The Light Wave Equation and the Special Relativity

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In a recent paper, it is asserted that the wave equation for the light in the vacuum cannot be used in the special relativity. However, it might be refuted.

Key words: light wave equation, special relativity.

We consider the following scenario. We have two reference systems, S and S', where S is at rest and S' is moving in the positive x coordinate direction with a constant speed v with respect to S. In addition, we have a wave ψ that propagates in the space, also in the positive x (coordinate) direction, with the same constant speed v with respect to S.

The so-called retarded time, t_r, is defined as follows. Let (x_1, t_1) and (x_2, t_2) be two points of S and ψ moves from the first point to the second one, then $t_2 - t_1 = (x_2 - x_1)/v$, and doing $t_r = t_1$, $t = t_2$ and $x = x_2 - x_1$, we have that $t_r = t - x/v$.

Then, with the retarded time, $t_r = t - x/v$, with the Galileo relativity, $x' = x - vt$, and with the Einstein special relativity (SR), $x' = \gamma(x - vt)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ and c is the speed of the light in the vacuum; we obtain (see appendix A) the wave equation in one dimension [1]:

$$\partial^2 \psi/\partial x^2 - (1/v^2)\partial^2 \psi/\partial t^2 = 0 \qquad (1)$$

And the wave equation for the light in the vacuum, which implies $v = c$, would be

$$\partial^2 \psi/\partial x^2 - (1/c^2)\partial^2 \psi/\partial t^2 = 0 \qquad (2)$$

But (2) cannot be used in the SR because $v = c$ implies $\gamma = \infty$.

This might be solved as follows. In the SR, for a particle, we have:

$E = \gamma mc^2$, where E is the energy and m the rest mass.

$p = \gamma mv$, where p is the momentum and v the speed.

From both equations it is obtained (see appendix B) that:

$$E^2 = p^2c^2 + m^2c^4 \qquad (3)$$

Then, for $v = c$, it is $p = \gamma mc$, and $pc = \gamma mc^2 = E$, which implies, from (3), $m = 0$.

Hence, for $v = c$, we have: $\gamma = \infty$ and $m = 0$.

On the other hand, for two inertial systems, S and S', it is: $f'/f = ((1 - v/c)/(1 + v/c))^{1/2}$ (from the relativistic Doppler effect), where f' and f are the frequencies of the light in the moving (S') and rest (S) frames, respectively, v being the moving speed of the primed frame. But also, $E'/E = ((1 - v/c)/(1 + v/c))^{1/2}$ (from the Lorentz transformation for the energy), then $E'/E = f'/f$, $E' = hf'$ and $E = hf$, which is the Planck-Einstein equation, h being the Planck constant [2] (pp. 142-143).

Hence, for the light, that is, for the photon it would be:

$\gamma = \infty$, $m = 0$, $E = pc$, $E = hf$, $p = hf/c$, and as $c = \lambda f$, where λ is the wavelength, then $p = h/\lambda$.

And also, as $\gamma mc^2 = E = hf$, then $\gamma m = hf/c^2$, which is the so-called "effective mass" of the photon, although $\gamma = \infty$ and $m = 0$.

Therefore, (2) would be valid also in the SR because of the effective mass of the photon.

Then, if we accept the SR, by virtue of it, (2) would be valid: 1) with the retarded time, $t_r = t - x/c$, for $v = 0$, where now v is the speed of the light source. 2) with the Galileo relativity, for $0 \leq v^2 << c^2$. And 3) with the SR, for $0 \leq v < c$.

In summary, although in a recent paper [1] it is asserted that the wave equation for the light in the vacuum cannot be used in the special relativity, this might be refuted.

Appendix A

From the retarded time, we have:

$\psi(x,t) = f(b) = f(t - x/v)$, where f is a function, with $b = t_r = t - x/v$
$\partial\psi/\partial x = \partial f/\partial x = (\partial f/\partial b)(\partial b/\partial x) = (\partial f/\partial b)(-1/v)$
$\partial\psi/\partial t = \partial f/\partial t = (\partial f/\partial b)(\partial b/\partial t) = (\partial f/\partial b)$, since $(\partial b/\partial t) = 1$
$\partial^2\psi/\partial x^2 = (-1/v)(\partial/\partial x)(\partial f/\partial b) = (-1/v)(\partial/\partial b)(\partial f/\partial x) = (1/v^2)\partial^2 f/\partial b^2$
$\partial^2\psi/\partial t^2 = (\partial/\partial t)(\partial f/\partial b) = (\partial/\partial b)(\partial f/\partial t) = (\partial/\partial b)(\partial f/\partial b) = \partial^2 f/\partial b^2$
$\partial^2\psi/\partial x^2 = (1/v^2)\partial^2 f/\partial b^2 = (1/v^2)\partial^2\psi/\partial t^2$
$\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = 0$

which is the wave equation in one dimension (1), with $0 < v < \infty$, and without using neither the Galileo transformation nor the Lorentz transformation.

From the Galileo transformation, we have:

$x' = x - vt$
$\psi(x,t) = f(x') = f(x - vt)$
$\partial\psi/\partial x = \partial f/\partial x = (\partial f/\partial x')(\partial x'/\partial x) = \partial f/\partial x'$, since $\partial x'/\partial x = 1$
$\partial\psi/\partial t = \partial f/\partial t = (\partial f/\partial x')(\partial x'/\partial t) = (\partial f/\partial x')(-v) = -v\partial f/\partial x'$
$\partial^2\psi/\partial x^2 = (\partial/\partial x)(\partial f/\partial x') = (\partial/\partial x')(\partial f/\partial x) = (\partial/\partial x')(\partial f/\partial x') = \partial^2 f/\partial x'^2$
$\partial^2\psi/\partial t^2 = (\partial/\partial t)(-v\partial f/\partial x') = -v(\partial/\partial x')(\partial f/\partial t) = -v(\partial/\partial x')(-v\partial f/\partial x') = v^2\partial^2 f/\partial x'^2$
$\partial^2\psi/\partial x^2 = \partial^2 f/\partial x'^2 = (1/v^2)\partial^2\psi/\partial t^2$

$\partial^2 \psi/\partial x^2$ - $(1/v^2)\partial^2 \psi/\partial t^2 = 0$

which is also the wave equation in one dimension, with $0 < v < \infty$.

And, from the Lorentz transformation, we have:

$x' = \gamma(x - vt)$, with $\gamma = (1 - v^2/c^2)^{-1/2}$
$\psi(x,t) = f(x') = f(\gamma(x - vt))$
$\partial\psi/\partial x = \partial f/\partial x = (\partial f/\partial x')(\partial x'/\partial x) = (\partial f/\partial x')\gamma = \gamma\partial f/\partial x'$
$\partial\psi/\partial t = \partial f/\partial t = (\partial f/\partial x')(\partial x'/\partial t) = (\partial f/\partial x')(-\gamma v) = -\gamma v\partial f/\partial x'$
$\partial^2 \psi/\partial x^2 = (\partial/\partial x)(\gamma\partial f/\partial x') = \gamma(\partial/\partial x')(\partial f/\partial x) = \gamma(\partial/\partial x')(\gamma\partial f/\partial x') = \gamma^2\partial^2 f/\partial x'^2$
$\partial^2 \psi/\partial t^2 = (\partial/\partial t)(-\gamma v\partial f/\partial x') = -\gamma v(\partial/\partial x')(\partial f/\partial t) = -\gamma v(\partial/\partial x')(-\gamma v\partial f/\partial x') = \gamma^2 v^2\partial^2 f/\partial x'^2$
$\partial^2 \psi/\partial x^2 = \gamma^2\partial^2 f/\partial x'^2 = (1/v^2)\partial^2 \psi/\partial t^2$
$\partial^2 \psi/\partial x^2 - (1/v^2)\partial^2 \psi/\partial t^2 = 0$

which is again the wave equation in one dimension, but now by virtue of the SR it would be $0 < v < c$, because $v = c$ would imply $\gamma = \infty$.

Appendix B

$E = \gamma mc^2$, $p = \gamma mv$ and $\gamma = (1 - v^2/c^2)^{-1/2}$

$E^2 = \gamma^2 m^2 c^4$ and $p^2 = \gamma^2 m^2 v^2$

$E^2 = (m^2c^4)/(1 - v^2/c^2) = c^2(m^2c^4)/(c^2 - v^2)$

$E^2/c^2 = (m^2c^4)/(c^2 - v^2)$

$p^2 = (m^2v^2)/(1 - v^2/c^2) = c^2(m^2v^2)/(c^2 - v^2)$

$E^2/c^2 - p^2 = (m^2c^4)/(c^2 - v^2) - c^2(m^2v^2)/(c^2 - v^2) = (m^2c^2)(c^2 - v^2)/(c^2 - v^2) = m^2c^2$

$E^2 = p^2c^2 + m^2c^4$

[1] José Francisco García Juliá, A Possible Refutation of the Relativity, viXra: 1409.0234 [Relativity and Cosmology].
http://vixra.org/abs/1409.0234

[2] James H. Smith, Introducción a la Relatividad Especial, Reverté, Barcelona, 1969. Original edition, Introduction to Special Relativity, Benjamin, New York.