{A12}$$

where for the mass of body **B** dependence on velocity was substituted the relation from Equation 5. More details about the SRT force transformation formulas can be found elsewhere (Rindler, 2001).