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PSEUDO LATTICE GRAPHS AND THEIR APPLICATIONS TO FUZZY AND NEUTROSOPHIC MODELS

Pseudo Lattice Graphs and their Applications to Fuzzy and Neutrosophic Models

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PREFACE

In this book for the first time authors introduce the concept of merged lattice, which gives a lattice or a graph. The resultant lattice or graph is defined as the pseudo lattice graph of type I. Here we also merge a graph with a lattice or two or more graphs which call as the pseudo lattice graph of type II. We merge either edges or vertices or both of a lattice and a graph or a lattice and a lattice or graph with itself.

Such study is innovative and these mergings are adopted on all fuzzy and neutrosophic models which work on graphs. The fuzzy models which work on graphs are FCMs, NCMs, FRMs, NRMs, NREs and FREs. This technique of merging FCMs or other fuzzy models is very advantageous for they save time and economy. Moreover each and every expert who works on the problems is given equal importance.

We called these newly built models as merged FCMs, merged NCMs, merged FRMs, merged NRMs, merged FREs and merged NREs.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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ILANTHENRAL K

Chapter One

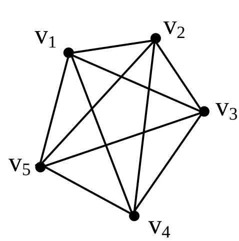
INTRODUCTON

In this chapter we just give some of the properties enjoyed by graphs and lattices. For in this book we obtain new classes of lattice-graphs by merging two lattices or by merging a lattice and a graph or a graph and a graph.

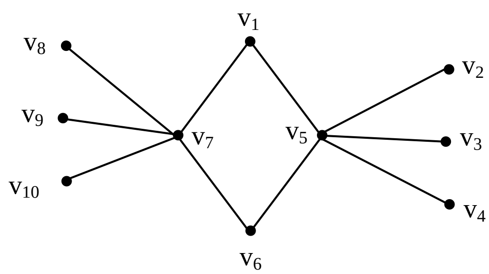
We use the term merging as follows. We may merge a vertex of a lattice L_1 with another vertex or L_2 or a edge and two vertices of a lattice L_1 with an edge and two vertices of a lattice L_2 or merge many vertices and many edges of a lattice L_1 with that of a lattice L_2.

Such study is new and innovative.

It goes without saying that every lattice is a connected graph but a graph in general is not a lattice, for



is a graph and not a lattice. Further



is a graph and not a lattice.

Now when in a lattice L_1 merged by a vertex or edge or both with another lattice L_2 we get the resultant graph which is termed as a pseudo lattice graph of type I it may be a lattice or a graph. Similarly using a lattice and a graph or a graph and a graph we get a graph termed as the pseudo lattice graph of type II.

This notion finds its applications in fuzzy and neutrosophic models which work on direct graphs like FCMs, NCMs, FRMs, NRMs, FREs and NREs [79, 90].

This book also studies about merging of neutrosophic lattices [87]. Thus these new type of merging may also find more applications in due course of time. Several open problems are suggested.

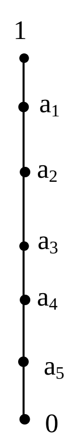
Chapter Two

PSEUDO LATTICE GRAPHS OF TYPE I

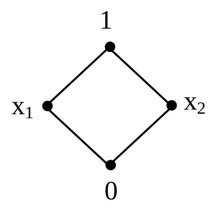
In this chapter we introduce a new mode of construction of graphs using lattices. We take two lattices merge one vertex or two vertices or three vertices or so on or merge one edge and two vertices or more edges and more vertices and arrive at a diagram. The resultant can be a graph or a lattice.

We will first illustrate all these situations by some examples.

Example 2.1: Let L_1 be the chain lattice C_7

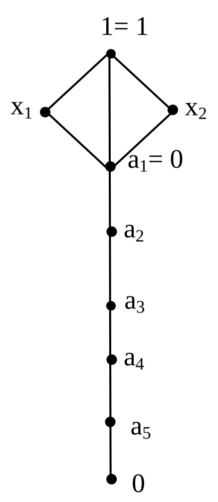


and L_2 =



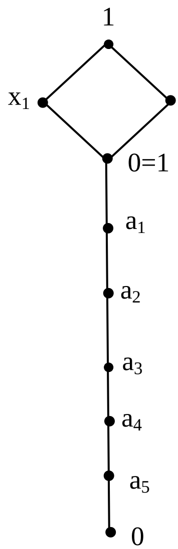
be the distributive lattice of order four.

We have the following ways of merging L_1 and L_2 and are denoted in the following.



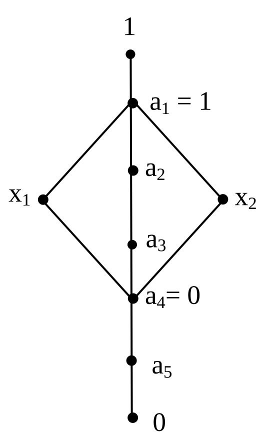
= M_1

Merging vertex 1 of L_1 with vertex 1 of L_2 and zero of L_2 with a_1 of L_1, we get the lattice given above. We can rename the vertices.



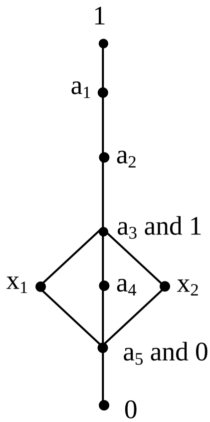
$= M_2$

Here vertex 1 of L_1 is merged with 0 vertex of L_2 and is denoted M_2.



$= M_3$

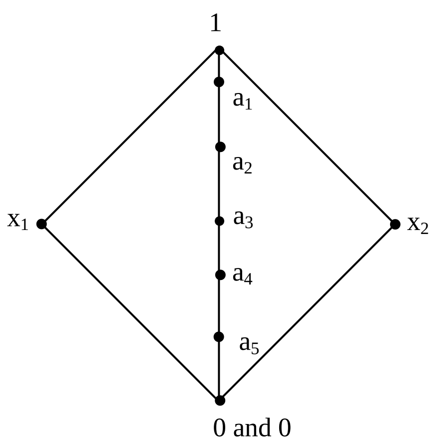
This sort of merging can be made which is self explanatory and the lattice is denoted by M_3.



= M_4

denoted by M_4.

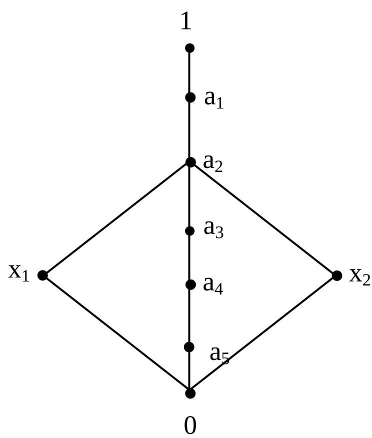
Let us merge in the following way 1 of L_1 is merged with 1 of L_2 and 0 of L_1 is merged with 0 of L_2.



= M_5

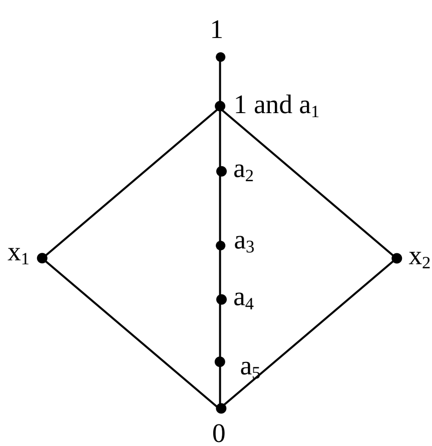
This merging is denoted by M_5.

This merging is denoted by M_6.



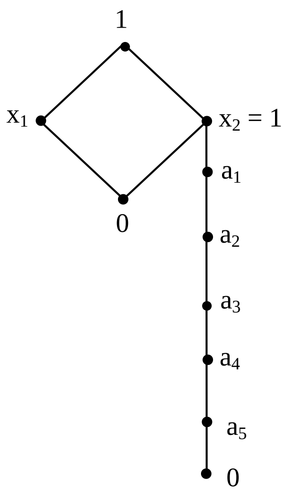
$= M_6$

This merging is denoted by M_7 .



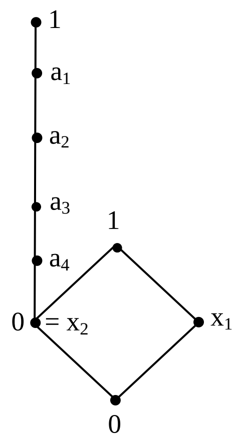
$= M_7$

Consider the merging of the vertices



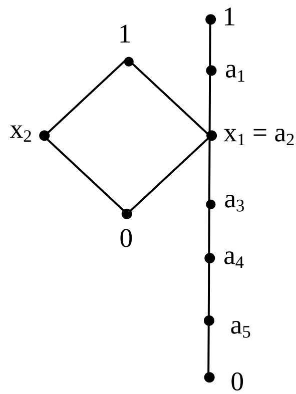
This is not a lattice only a graph.

Now consider the merging of 0 with x_2 which is as follows:



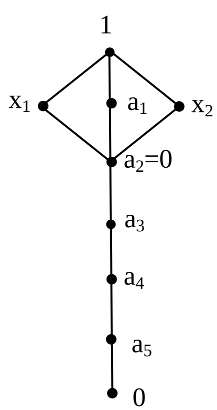
This is also a pseudo lattice graph which is not a lattice.

Now we merge a_2 with x_1 which is as follows. The resultant is not a lattice only a graph.



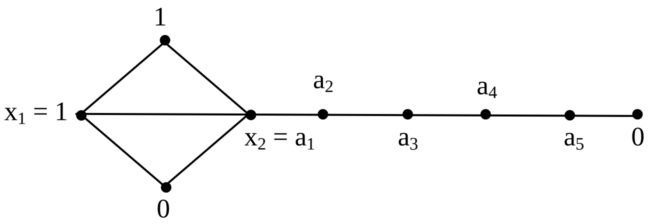
We can merge a_1 with x_1 or a_2 with x_1 or a_3 with x_1 or a_4 and in all cases we get only a graph and not a lattice.

We see merging 1 with 1 and a_2 with zero and we get a lattice which is modular. This is as follows:

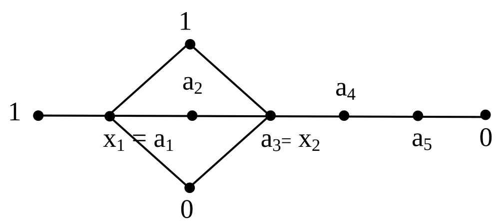


Clearly the above figure is a lattice and is a modular lattice.

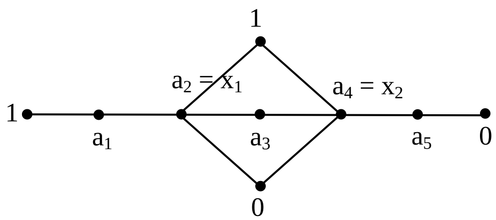
Suppose x_1 is merged with 1 and x_2 is merged with a_1 we get the following graph.



Clearly this is not a lattice.

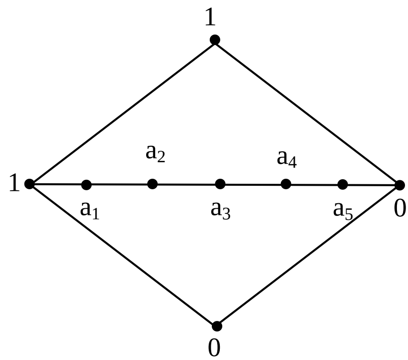


We can also merge x_1 with a_1 and x_2 with a_3 and get a graph which is not a lattice. We can merge a_2 with x_1 and a_4 with x_2 which is also follow:

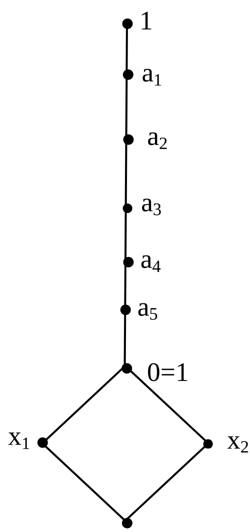


This is also only a graph and is not a lattice.

Finally we can merge 0 with x_2 and 1 with x_1 which is as follows:



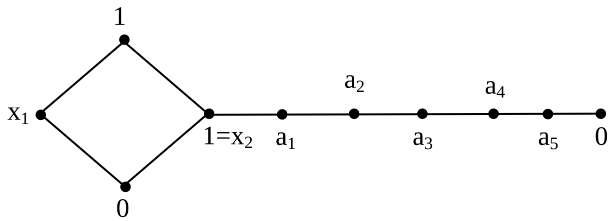
Clearly the resultant is graph and not a lattice.



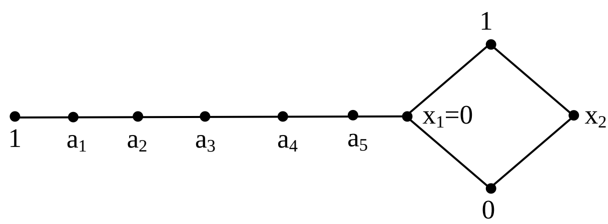
The merging of 0 of C_6 with 1 of lattice we get the resultant is a lattice which is distributive.

We see when we merge two distributive lattice we can get the resultant as a distributive lattice or a modular lattice or only a graph which is not a lattice.

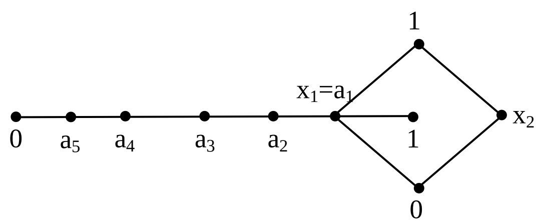
We can also merge x_2 with 1 and get the following graph.



or merge 0 with x_1 we get the following graph.

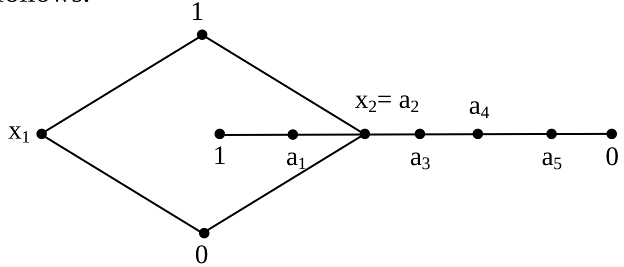


We can also merge x_1 with a_1 which is as follows:

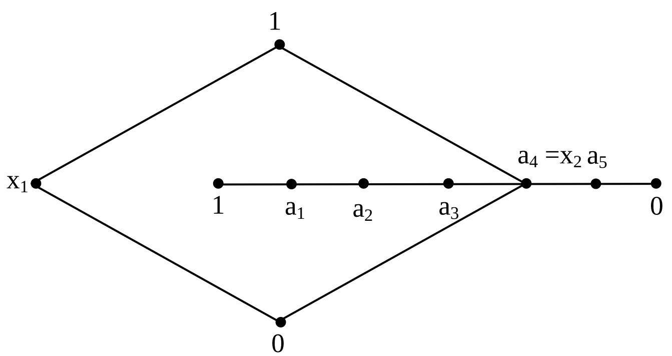


Clearly this is also a graph and is not a lattice.

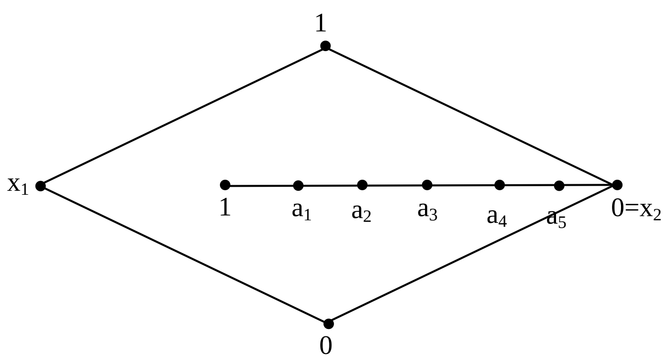
We can merge x_2 with a_2 and get the following graph which is as follows.



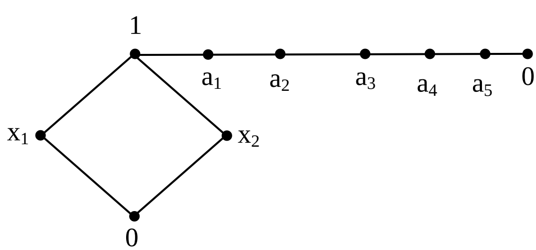
We see if we merge a_4 with x_2 we get the following graph



We get if we merge 0 with x_2 then we get the following graph.

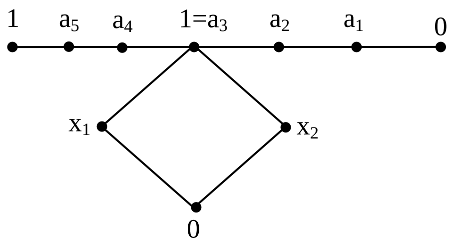


We also merge 1 with 1 horizontally.

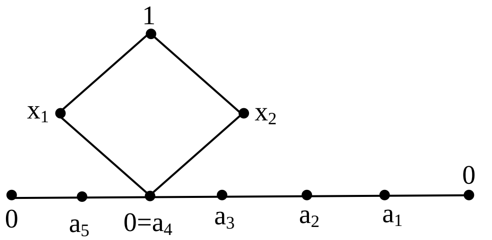


This is also only a graph and not a lattice.

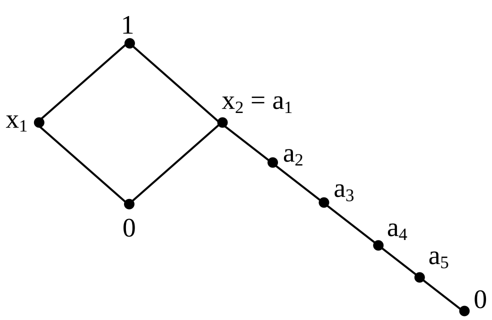
We can merge a_3 with 1 horizontally and get the following graph.



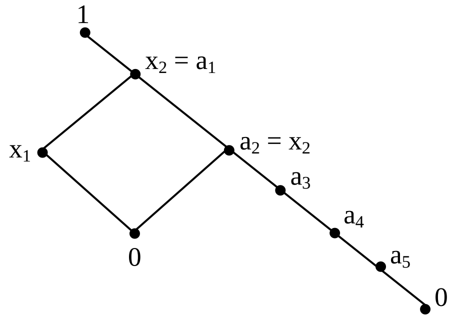
We can also merge 0 with a_4 and get the following graph.



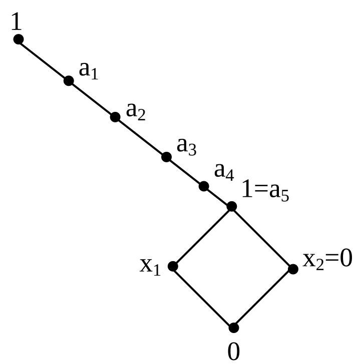
Now if we can merge one edge with another edge we get only a graph which is as follows:



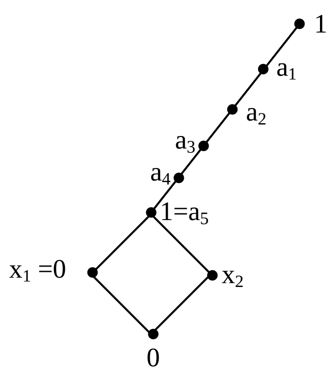
We can merge 1 with a_1 and x_2 with a_2 which is as follows.



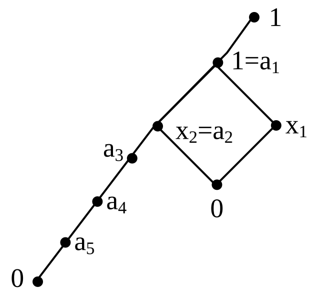
We can merge 1 with a_4 and x_2 with 0 which is the following graph.



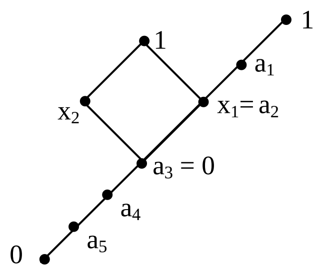
We can also merge x_1 with 0 and 1 with a_5 and the edge 1 x_1 with $0a_5$ given by the following graph.



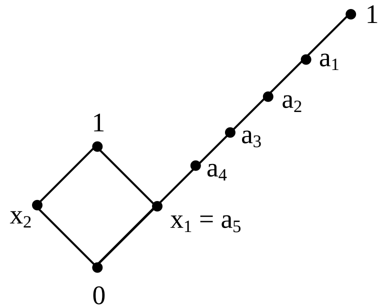
We can also merge x_2 with a_2 and $1 = a_1$ and the edge $1x_2$ with a_2a_1 and get the related graph that is as follows:



Now we can merge the edge $0x_1$ with edge a_3a_2 so that



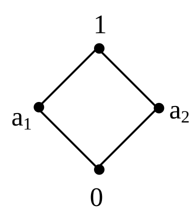
Clearly this is not a lattice only a graph.



The following observations are to be made while merging a vertex of two lattices or merging only two vertices and not an edge of two lattices or merging an edge and two vertices of the lattices. From the example one it is clear that when we had used lattices both of which are distributive we may get a distributive lattice or a modular lattice or a graph which is not a lattice.

However after giving one to two more examples we proceed onto define the concept mathematically.

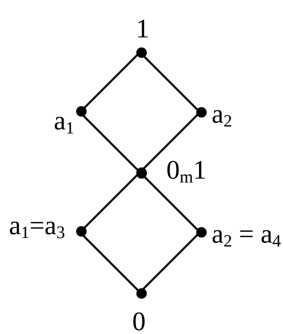
Example 2.2: Let L be the lattice



We merge the vertices of L with L or edges and vertices of L with L.

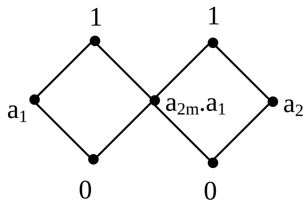
Some of the merging are described in the following.

Merge vertex 0 with vertex 1 of L.



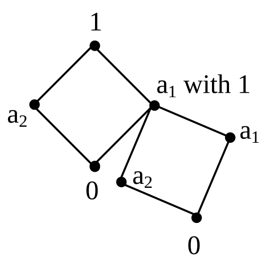
This new graph is a lattice which is distributive.

We merge two lattices we can rename the vertices 1_m0 or 0_m1 means zero is merged with one or equivalently one is merged with 0.

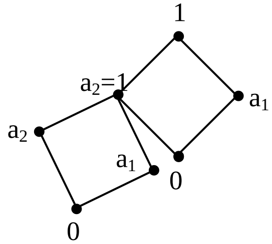


a_2 is merged with a_1 the resultant is not a lattice only a graph.

We can also merge a_1 with 1 of L. The resulting graph is as follows.

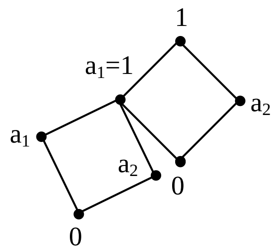


a_2 merged with 1.

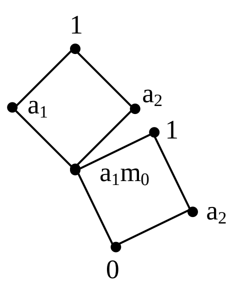


is only a graph not a lattice.

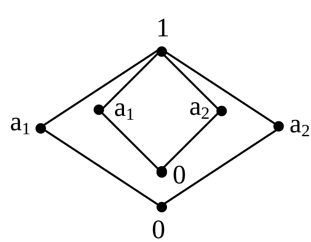
Merging 1 with a_1



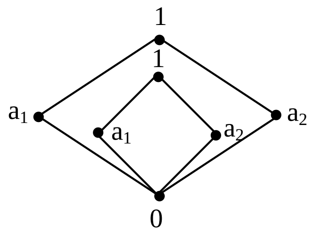
Merging a_1 with 0 of the lattices



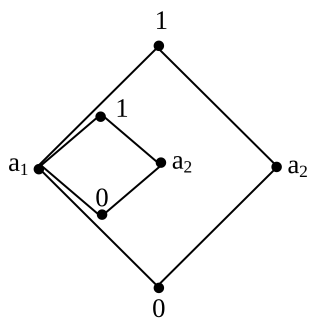
Now we can merge one with one.



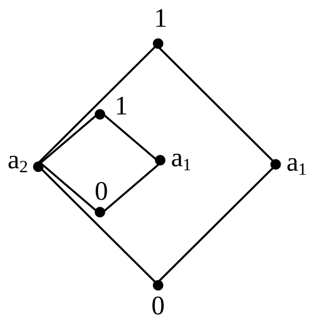
is only a graph.



Merging 0 with 0 of the lattice L with L.

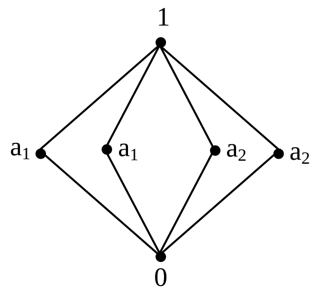


Merging a_2 with a_2 we get the following graphs.

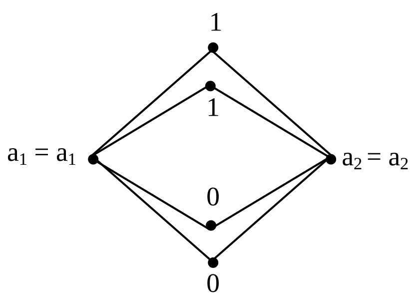


Now we can also merge only two vertices of L and get the graphs.

Some of them are given in the following:

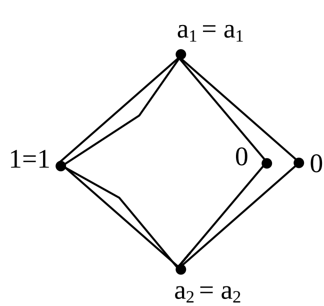


The resulting diagram is a lattice which is modular.

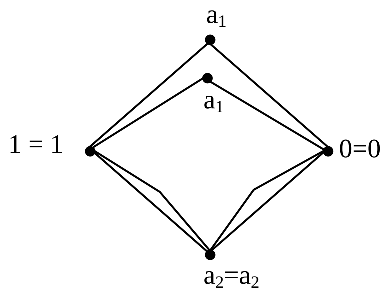


The resulting diagram is only a graph and not a lattice.

Thus using a distributive lattice we may get after merging its vertices a distributive lattice or a modular lattice or a graph. Now suppose we merge three vertices and not their respective edges we can get the following graphs

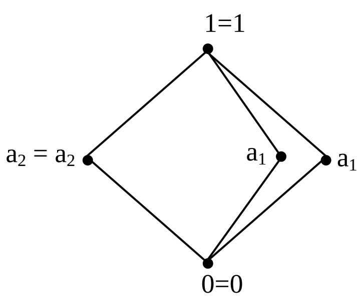


or



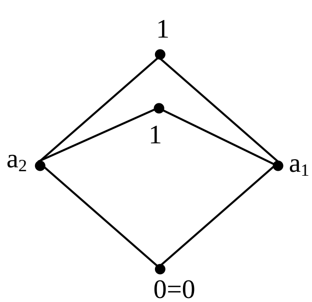
in both cases we get the same graph.

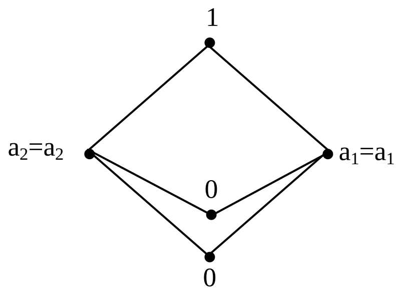
Now we can also get the graph by merging 3 of its vertices and two its edges.



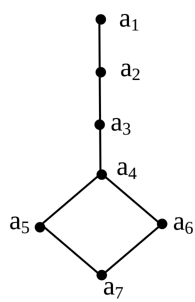
This is clearly a modular lattice.

The other way of merging the vertices with the edges is as follows.



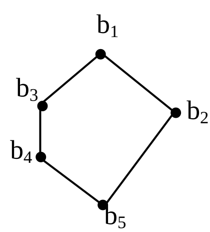


Example 2.3: Let L_1 =



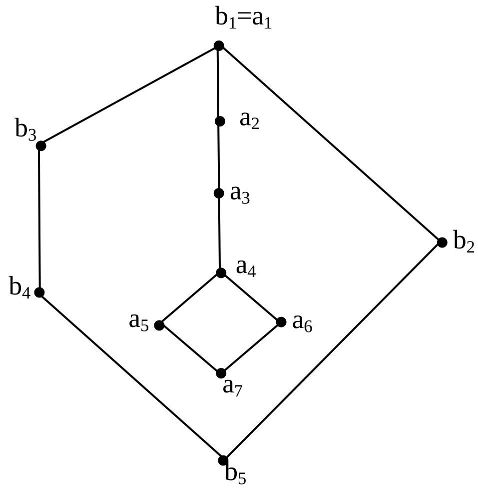
be a lattice with a_7 the least element and a_1 the greatest element.

L_2 =

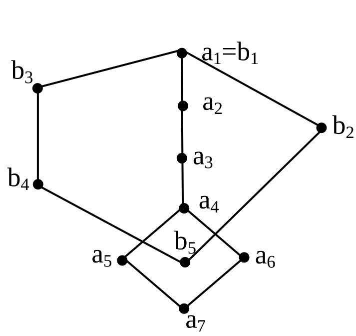


is a pentagon lattice.

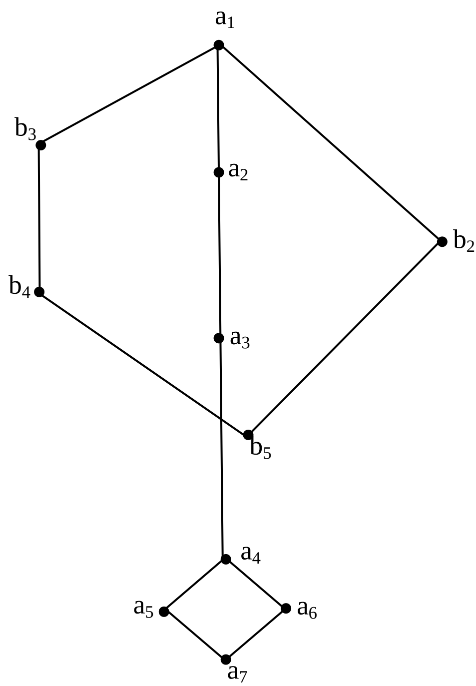
We first merge the vertex a_1 with b_1 we have the following graphs.



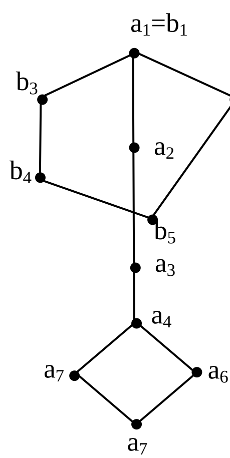
or

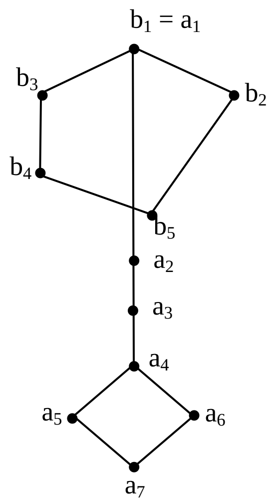


or



or



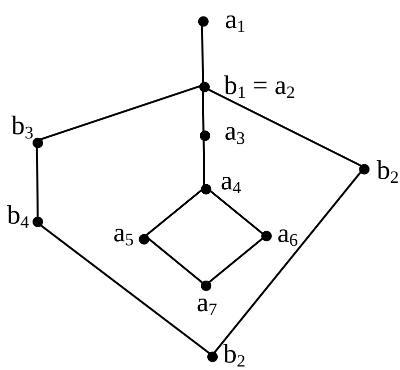


In all these cases we see the resultant is only a graph and not a lattice.

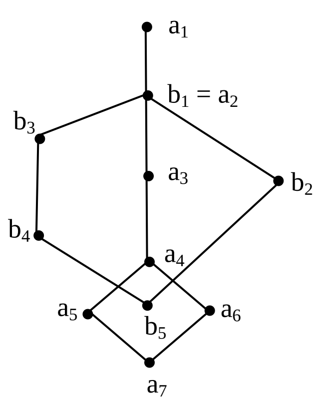
We define a pseudo lattice graph of type I as the lattice or a graph got by merging two lattices by a vertex or vertices an edge or edges or both.

We see we have at least two lattices built using the lattices L_1 and L_2 and both of them are non distributive and non modular as they contain a pentagon lattice as a sublattice which is both non modular and non distributive.

We can also merge vertex b_1 with a_2 and obtain the following.

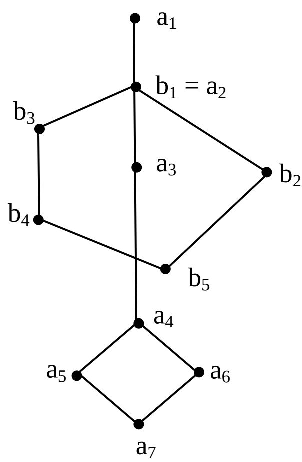


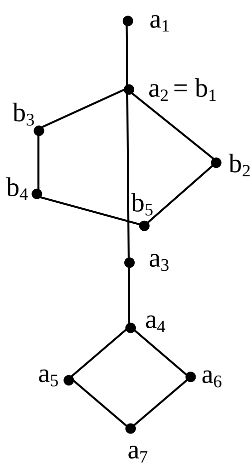
or



We see both the graphs are not lattices.

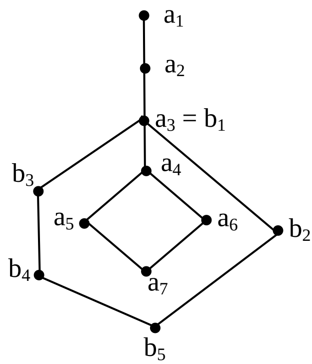
They are a special type of graphs with same number of vertices and edges.



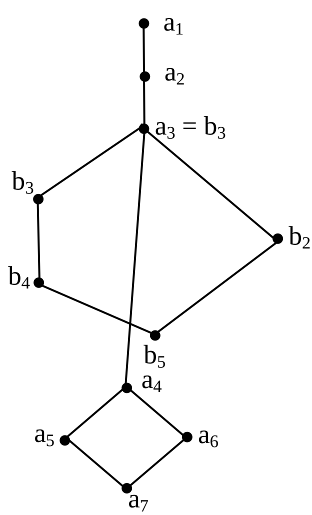


are all pseudo lattice graphs of type I which are only graphs and not lattices.

On similar lines we can merge vertex b_1 with a_3 and we have the following graphs.

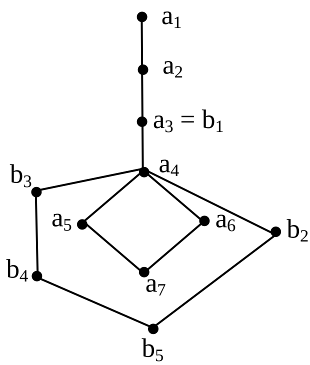


or

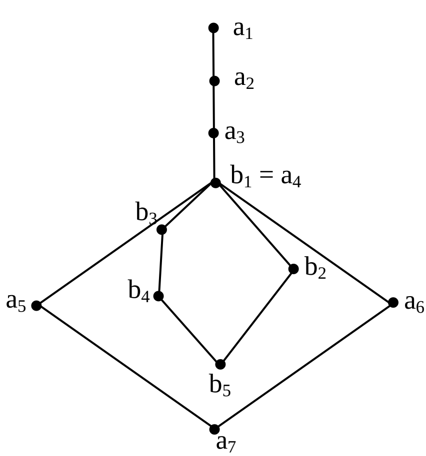


are pseudo lattice graphs of type I which are only graphs and not lattices.

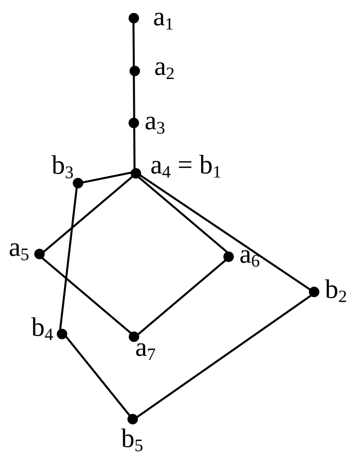
By merging the vertex a_4 with b_1 we get the following four types of graphs.



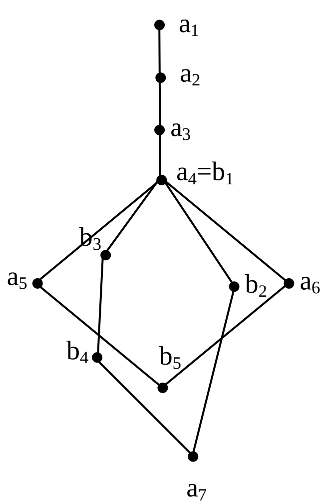
or



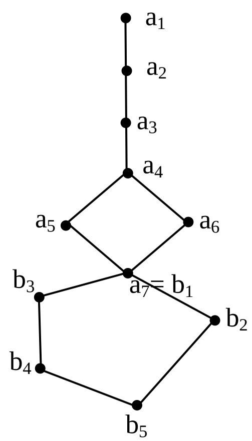
or



or

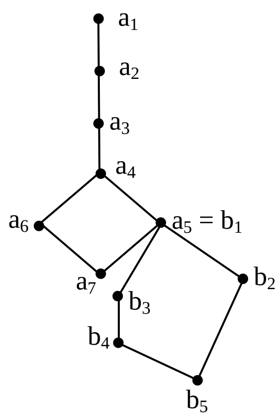


All of them are pseudo lattice graphs of type I which are not lattices only graphs.

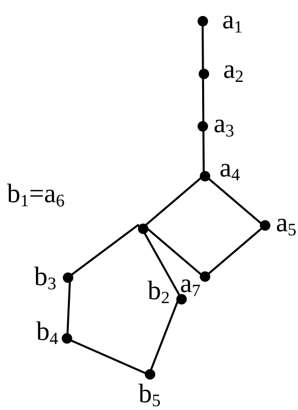


is a lattice which is not modular.

We can merge the vertex b_1 with a_5 or a_6 and get pseudo lattice graphs of type I which are only graphs.

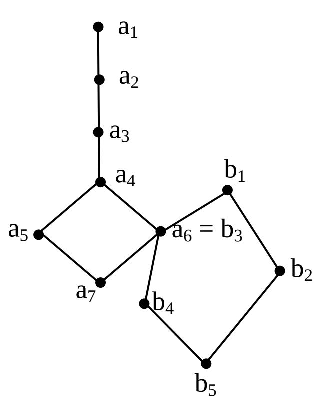


or

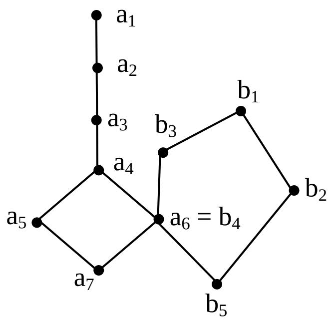


are only graphs.

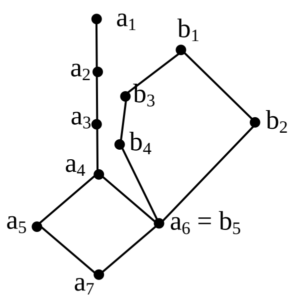
Now by merging the vertex a_6 or a_5 with b_2 or b_3 or b_5 or b_4 we get the following pseudo lattice graphs of type I.



or

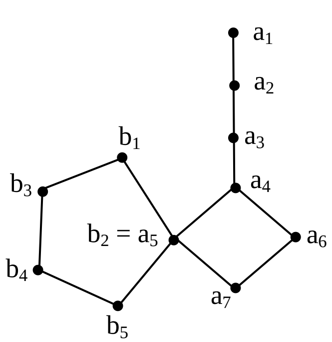


or



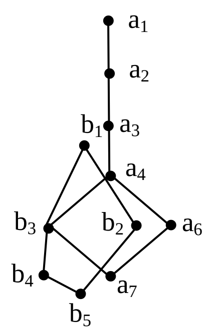
is again a pseudo lattice graph of type I which is a graph and not a lattice.

Let us merge the vertex b_2 with a_5 we get



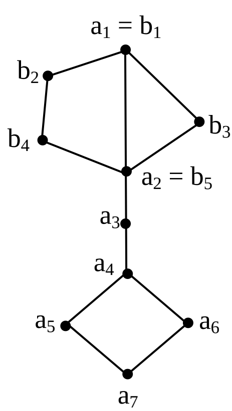
a pseudo lattice graph of type I which is only a graph.

We can merge b_4 with a_5 and get the following pseudo lattice graph.

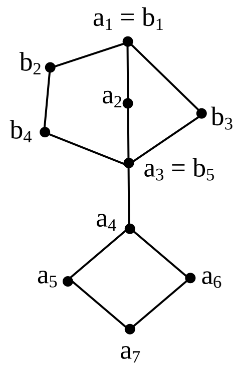


This is only a graph and not a lattice and so on.

Next merge only two vertices to get the pseudo lattice graph of type I.

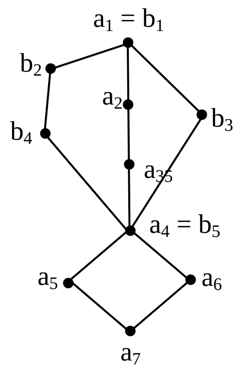


The above pseudo lattice graph of type I is a lattice which is non distributive and non modular.



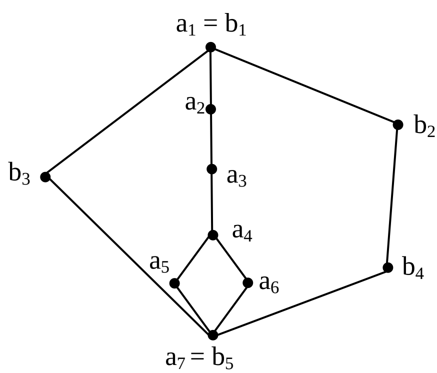
is a pseudo lattice graph of type I which is again a non distributive and non modular lattice.

Now



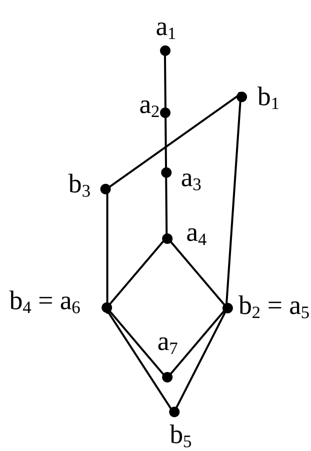
is a pseudo lattice graph of type I which is a non distributive and non modular lattice.

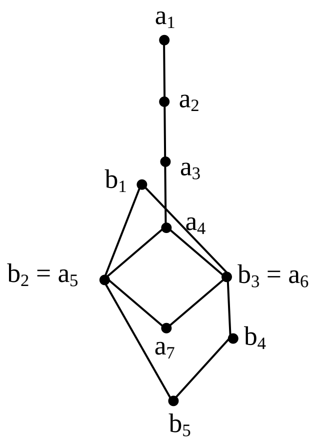
Now



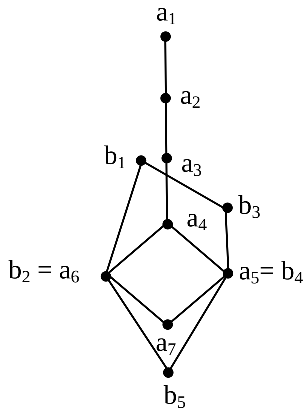
is a pseudo lattice graph of type I.

Now consider

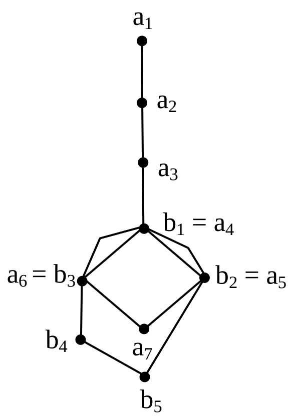




or

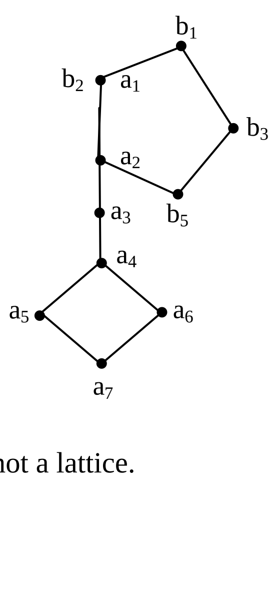


We can also have the merging of three vertices



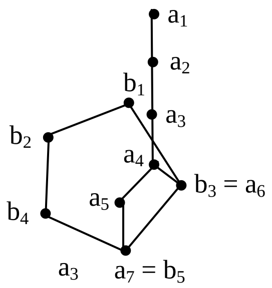
and so on.

We give a few illustration of merging edges as well as vertices



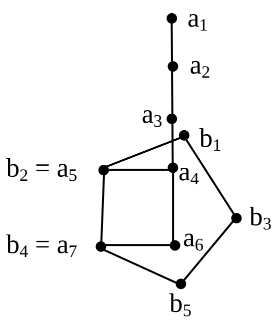
is only a graph not a lattice.

We get a pseudo lattice graph by merging edges b_3b_5 with a_6a_7 which is as follows.

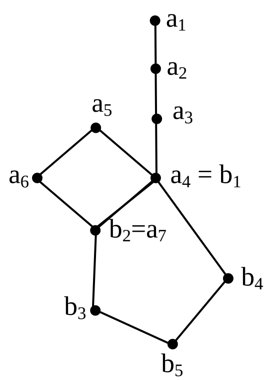


is not a lattice only a graph.

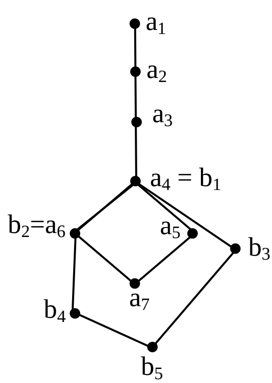
We can merge a_5a_7 with b_4b_5 and get the pseudo lattice graph which is as follows:



We can also merge edges b_1b_2 with a_6a_7 and get the following pseudo lattice graph of type I.

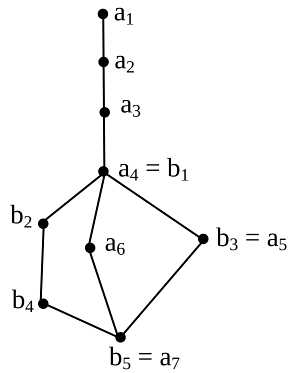


We can also merge edges a_4a_6 with b_1b_2 and get the following pseudo lattice graph which is only a graph and not a lattice.



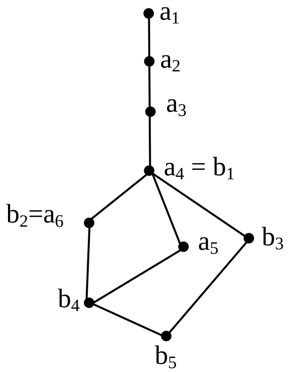
Likewise we can merge an edge and a vertex and get a pseudo lattice graph of type I.

We can merge 2 edges and three vertices and get the following few pseudo lattice graphs of type I.



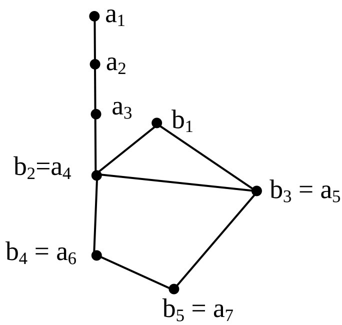
This is a lattice.

We can also merge edges b_1b_2 and b_2b_4 with a_4a_6 and a_6a_7 respectively.

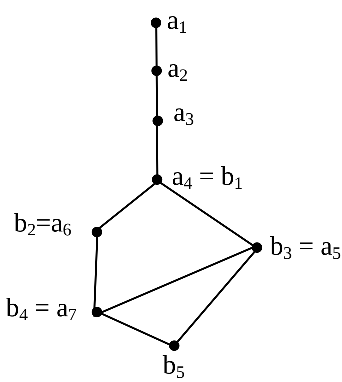


This is also pseudo lattice graph of type I which is also a lattice.

We can also merge four vertices and three edges which is a pseudo lattice graph of type I which is as follows.

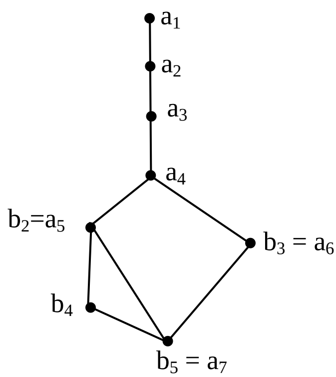


We can merge three edges with four vertices in the following way the get the pseudo lattice graph of type I.



This is a lattice which is both non distributive and non modular.

We can also merge four vertices and three edges and obtain the following pseudo lattice graph of type I which is as follows.



This is a lattice which is not modular and non distributive. We can also still differently obtain several other graphs.

Thus we propose the following open problems.

1. Given two lattices L_1 and L_2 how many pseudo lattice graphs of type I can be got ($|L_1| = n$, $|L_2| = m$ that is number of vertices of L_1 is n and that of L_2 is m and number of edges of L_1 is s and that of L_2 is t)?

2. How many of the pseudo lattice graphs of type I are lattices?

3. If both L_1 and L_2 are distributive lattices, can the pseudo lattice graph which is a lattice be a modular lattice?

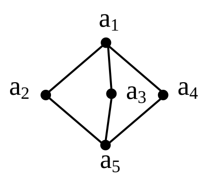
4. How many pseudo lattice graphs of type I that is obtained by merging one vertex is a lattice?

5. How many pseudo lattice graphs of type I can be obtained by merging two vertices but not an edge is a lattice?

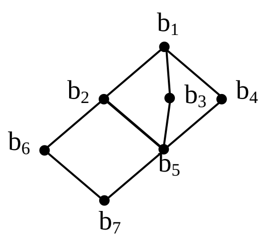
6. How many pseudo lattice graphs of type I can be obtained by merging two vertices and an edge of lattices?

   Since this study is very new the six problems proposed above can be realized as open conjectures.

Example 2.4: Let L_1 =

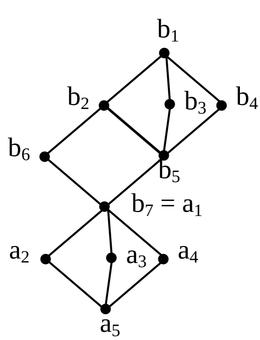


and L_2 =

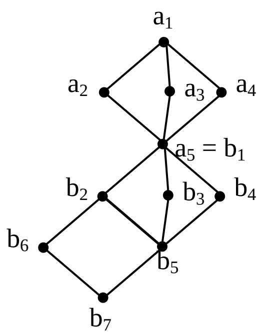


be the two lattices.

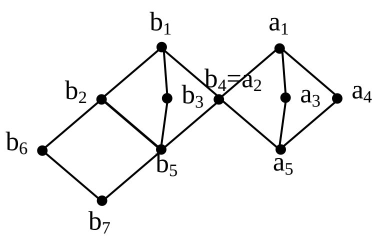
Suppose we merge vertex b_7 with vertex a_1 then we get the following pseudo lattice graph of type I.



If we merge b_1 with a_5 we get the following pseudo lattice graph of type I.

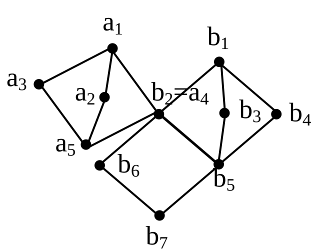


Suppose we merge the vertex b_4 with a_1 we get the following pseudo lattice graph of type I.



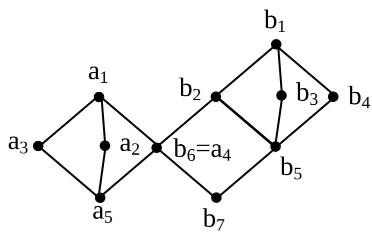
This is not a lattice only a graph.

By merging vertex b_5 with a_1 we get the following pseudo lattice graph of type I.

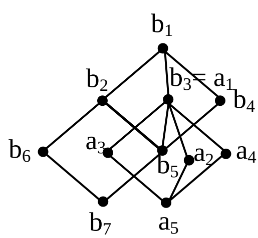


This is only a graph and not a lattice.

By merging vertex b_6 with vertex a_4 we get the following pseudo lattice graph of type I.

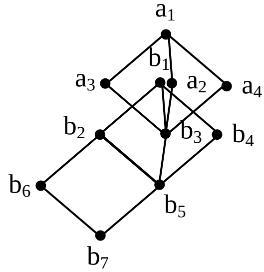


This is a graph and not a lattice. Let us merge b_3 with a_1;



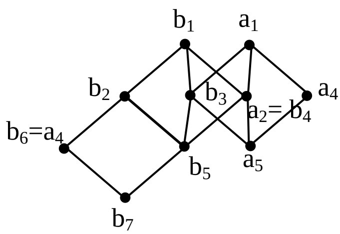
is the pseudo lattice graph of type I which is only a graph and not a lattice.

Merging of a_5 with b_3 gives the following pseudo lattice graph of type I.

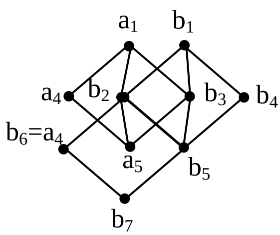


This is only a graph and not a lattice we can get several such pseudo lattice graphs of type I.

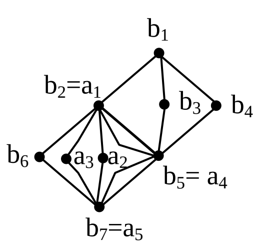
We can give a few pseudo lattice graphs of type I by merging only two vertices and not an edge.



the vertex a_3 and b_3 and a_2 and b_4 are merged.

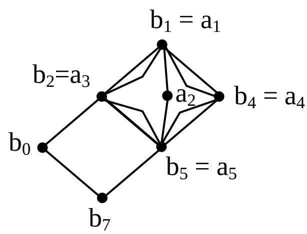


This is also a graph. So the pseudo lattice graph got by merging the vertex a_3 with b_2 and vertex b_3 with a_4 is only a graph and not a lattice.

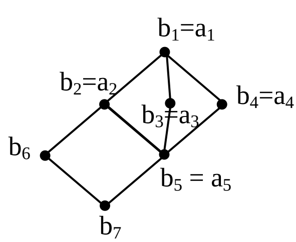


The pseudo lattice graph got by merging the vertices b_2 with a_1, a_4 with b_5 and b_7 with a_5 we get the graph which is not a lattice.

We can merge four vertices and get the following pseudo lattice graph.



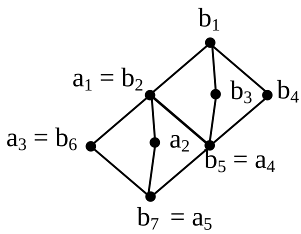
This pseudo lattice graph is obtained by merging the vertices a_1 with b_1, a_4 with b_4, a_5 with b_5 and edge b_1 b_4 with edge a_1a_4 and edge b_4b_5 with edge a_4a_5. This pseudo lattice graph of type I is a lattice which is modular.



Thus using these two modular lattices we get pseudo lattice graph of type I which is a modular lattice in some cases and just only graphs in many cases.

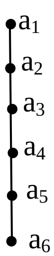
We see by merging the vertices a_1 with b_1, a_2 with b_2, a_3 with b_3, a_4 with b_4 and a_5 with b_5 and edges a_1a_2 with b_1b_2 a_1a_3 with b_1b_3, a_1a_4 with b_1b_4; b_2b_5 with a_2a_5 and b_3b_5 with a_3a_5 we get the pseudo lattice graph of type I is the lattice L_2.

We can also merge in this form

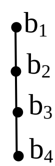


to get a pseudo lattice graph of type I where the vertex a_1 is merged with the vertex b_2, vertex a is merged with vertex b_6, vertex b_5 is merged with vertex a_4 and vertex b_7 is merged with vertex a_5, the corresponding edges are also merged. The resultant, pseudo lattice graph of type I is a modular lattice.

Example 2.5: Let L_1 =



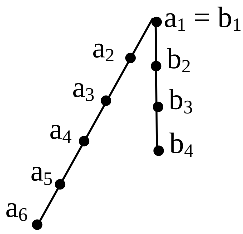
and



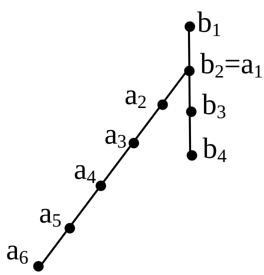
be two chain lattices.

Only in one case when vertex a_1 is merged with vertex b_4 or vertex a_6 is merged with vertex b_1 we get the pseudo lattice graph of type I to be a chain lattices in all other the pseudo lattice graph of type I is only a graph more so it is a tree.

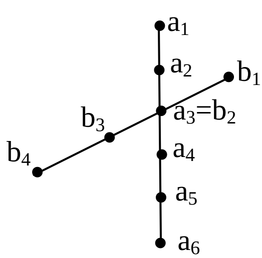
A few merging of vertices and edges of L_1 with L_2 is given in the following.



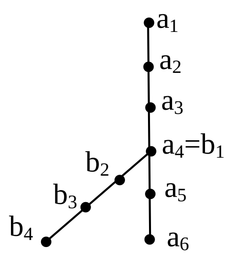
is a tree.



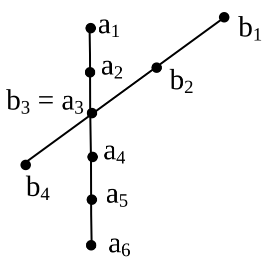
The pseudo lattice graph of type I is a tree got by merging vertex a_1 with vertex b_2.



The pseudo lattice graph of type I is a tree got by merging vertex a_3 with vertex b_2.

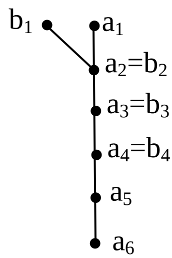


The pseudo lattice graph of type I got by merging vertex a_4 with vertex b_1.

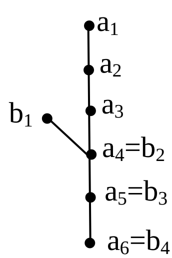


is a pseudo lattice graph of type I got by merging vertex a_3 with b_3.

We can merge a maximum of 3 edges and 3 vertices in which case we get a chain lattice.

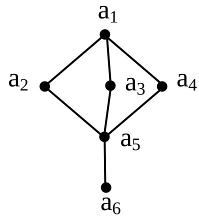


is a true which is a pseudo lattice graph of type I got by merging the vertices a_2 with b_2, a_3 with b_3 and a_4 with b_4.

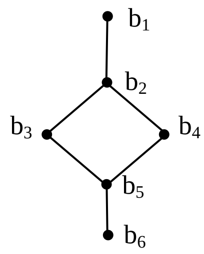


the pseudo lattice graph of type I by merging vertices a_4 with b_2, a_5 with b_3 and vertex b_4 with a_6. This is again a tree.

Example 2.6: Let L_1 be a lattice.

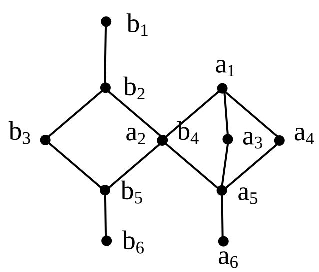


and L_2 a lattice



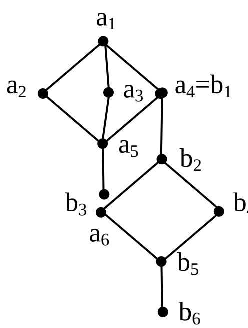
L_1 is a modular lattice and L_2 is a distributive lattice.

We can merge vertex b_4 with vertex a_2 and get the following pseudo lattice graph of type I.

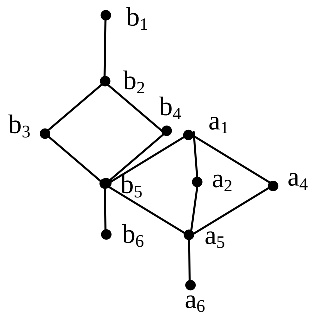


This is a graph and not a lattice.

We can merge vertex a_4 with b_1 and get the following pseudo lattice graph of type I.



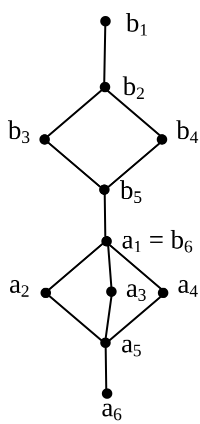
This is a graph and not a lattice.



Let the vertex b_5 be merged with a_2.

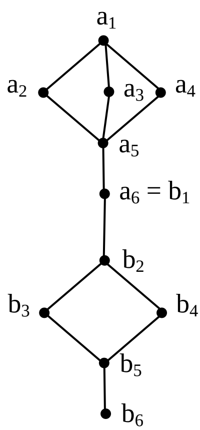
The pseudo lattice graph of type I is only a graph.

We can merge vertex b_6 with vertex a_1 and get the following pseudo vertex graph of type I.



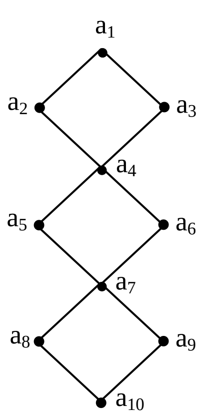
This is a lattice.

We can merge vertex a_6 with b_1 which is a pseudo lattice graph of type I which is the following lattice.

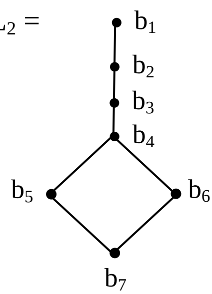


Thus we can get pseudo lattice graph of type I using the lattices L_1 and L_2.

Example 2.7: Let L_1 =

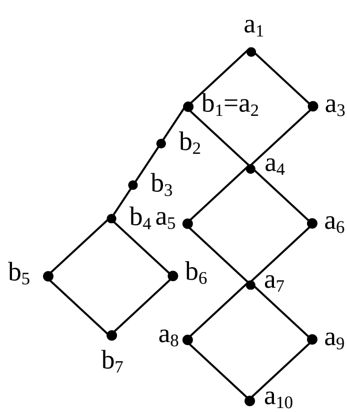


and L_2 =



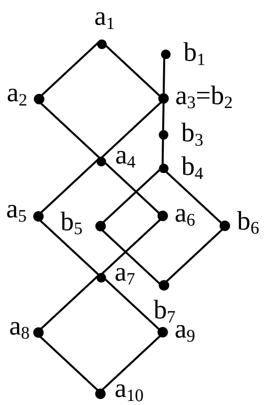
be any two lattices.

We can merge vertex b_1 with a_2 and get the following pseudo lattice graph of type I.



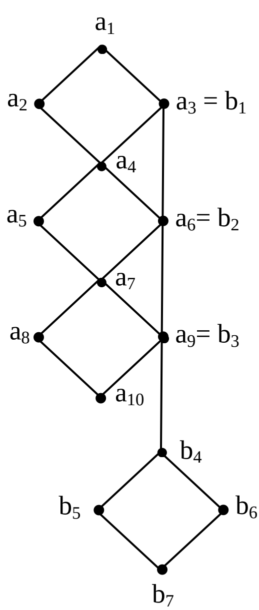
This is only a graph and is not a lattice.

Let us merge vertices a_3 with b_2 and b_3 with a_5 we get the following pseudo lattice graph of type I.



We see this not a lattice only a graph.

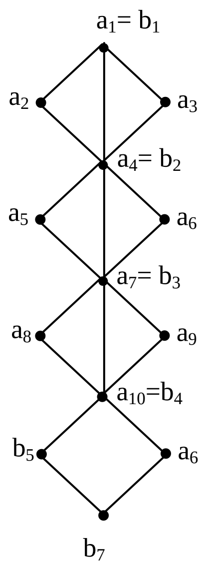
We can also merge only three vertices and get the pseudo lattice graph of type I which is as follows.



This is not a lattice only a graph.

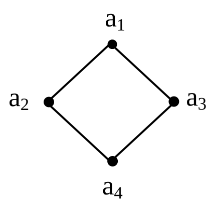
Thus by this new method we get several types of graphs which are new and enjoy properties like being a lattice and so on.

We can also merge only four of the vertices and get the following pseudo lattice graphs of type I.



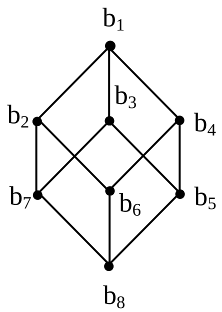
which is a graph.

Example 2.8: Let us consider two Boolean algebras.

B_1 = 

and

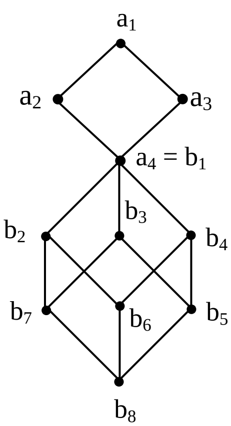
B_2 =



We find the pseudo lattice graphs of type I by merging the vertices or edges or both. It is important to keep in record is that it will not give a new Boolean algebra.

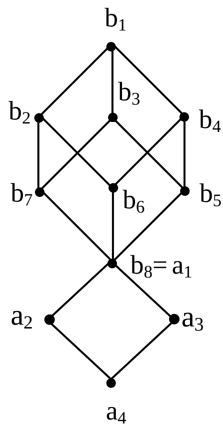
The pseudo lattice graph of type I may be a lattice or a graph a Boolean algebra when the Boolean algebra B_1 is merged with B_2 in such a manner that all the four vertices are merged with four vertices and four edges are also merged with four edges. Then we get the Boolean algebra B_2 only.

By merging vertex a_4 of with vertex b_1 of b_2 we get the pseudo lattice graph of type I which is as follows:

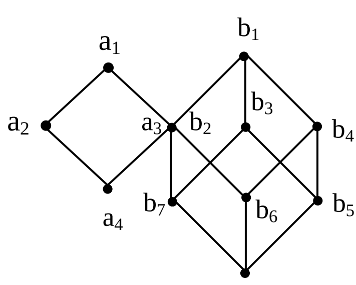


This is a lattice which is distributive and not a Boolean algebra.

By merging a_1 with b_8 we get the following pseudo lattice graph of type I. This is also not a Boolean algebra only a lattice.



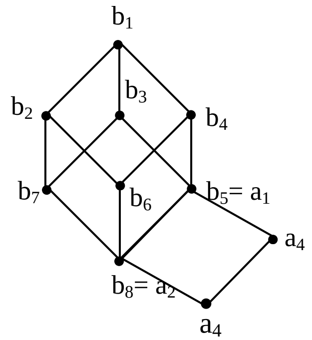
Let us merge vertices a_3 with b_2 and get the following pseudo lattice graph of type I.



Clearly this is only a graph and not a lattice.

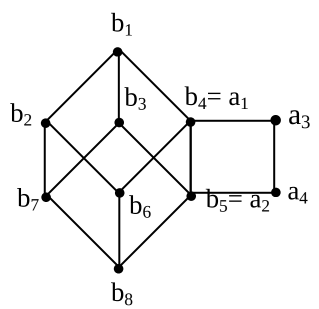
The merging of vertices can be b_5 and a_2 or b_4 a_2 or b_7 and a_2 so on.

Now we can also merge the edges a_1a_2 with b_5b_8 and vertices a_1 with b_5 and a_2 with b_8 which is as follows:

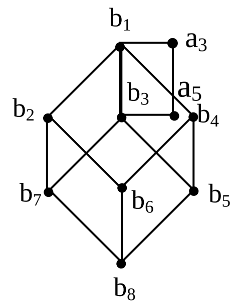


The pseudo lattice graph of type I is a lattice.

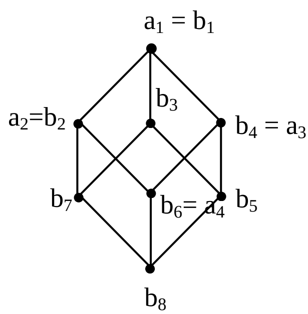
Suppose vertex b_4 is merged with a_1 and b_5 with a_2 we get the following pseudo lattice graph of type I.



Now we can merge edge b_1b_3 with a_1a_2 in the following way and get a pseudo lattice graph of type I.



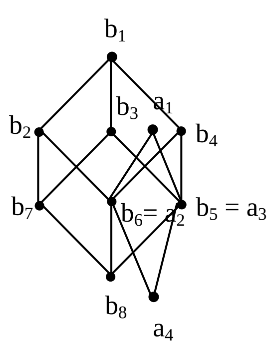
Finally we can merge four vertices and four edges in the following way to get the pseudo lattice graphs of type I.



The resultant is a Boolean algebra B_2.

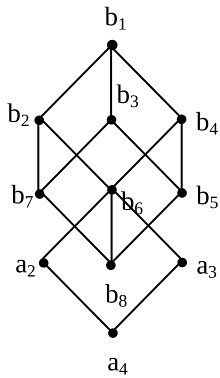
Thus by merging like this in six ways we get only a Boolean algebra of order 8 that is B_2 itself.

Now we can merge only two of the vertices b_6 with a_2 and b_5 with a_3 and

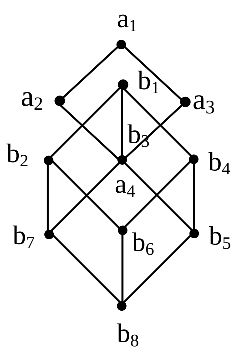


The pseudo lattice graph of type I is not a lattice it is only a graph.

We can merge vertex b_6 with a_1 and get the following pseudo lattice graph of type I.

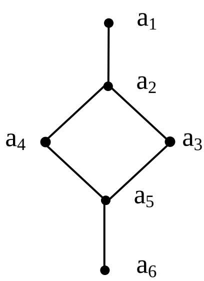


Likewise we can merge vertex b_3 with vertex a_4 which gives the following pseudo lattice graph of type I.

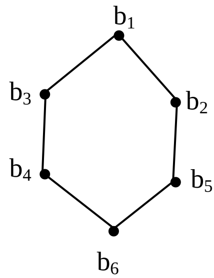


Example 2.9: Let us consider two lattices

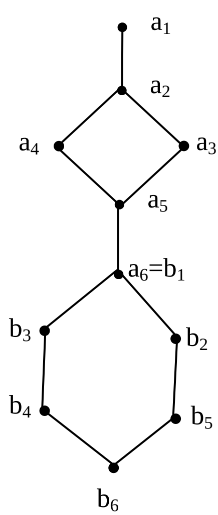
$L_1 =$



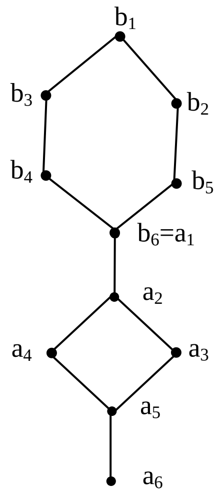
L_2 =



We can adjoin the vertex a_6 with b_1 which is as follows:

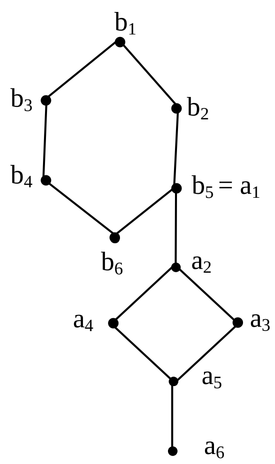


The pseudo lattice graph is a lattice which is both non distributive and non modular.

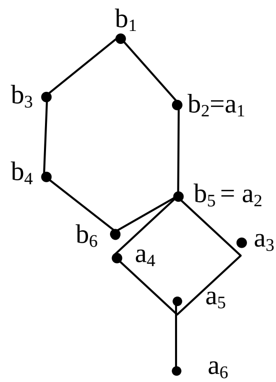


The pseudo lattice graph is a distributive lattice.

However by merging vertex a_1 with vertex b_5 we get the following pseudo lattice graph of type I.

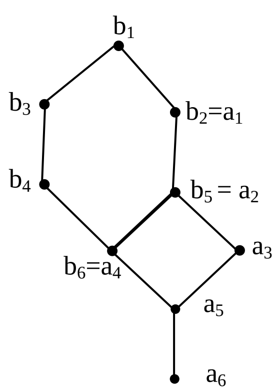


This is only a graph and not a lattice.
We can merge edge a_1a_2 with b_2b_5 which is as follows:

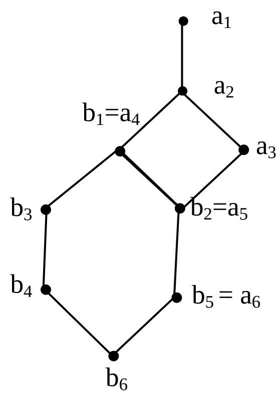


This is a pseudo lattice graph of type I which is not a lattice.

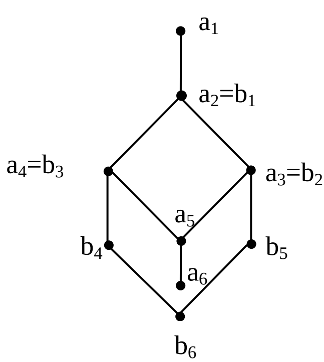
We can merge the edge a_1a_2 with b_2b_5 and edge a_2a_4 with b_5b_6 which is as follows:



Clearly the pseudo lattice graph of type I is a lattice. We can merge edges a_5a_6 with b_2b_5 and edge a_6a_5 with edge b_1b_2 and get the following pseudo lattice graph of type I.

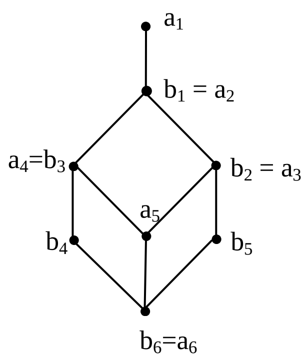


This is also a lattice. We can also merge b_1b_3 with a_2a_4 and b_1b_2 with edge a_2a_3 and get the following pseudo lattice graph of type I.



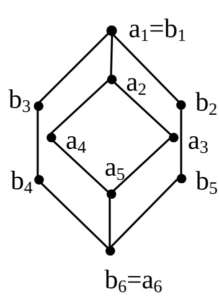
The resulting pseudo lattice graph of type I is only a graph and not a lattice.

In the same pseudo lattice graph of type I we can merge also vertex a_6 with vertex b_6 and get the following pseudo lattice graph of type I.

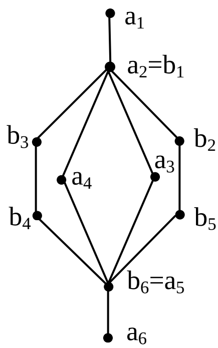


This is a lattice or order 8.

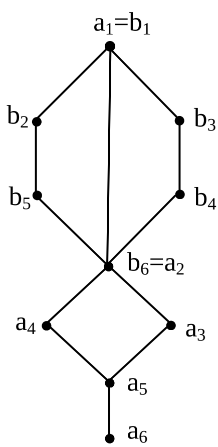
We get the following pseudo lattice graph of type I.



Thus this is only a graph and not a lattice.

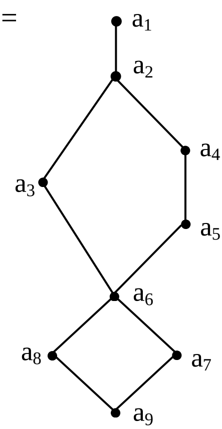


This pseudo lattice graph of type I is a lattice.

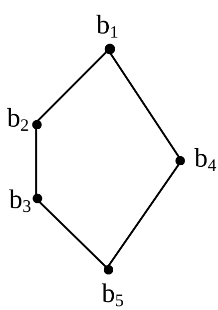


This is a lattice as well as a graph.

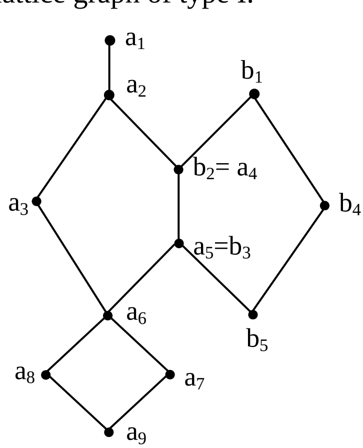
Example 2.10: Let L_1 =



and L_2 =



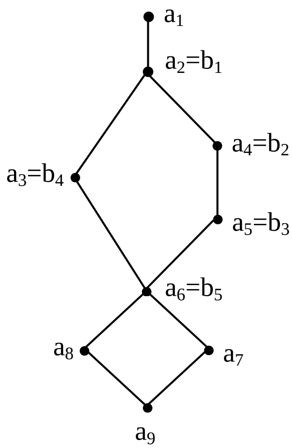
be two lattices. We can merge a_4a_5 with b_2b_3 and get the following pseudo lattice graph of type I.



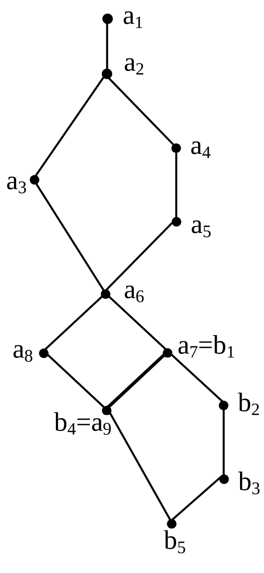
This is only a graph and not a lattice.

We can also merge a_2a_3 $a_4a_5a_6$ part with b_1b_4 $b_2b_3b_5$ and get a pseudo lattice graph which is as follows:

It is clearly a lattice and that lattice is L_1.

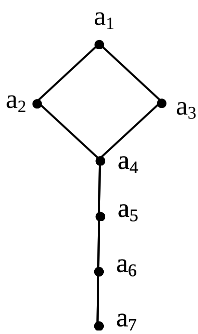


We can also merge the vertex a_7 with v_1 and edge a_7a_9 with b_1b_4 and b_4 with a_9 and obtain a pseudo lattice graph of type I which is as follows:

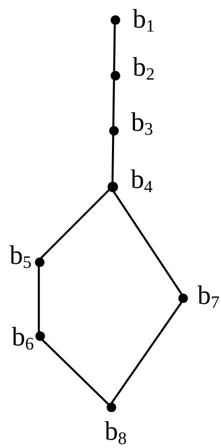


This pseudo lattice graph of type I is a lattice.

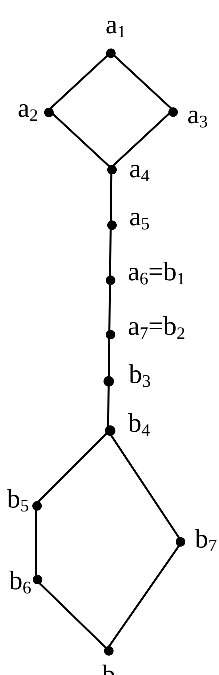
Example 2.11: Let L_1 and L_2 be the two lattices given by



and

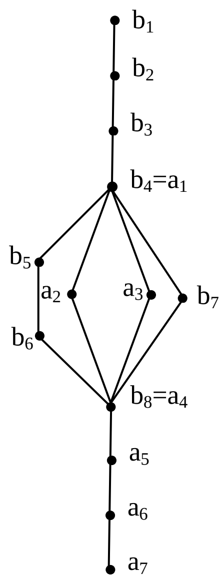


We can merge the edge a_6a_7 with edge b_1b_2 and obtain the following pseudo lattice graph of type I.



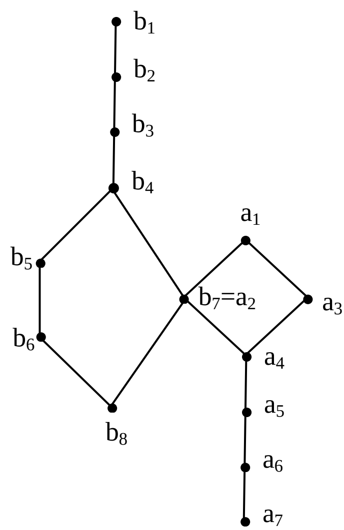
Clearly this is a lattice which is not distributive and not modular.

Now we merge b_4 with a_1 and b_8 with a_4 and obtain the pseudo lattice graph of type I.



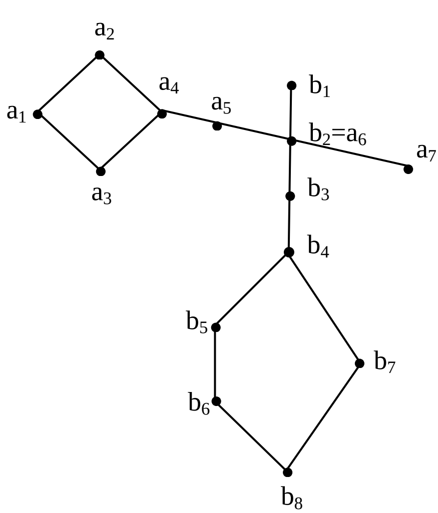
Clearly the resultant is a lattice and not a modular or distributive lattice.

We can merge the vertex a_2 with b_7 and obtain the pseudo lattice graph of type I which is as follows:



Clearly the pseudo lattice graph is a lattice.

We can merge vertex b_2 with a_6 and get the following pseudo lattice graph.



The resultant is only a graph and not a lattice.

Now having seen several examples of pseudo lattice graphs we define substructure in them.

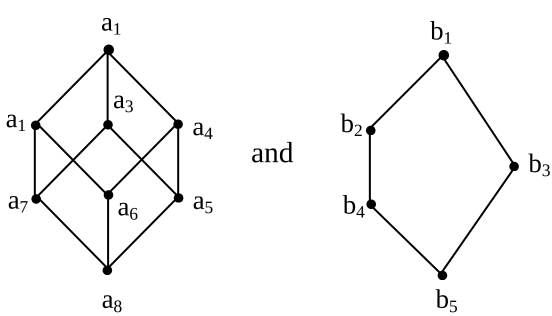
DEFINITION 2.1: *Let L_1 and L_2 be two lattices. S be the pseudo lattice graph of type I obtained by merging a vertex or more vertices or merging edges and vertices. Let P be the subgraph of S; subgraph defined in the usual way, then*

(i) P can be lattice or
(ii) P can be a graph.

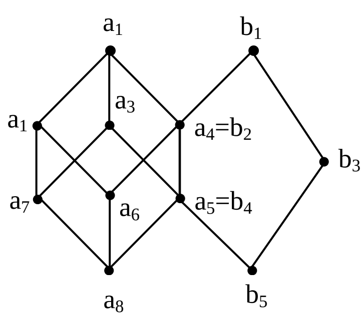
P is defined as the pseudo lattice subgraph of S of type I.

We will illustrate this situation by an example or two.

Example 2.12: Let L_1 and L_2 be the following lattices.

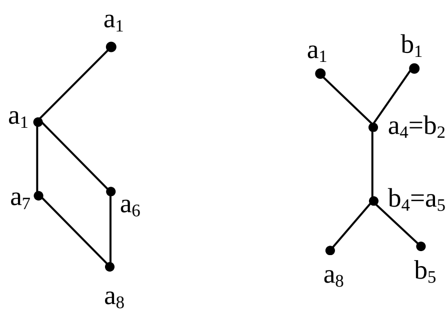


Let S be the pseudo lattice graph of type I obtained by merging a_4 with b_2 and b_4 with a_5.



Clearly S is not a lattice S has several subgraphs that are subgraphs and not lattices and a few lattices.

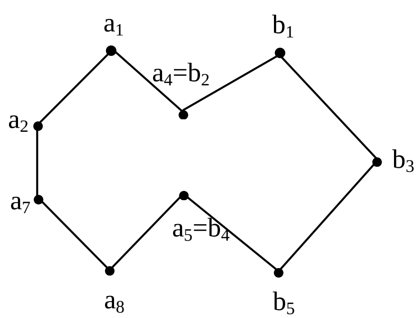
We will illustrate this in the following.



P_1 is a subgraph of S which is not a lattice.

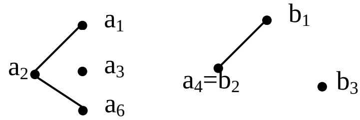
P_2 is also a subgraph of S which is not a lattice P_2 is a tree.

Consider P_3 the pseudo lattice graph of type I which is as follows.



is only a subgraph which is a connected subgraph of S.

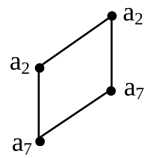
Let P_4



be the pseudo lattice subgraph of type I.

P_4 is a graph and it is not connected.

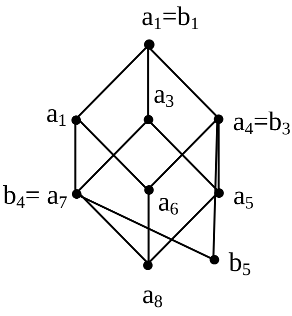
Let P_5



be the pseudo lattice graph of type I. Clearly P_5 is a lattice.

Thus with S got by merging an edge of the two lattices L_1 and L_2 we got 5 subgraphs of S.

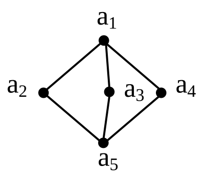
Now we obtain a pseudo lattice graph of type I by merging the edges a_2a_7 with b_2b_4 edge a_1a_2 with edge b_1b_2 and edge a_1a_4 with a_1b_4 which is denoted by S_1 is as follows:

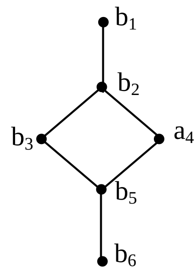


Clearly S_1 is not a lattice only a graph 9 vertices and 14 edges.

However this is only a graph.

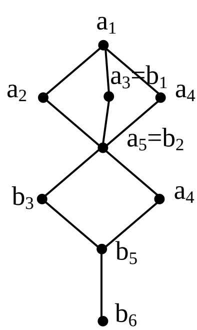
Example 2.13: Let L_1 and L_2 be two lattices given in the following.

$L_1 =$ 

$L_2 =$ 

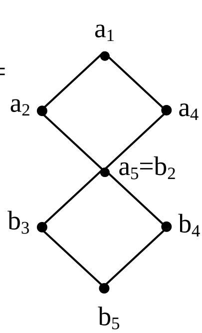
Let S_1 be the pseudo lattice graph obtained by merging the edges a_3a_5 with b_1b_2

$S_1 =$



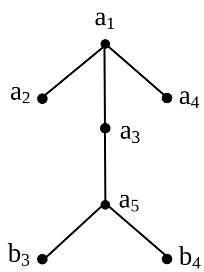
Clearly the pseudo lattice graph of type I. S_1 is a lattice.

S_1 is modular and non distributive.

$P_1 =$ 

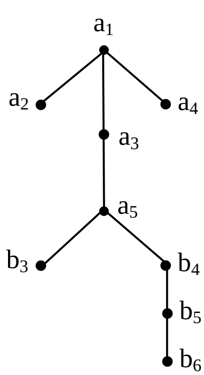
is a subgraph which is distributive lattice.

$P_2 =$



is a subgraph which is a tree. P_2 is not a lattice.

Consider P_3

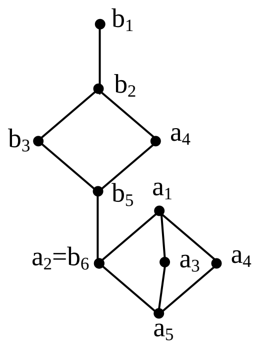


a pseudo lattice subgraph of S_1. P_3 is not a lattice only a graph which is a tree.

Thus S_1 has several subgraphs which are lattices or graphs which are trees or otherwise.

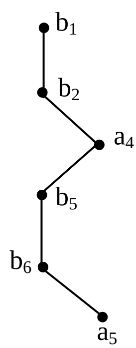
Let us consider S_2 the pseudo lattice graph given by the following.

$S_2 =$

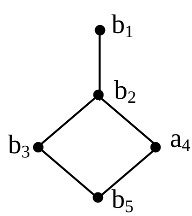


S_2 is got by merging a_2 with b_6. We see S_2 has subgraphs some of which are sublattices and some of them are subgraphs.

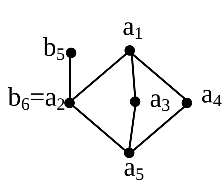
$P_1 =$



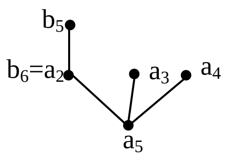
$P_2 =$



$P_3 =$



pseudo lattice subgraphs some of which are sublattices and some are subgraphs.

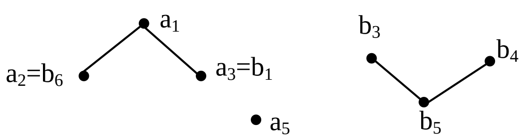


is a pseudo lattice subgraph which is a subgraph and not a lattice.

Infact P_3 can be realized as a semi lattice.

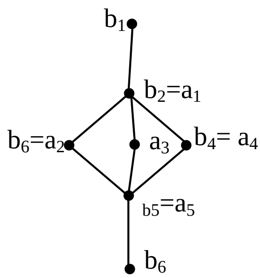
However S_3 is connected.

P_4 =

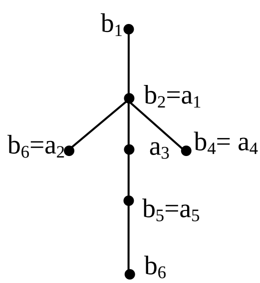


be a pseudo lattice subgraph of S_2. P_4 is only a subgraph which is not connected.

Let M be a pseudo lattice graph got by merge edge a_1a_2 with edge b_2b_3, edge a_1a_4 with b_2b_4 and edge a_2a_5 with b_3b_5 and edge a_4a_5 with b_4b_5 and get the following graph;

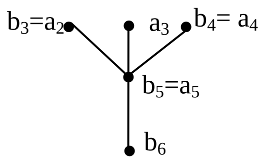


This is a lattice we have the following pseudo lattice subgraph of M.



which is a tree and not a lattice.

Consider the subgraph



which is also a subgraph is a tree.

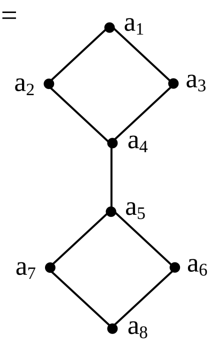
Now having seen merging vertices or edges of two lattice we now proceed onto merge three vertices of three lattices as a single point or merging vertices taken two by two of lattices, the same is true for edges.

This merging will result in a graph or a lattice known as multi pseudo lattice graph.

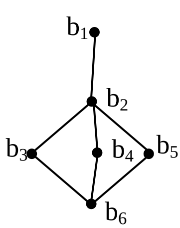
This will be illustrated by the following examples.

Example 2.14: Let

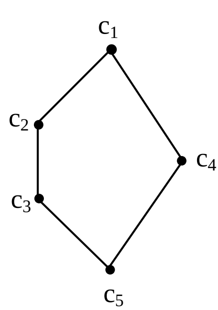
L_1 =



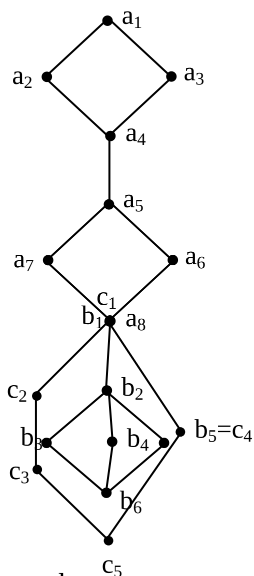
L_2 =



and L_3 =

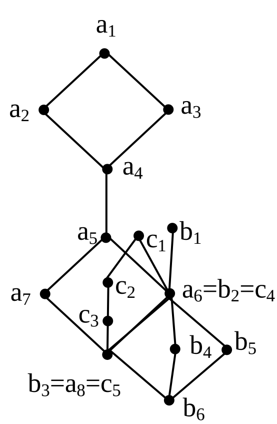


be three lattices by merging b_1 with c_1 and a_8 vertices we have the following pseudo lattice graph of type I.



is not a lattice only a graph.

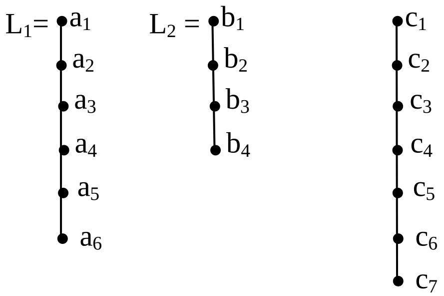
Now we merge the three edges a_6a_8, b_2b_2 and c_4c_5 and get the following pseudo lattice graph of type I.



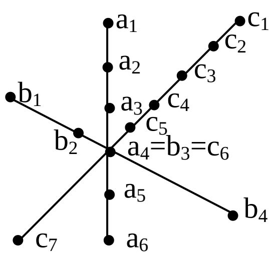
Clearly the resultant is only a multi pseudo lattice graph.

We can by this way build several such graphs.

Example 2.15: Let us consider the three chain lattices.

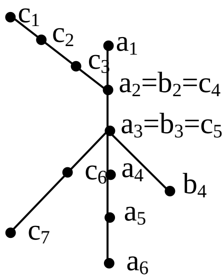


By merging the vertices a_4, b_3 and c_6 we get the following multi pseudo lattice graph of type I.

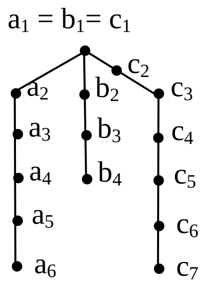


is not a lattice only a graph in fact a tree.

We can also merge in the following way.



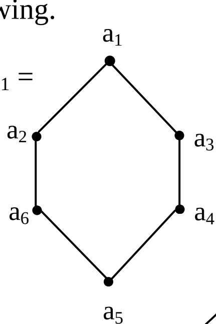
We can merge vertices a_1, b_1 and c_1 together



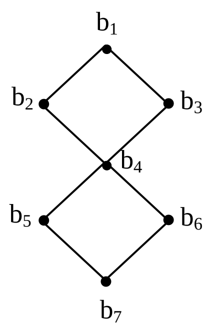
The pseudo lattice graph of type I is not a lattice only a graph which is a tree.

Example 2.16: Let L_1, L_2 and L_3 be three lattices given in the following.

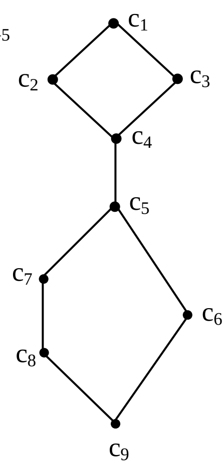
L_1 =



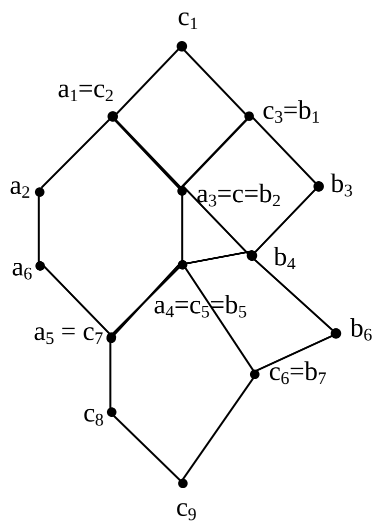
L_2 =



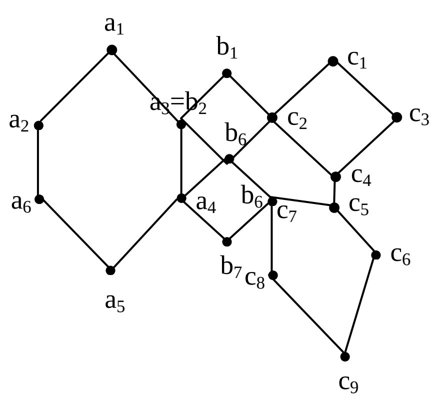
and L_3 =



We merge edges a_3a_4, c_4c_5, a_1a_3 with c_2c_4 and a_4a_5 with c_4c_7 and vertices b_2 with a_3 and b_5 with a_4 and get the following multi pseudo lattice graph of type I.

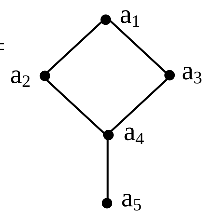


We can merge vertices a_3 with b_2 and a_4 with b_5 and b_3 with c_2 and b_6 with c_7 and get the pseudo lattice graph of type I.

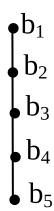


The resultant graph is only a graph and not a lattice. However it is a connected graph.

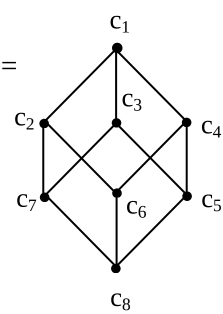
Example 2.17: Let L_1 =



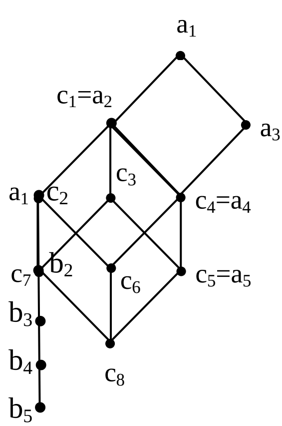
L_2 =



L_3 =



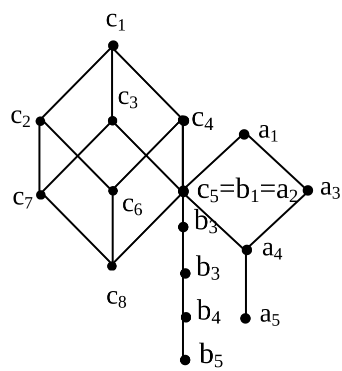
be three lattices let us merge b_1b_2 with c_2c_7 and c_4c_5 with a_4a_5 c_1c_4 with a_2a_4 and the resulting pseudo lattice graph of type I, S_1 is as follows.



Let us merge b_1b_2 with c_2c_7 and c_4c_5 with a_4a_5, c_1 c_4 with a_2a_4 and the resulting pseudo lattice graph of type I, S_1 is as follows.

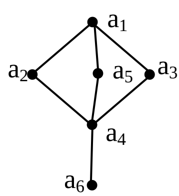
Clearly S_1 is not a lattice only a graph.

We can merge the 3 vertices b_1, c_5 and a_2 and get the following multi pseudo lattice graph of type I.

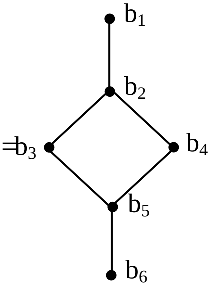


This is only a graph and not a lattice.

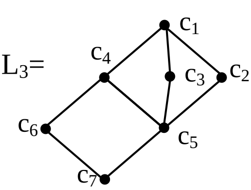
Example 2.18: Let L_1 =



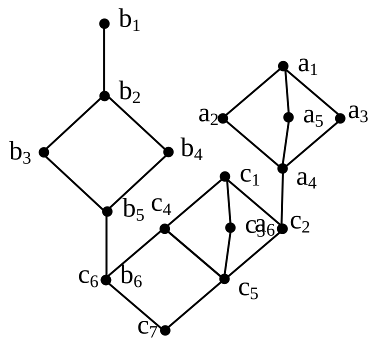
L_2 =



L_3=

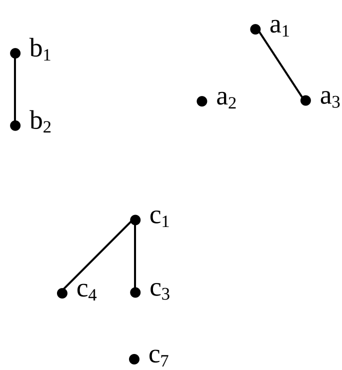


Merge vertices c_6 with b_6 and c_2 with a_6, we get the following multi pseudo lattice graph of type I which is as follows.

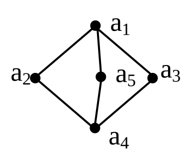


The resulting diagram is only a graph and not a lattice.

Now we can for all these pseudo lattice graphs of type I find subgraphs and the study the property of connected ness and so on.



is a subgraph which is not connected.



a subgraph which is a lattice which is connected.

Now having seen examples of pseudo lattice graphs of type I and their substructures we proceed onto define pseudo lattice graphs of type II using a lattice and a graph.

Chapter Three

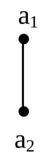
PSEUDO LATTICE GRAPHS OF TYPE II

In this chapter the new notion of merging one or more vertices of a lattice with that of a graph or one or more edges of a lattice with a graph is carried out. This study is new and innovative. The resultant graph (or lattice) is defined as the pseudo lattice graph of type II.

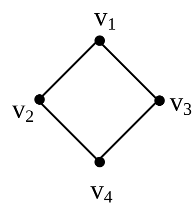
In the earlier chapter merging of a vertex or more vertices or one edge or more edges of lattices was carried out. Those resulting graphs or lattices were defined as pseudo lattice graph of type I. Several interesting features about these pseudo lattice graphs of type I was systematically defined and developed.

Before we make the definition of pseudo lattice graph of type II we will first illustrate the situation by an example or two.

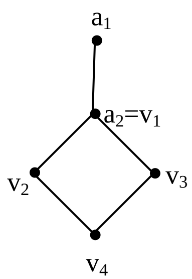
Example 3.1: Let L be the chain lattice of order two;

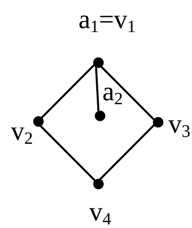


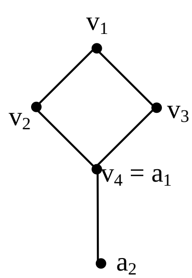
Let G be the graph

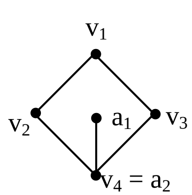


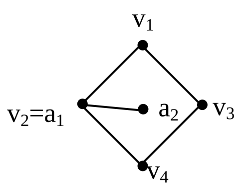
We give some of the possible merging of the vertices.

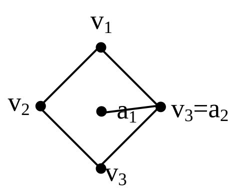


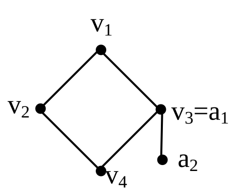


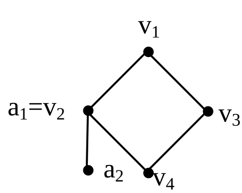


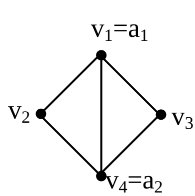


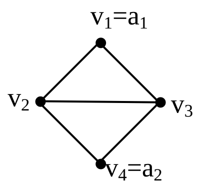

v_1
$v_2=a_1$
a_2
v_3
v_4


v_1
v_2
a_1
$v_3=a_2$
v_3


v_1
v_2
$v_3=a_1$
a_2
v_4


v_1
$a_1=v_2$
v_3
a_2
v_4

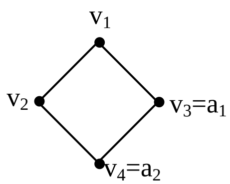

$v_1=a_1$
v_2
v_3
$v_4=a_2$



The last two pseudo graphs are got by merging two vertices.

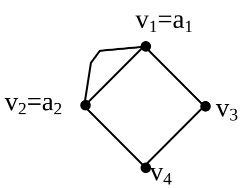
Finally we can merge an edge and two vertices.

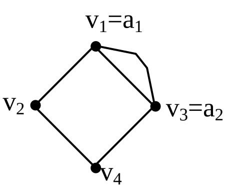
So that we get



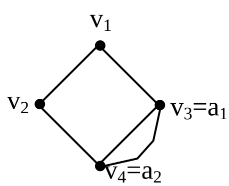
Such merging will be called as special trivial merging for the resultant gives the graph G (it may give the lattice L).

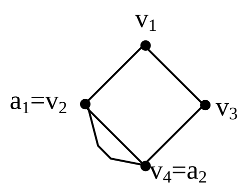
Finally we can merge two vertices so that the graph has the following form



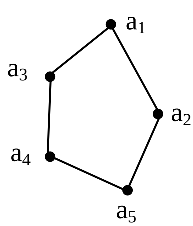


Thus we can get several pseudo lattice graphs of type II.

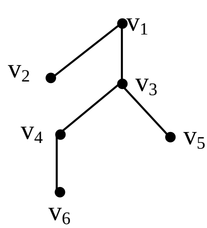




Example 3.2: Let L be a lattice given by

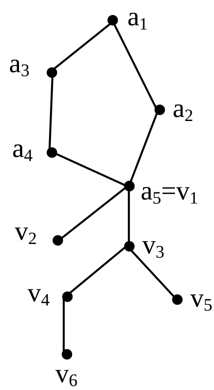


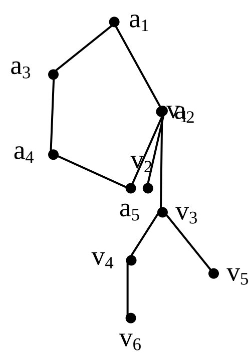
and G be the graph given in the following.

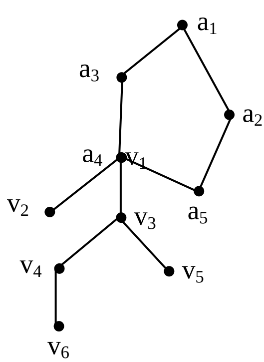


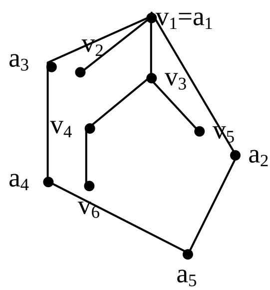
We find pseudo lattice graphs of type II.

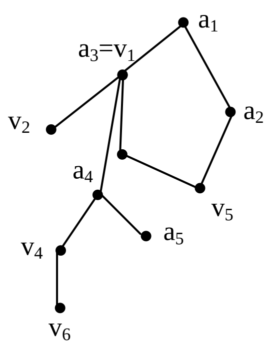
Merging of one vertex of L with one vertex of a graph G.


a1
a3
a2
a4
a5=v1
v2
v3
v4
v5
v6

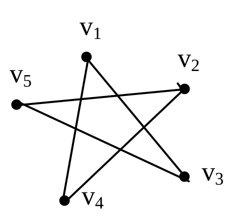

a1
a3
v1a2
a4
v2
a5
v3
v4
v5
v6

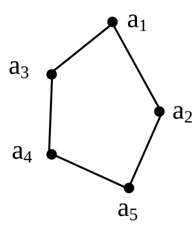

a1
a3
a2
a4 v1
v2
v3
a5
v4
v5
v6



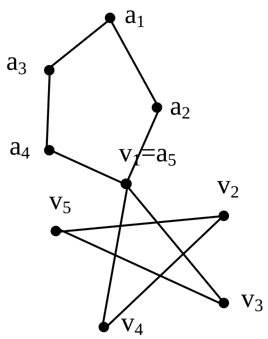


Example 3.3: Let

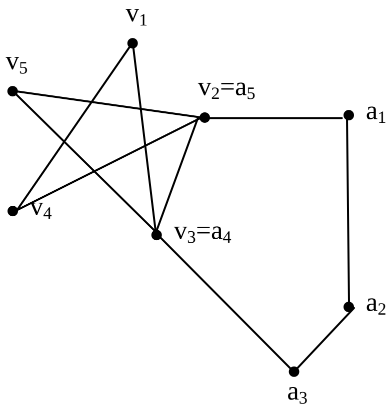
G =  be a graph

and L = 

be a lattice. The pseudo lattice graph of type II got by merging v_1 with a_5 is follows:

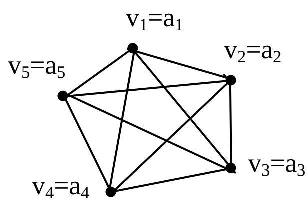


The resultant is a graph we merge the vertices v_2 with a_5 and v_3 with a_4 and obtain the pseudo lattice graph of type II.



The resultant is only a graph.

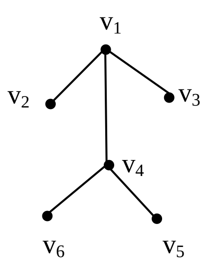
We see both the graphs are distinct. We can merge the 5 vertices v_1, v_2, v_3, v_4 and v_5 with a_1, a_2, a_3, a_4, a_5 which is as follows.



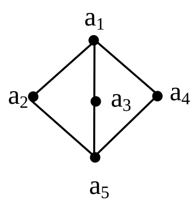
This is a graph.

Thus the pseudo lattice graph of type II got by using L with G is a complete graph.

Example 3.4: Let G =

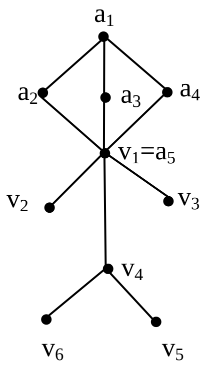


and L =



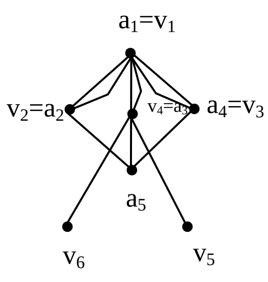
be the graph and lattice respectively.

We can merge vertices a_5 with v_1 and obtain the following pseudo lattice graph of type II.

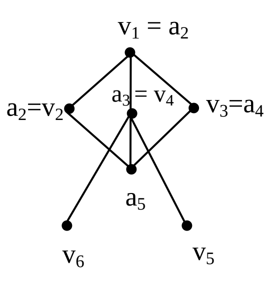


This is only a graph.

Now we merge v_1 with a_1, v_2 with a_2, v_3 with a_4, v_4 with a_3 and obtain the following pseudo lattice graph of type II.

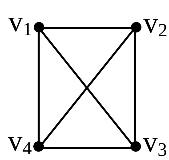


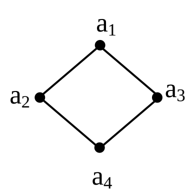
The resultant is again a graph. We can also get the pseudo lattice graph of type II by merging edges v_1v_2 with a_1a_2 and v_1v_3 with a_1a_4 and v_1v_4 with a_1a_3 and is as follows.



The resultant is only a graph and not a lattice.

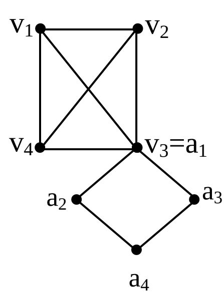
Example 3.5: Let

G = 

and L = 

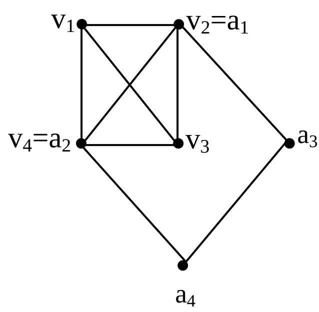
be a graph and lattice respectively.

By merging vertex a_1 with vertex v_3 we get

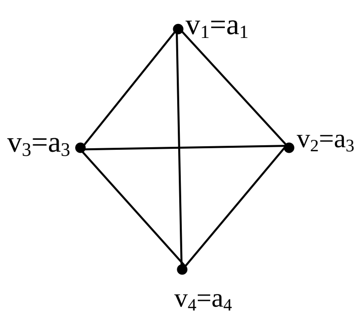


The pseudo lattice graph of type II is only a graph.

We can merge the edge v_2v_4 with a_1a_2 and obtain the following pseudo lattice graph of type II.

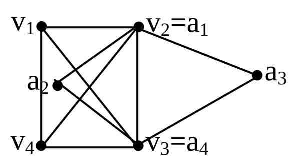


This is only a graph. We can merge the four edge and four vertices and get the following pseudo lattice graph of type II.



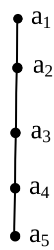
which is nothing but the graph G.

By merging vertices v_1 with a_1 and v_3 with a_4 we get the following pseudo lattice graph of type II.



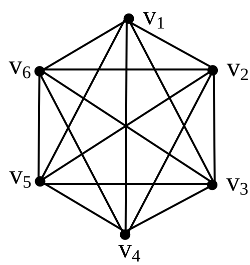
The resultant graph is only a graph.

Example 3.6: Let L =



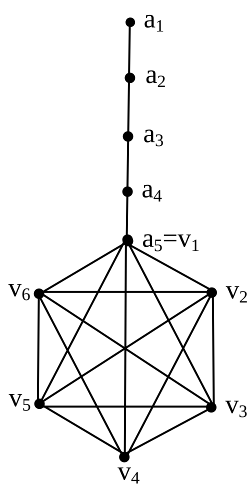
and

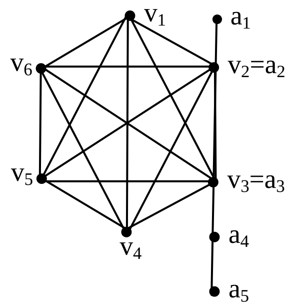
G =



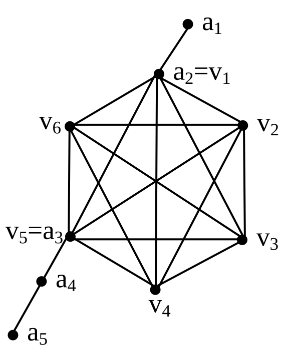
be a lattice and a graph respectively.

We get the following pseudo lattice graphs of type II.



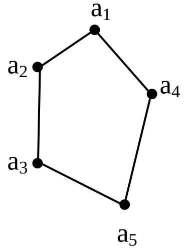
P_2 = 

and

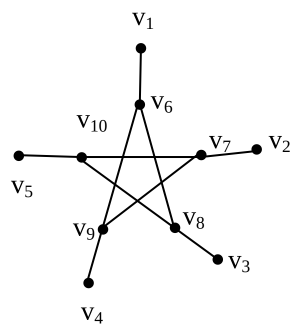
P_3 = 

All of them are only graphs.

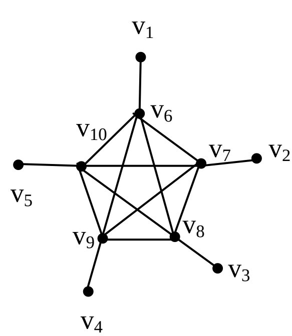
Example 3.7: Let



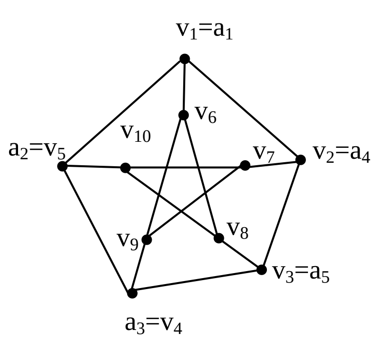
be the pentagon lattice and



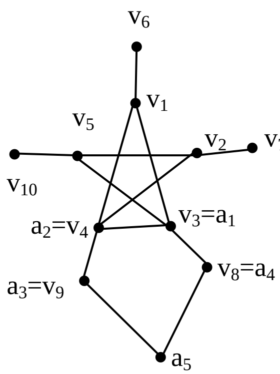
be a graph. We can get several pseudo lattice graphs of type II which are as follows:



is a graph.

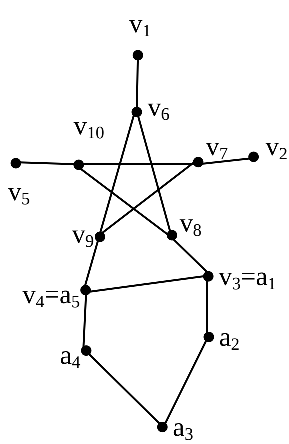


We get the pseudo lattice graph of type II to be a graph which is the Peterson graph.



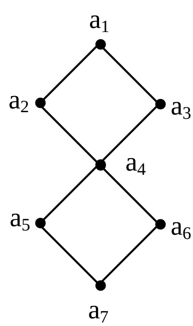
is a pseudo lattice graph of type II obtained by merging the vertices a_1 with v_3, a_4 with v_8, a_8 with v_9 and a_2 with v_4 and merging the edges v_1 v_9 with a_2 a_3 and v_3 v_8 with a_1 a_4.

The resultant is only a graph.

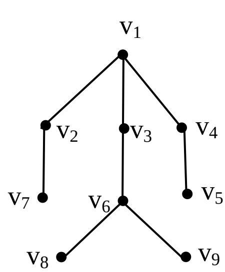


The merging of the vertices v_3 with a_1 and v_4 with a_5 results in a pseudo lattice graph of type II which is a graph.

Example 3.8: Let

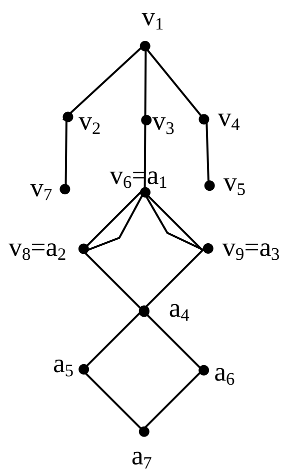
L = 

and

G = 

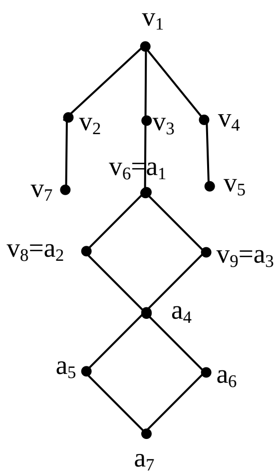
be a lattice and a graph respectively.

We can merge vertex v_8 with a_2 v_6 with a_1 and v_9 with a_3 and obtain the following pseudo lattice graph of type II.

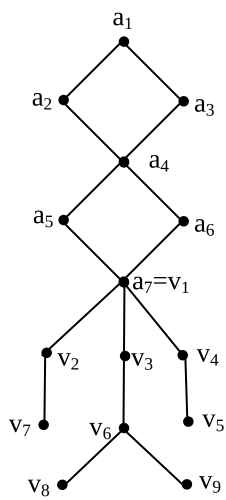


Clearly this is not a lattice only a graph.

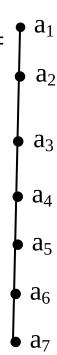
We can also merge in the above the edges a_1a_2 with v_6v_8, a_1a_3 with v_6v_9 and get the following pseudo lattice graph of type II.



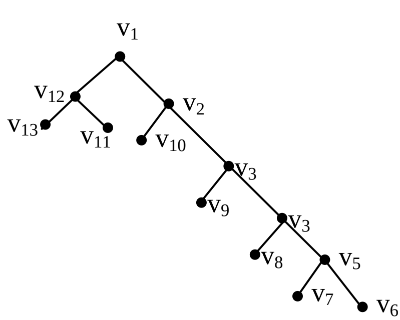
We can merge only the vertex a_7 with v_1 and get the following pseudo lattice graph of type II.



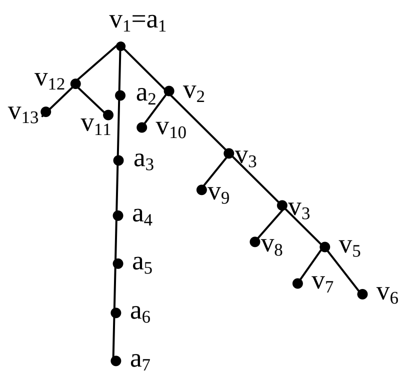
Example 3.9: Let L =



and G =

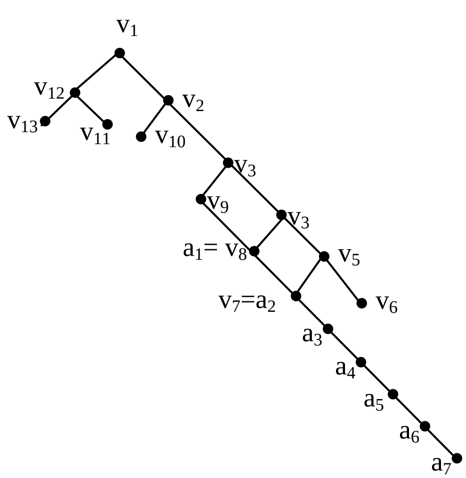


be a lattice and graph respectively where ever adjoin a vertices of G with L we will get only the pseudo lattice graph of type II to be only a tree.



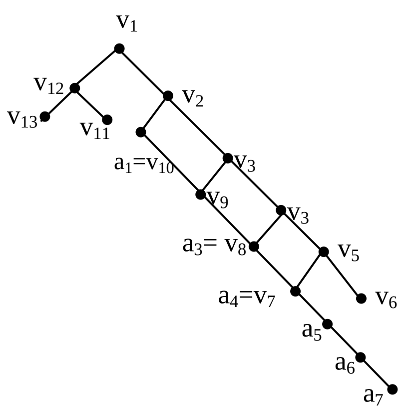
However if we merge two vertices of the graph with the lattice we may not in general get a tree.

This is illustrated by the following.



Clearly this pseudo lattice graph of type II is not a tree only a graph.

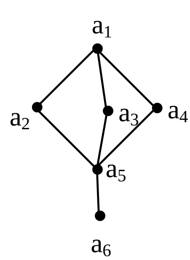
Let us merge the vertices v_{10} with a_1, v_9 with a_2, v_8 with a_3 and v_7 with a_4. The following pseudo lattice graph of type II is not a tree.



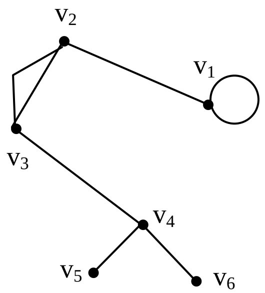
Clearly this is only graph and not a tree.

So merging a tree with a chain lattice may not in general give a pseudo lattice graph of type II which is a tree.

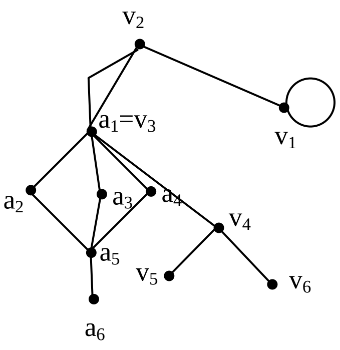
Example 3.10: Let L =



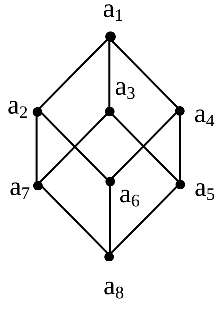
and

G = 

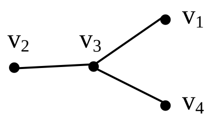
be the lattice and graph respectively. We can merge v_3 vertex with a_4 and obtain the pseudo lattice graph of type II.



This is only a graph.

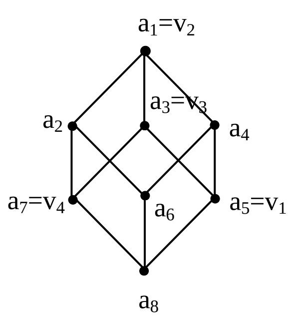
Example 3.11: Let L = 

and

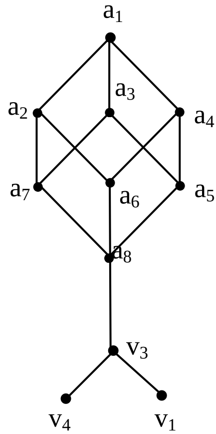
G = 

be a lattice and graph respectively.

By merging vertices v_3 with a_3 and edge v_2v_3 with a_1a_3 v_1v_3 with a_3a_5 and v_3v_4 with a_3a_7 we get the following pseudo lattice graph of type II which is as follows:

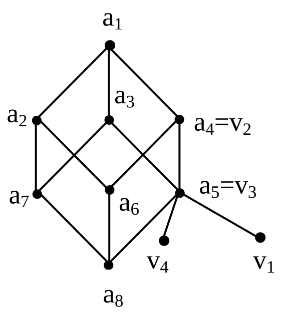


This is a lattice which is a Boolean algebra.



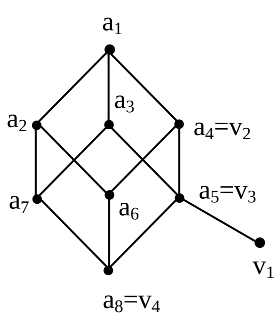
The pseudo lattice graph of type II is only a graph and not a lattice.

By merging the edge a_4a_5 with v_2v_3 we get the following pseudo lattice graph of type II.



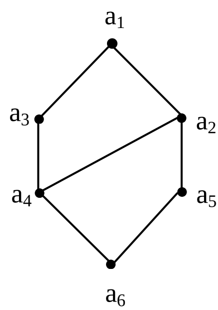
This is only a lattice.

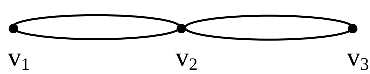
Suppose a_5a_8 with edge v_3v_4 in addition to merging the edge a_4a_5 with v_2v_3 we get the following pseudo lattice graph of type II.



This is a graph and not a lattice.

Example 3.12: Let L be the lattice

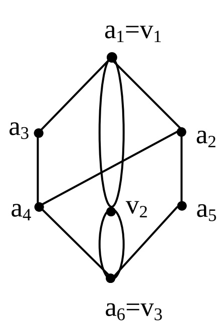


G = 

and G the graph.

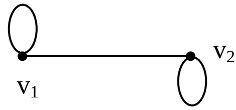
What ever be the merging a vertex or two vertices we will get only the pseudo lattice graph of type II to be a graph.

We merge vertex a_1v_1 and a_6 and v_3 and obtain the following pseudo lattice graph of type II.

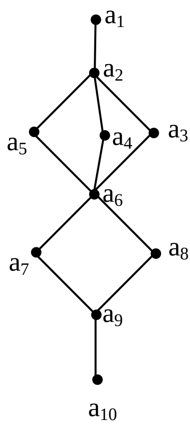


This is only a graph and not a lattice.

Example 3.13: Let

G = 

be a graph and L is a lattice given in the following.



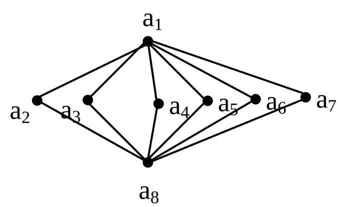
Any pseudo lattice graph of type II by merging any of the vertices or edges is only a graph and never a lattice as the graph has self loops.

In view of this we have the following theorem.

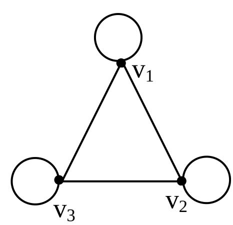
THEOREM 3.1: *Let L be any lattice and G a graph with a loop. The pseudo lattice graph of type II using this L and G is never a lattice.*

Proof: Follows from the fact G is a graph with a loop we see so the pseudo lattice graph of type II can never be a lattice.

Example 3.14: Let L be a lattice whose Hasse diagram is as follows:

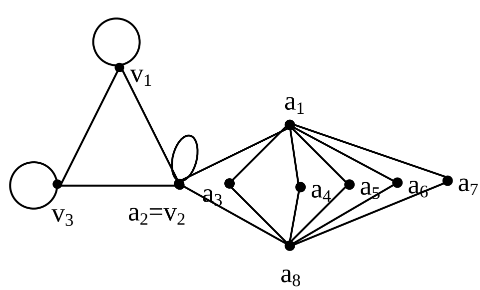


and G be the following graph.

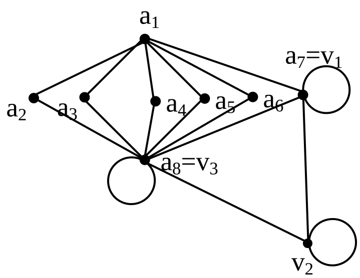


Let the pseudo lattice graph of type II be obtained by merging vertices a_2 with v_2.

We get the following graph.



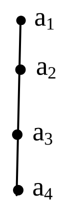
We can merge the edges a_8a_7 with v_1v_3 and get the following pseudo graph of type II.



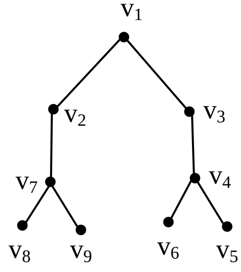
Clearly this is only a graph.

Example 3.15: Let L be a chain lattice

C_4 =

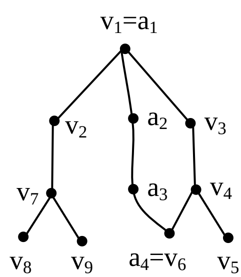


and G be a tree



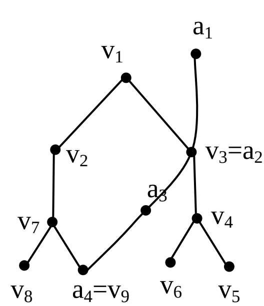
Lattice graph of type II is a tree in some cases only not in all cases.

Suppose we merge vertices v_1 with a_1 and a_4 with v_6 we get the following pseudo lattice graph of type II.



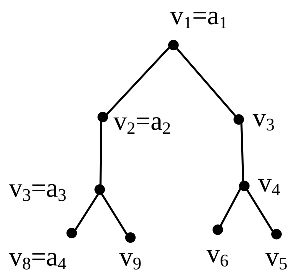
The resultant graph is only a graph and not a tree.

Let us merge vertex v_3 with a_2 and v_9 with a_4 and get the pseudo lattice graph of type II which is as follows:



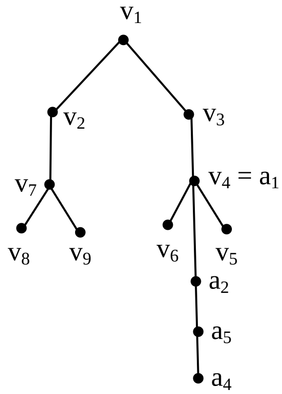
We get a graph which is not a tree.

Let the pseudo lattice graph of type II of L and G got by merging the vertices v_1 with a_1 v_2 with a_2, v_7 with a_3 and v_8 with a_4 be obtained which is given in the following.



The resultant is a graph identical with G hence a tree.

We will merge vertex v_4 with a_1 the resultant pseudo lattice graph of type II is as follows:



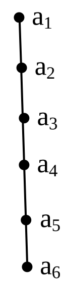
We see the resultant is a tree different from G.

Now we will see the substructures of the pseudo lattice graphs of type II.

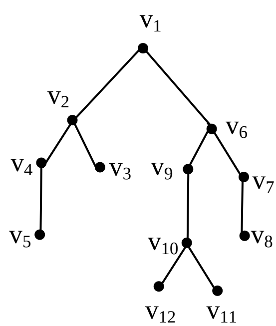
This is illustrated by the following examples.

Example 3.16: Let

L =



a chain lattice and G be the graph

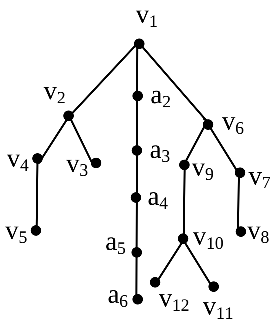


be a tree.

The pseudo lattice graph of type I in general is not a lattice or a tree only a graph.

We find subgraphs of the pseudo lattice graphs of type II.

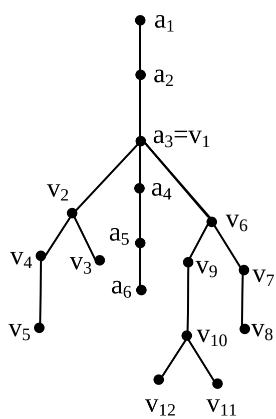
$P_1 =$



This is a tree and every subgraph of this P_1 is also tree.

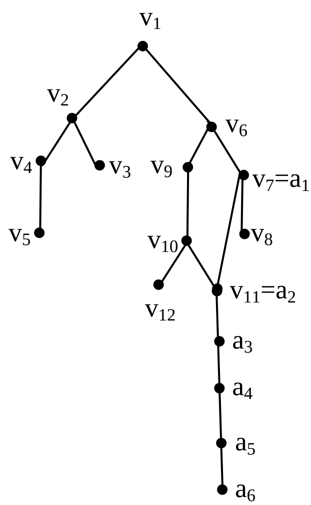
We have the following pseudo lattice graph of type II.

$P_2 =$



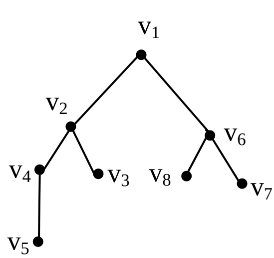
All subgraphs of P_2 are only trees in this case also.

Let us merge vertices v_8 with a_1 and v_{12} with a_2 and get the pseudo lattice graph of type II which is as follows:

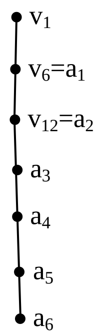


We see this is not a lattice a graph which is not a tree.

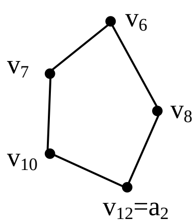
This has subgraphs which are trees for instance



Subgraphs which are chain lattices viz.

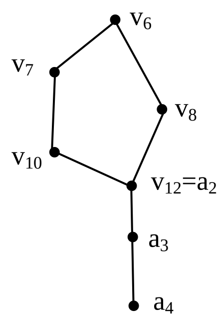


Now consider the subgraph



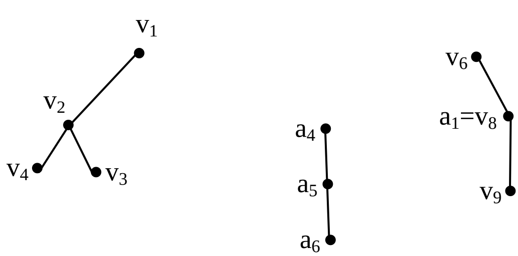
This subgraphs is non modular and a non distributive lattice.

The subgraph



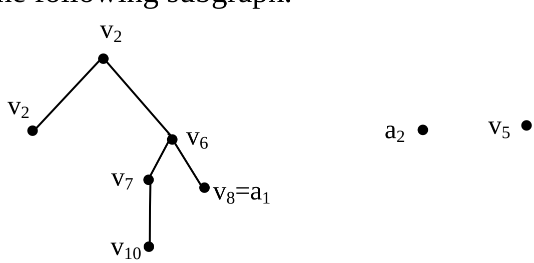
is again a non modular non distributive lattice. All these subgraphs are connected.

We have subgraphs which are disconnected also. These are illustrated in the following.



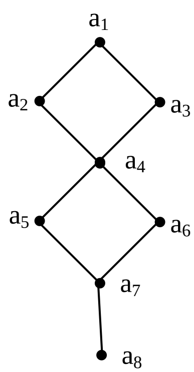
The above subgraph is a not connected subgraph.

Consider the following subgraph.

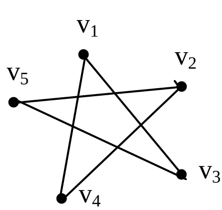


This is also a subgraph which is not connected.

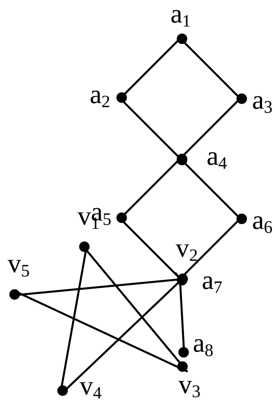
Example 3.17: Consider the following lattice



and the graph



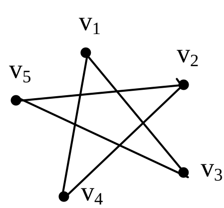
The pseudo lattice graph of type II of the lattice and graph by adjoining two vertices is as follows:



This is only a subgraph and not a lattice. This has subgraphs which are lattices as well as graphs.

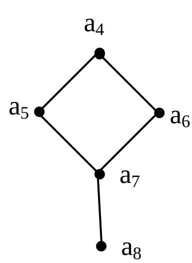
Consider,

$P_1 =$



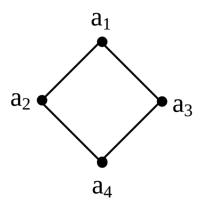
P_1 is a subgraph of S_1.

$P_2 =$



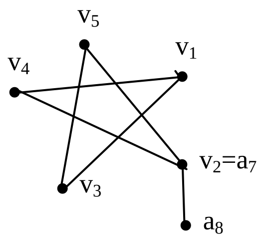
P_2 is a subgraph which is a distributive lattice.

$P_3 =$



is a subgraph which is a sublattice also Boolean algebra of order 2^2.

$P_4 =$



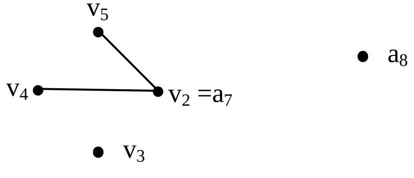
is a subgraphs which is not a lattice.

All the subgraphs P_1, P_2, P_3 and P_4 are connected subgraphs of the pseudo lattice graph of type II.

However all lattices are always connected graphs.

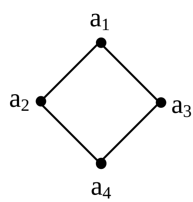
We also have subgraphs which are not connected which are as follows:

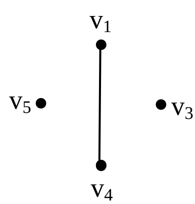
$T_1 =$



The above subgraph T_1 is not connected.

T_2 =





The subgraph T_2 is also not connected.

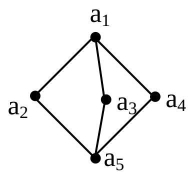
In view of all these we have the following theorem.

THEOREM 3.2: *Let P_{GL} = {Collection of all pseudo lattice graphs of type II merging vertices / edges of the graph G and lattice L}.*

(i) G is a subgraph of every P in P_{GL}.
(ii) L is a subgraph which is a lattice in every P in P_{GL}.
(iii) Every subgraph of P_{GL} need not in general be a connected graph.
(iv) $P \in P_{GL}$ has connected subgraphs.

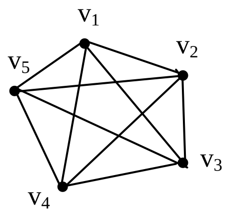
The proof is direct and hence left as an exercise to the reader.

Example 3.18: Let L =



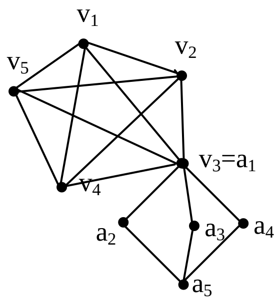
and

G =



be a lattice and a graph respectively.

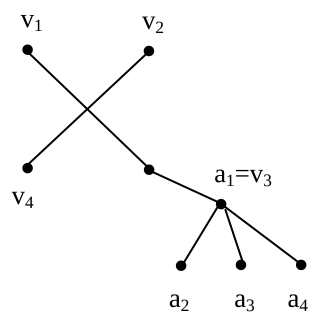
P =



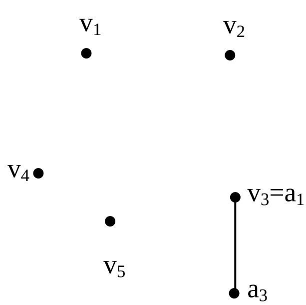
This P is a pseudo lattice graph of type II which is only a graph.

This has subgraphs which are lattices as well as subgraphs some are connected subgraphs. Some disconnected subgraphs of P exist.

For all vertices alone is a subgraph of P which is totally disconnected.

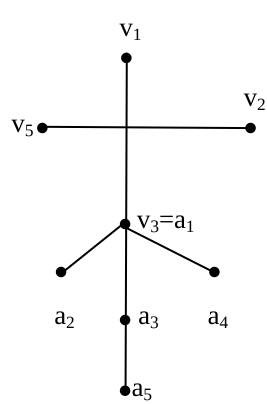


is a subgraph of P which is disconnected.



is again subgraph which is disconnected.

Now consider the pseudo subgraph is as follows.

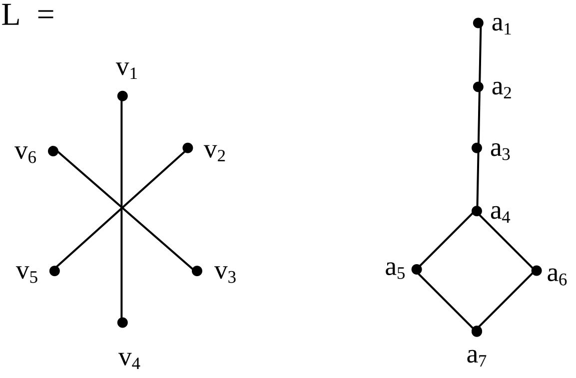


This is only a subgraph and not a lattice.

We can have connected and not connected subgraphs of P.

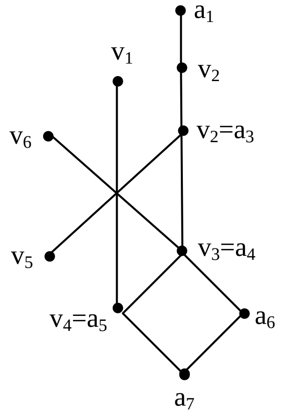
Example 3.19: Let

L =



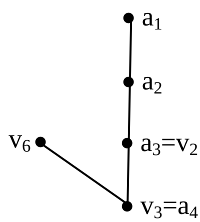
be a lattice and a graph. We can get several such pseudo lattice graphs of type II.

We get a pseudo lattice graph of type II.



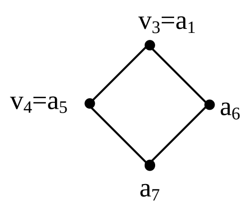
This is only a graph and not a lattice.

S_1 =



is a graph and not a lattice.

Consider



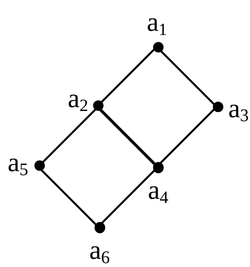
a subgraph which is lattice infact a Boolean algebra of order four.

We have seen subgraphs of the pseudo lattice graphs of type II.

Consider the following example.

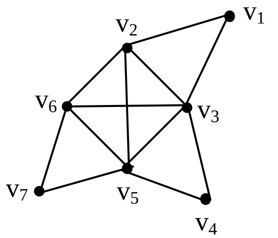
Example 3.16: Let

L_1 =



and

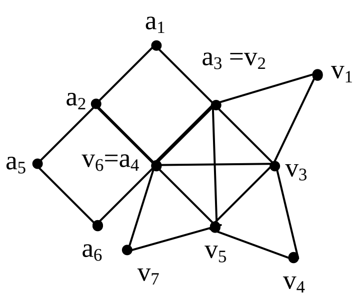
G =



be a lattice and graph respectively.

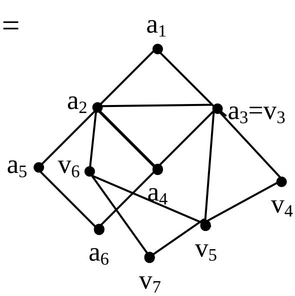
We can have several pseudo lattice graphs of type II got by merging vertices or edges or both.

P_1 =



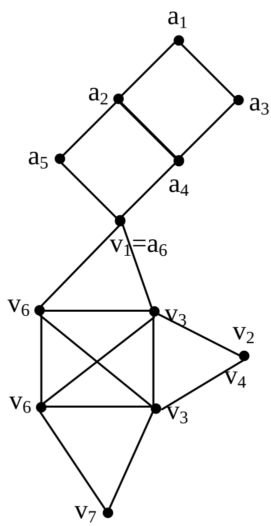
This is a pseudo lattice graph of type II which is only a graph and not a lattice.

Consider P_2 =



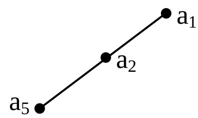
The pseudo lattice graph P_2 of type II is only a graph different from P_1.

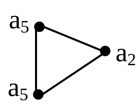
P_3 =



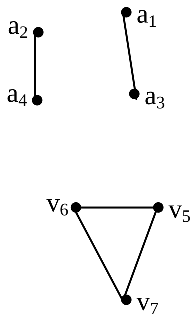
The pseudo graph of type II; the resultant is only a graph and not a lattice different from P_1 and P_2.

We have several subgraphs.

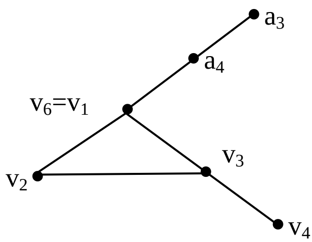




is a subgraph.



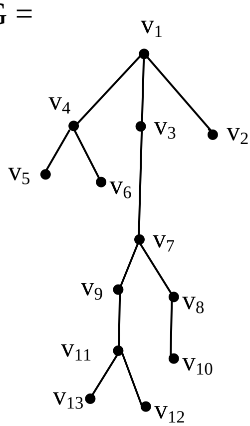
a_1 a_3 a_2 a_4 v_6 v_5 and v_7 are the vertices of the subgraph.



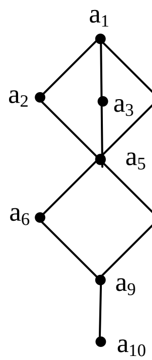
is again a subgraph.

Example 3.21: Let

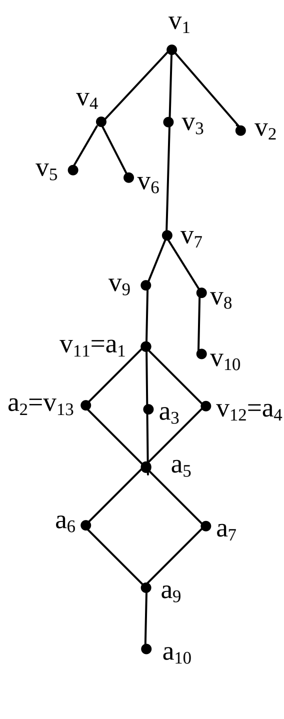
G =



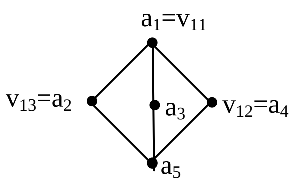
and L =



be the graph and lattice respectively. We can get several pseudo lattice graphs of type II using them. They are as follows:

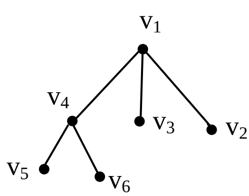


We see this pseudo lattice graph of type II is not a lattice.

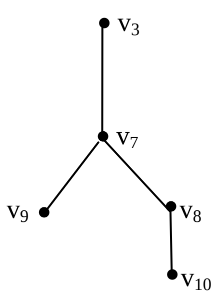


is a subgraph which is sublattice.

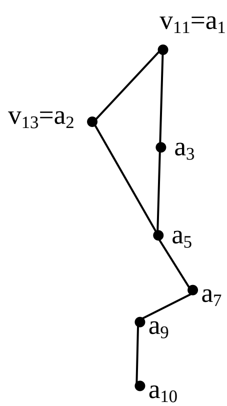
W =



is a subgraph which is a tree.



= B



B is a subgraph which is not a connected subgraph.

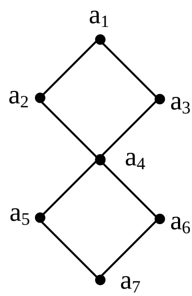
We can consider number of lattices and graphs (say t lattices t ≥ 1 and n – t number of graphs) and merge n of the vertices or n of the edges or merge say some r of the vertices so that all of them are merged in some way or other, that is the merging is done in such a way that no lattice lattice or graph is left out, without being merged with another graph so that an unbroken cycle is set.

We will illustrate this situation by the following examples.

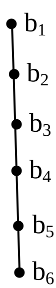
We call the resultant graph as the pseudo lattice graph of type II.

Example 3.22: Let L_1, L_2, L_3 and G_1, G_2 be the lattices and graphs which are as follows:

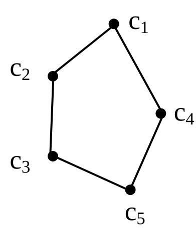
L_1 =



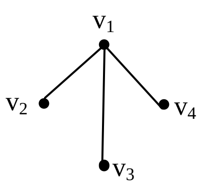
L_2 =



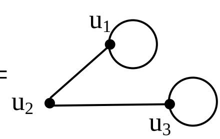
and L_3 =



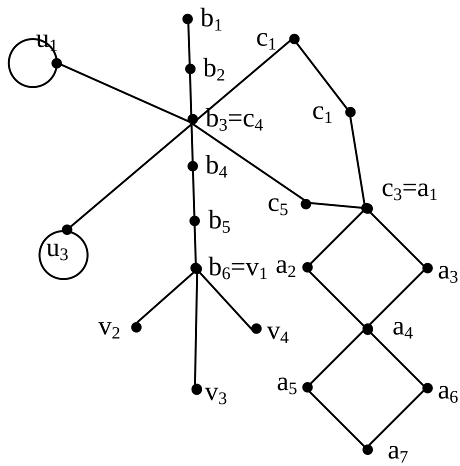
G_1 =



and G_2 =

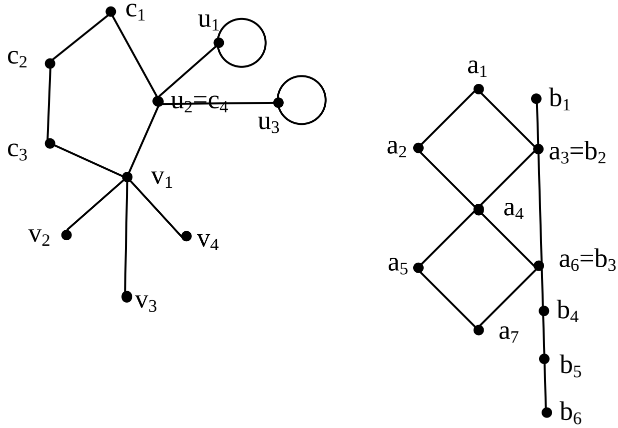


We give a pseudo lattice graph of type II which is as follows:



We see B is a pseudo lattice graph of type II. This is only a graph and not a lattice.

Consider the following graph S.



Clearly S is not a pseudo lattice graph of type II for they are in two disjoint representation where only one of them is a pseudo lattice graph of type II and another is only a pseudo lattice graph of type I.

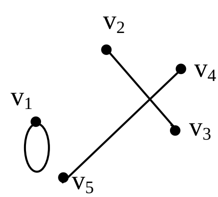
Thus we see every lattice or graph should be merged so that they are not two separate entitles.

Thus there is atleast one graph or lattice which is merged with more than one graph or lattice.

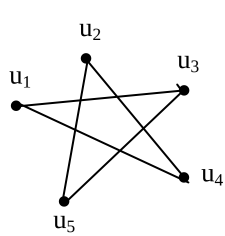
Unless this is done the resultant graph is not a pseudo lattice graph of type II.

Example 3.19: Let

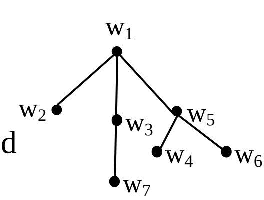
G_1 =



G_2 =

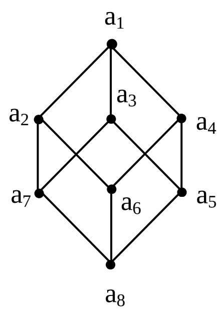


and

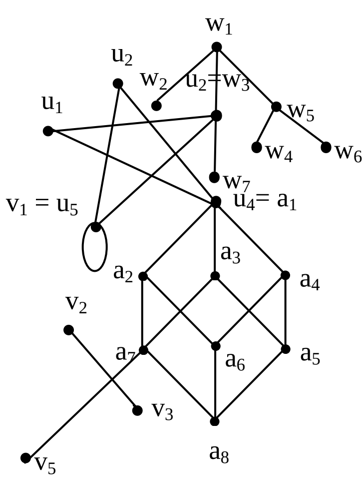


be 3 graphs and

$L_1 =$



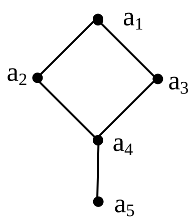
be a lattice. We have the following pseudo lattice graphs of type II.

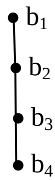


This a pseudo lattice graph of type II which is a graph.

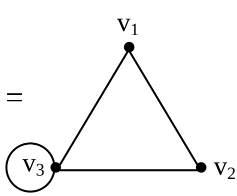
The graph G_2 is merged with all the graphs and lattice, however the graph G_3 is merged only with G_2 however G_2 is merged with G_1.

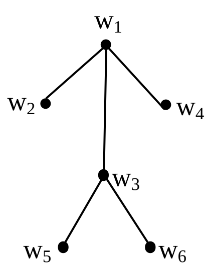
Example 3.20: Let

L_1 = 

and L_2 = 

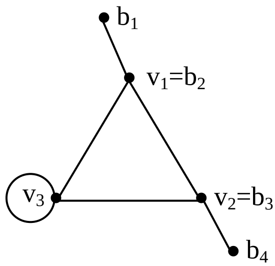
be two lattices.

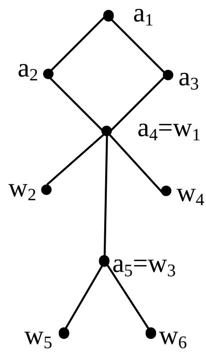
G_1 = 

and G_2 = 

be two graphs.

Using the method of merging of the vertices and or edges we get a pseudo lattice graph of type II.





Clearly the resultant is not a pseudo lattice graph of type II.

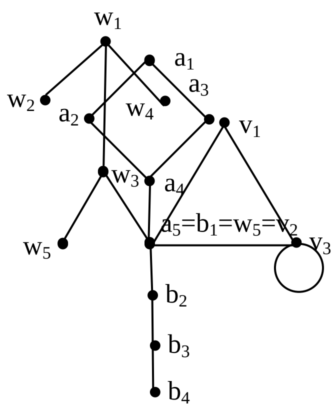
Infact we have only two pseudo lattice graphs of type I for we see they are not merged as per the definition of pseudo lattice graph of type II.

This example is mainly to show that it is mandatory for all the lattices and graphs to be merged with each other so that it is not like the Example 3.24.

Consider the merging of vertices b_1 with a_5, v_2 and w_5.

We get a pseudo lattice graph of type II which is as follows.

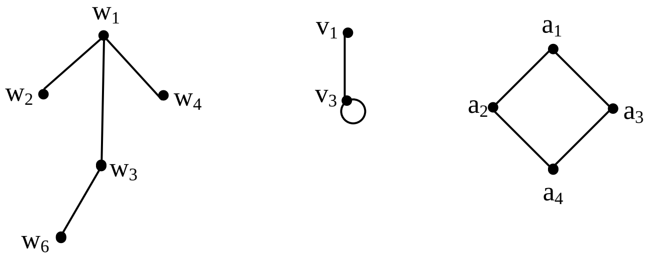
S =

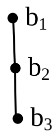


This is a graph and when a vertex of every graph is merged we call such pseudo lattice graphs of type II as strongly merged graph.

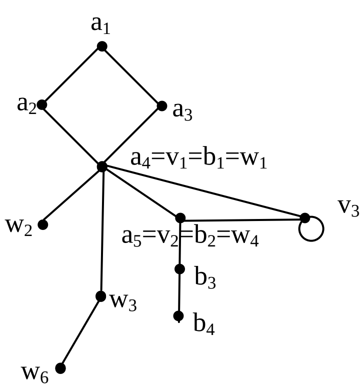
If that point or vertex is removed we call the resultant graph as the dismantled graph.

For we see if the vertex b_1 (a_5, v_2 and w_5) is removed the resultant is four disjoint or non connected subgraphs which is as follows:

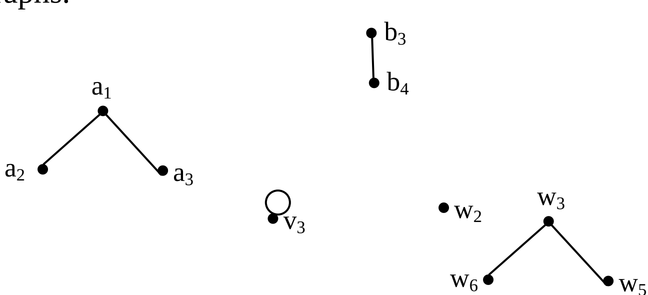


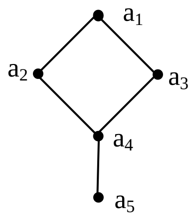


Thus we see in the subgraphs, three are lattices and one is a semilattice. Such merging (or bonding of a single vertex is a strong vertex merge, but one can easily dismantle that graph also. We can also get the strongly edge merged graphs which is as follows:

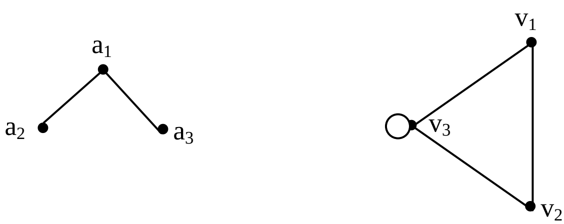


The resultant pseudo lattice graph of type II is a strongly edge merged pseudo lattice graph of type II. The removal of that edge dismantles the graph leading to four subgraphs.



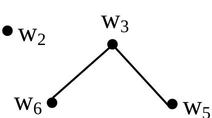


removed of edge a_4a_5 results in



Removal of edge v_1v_2 results in 

Removal of edge w_1w_4 results in



Removal of edge b_1 b_2 results in

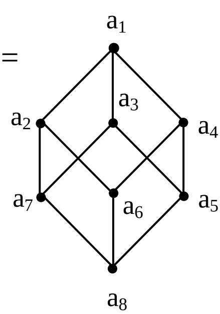
Thus we get the dismantled subgraph which is entirely different. We see strong merging by vertices or edges becomes dismantled if that vertex or edge is moved.

If in case of n_1 lattices and m_1 graphs is a weakly merged pseudo lattice graph if we have maximum number of merging vertices or edges is two that for a lattice or graph is merged to a maximum of two lattice or graph.

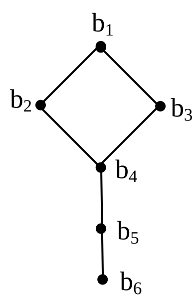
This will be illustrated by the following examples.

Example 3.25: Let

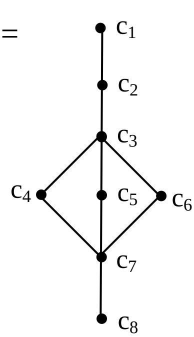
L_1 =



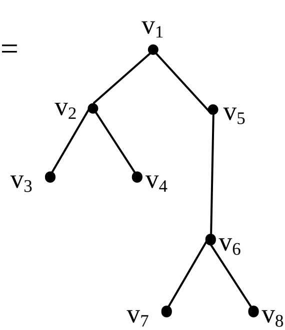
L_2 =



L_3 =

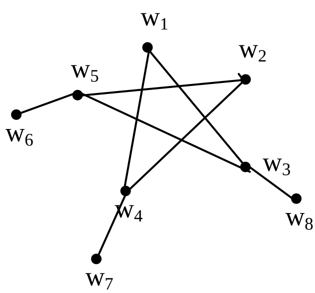


G_1 =



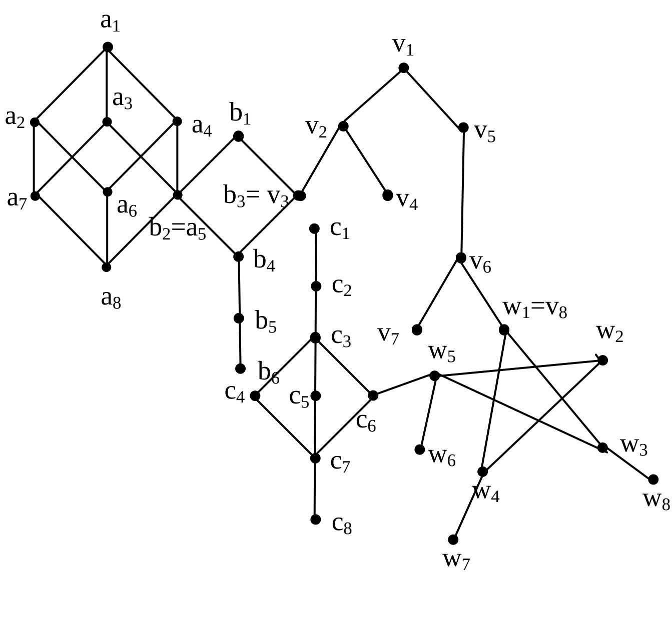
and

G_2 =



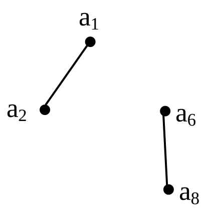
the lattices and graphs.

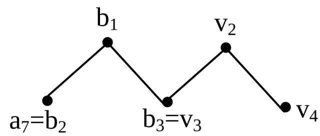
We get the following pseudo lattice graphs.

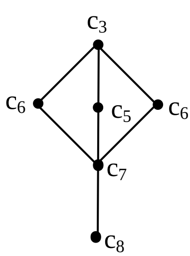


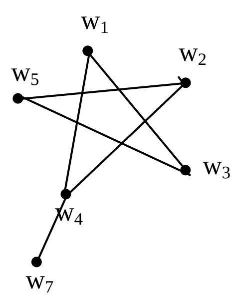
We see each of the lattices or graphs are maximum merged in twos.

The subgraphs are as follows:

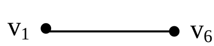




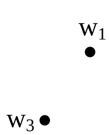


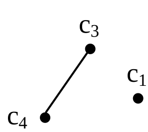


We can have subgraphs of order one $\bullet$ w_4 viz. one vertex, subgraphs of order two viz. two vertices or an edge connecting the vertices.

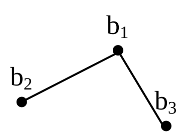


We can have three vertices or three vertices and an edge or three vertices and two edges.



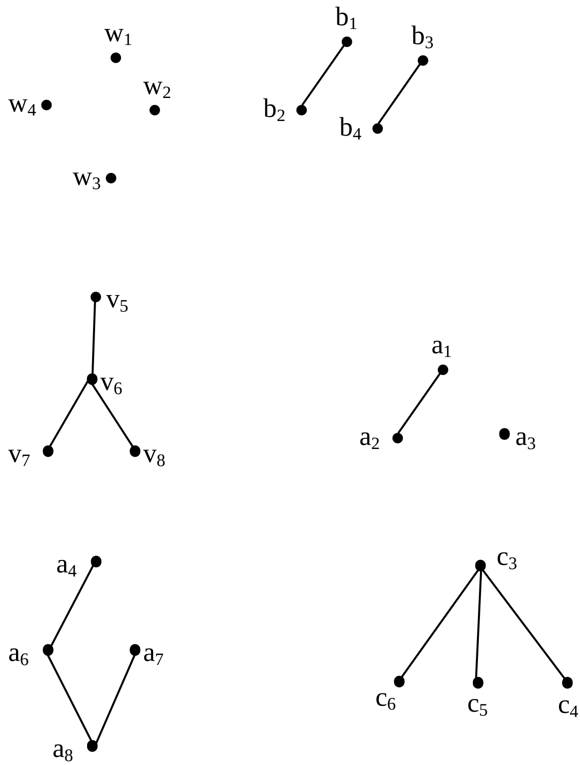


and



We work with subgraphs and get subgraphs of very many different orders.

We can have order four subgraphs which are as follows:



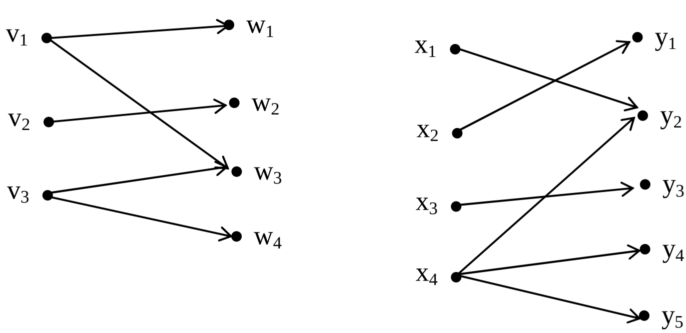
and so on.

These merging of graphs can play a vital role while working with merging of a node or concept in FCMs (Fuzzy Cognitive Maps) model. So that the merging concept will give a larger dynamical system. Likewise merging of a graph with two vertices and an edge will result in a some way or other a partially combined FCMs. So one can think of getting more and more FCMs models by this method.

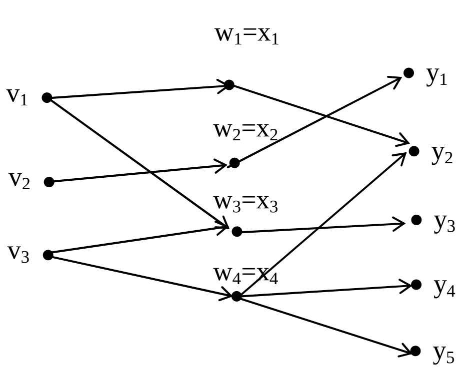
Thus at this juncture we see this merging of graphs also has more applications in both FCMs model, FRMs (Fuzzy Relational Maps) model and FREs (Fuzzy Relational Equations) models.

Since we can with out loss of generality assume all lattices are trivially or obviously graphs we have no problem in merging a graph with a graph by a vertex or an edge or both or by collection of vertices or several vertices and edges.

Let us consider two directed graphs.



We see the two graphs are merged in this manner w_i is merged with x_i, i = 1, 2, 3, 4.



This pseudo lattice graph of type II will also be known as the linked graph.

This is the type of graphs associated with FRMs or FREs.

Likewise we can merge one vertex or an edge or more vertex and get a merged FCM. Both these concepts will be defined and developed in this chapter.

Let us suppose we have two experts working on the same problem.

However both the experts work with a different set of concepts but they have some nodes to be in common. In regards of some common nodes / concepts they have some edges also to be common.

Now if the two direct graphs G_1 and G2 be given by the two experts. We can take the directed graph of the experts and merge the common nodes / edges get a new graph the pseudo graph and now using this pseudo graph we can analyse the problem.

We define the FCMs which has merged directed graphs will be defined as the merged or glued FCMs. Such study is interesting and leads to many results in FCM models as they are not combined FCM but some what merged FCMs.

This model will be illustrated by the following examples.

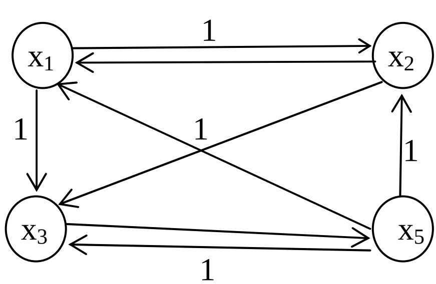
Example 3.26: Let us consider a study of any nations political situation, that is the prediction of electoral winner or how people tend to prefer a particular politician and so on and so forth involves not only a lot of uncertainly for this no data is available.

They form an unsupervised data. Hence we are at the outset justified in using FCM model.

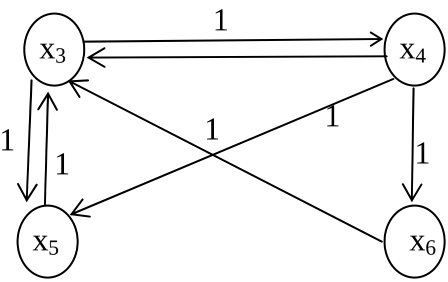
Suppose it is from India the Indian politics is analysed using six nodes.

x_1 - Languages
x_2 - Community
x_3 - Service to people public figure configuration and personality ad nature
x_4 - Finance and media
x_5 - Party's strength and opponents strength
x_6 - Working member for the party.

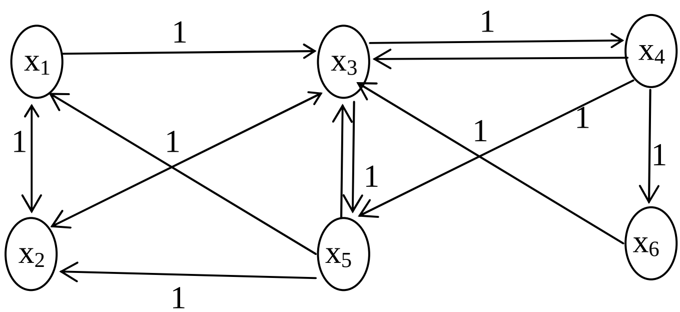
Suppose we have two experts. Experts one E_1 uses the four nodes x_1, x_2, x_3 and x_5 and expert to E_2 uses the four nodes x_3, x_4, x_5 and x_6. The directed graph given by experts E_1 one is



The directed graph given by expert E_2 is as follows.



The merged two edges $x_3\ x_5$ of E_1 and E_2 is as follows:



Now the above graph is a merged graph.

The connection matrix of the MFCM (Merged Fuzzy Cognitive Maps) is as follows.

$$M = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array}\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now using this merged FCM we are in a position to get the merged opinion of the two experts. This is not the combined opinion only a merged opinion.

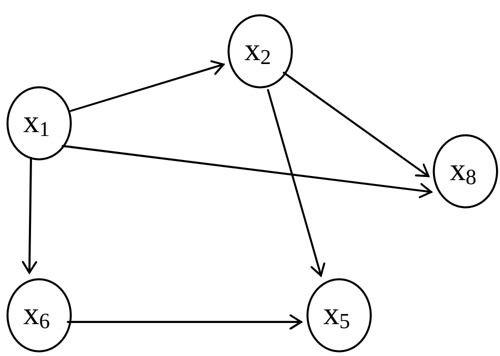
We can use these and get the merged opinion of both the experts. This saves times and also gives equal importance to both the experts.

We will give one more example of them.

Example 3.27: Let $X_1, X_2, \ldots, X_6, X_7, X_8$ and X_9 be seven attributes / nodes associated with the problem.

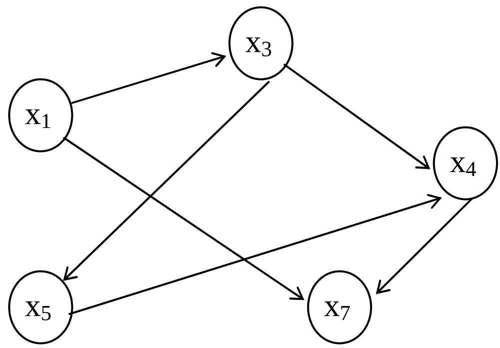
Let the first expert works with the nodes X_1, X_2, X_8, X_5, X_6 and the second expert works with the second expert nodes X_1, X_5, X_4, X_3 and X_7.

The directed graph given by the

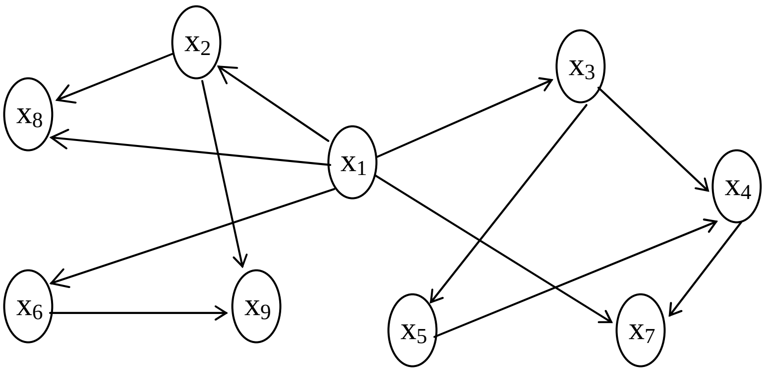


First expert using the nodes X_1, X_2, X_9, X_8 and X_6.

The directed graph using the nodes X_1, X_5, X_4, X_3 and X_7 is given by the second expert is as follows:



We now merge the vertex X_1 of the two graphs.



So that we get the over all model using all the nine nodes.

Thus by this method we get the Merged FCM (MFCM).

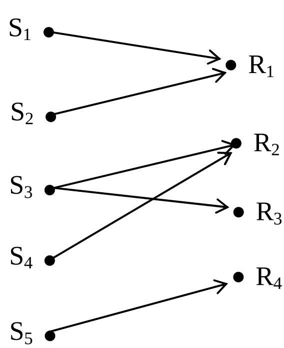
Such applications are very useful in the study of fuzzy models.

We now show by examples how merging of graphs give new models in case of FRM and FRE.

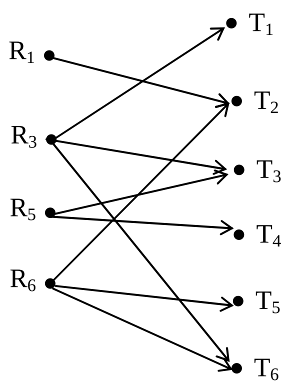
Example 3.28: Let us consider FRM given by two experts working with only one common set of concepts; how to relate by merging them.

Let us consider three sets of attributes S_1, S_2, S_3, …, S_5 and R_1, R_2, R_3, R_4 used by expert one and R_1, R_3, R_5, R_6 and T_1, T_2, T_3, T_4, T_5 and T_6 are the attributes worked by the second expert.

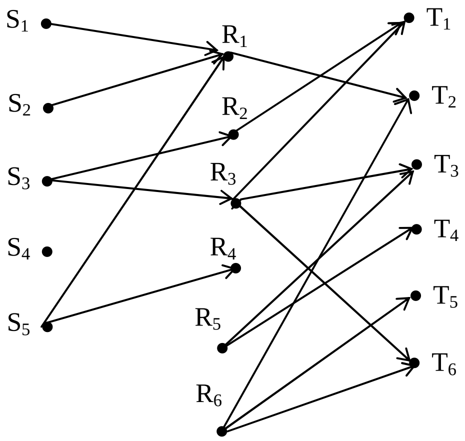
We give the Fuzzy Relational Maps (FRMs) directed graph of the first expert is as follows:



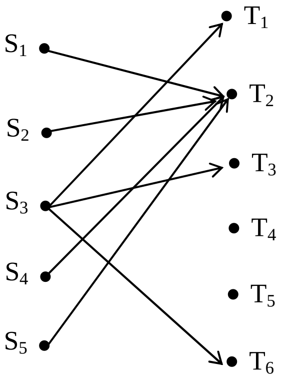
The directed graph of the second expert is as follows:



We can now get the second expert opinion.



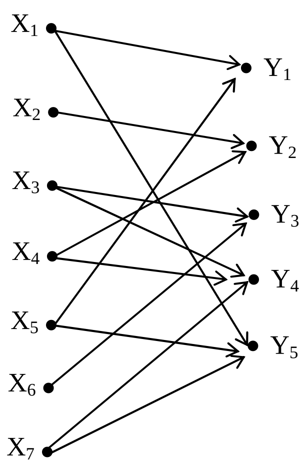
We get the following directed graph.



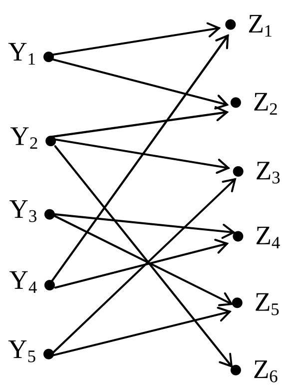
This is the way merging results in a new graph and hence new FRMs. On similar lines we can have FREs whose bigraphs can be merged at one or more vertices.

Let us consider $X_1, X_2, \ldots, X_7$ some 7 concepts related with a problem $Y_1, Y_2, \ldots, Y_5$ be some five concepts related with the problem. If $X_1, X_2, \ldots, X_7$ is taken as the domain space and $Y_1, Y_2, \ldots, Y_5$ as the range space of the

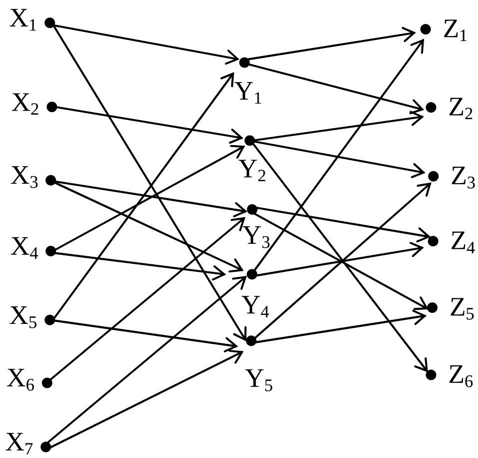
FRM then we get the following bigraph related with the FRMs.

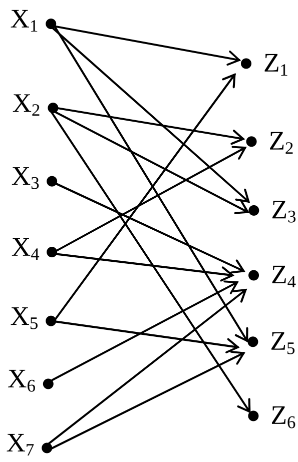


Suppose another expert works with $Y_1, Y_2, \ldots, Y_5$ as the domain space and say $Z_1, Z_2, \ldots, Z_6$ as the range space we get the following bigraph



Now be merge the two bigraphs on the five vertices we get the pseudo graph which is as follows.





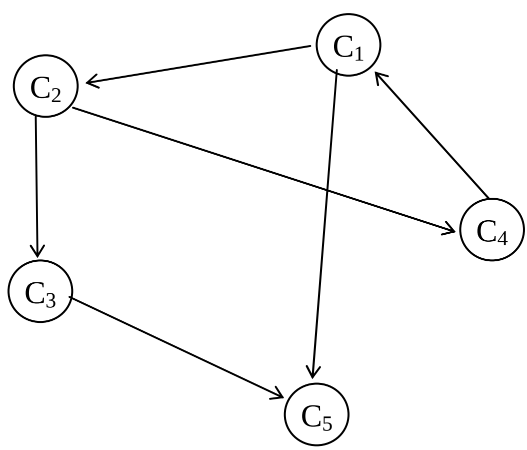
Thus merging FRMs leads to or works like linked FRMs. On similar lines we can use the bigraphs of FREs and merge them to get a bigraph.

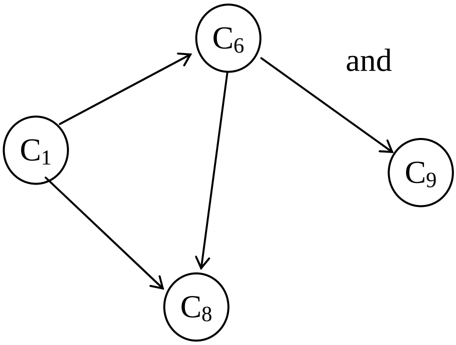
Thus merging of vertices or edges graphs of fuzzy models gives us the merged fuzzy model.

This newly constructed merged fuzzy model like merged Fuzzy Cognitive Maps, merged Fuzzy Relational Maps and merged Fuzzy Relational Equation play a vital role in studying social problems in studying social problems and interlinking or merging the attributes resulting new results.

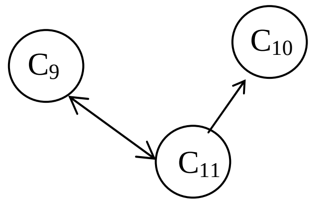
Thus we can using the concept of merging of graphs construct new merged fuzzy models.

Infact we can also merge more than 3 graphs of 3 fuzzy models working on the same problem and get a new merged model and so on and so forth





and



be three directed graphs given by three different experts.

We see the connection matrices H given by the three graphs are as follows.

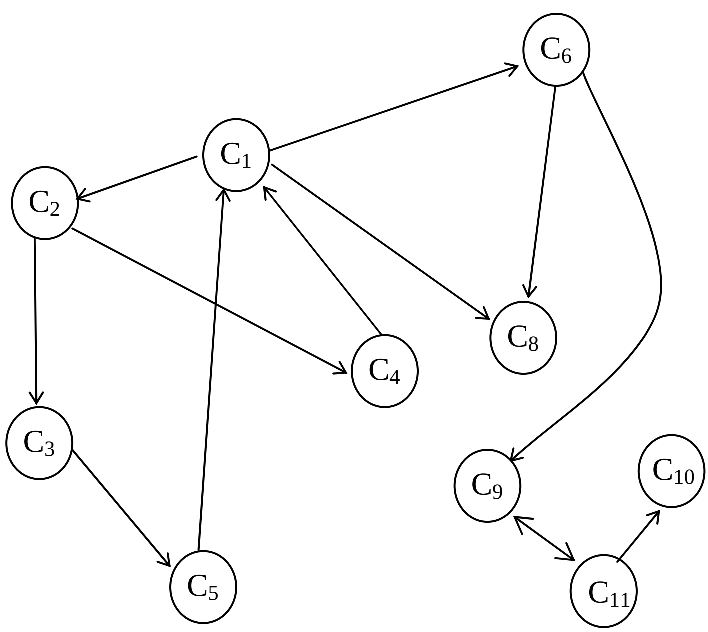
$$E_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$E_2 = \begin{array}{c} \\ c_1 \\ c_6 \\ c_8 \\ c_9 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_6 & c_8 & c_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\text{and } E_3 = \begin{array}{c c} & \begin{array}{ccc} c_9 & c_{10} & c_{11} \end{array} \\ \begin{array}{c} c_9 \\ c_{10} \\ c_{11} \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

We see the graphs of the first and second expert have the vertex C_1 to be the common vertex to be merged.

For the expert two and three C_9 is the common vertex which is to be merged.



Now we using this merged graph obtain the connection matrix of the merged model.

$$E = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \end{array} \begin{array}{c} \begin{array}{cccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_8 & c_9 & c_{10} & c_{11} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Now E_1 is the merged FCM and the consolidated one will give the opinion of all the three experts.

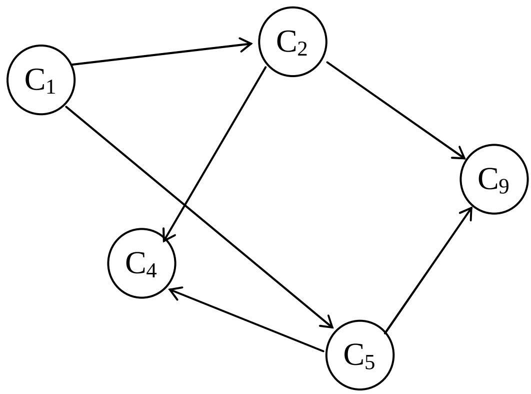
However it is distinctly different from the combined FCM.

Thus this new merged model can at a time give the hidden pattern in a consolidated way their by saving time and economy.

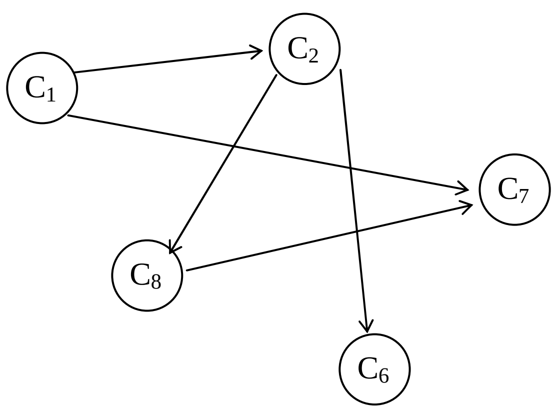
We will some more illustrations of them.

Suppose one works with a problems with nodes C_1, C_2, …, C_{10}. Three experts work on the problem and two of them have the node C_1 and C_2 in common and other two them have the node C_7 and C_8 in common.

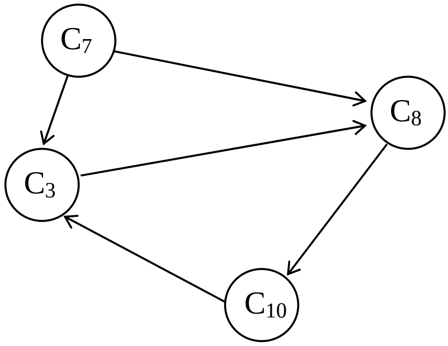
The directed graph given by the three experts are as follows.



The above is the directed graph given by the first expert. The directed graph given by the second expert is as follows.



The direct graph given by the third expert is as follows:



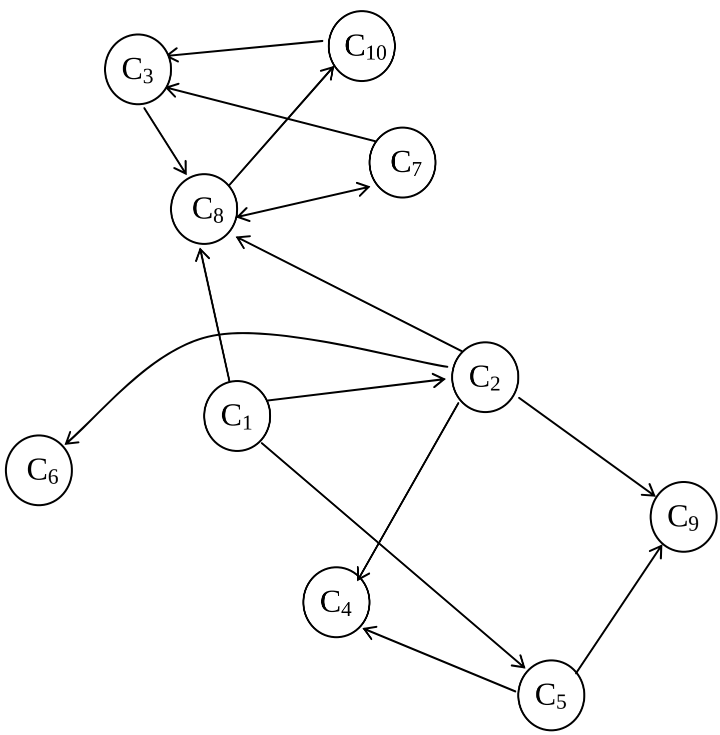
The connection matrices of the following three directed graphs E_1, E_2 and E_3 are as follows:

$$E_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_4 \\ c_5 \\ c_9 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_4 & c_5 & c_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$E_2 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_6 & c_7 & c_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\text{and } E_3 = \begin{array}{c} \\ c_3 \\ c_7 \\ c_8 \\ c_{10} \end{array} \begin{array}{c} \begin{array}{cccc} c_3 & c_7 & c_8 & c_{10} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We get the merged graph which is as follows:



Let E be the merged connection matrix of the merged graph which is as follows:

$$E = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{array} \begin{array}{c} \begin{array}{cccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} \end{array} \\ \left[\begin{array}{cccccccccc} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Using this matrix E as the merged dynamical system one can work with the fuzzy models.

In the same way merged FRMs and merged FREs are constructed. Thus the merged graphs play a vital role in this study.

Chapter Four

PSEUDO NEUTROSOPHIC LATTICE GRAPHS OF TYPE I AND TYPE II

In this chapter we define the concept of pseudo neutrosophic lattice graphs of type I using two lattices in which atleast one should be a neutrosophic lattice. We also define pseudo neutrosophic lattice graph of type II in which atleast one of the lattice or the graph must be neutrosophic.

In the case of type II we also make use of both graphs where atleast one of them is a neutrosophic graph. Finally we give the applications of these pseudo neutrosophic lattice graphs of type II when two graphs are used in fuzzy neutrosophic models. These new fuzzy neutrosophic models are termed as merged fuzzy neutrosophic models.

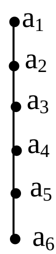
For definition of neutrosophic graphs refer [79, 89]. For the concept of lattices and neutrosophic lattices refer [87].

DEFINITION 4.1: *Let L_1 and L_2 we any two neutrosophic lattices we can merge the vertices or edges or both and get a pseudo neutrosophic lattice of type I. This can be extended to any number of neutrosophic lattices $L_1, L_2, \ldots, L_n$; $n < \infty$.*

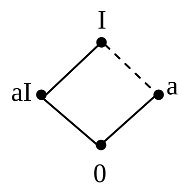
It is pertinent to keep on record that all lattice need not be neutrosophic but atleast one lattice must be a neutrosophic lattice.

We will first illustrate this by some examples.

Example 4.1: Let L_1 =



and L_2 =

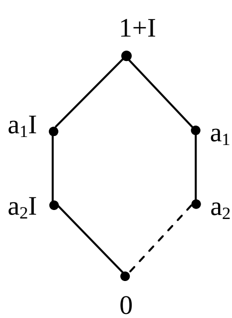


be two lattices a neutrosophic lattice L_2 and a lattice L_1. We can merge vertices of L_2 with any of the vertices of L_1.

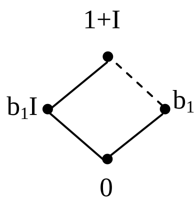
The resultant graph is defined as the neutrosophic pseudo lattice graph of type I.

More merging of vertices is possible only when we take both the lattices to be neutrosophic lattices.

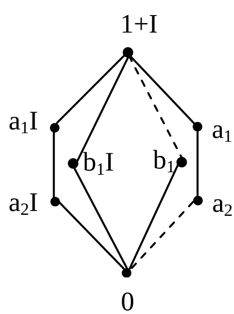
Example 4.2: Let L_1 =



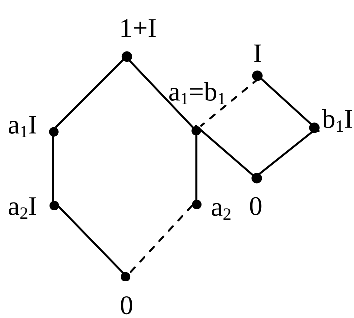
and L_2 =



be any two neutrosophic lattices. We can merge 1 + I with 1 + I and zero with zero and rest no other merging.



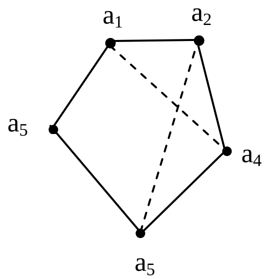
be the neutrosophic pseudo lattice graph of type I.



This is a neutrosophic pseudo lattice graph of type I which is only a neutrosophic graph.

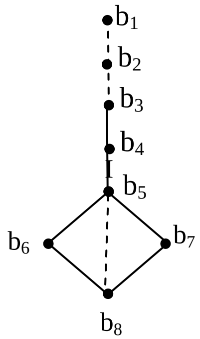
Example 4.3: We can also have neutrosophic lattices with both vertices and edges to be neutrosophic. Then we can have merging of the real edges or merging of the neutrosophic edges resulting in pseudo neutrosophic lattices.

Let



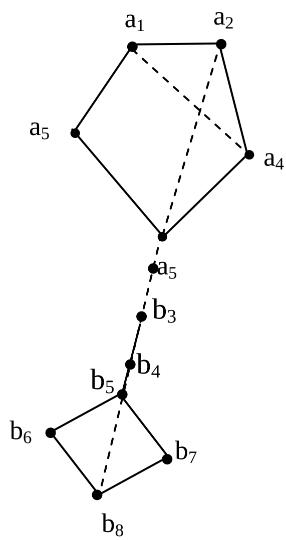
be the edge neutrosophic lattice L_1 and

L_2 =



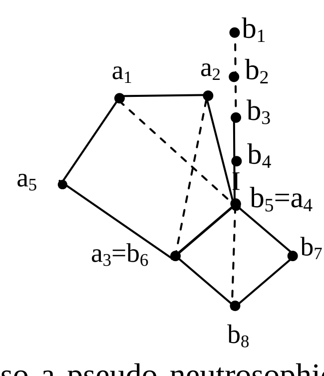
be any edge neutrosophic lattice.

We can merge edge a_1a_4 with b_2b_3 and obtain the pseudo neutrosophic lattice graph which is as follows:

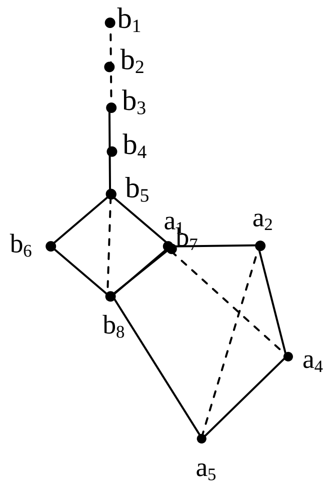


The resultant is only a neutrosophic graph. We have merged the neutrosophic edge with a neutrosophic edge.

We can also merge b_5b_6 with a_4a_3 and get a pseudo neutrosophic lattice graph which is as follows:

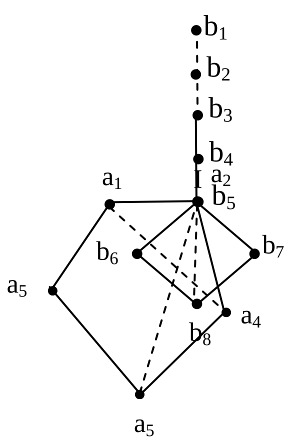


This is also a pseudo neutrosophic lattice graph which is only a neutrosophic graph which is not a lattice. We can get several such pseudo neutrosophic lattice graphs.



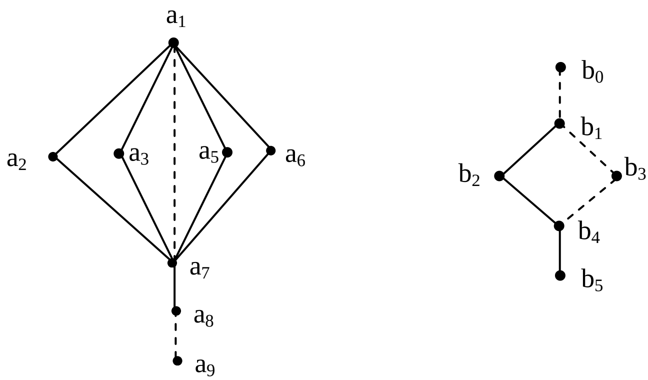
This is again a pseudo neutrosophic lattice graph of type I which is also a neutrosophic lattice.

We can get this type of pseudo neutrosophic lattice graphs of type I.

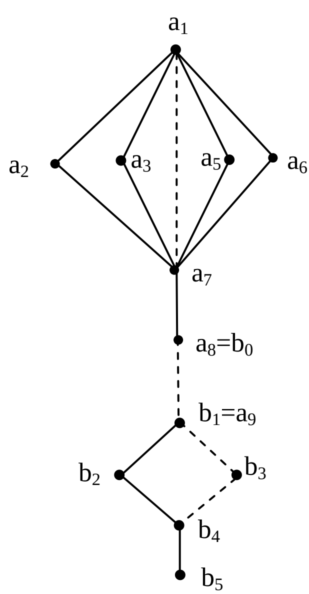


This is a yet another pseudo neutrosophic lattice graph of type I by merging the vertices a_2 with b_5.

Example 4.4: Let us consider the following two neutrosophic lattices L_1 and L_2.

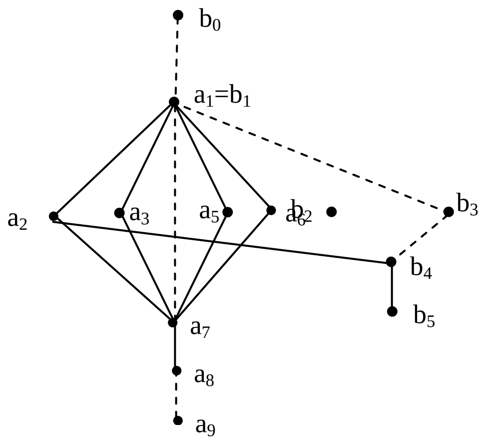


We can merge the neutrosophic edges a_8a_9 with b_0b_1 and get the following pseudo neutrosophic lattice graphs of type I.



This is again a neutrosophic lattice.

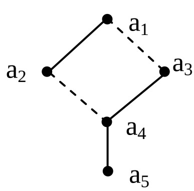
We can merge the edges b_1b_2 with edge a_1a_2 and get the pseudo neutrosophic lattice graph of type I which is as follows:



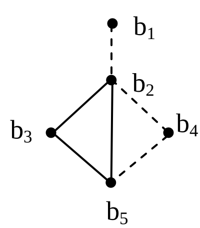
We see it is not a neutrosophic lattice which is also a neutrosophic graph.

Example 4.5: Let us consider the following three neutrosophic lattices

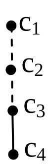
L_1 =



L_2 =

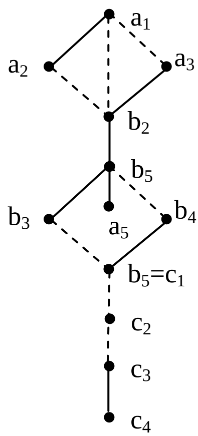


and L_3 =



.

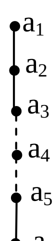
We now merge edges a_4a_5 with b_2b_5 and merge vertex b_5 with c_1.



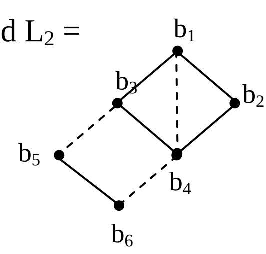
The neutrosophic pseudo lattice graph of type I is only a neutrosophic graph.

Thus we can merge more number of neutrosophic lattices and obtain a pseudo neutrosophic lattice graph of type I. Interested reader can construct more of them we can also as in case of usual pseudo lattice graphs of type I find substructure in case of neutrosophic pseudo lattice graphs of type I which will be illustrated in an example or two.

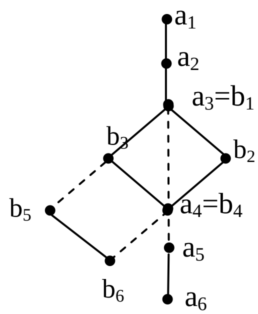
Example 4.6: Let $L_1 =$



and $L_2 =$

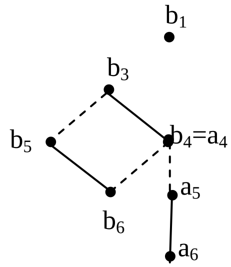


be the two neutrosophic lattices. We can merge edge a_3a_4 with b_1b_4 and get the neutrosophic pseudo lattice graph which is as follows.



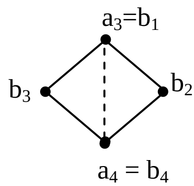
Clearly the neutrosophic pseudo lattice graph of type I is not a neutrosophic lattice only a neutrosophic graph.,

Consider

$P_1 =$ 

P_1 is a neutrosophic subgraph of type I.

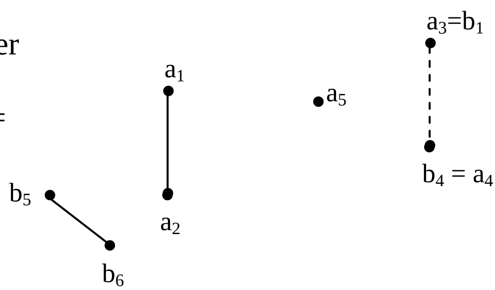
P_2 =



P_2 is the neutrosophic subgraph which is also a neutrosophic lattice.

Consider

P_3 =

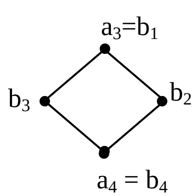


be the subgraph which is only a subgraph and not a sublattice.

We can have several such subgraphs. We see P_1, P_2 and P_3 happen to be neutrosophic subgraphs.

We can also have subgraphs which are not neutrosophic for

P_4 =

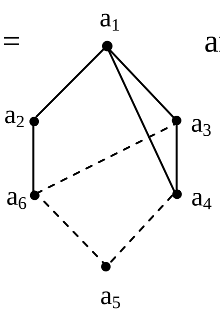


P_4 is a subgraph which is not neutrosophic, it is also a sublattice which is not neutrosophic.

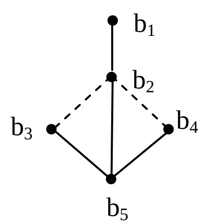
Thus we see in general a neutrosophic pseudo graph lattice can have a subgraph which is a neutrosophic lattice or which is not a neutrosophic lattice or a graph which is a neutrosophic graph or not a neutrosophic graph so this neutrosophic lattice graph of type I can have four types of substructures.

Example 4.7: Let L_1 and L_2 be any two neutrosophic lattices which is as follows:

L_1 =

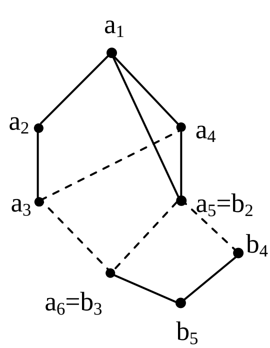


and L_2 =



We can merge edges a_4a_5 with b_1b_2 and b_2b_3 with a_5a_6.

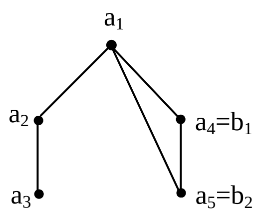
We get S the following neutrosophic pseudo lattice graph of type I.



We see the resultant is a neutrosophic lattice.

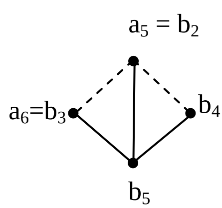
This S has all the four types of subgraphs which are described in the following:

$P_1 =$



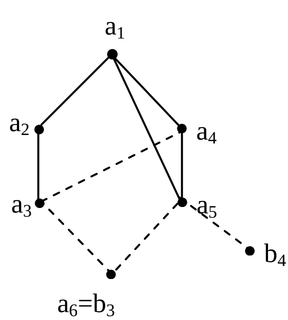
is a subgraph which is not a neutrosophic subgraph.

$P_2 =$

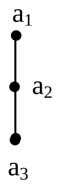


P_2 is a subgraph which is a neutrosophic sublattice of S.

$P_3 =$



P_3 is a subgraph which is a neutrosophic subgraph of S.

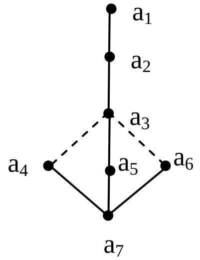


= P_4. P_4 is a subgraph which is a lattice which is not neutrosophic. We can also have cranky or unnatural merging in neutrosophic graphs. That is we try to merge a neutrosophic vertex with a non neutrosophic vertex or a neutrosophic edge with a non neutrosophic edge.

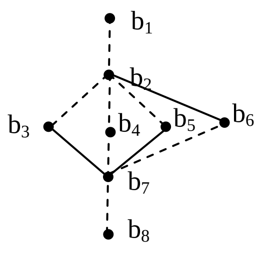
We call the merged neutrosophic lattice as neutrosophic cranky pseudo lattice graphs.

We will give examples of cranky neutrosophic pseudo lattice graphs.

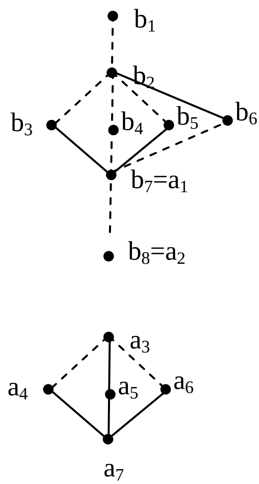
Example 4.8: Let L_1 =



and L_2 =



be any two neutrosophic lattices. We merge edge a_1a_2 of L_1 with the neutrosophic edge of b_7b_8. Let S be the resultant cranky neutrosophic pseudo lattice graph of type I whose graph is as follows:



S is a neutrosophic lattice.

Now when a neutrosophic edge is merged with the real edge we always make it only as a neutrosophic edge. This is the assumption or definition made in this book.

Interested reader can give more examples of such cranky pseudo neutrosophic lattice graphs. However we leave the following theorem for the reader.

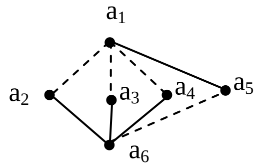
THEOREM 4.1: *Let L_1 and L_2 be two neutrosophic lattices. A cranky pseudo neutrosophic lattice graph of type I of L_1 and L_2 got by merging a neutrosophic edge or vertex with a real edge or vertex respectively is always a neutrosophic lattice graph of type I.*

Now we proceed onto define neutrosophic pseudo lattice graph of type II.

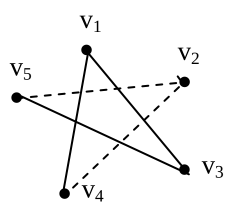
Let G be a neutrosophic graph and L be a neutrosophic lattice or one of G or L alone is neutrosophic then if we merge the edges of them or merge the vertices of them we define the resultant graph to be a pseudo neutrosophic lattice graph of type II.

We will illustrate this situation by some examples.

Example 4.9: Let L_1 =

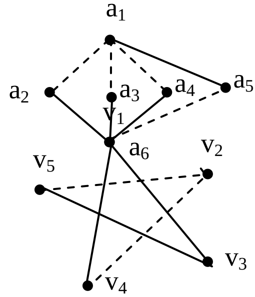


G =



be the neutrosophic lattice and G be the neutrosophic graph. Let us merge edge v_1 and a_6; we obtain the pseudo neutrosophic lattice graph of type II which is as follows:

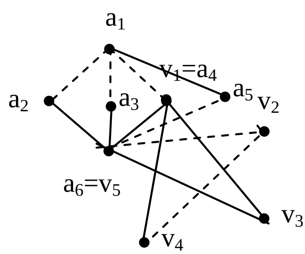
P_1 =



P_1 is a graph and not a lattice.

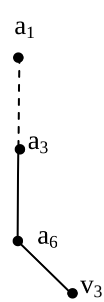
Consider

P_2 =



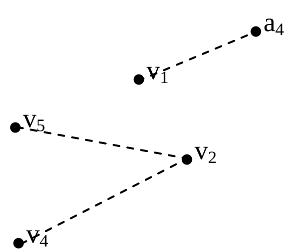
Clearly P_2 the pseudo neutrosophic lattice graph of type II which is not a neutrosophic lattice which is not a neutrosophic graph. Now we will find subgraphs of P_1 and P_2 in the following

S_1 =



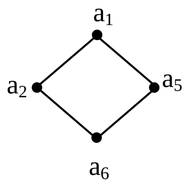
is a sublattice of P_1 which is neutrosophic.

S_2 =



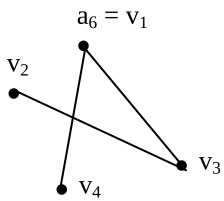
is a subgraph which is neutrosophic and is only a subgraph not a sublattice.

S_3 =



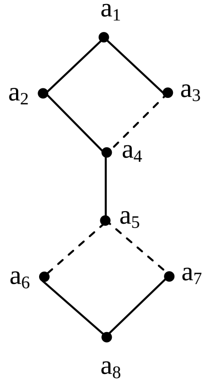
is a subgraph which is a sublattice which is not neutrosophic.

S_4 =

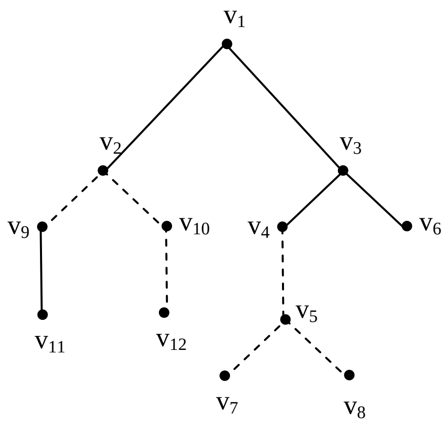


is a subgraph which is not a neutrosophic subgraph and is not a lattice.

Example 4.10: Let L =

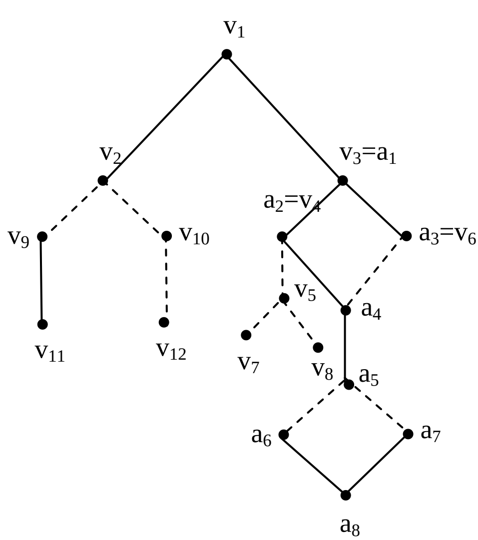


be the neutrosophic lattice and G =



be the neutrosophic graph.

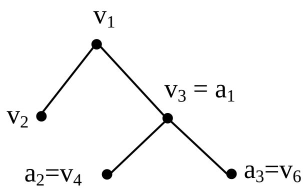
Suppose we merge a_1 with v_3 edge v_3v_4 with a_1a_2, edge v_3v_6 with a_1a_3 then we get the following pseudo neutrosophic lattice graph of type II say S.



Clearly this is not a neutrosophic lattice but only a neutrosophic graph.

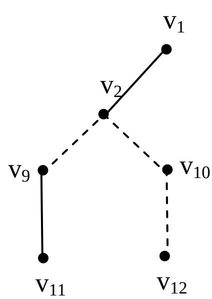
We can have sublattices, subgraphs, neutrosophic sublattices and neutrosophic subgraphs which are as follows:

$P_1 =$



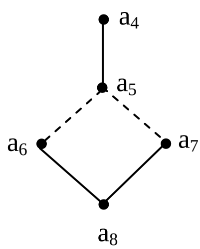
is a subgraph of S which is not neutrosophic.

$P_2 =$

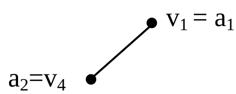


is a subgraph which is a neutrosophic subgraph of S.

Consider

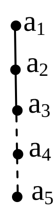


is a subgraph which is a neutrosophic lattice.



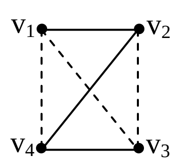
is a subgraph which is a non neutrosophic sublattice of order two.

Example 4.11: Let L =



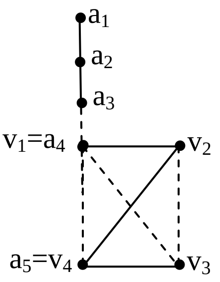
be a neutrosophic lattice.

G_1 =



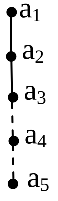
be a neutrosophic graph.

We can merge the edges the v_1v_4 to a_4a_5 and obtain the neutrosophic pseudo lattice graph of type II which is denoted by S.



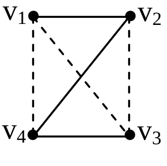
Clearly S is a pseudo neutrosophic graph which is not lattice.

$B_1 =$



is a subgraph of S which is a neutrosophic lattice.

$B_2 =$



is a subgraph which is a neutrosophic subgraph and not a lattice.

In view of this we have following theorem.

THEOREM 4.2: *Let L be a neutrosophic lattice and G be a neutrosophic graph. S be the pseudo neutrosophic lattice graph of type II got by merging vertices or edges or both.*

B be the cranky pseudo neutrosophic lattice graph of type II got by merging real edges with neutrosophic edges of neutrosophic vertices with real vertices.

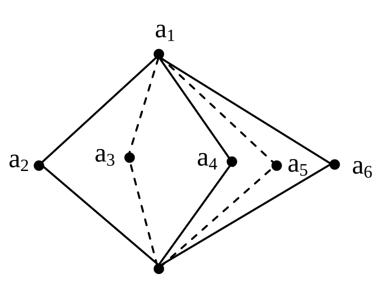
(1) S has L to be neutrosophic sublattice and G to be a neutrosophic subgraph.
S has also sublattices and subgraphs which are not neutrosophic.

(2) B has L to be a neutrosophic sublattice and G to be a neutrosophic subgraph. B has cranky subgraphs and sublattices.

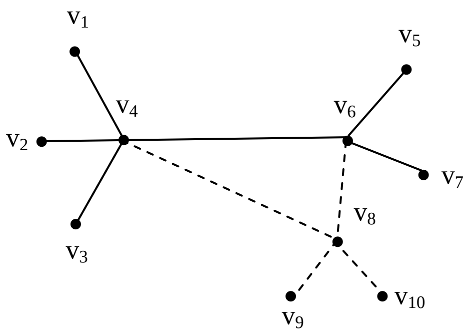
The proof follows from the fact B is a cranky pseudo neutrosophic lattice so S has sublattices and subgraphs which are not neutrosophic. Hence the claim.

The rest can be proved by any interested reader.

Example 4.12: Let L =

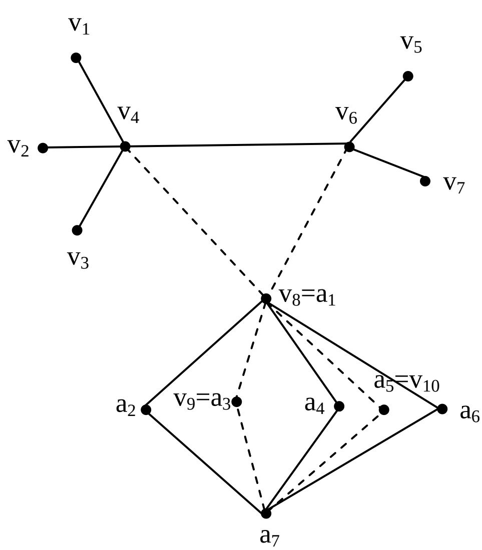


be a neutrosophic lattice.



be a neutrosophic graph.

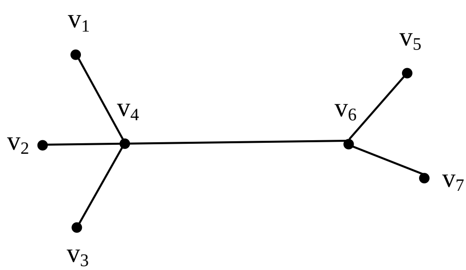
We can merge edge a_1 a_3 with v_8 v_9 and edge a_1a_5 with v_8v_{10} and get the following neutrosophic pseudo lattice graphs of type II.



Clearly S is not a neutrosophic lattice only a neutrosophic graph.

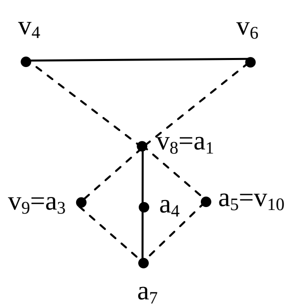
This has subgraphs which are neutrosophic lattices, non neutrosophic lattices, neutrosophic graphs and non neutrosophic graphs.

We see P_1



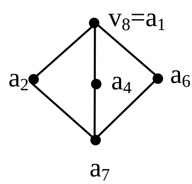
is a subgraph of S which is only a graph and not a neutrosophic graph.

P_2 =



is a subgraph which is a neutrosophic subgraph of S.

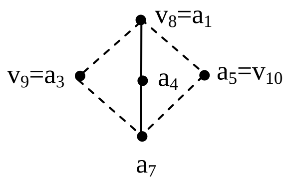
P_3 =



P_3 is a subgraph of S which is not neutrosophic.

Consider

P_4 =

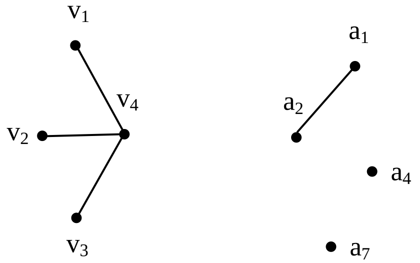


a subgraph which is a neutrosophic sublattice of S.

Interested reader can construct substructures. All the substructures P_1, P_2, P_3 and P_4 given here are connected.

Without loss of generality we can also have substructures which are not connected and they are subgraphs neutrosophic or otherwise

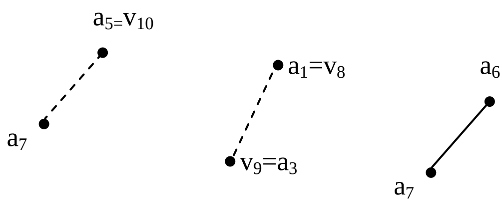
T_1 =



T_1 is a subgraph of S. T_1 is a subgraph which is not neutrosophic.

However T_1 is not a lattice but T_1 is only a subgraph which is not connected.

Let T_2 =



be a subgraph.

T_2 is not a lattice T_2 is a subgraph of S which is neutrosophic and is not connected.

However all lattices are connected so we cannot have sublattices which are not connected neutrosophic or otherwise.

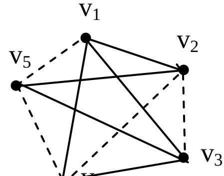
Now interested reader can study this situation.

Finally we can also merge the vertices or edges or both of neutrosophic graphs which we choose to call only as pseudo neutrosophic lattice of type II.

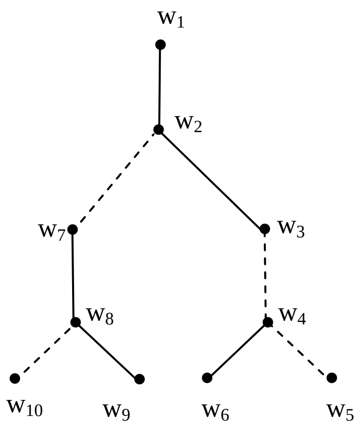
Now we will first illustrate this by some examples.

Example 4.13: Let

G_1 =

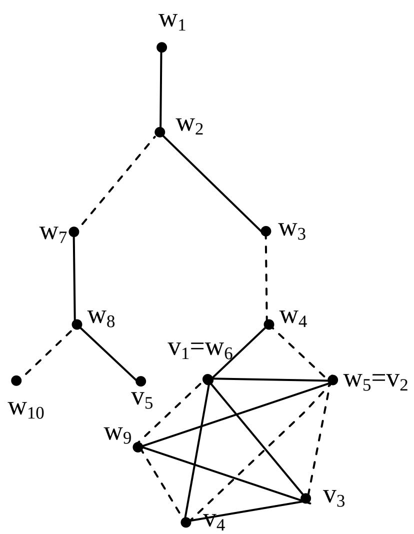


and G_2 =



be two neutrosophic graphs.

Let us merge vertices w_6 with v_1 and w_5 with v_2. We get the neutrosophic pseudo lattice graph of type II which is as follows:

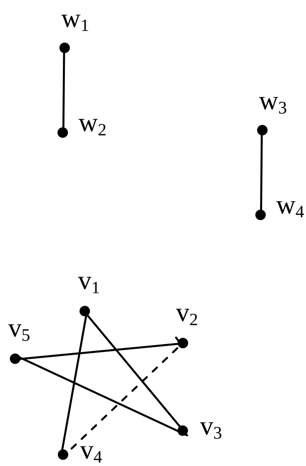


Clearly S is a neutrosophic pseudo lattice graph of type II.

Several such pseudo neutrosophic lattice graphs of type II can be got by merging vertices or edges or both.

We give a few substructures of them in the following.

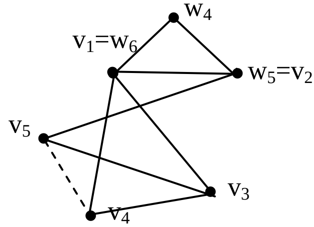
P_1 =



is a subgraph of S which is not connected and it is a neutrosophic subgraph of S.

Now consider P_2 a neutrosophic subgraph.

P_2 =



P_2 is a subgraph of S which is not neutrosophic but is connected.

$P_3 =$

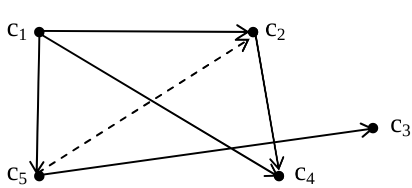
is a again a subgraph which is neutrosophic but is not connected.

Thus we find these pseudo neutrosophic lattice graphs of type II when two neutrosophic graphs are used find applications in neutrosophic fuzzy models like Neutrosophic Cognitive Maps (NCMs) models, Neutrosophic Relational Maps (NRMs) models and Neutrosophic Relational Equations (NREs) model as all these three models function on neutrosophic directed graphs.

These will be illustrated by the following examples.

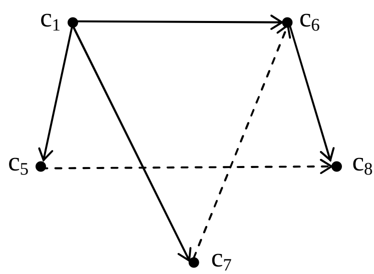
Example 4.14: Let G_1 and G_2 be two neutrosophic directed graphs associated with the Neutrosophic Cognitive Maps (NCMs) model of two experts who work on the same problem.

$G_1 =$



be the neutrosophic directed graph given by the first expert for the NCMs.

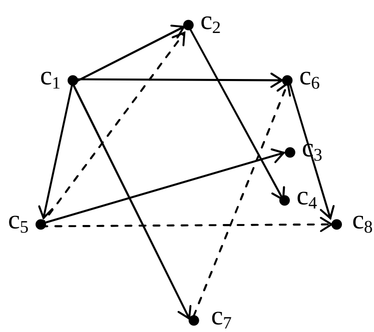
Let G_2 =



be the neutrosophic directed graph given by the second expert using the same NCMs node for the same problem.

Now we see both the graphs G_1 and G_2 have the vertices C_1 and C_5 in common and C_1C_5 edge is also common.

Thus we can merge these two directed graphs of the NCMs. By merging C_1C_5 of them we get the following directed neutrosophic graph S.



The neutrosophic connection matrix of the neutrosophic graph given by the first expert is as follows:

$$M_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The connection neutrosophic matrix of the neutrosophic directed graph given by the second expert is as follows:

$$M_2 = \begin{array}{c} \\ c_1 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_5 & c_6 & c_7 & c_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we obtain combine neutrosophic connection matrix of the pseudo neutrosophic lattice graph S of type II.

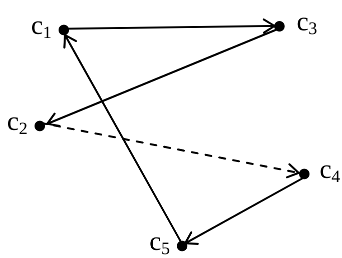
$$M = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 1 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now M gives the merged dynamical system of the two experts opinion of the NCMs. This new matrix functions for both the experts opinion in a merged way. This application has two advantages.

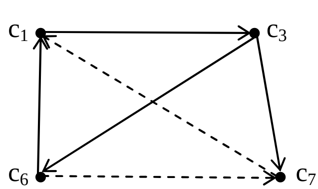
In the first place it gives equal importance to both the experts and secondly we work with the single dynamical system time which can save time and economy.

Example 4.15: Let us consider three directed neutrosophic graphs of NCMs related with the same problem.

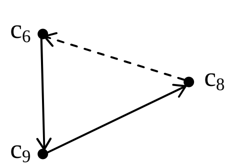
G_1 =



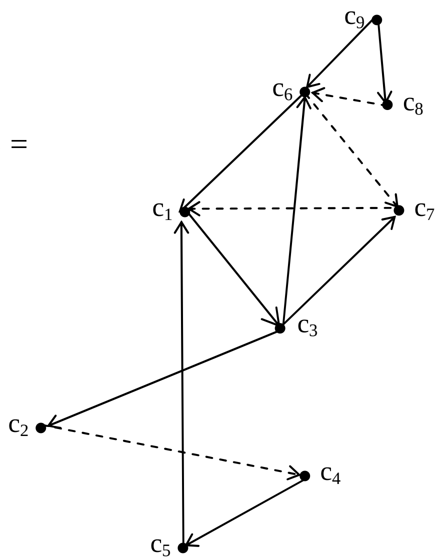
G_2 =



and G_3 =



We see these three graphs can only be merged in a unique way that is one and only way which is as follows:

S = 

We give the connection neutrosophic matrices of all the three directed graphs.

M_1 is the connection neutrosophic matrix of graph G_1,

$$M_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The neutrosophic connection matrix M_2 of the graph G_2 is as follows:

$$M_2 = \begin{array}{c} \\ c_1 \\ c_3 \\ c_6 \\ c_7 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_3 & c_6 & c_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & I \\ I & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The neutrosophic connection matrix of the graph G_3 is as follows:

$$M_3 = \begin{array}{c} \\ c_6 \\ c_8 \\ c_9 \end{array} \begin{array}{c} \begin{array}{ccc} c_6 & c_8 & c_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ I & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Now we get the connection neutrosophic matrix S of the pseudo neutrosophic lattice graph of type II got after merging the edge c_1c_3 of G_1, with the edge c_1c_3 of G_2 and the vertex c_6 of G_2 with c_6 of G_3.

$$M = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Thus by working with this M as the dynamical system time is saved and all the work (that is experts opinion) is consolidated as a single system.

Thus we get the merged neutrosophic cognitive maps model which is better than the combined NCM model.

Now we describe this suppose G_1, G_2, …, G_n are the neutrosophic directed graphs given by n experts who work on the same problem. We have every graph has atleast a common edge or a common vertex.

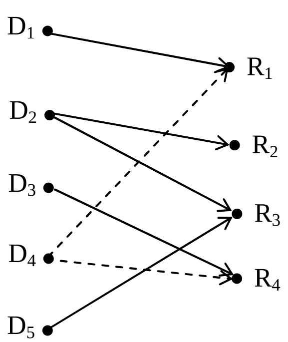
Thus we merge all the n-graphs together to obtain a neutrosophic pseudo lattice graph as the merged opinions of the experts we work with the merged NCMs.

This study is new and interesting. Next we describe the merged Neutrosophic Relational Maps model and Neutrosophic Relational Equations model. Such study has been carried out in chapter III for FRMs and FREs models.

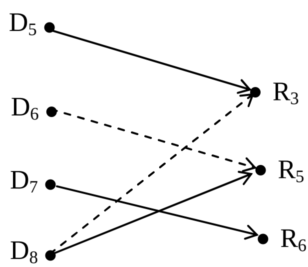
We will illustrate this situation by some examples.

Example 4.16: Let us consider the neutrosophic directed graphs H_1 and H_2 given by two experts studying the same problem using the Neutrosophic Relational Maps model.

H_1 =

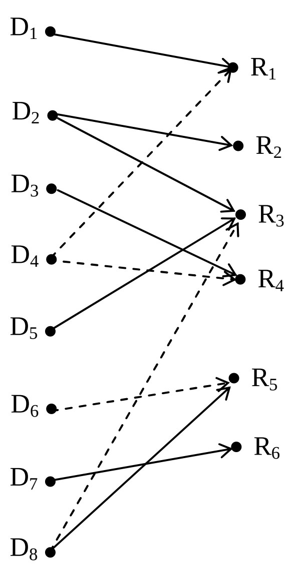


neutrosophic bipartite graph given by the first expert.
Let



neutrosophic directed bigraph of the NRMs given by the second expert.

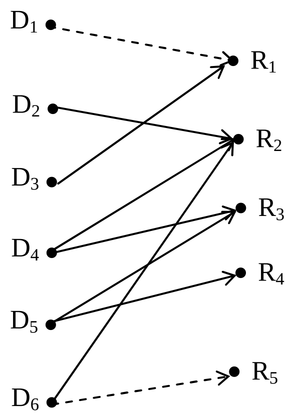
We see in the graphs H_2 and H_1 only the edge D_5R_3 is common. By merging D_5R_3 of H_1 and H_2 we get the following neutrosophic pseudo lattice graph H of type II.



H is the merged neutrosophic directed graph of the NRM. However H is not a linked merged NRM.

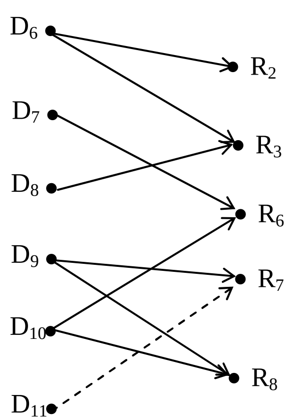
Example 4.17: Let H_1 and H_2 be any two neutrosophic directed graphs of a NRMs given by two experts on the same problem.

H_1 =



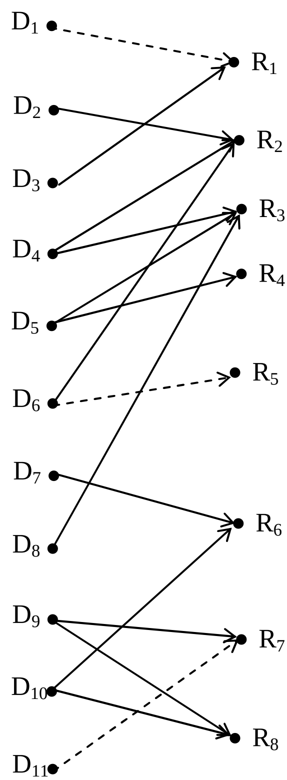
and

H_2 =



We see the two graphs can only be merged in a unique way to get at the merged NRM. The merged graph is as follows:

H =

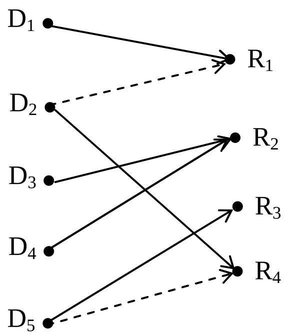


Using this directed graph H we can obtain the merged connection matrix of the merged graph H.

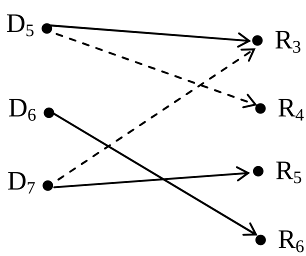
Using H analysis of the problem can be made.

Interested reader can study such merged NRMs model. The main advantage is it saves time and economy we can also obtain a merged NRMs models using three experts or more. Just for the sake of the simplicity we give an example where 3 experts opinion on an NRM model; say the graphs of the NRM model given by the three experts be G_1, G_2 and G_3 where

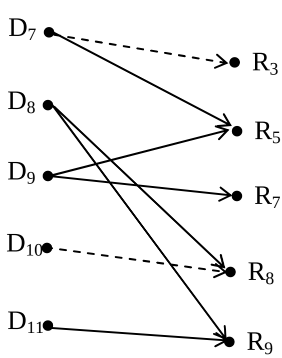
G_1 =



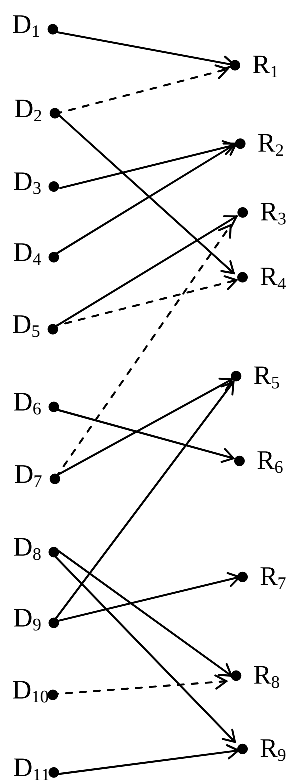
G_2 =



G_3 =



Now we find the merged neutrosophic graph (that the pseudo neutrosophic lattice graph of type II. We see for the neutrosophic graphs G_1 and G_2 the edge $D_5 \to R_3$ and the neutrosophic edge D_5R_4 are to be merged and for the neutrosophic graphs G_2 and G_3 the edge D_7R_3 are merged. Now we have H_1 the pseudo neutrosophic lattice graph of type II.



H gives the merged neutrosophic graph which can serve as the merged Neutrosophic Relational Maps directed graph of the model. We can use the graph and get the merged connection matrix.

It is pertinent to keep on record that we can get any number of experts opinion as the graph and merged connection matrix.

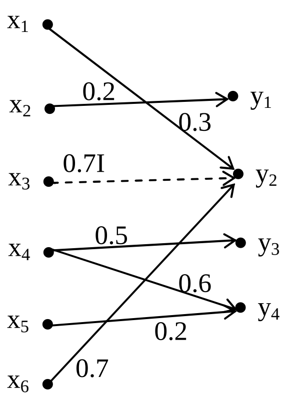
It is pertinent to keep on record that we can get any number of experts opinion as the graph and merged appropriate and get the merged NRMs model.

Such study is time saving and innovative.

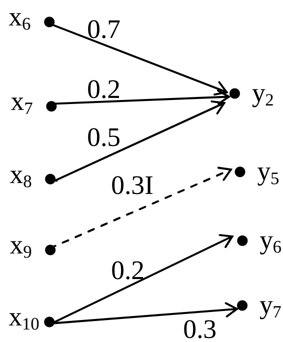
Now we give an illustration of how merged graph of two neutrosophic weighted directed graph of the Neutrosophic Relational Equations NREs models can be constructed.

Example 4.18: Let G_1 and G_2 be two neutrosophic weighted directed graphs associated with the problem given by the experts.

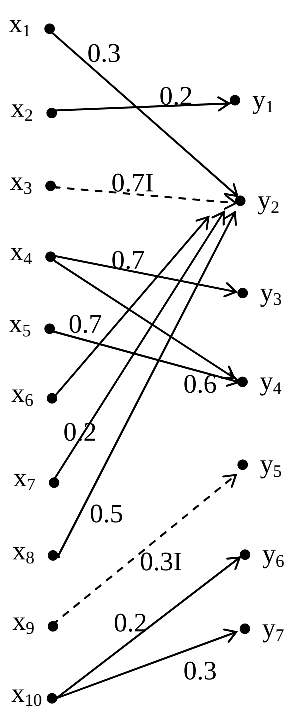
$G_1 =$



and G_2 =



Now we can merge only in way to get the pseudo neutrosophic lattice graph of type II. The merged graph is as follows:



Using this graph we can get the merged NREs model. Such study is time saving and annuls any form of discrimination among experts.

Interested reader can merge more than two NREs graph and obtain the merged NREs model. Now this is the first time such new study is carried out.

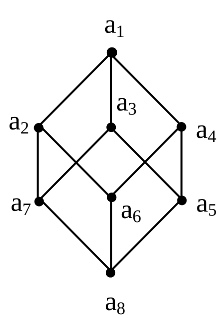
An entire chapter is devoted to problems which deal with all types of merging of different models.

Chapter Five

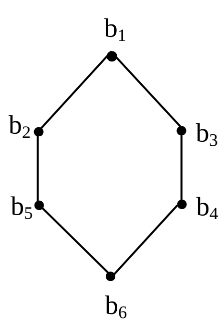
SUGGESTED PROBLEMS

In this chapter we suggest problems some of which are open conjectures and some of them are difficult problems.

1. Let L_1 =



and

L_2 =  be any two lattices.

(i) How many single vertex merging pseudo lattice graphs of type I can be obtained?

(ii) How many of them in question (i) are lattices?

(iii) How many pseudo lattice graphs of type I can be got merging only two of the vertices and not the edges?

(iv) How many of them in question (iii) are lattices?

(v) How many pseudo lattice graphs can be got by merging a edge and the two vertices?

(vi) How many of them in question (v) are lattices?

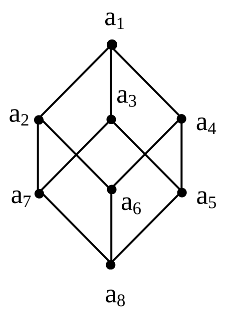
(vii) How many pseudo lattice graphs of type I can be got by merging only three vertices?

(viii) How many pseudo lattice graphs of type I can be got by merging three vertices and two edges?

(ix) How many of them are lattices in question (viii).

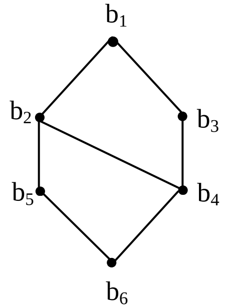
2. Obtain some special and interesting features enjoyed by pseudo lattice graphs of type I.

3. Let L_1 =



and

L_2 =



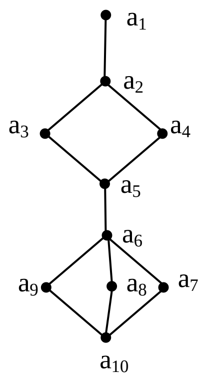
be any two lattices.

(i) Study questions (i) to (ix) of problem (1) for the pseudo lattice graphs of type I using L_1 and L_2.

(ii) Is the lattice of the pseudo lattice graphs of type I distributive?

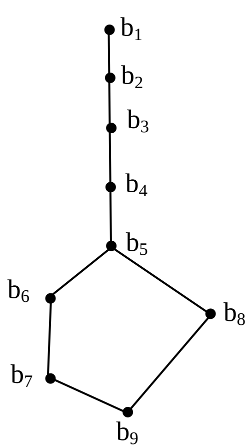
(iii) Can we have a pseudo lattice graph of type I lattice to be non modular in this problem?

4. Let L_1 =



and

L_2 =



be two lattices.

(a) Study questions (i) to (ix) of problem (1) for this pseudo lattice graph of type one got using lattices L_1 and L_2.

   (i) Find all subgraphs of the P the pseudo lattice graph of type I obtained by merging vertex a_1 with vertex b_7.

   (ii) Can any of these subgraphs of P be lattices?

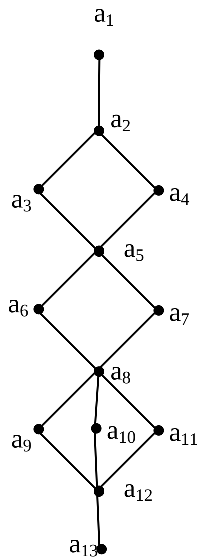
   (iii) How many of these subgraphs of P be connected?

   (iv) Find the total number of subgraphs of P.

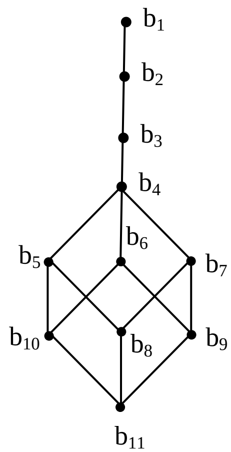
b. Study question (i) to (iv) for the B pseudo lattice subgraph of type I where B is obtained by merging one the vertices a_1 with b_1, a_2 with b_2, a_5 with b_3, a_6 with b_4, a_8 with b_5 and a_{10} with b_9.

5. Let

$L_1 =$



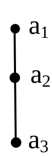
and L_2 =



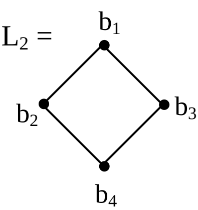
be any two lattices.

(i) In how many ways can L_1 and L_2 be merged vertices or edges or both to get pseudo lattice graphs of type I?
(ii) How many of the pseudo lattice graphs of type I got using L_1 and L_2 are lattices?
(iii) Find all subgraphs of these pseudo lattice graphs of type I which are sublattices.
(iv) Is every pseudo lattice graph of type I got using L_1 and L_2 connected?
(v) Does the number pseudo lattice graphs of type I dependent on the number of edges and vertices of the lattices L_1 and L_2.

6. Let L_1 =

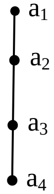


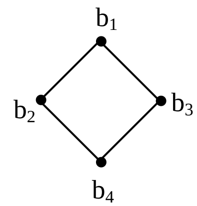
and L_2 =



be two lattices.

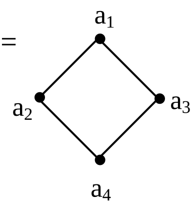
Study questions (i) to (v) for this pair of lattices.

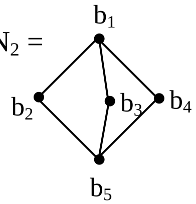
7. Let M_1 = 

and M_2 = 

be any two lattices.

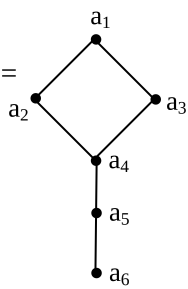
(i) Study questions (i) to (v) for this pair of lattices.
(ii) Compare this pair with the pair in problem (6)

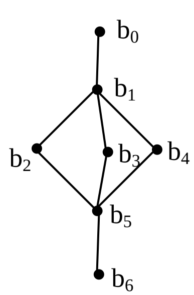
8. Let N_1 = 

and N_2 = 

be any two lattices.

(i) Study questions (i) to (v) of problem 6 for this pair of lattices.
(ii) Compare the pairs in problem 6 and 7 with this pair.

9. Let L_1 = 

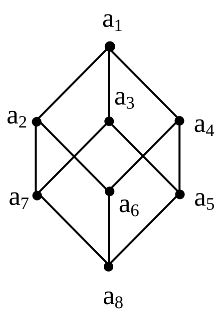
and L_2 = 

be any two lattices.

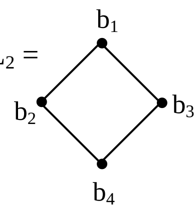
Study questions (i) to (v) for problem 6 for this pair of lattices L_1 and L_2.

Compare this pair with M_1 and M_2 the pair of problem 7.

10. Let L_1 =



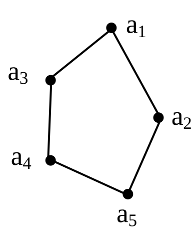
and L_2 =



be any two lattices.

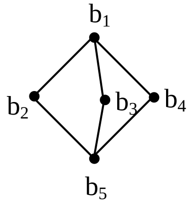
(i) Study questions (i) to (v) for problem 6 for this pair.
(ii) Does this have any impact on Boolean algebras?

11. Let L_1 =



and

L_2 =

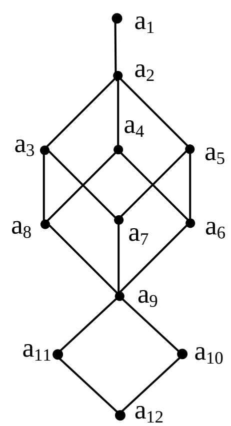


be a pair of lattices.

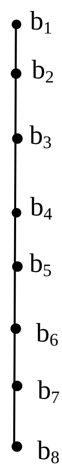
(i) Study questions (i) to (v) of problem 6 for this pair.
(ii) Does this have any impact on the non modularity of lattices or modularity of lattices?

12. Obtain some special and interesting features associated with pseudo lattice graphs of type I.

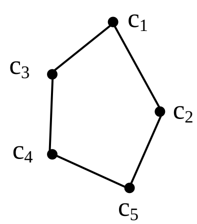
13. Let B_1 =



B_2 =



and B_3 =



be three lattices.

(i) How many pseudo lattice graphs of type I can be obtained;

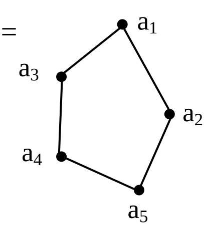
(a) by merging only one vertex to each?

(b) by merging two vertices or an edge with two vertices each.

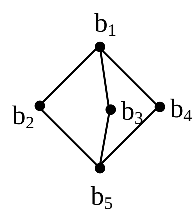
(c) How many pseudo lattice graphs of type I can be got by merging in all possible ways?

(ii) How many of the pseudo lattice graphs of type I using B_1, B_2 and B_3 are lattices?

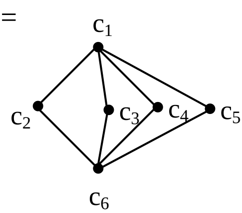
14. Let C_1 =



C_2 =



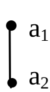
and C_3 =



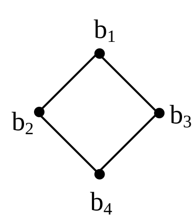
be three lattices.

Study questions (i) to (ii) of problem 13 for these three lattices C_1, C_2 and C_3.

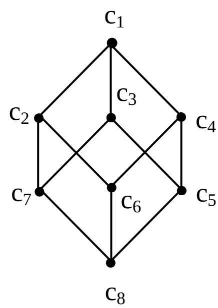
15. Let B_1 =



B_2 =



$B_3 =$

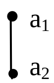


be three lattices.

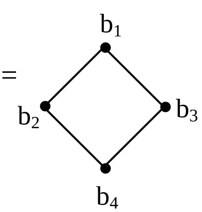
(i) Study questions (i) to (ii) of problem 13 for these three lattices.

(ii) Does this happen to have a flavour of Boolean algebras?

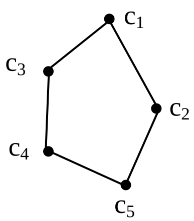
16. Let $L_1 =$



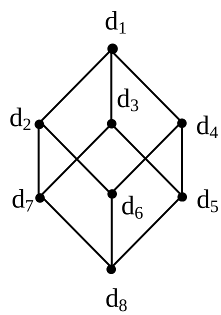
and $L_2 =$



$L_3 =$



$L_4 =$



be four lattices.

Study questions (i) to (iii) of problem 13 for this pair of lattices L_1, L_2, L_3 and L_4.

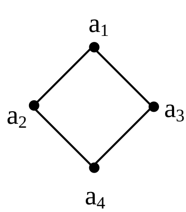
17. Suppose we have n finite lattices $L_1, L_2, \ldots, L_n$.

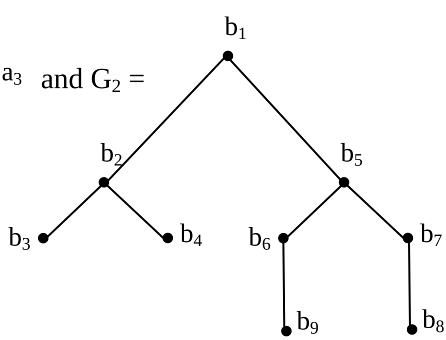
    (i) How many pseudo lattice graphs of type I can be constructed?
    (ii) Study questions (i) to (ii) of problem 13 for these lattices.
    (iii) How many of these are lattices?

18. Obtain some special features enjoyed by pseudo lattice graphs of type II.

19. Find the major difference between pseudo lattice graphs of type I and type II.

20. Let

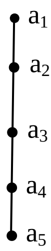
L_1 = 

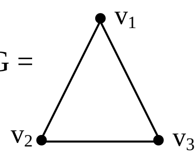
and G_2 = 

be a lattice and a graph respectively.

(i) Let S be the collection of all pseudo lattice graphs of type II. What is the cardinality of S?
(ii) How many graph in S are lattices?
(iii) Can S have semilattices?
(iv) Find all special properties enjoyed by the elements of S.

21. Let

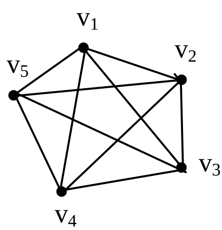
L = 

and G = 

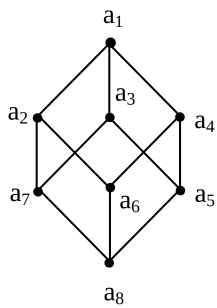
be a lattice and a graph respectively.

Study questions (i) to (iv) of problem 20 for this G and L.

22. Let

G = 

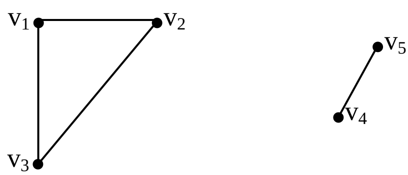
be a graph and

L = 

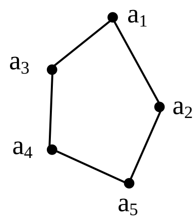
be a lattice.

Study questions (i) to (iv) of problem 20 for this G and L.

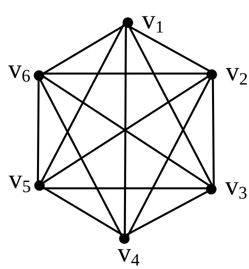
23. Let

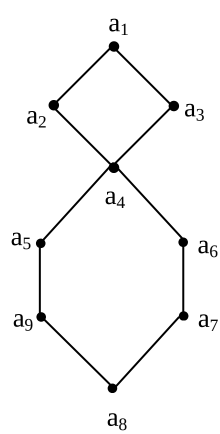
G = 

be a graph and

L = 

be a lattice. Study questions (i) to (iv) of problem 20 for this G and L.

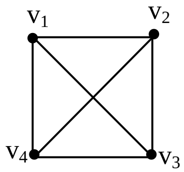
24. Let G = 

and 

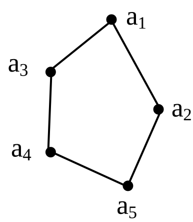
be a graph and a lattice respectively.

Study questions (i) to (iv) of problem 20 for this G and L.

25. Let

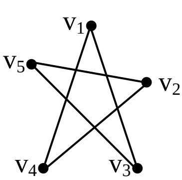
G = 

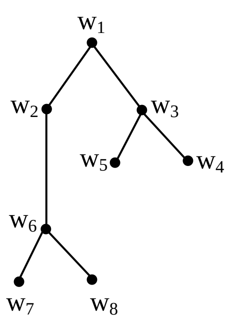
be a graph and

L = 

be a lattice.
S = {Collection of all pseudo lattice graphs of type II}. Study questions (i) to (iv) of problem 20 for this L and G.

26. Let

G_1 = 

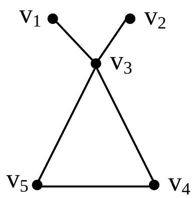
and G_2 = 

be two graphs. B = {Collection of all pseudo lattice graph of type II}.

(i) Find o(B).
(ii) Find all lattices in B.
(iii) Is it possible B contains lattices?
(iv) How many subgraph of graphs in B are lattices?
(v) Is it possible for B to have semi lattices?

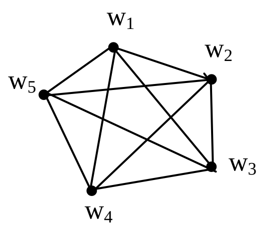
27. Let

$G_1 =$



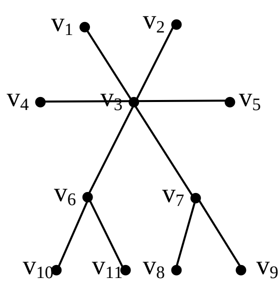
and

$G_2 =$

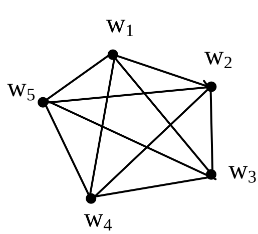


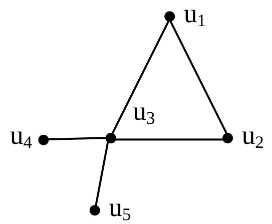
be two graphs. T = {Collection of pseudo lattice graphs of type II got by merging differently G_1 with G_2}. Study questions (i) to (v) of problem 26 for this T.

28. Let $G_1 =$



$G_2 =$



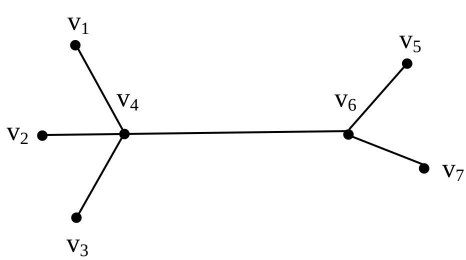
G_3 = 

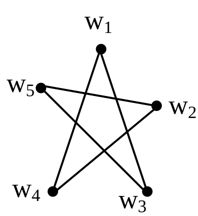
be three graphs.

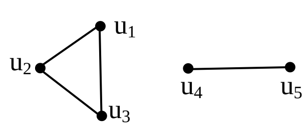
P = {Collection of all pseudo lattice graphs of type II obtained by merging vertex or edge or both or vertices and edges of the three graphs}

(i) Study questions (i) to (v) of problem 26 for this P.
(ii) Compare T of 27 with this P.

29. Let

G_1 = 

G_2 = 

G_3 = 

be the three graphs.

S = {Collection of all pseudo lattice graphs of type II got by merging vertices or edges or both of these graphs}.

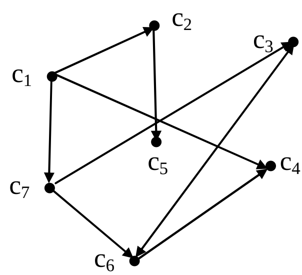
Study questions (i) to (v) of problem 26 for this S.

30. Let $G_1, G_2, G_3, \ldots, G_n$ be n graphs.
    Suppose T = {Collection of all pseudo lattice graphs of type II got by merging vertices or edges or both}.

    (i) Study all properties associated with T.
    (ii) Study questions (i) to (v) of problem 26 for this T.

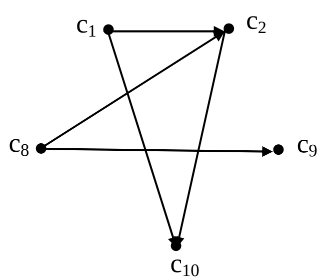
31. Let G_1 and G_2 be two directed graphs associated with FCMs associated a same problem.

G_1 =



and

G_2 =

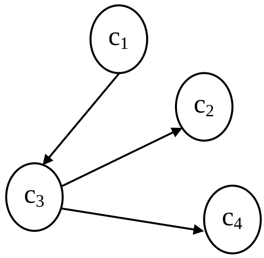


We get S is the only pseudo lattice graph of type II obtained by merging c_1c_2 of G_1 with c_1c_2 of G_2.
The resultant S gives the FCMs model which is the merged FCMs.
Study the merged FCMs in case of any real world problem using 2 or more experts opinion on a same problem.

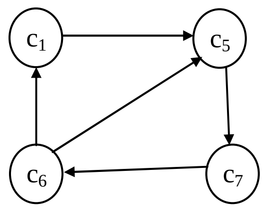
32. Study the merits of using merged FCMs of several FCMs of a problem.

33. Let

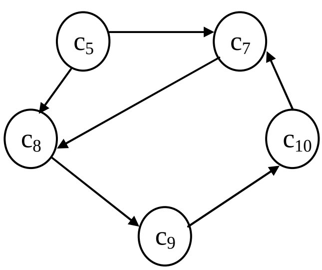
G_1 =



G_2 =



G_3=



be the three directed graphs of the same problem given by three experts.

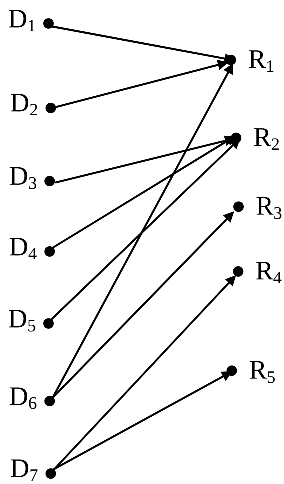
Find the merged FCMs of the three directed graphs.

34. If G_1, G_2, G_3, …, G_n be the directed graphs of FCMs related with a problem given by n experts.

Prove there exists more than one merged FCM which can simultaneously give the opinion of the n experts.

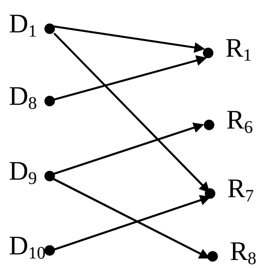
35. Let

$G_1 =$



and

$G_2 =$



be the directed graphs related to FRMs given by two different experts.

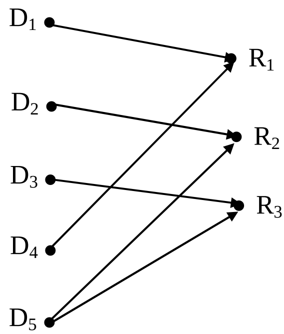
(i) Prove there exists a unique merged graph which gives the merged FRMs.

36. Find / Give some important properties enjoyed by the merged FRMs of n-different experts.

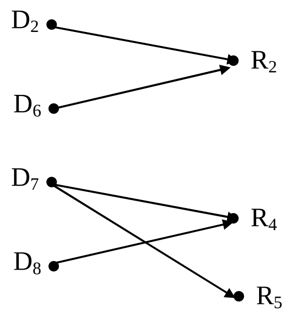
37. Prove such merged FRMs are better than studying n-experts FRMs separately.

38. Let

G_1 =

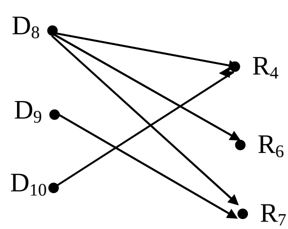


G_2 =



and

G_3 =



be the 3 directed graphs given by three different experts on the same problem using FRMs.

(i) Show there exist one and only one merged FRM of G_1, G_2 and G_3.

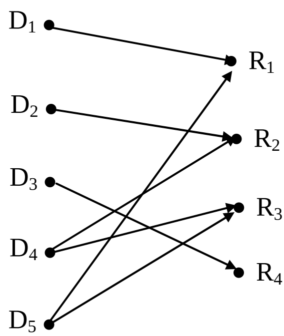
(ii) Prove this merged FRM is a powerful tool which saves money and economy.

39. Find some special features enjoyed by merged linked FRMs.

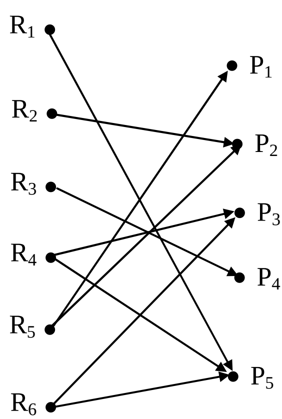
    Show merged linked FRMs are different from the merged FRMs and linked FRMs.

40. Let

$G_1 =$



$G_2 =$



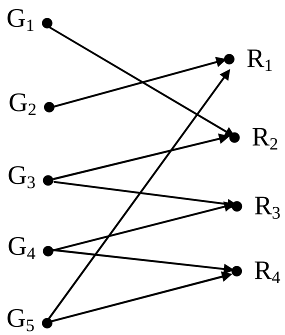
be the two directed graphs of FRM on the same problem given by two different experts.

Find the merged linked FRMs and its directed graph. Show the merged graph is unique.

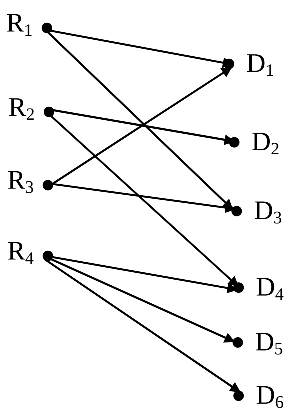
41. Let H_1 and H_2 be the two bipartite graph given by two experts for the FRM model.

Let

H_1 =



H_2 =



Find the linked graph of G_1, G_2 got by merging the vertices R_1, R_2, R_3 and R_4 of G_1 with G_2.
Show such merging is unique.

42. Obtain some special features enjoyed by merged FRMs.

43. Give by an example 3 or more directed graphs of a FRM can be merged to get a merged FRM.

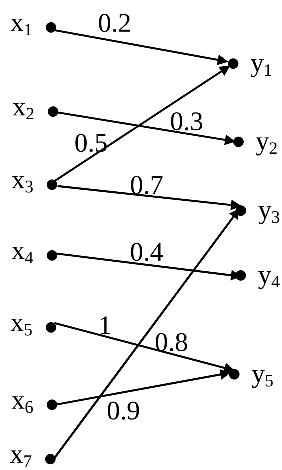
Show this new merged FRM is a best special type of combined FRM model which save time and money.

44. Study merged Fuzzy Relational Equations (FREs) model.

45. Give some examples of merged FREs of more than two experts.

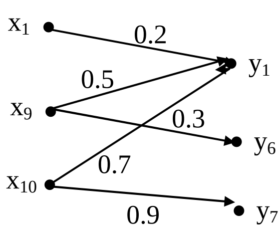
46. Let

G_1 =



and

G_2 =



be the directed graphs associated with the Fuzzy Relational Equations (FRE) model given by two experts who work on the same set of constraints.
Prove there exists a unique merged FRE model.

47. Study the special features of merged FRE model.

48. Compare a merged FRE model with a linked FRE model.

49. Give one real world problem illustration of merged FRE model.

50. Prove merged FRE models of more than 3 experts by illustrative examples.

51. If G_1, G_2, …,. G_n are n directed graphs of the FRMs of n experts opinion on a problem.

    (i) Prove there exists many merged FRMs.
    (ii) Prove this merged FRMs is better than using n- FRMs.
    (iii) Prove this gives equal importance to all the n experts.

52. Obtain some special properties enjoyed by pseudo neutrosophic lattice graphs of type I.

53. Compare the pseudo neutrosophic lattice graphs of type I with that of the pseudo lattice graphs of type II.

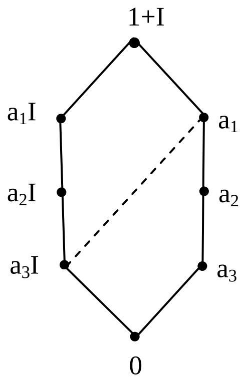
54. Enumerate some new and innovative applications of pseudo neutrosophic lattice graphs of type I.

55. Can we merge a real vertex of a lattice with the neutrosophic vertex of another lattice?

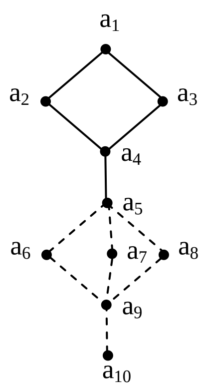
56. Can the real edge of a lattice L_1 be merged with the neutrosophic edge of the lattice L_2?

57. What is the specialty in the case of problems 55 and 56?

58. Let

$L_1 =$ 

and

$L_2 =$ 

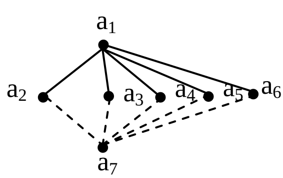
be any two neutrosophic lattices.

(i) S = {Collection of all pseudo neutrosophic lattice graphs of type I} find o(S).

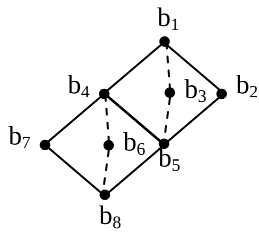
(ii) How many of these A ∈ S are lattices?

(iii) Is every A ∈ S a connected graph?

59. Let

$L_1 =$ 

and

$L_2 =$ 

be any two neutrosophic lattices.

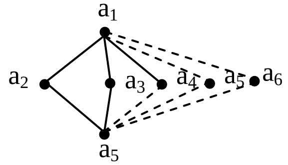
(i) P = {Collection of all pseudo neutrosophic lattice graphs of type I}; find o(P).

(ii) Find all neutrosophic lattices in P.

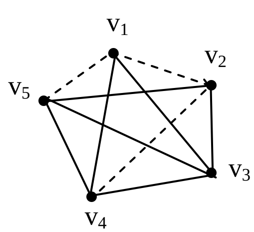
60. Obtain some special features enjoyed by pseudo neutrosophic lattice graphs of type II.

61. Is it possible that by merging vertices or edges or both of two neutrosophic lattices? The resultant pseudo neutrosophic lattice graph of type II has no lattice and only graphs?

62. Let

$L_1 =$ 

and

$G =$ 

be a neutrosophic lattice and a neutrosophic graph respectively.

(i) Find order of S = {Collection of a pseudo neutrosophic lattice graphs of type II}.

(ii) How many elements in S are neutrosophic lattices?

63. Let L_1 and L_2 be any two finite neutrosophic lattices. S = {Collection of all edge or vertex or merging of both of the lattices L_1 and L_2}.
= {Collection of all pseudo neutrosophic lattice graphs of type I}.

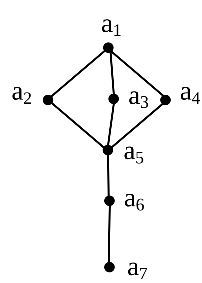
(i) Find o(S).

(ii) Does order of S depend on the number of vertices and edges of L_1 and L_2?

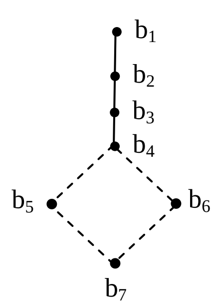
(iii) Obtain some special features enjoyed by S.

(iv) Prove S can have subgraphs / sublattices which are not neutrosophic.

64. Let

$L_1 =$ 

and

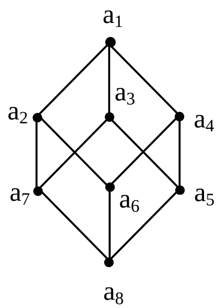
$L_2 =$ 

be neutrosophic lattices. S = {Collection of all pseudo neutrosophic lattice graphs of type I got by merging the vertices or edges or both}.

(i) Find o(S).
(ii) Find all lattices in S.
(iii) Find sublattices A in S.

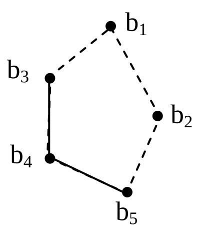
65. Let

$L_1 =$



and

$L_2 =$

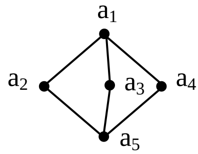


be any two lattices.
S = {Collection of all pseudo neutrosophic lattice graphs of merging vertices or edges of L_1 with L_2}.

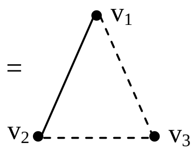
(i) Find o(S).
(ii) Prove S has sublattices.
(iii) Can A ∈ S be a lattice?

66. Let

$L =$



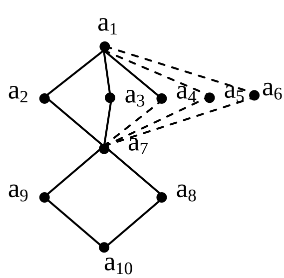
and

G = 

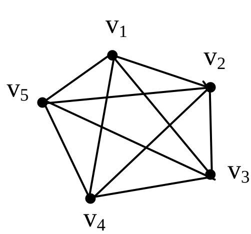
be the lattice and the neutrosophic graph.
S = {Collection of all pseudo neutrosophic lattice graphs of type II}.

Study questions (i) to (iii) of problem 65 for this S.

67. Let

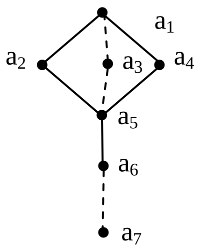
L = 

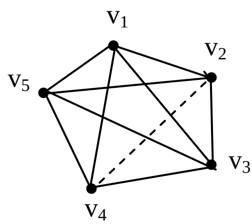
and

G = 

be a neutrosophic lattice and graph.
Study questions (i) to (iii) of problem 65 for this S.

68. Let L and G be a neutrosophic lattice and a graph respectively given in the following.

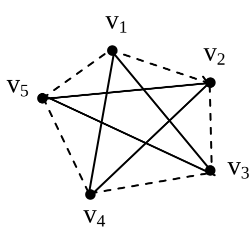




Study questions (i) to (iii) of problem (59) for this P = {Collection of all pseudo lattice graphs of type II using L and G.

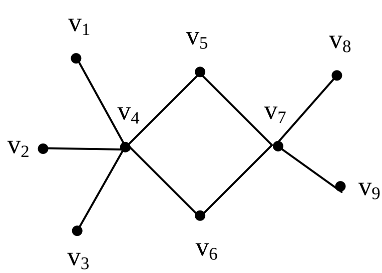
69. Let G and G_1 be two graphs where G_1 is the usual graph.

G_1 =



and

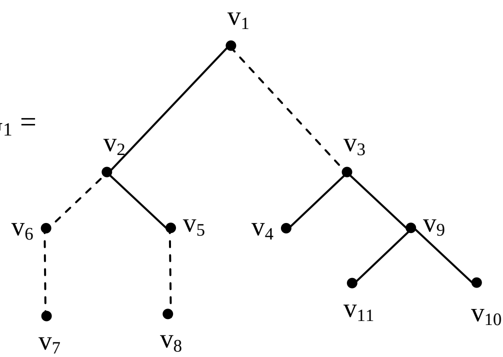
G_2 =



M = {Collection of all pseudo lattice graphs of type II using G_1 and G_2}.
Study question (i) to (iii) of problem 65 for this M.

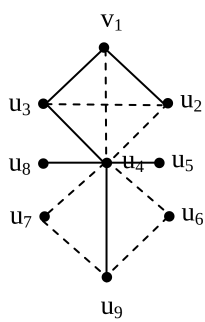
70. Let

G_1 =



and

G_2 =



be two neutrosophic graphs.
S = {Collection of all pseudo lattice graphs of type II}.

Study question (i) to (iii) of problem 65 for this S.

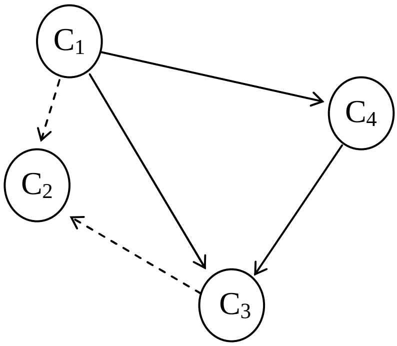
71. Show pseudo neutrosophic lattice graphs of II are used in neutrosophic fuzzy models like NCMs (Neutrosophic Relational Maps) and NREs (Neutrosophic Relational Equations).

72. Give one example from real word problem where the use of NCMs directed graphs are merged to get the merged neutrosophic cognitive Maps model.

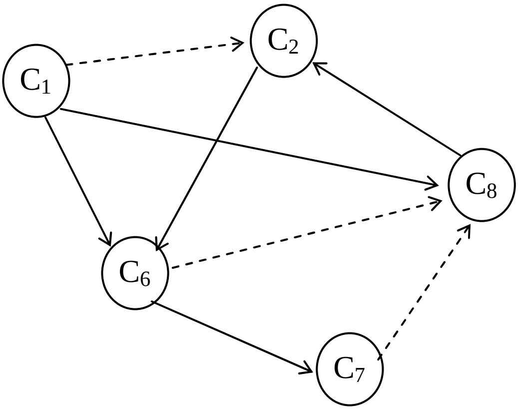
73. Explain by illustration the merged NRMs model.

74. Describe by an example the merged NRE model.

75. Let G_1 =



and G_2 =



be the direct graphs of the FCM and NCM respectively given by two experts on the same problem.

We can merge G_1 and G_2 only in one way by merging the edge C_1C_3. The resultant gives the merged NCM.

Study the using the merged graphs connection matrix the merged NCM model.

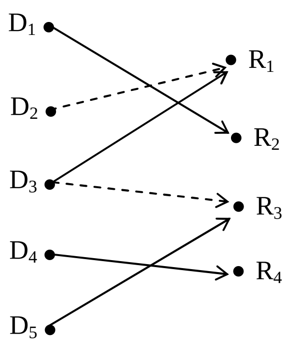
76. Let G_1 and G_2 be two directed neutrosophic graphs associated with the NCMs model for the same problem.

    Prove the merged directed graph is unique and can be got only by merging C_1 node of G_1 with C_1 node of G_2.

    Study the merged NCMs model.

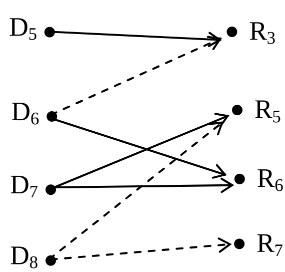
77. Let

G_1 =



and

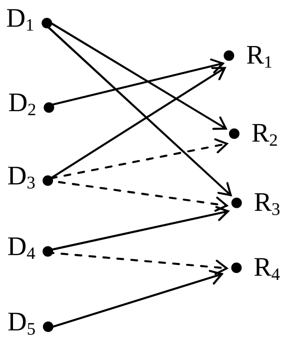
G_2 =

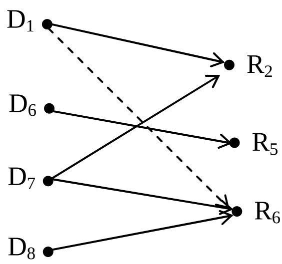


be the neutrosophic directed bipartite graphs of the NRMs model given by two experts on the same problem.

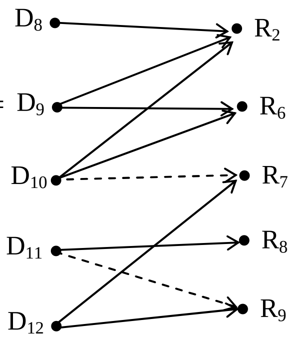
(i) Show we have only one merged NRM.
(ii) We can have one and only pseudo lattice graph of type II.
(iii) Study the merged NRM model.

78. Let

G_1 = 

G_2 = 

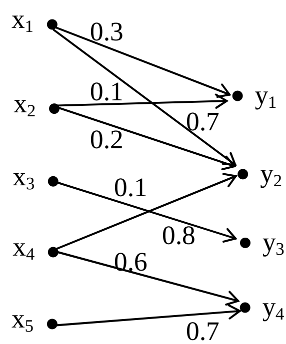
and

G_3 = 

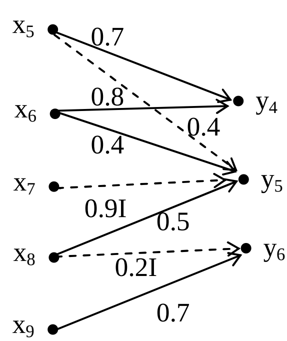
be any three directed neutrosophic bipartite graphs of a NRM model which is given above.

(i) Show the merged NRM is unique.
(ii) Study the merged NRM model.
(iii) What are the merits of merged NRMs model?
(iv) Show it is different from combined NRMs model and linked NRMs model.

79. Let

G_1 = 

and

G_2 = 

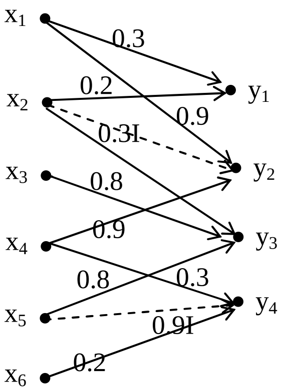
be the bipartite graph of the FRE and NRE respectively given in the following.

(i) Show there exist one and only one merged NRE using G_1 and G_2.
(ii) Find some special features about these merged NREs.

(iii) What are the basic advantages in using the merged NRE models.

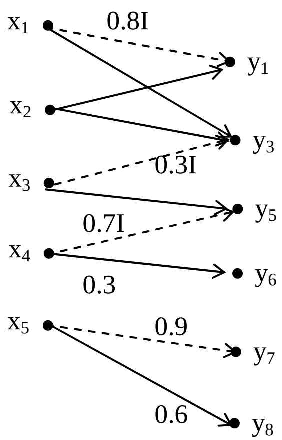
74. Let

G_1 =



and

G_2 =



be the neutrosophic bipartite graphs of two NREs which are related with the same problem.

(i) Prove there exists only one merged neutrosophic graph describing the merged NRE model.

(ii) Study this NRE model
(iii) Spell out the advantages of using the merged NREs model.

81. Give a real world problem in which merged NREs model is used.

82. Enumerate the general merits of using merged fuzzy or neutrosophic models.

83. Differentiate the merged FCMs models and the combined FCMs models.

84. What is the difference between the merged FRMs model and linked FRMs model?

    Study the above question in NRM models.

85. Distinguish the properties between the merged NRMs model and the combined NRM model.

86. Give some interesting applications of pseudo lattice graphs of type II.

87. Show by the method of merging models one can merge more than two graphs to get the merged model if the necessary conditions are satisfied.

88. Give a real world problem in which the three graphs of a FCMs model are merged on in the concepts and not on the edges. Study the same problem in case of NCMs.

89. Can problem 85 be true in case of related graphs of FRMs and NRMs?

90. Give a real world problem illustrated in which four appropriate graphs of NREs / FREs are merged to get the new merged NRE model.

(i) Study the advantages.
(ii) What can be the probable disadvantages in using merged fuzzy models?

91. Can the pseudo lattice graphs of type I or type II which are trees be helpful in data mining?

92. Obtain a sufficient and necessary condition for the pseudo lattice graph to be a lattice?

93. Is it possible if two lattices are used then no pseudo lattice graph of type I will be a lattice?

94. Suppose we use a merged FCM (or FRM or FRE) model using the merging of their respective graph.

    (i) Can we say certain special subgraphs of the merged graphs result in submodels?

    (ii) Can we always say subgraphs of every experts can be got from the merged model?

    (iii) Prove use of merged models save time and economy.

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In this book authors for the first time have merged vertices and edges of lattices to get a new structure which may or may not be a lattice but is always a graph. This merging is done for graphs also, which will be used in the merging of fuzzy models. Further merging of graphs leads to merging of matrices both these concepts play a vital role in merging of fuzzy and neutrosophic models. Several open conjectures are suggested.

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