Correlation of Neutrosophic Sets in Probability Spaces

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Abstract

In this paper, we introduce and study the concepts of correlation and correlation coefficient of neutrosophic data in probability spaces. In the case of finite spaces, these results give the results due to Salama et al. [16, 17, 18] and study some of their properties.

Keywords: Correlation Coefficient, Fuzzy Sets, Neutrosophic Sets, Intuitionistic Fuzzy Sets.

1. INTRODUCTION

In 1965 [26], Zadeh first introduced the concept of fuzzy sets. Fuzzy set is very much useful and in this one real value $\mu_A(x) \in [0,1]$ is used to represent the grade of membership of a fuzzy set *A* defined on the crisp set *X*. After two decades Atanassov [19, 20, 21] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Intuitionistic fuzzy sets handle incomplete information i.e., the grade of membership function and non-membership function but not the indeterminate information and inconsistent information which exists obviously in belief system. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. Smarandache [11, 12, 13, 14, 15] introduced another concept of imprecise sets called neutrosophic sets. Salama et al. [1, 2, 3, 4, 5, 6, 7, 16, 17, 18] introduced and studied the operations on neutrosophic sets. In statistical analysis, the correlation coefficient plays an important role in measuring the strength of the linear relationship between two variables. As the correlation coefficients defined on crisp sets have been much discussed, it is also very common in the theory of fuzzy sets to find the correlation between fuzzy sets, which accounts for the relationship between the fuzzy sets. In this paper we discuss and derived a formula for correlation coefficient, defined on the domain of neutrosophic sets in probability spaces.

2. TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [11, 12, 13, 14, 15] and Salama et al. [1, 2, 3, 4, 5, 6, 7, 16, 17, 18].

10.2478/jamsi-2014-0004

Smarandache introduced the intuitionistic components T, I, and F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^{-}0\ ,1^{+}\right[$ is non- standard unit interval.

DEFINITION 2.1 [26]. Let *X* be a fixed set. A fuzzy set *A* of *X* is an object having the form $A=\{(x,\mu_A(x)),x\in X\}$ where the function $\mu_A : X\rightarrow[0,1]$ define the degree of membership of the element $x\in X$ to the set *A*, which is a subset of *X*.

DEFINITION 2.2 [19], [20], [21]. Let *X* be a fixed set. An intuitionistic fuzzy set *A* of *X* is an object having the form:
$A=\{(x,\mu_A(x),\nu_A(x)),x\in X\}$, Where the function: $\mu_A : X\rightarrow[0,1]$ and $\nu_A : X\rightarrow[0,1]$ define respectively the degree of membership and degree of non-membership of the element $x\in X$ to the set *A*, which is a subset of *X* and for every $x\in X$, $0\le\mu_A(x)+\nu_A(x)\le 1$.

DEFINITION 2.3 [1, 2]. Let *X* be a non-empty fixed set. A neutrosophic set (NS) *A* is an object having the form: $A=\{(x,\mu_A(x),\gamma_A(x),\nu_A(x)),x\in X\}$ where $\mu_A(x),\gamma_A(x)$ and $\nu_A(x)$ represent the degree of membership function, the degree of indeterminacy, and the degree of non membership function respectively of each element $x\in X$ to the set *A*.

In 1991, Gerstenkorn and Manko [24] defined the correlation of intuitionistic fuzzy sets *A* and *B* in a finite set $X=\{x_1,x_2,\ldots,x_n\}$ as follows:

$$C_{GM}(A,B)=\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)) \tag{2.1}$$

and the correlation coefficient of fuzzy numbers *A*,*B* was given by:

$$\rho_{GM}=\frac{C_{GM}(A,B)}{\sqrt{T(A)\cdot T(B)}} \tag{2.2}$$

where

$$T(A)=\sum_{i=1}^{n}(\mu_A^2(x_i)+\nu_A^2(x_i))\,. \tag{2.3}$$

Yu [9] defined the correlation of *A* and *B* in the collection *F*([*a*,*b*]) of all fuzzy numbers whose supports are included in a closed interval [*a*,*b*] as follows:

$$C_Y(A,B)=\frac{1}{b-a}\int_a^b \mu_A(x)\mu_B(x)+\nu_A(x)\nu_B(x)dx\,, \tag{2.4}$$

where $\mu_A(x)+\nu_A(x)=1$ and the correlation coefficient of fuzzy numbers *A*, *B* was defined by

$$\rho_Y = \frac{C_Y(A,B)}{\sqrt{C_Y(A,A) \times C_Y(B,B)}} \,. \tag{2.5}$$

In 1995, Hong and Hwang [10], 25] defined the correlation of intuitionistic fuzzy sets *A* and *B* in a probability space (*X*, *B*, *P*) as follows:

$$C_{HH}(A,B) = \int_X (\mu_A \mu_B + \nu_A \nu_B) dP \tag{2.6}$$

and the correlation coefficient of intuitionistic fuzzy numbers *A*, *B* was given by

$$\rho_{HH} = \frac{C_{HH}(A,B)}{\sqrt{C_{HH}(A,A) \cdot C_{HH}(B,B)}} \,. \tag{2.7}$$

Hung and Wu [25] introduce the concept of positively and negatively correlated and used the concept of centroid to define the correlation coefficient of intuitionistic fuzzy sets which lies in the interval [-1,1], and the correlation coefficient of intuitionistic fuzzy sets *A* and *B* was given by:

$$\rho_{HW} = \frac{C_{HW}(A,B)}{\sqrt{C_{HW}(A,A) \cdot C_{HW}(B,B)}} \,, \tag{2.8}$$

where

$$C_{HW} = m(\mu_A)m(\mu_B) + m(\nu_A)m(\nu_B) \tag{2.9}$$

$$m(\mu_A) = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \qquad m(\nu_A) = \frac{\int x\nu_A(x)dx}{\int \nu_A(x)dx}$$

$$m(\mu_B) = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx} \qquad m(\nu_B) = \frac{\int x\nu_B(x)dx}{\int \nu_B(x)dx}$$

Hanafy and Salama [16, 17, 18] defined the correlation of neutrosophic data in a finite set $X = \{x_1, x_2, \ldots, x_n\}$ as follows:

$$C_{HS}(A,B) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \gamma_A(x_i)\gamma_B(x_i)) \tag{2.10}$$

and the correlation coefficient of fuzzy numbers *A*,*B* was given by:

$$\rho_{HS} = \frac{C_{HS}(A,B)}{\sqrt{T(A) \cdot T(B)}} \tag{2.11}$$

where

$$T(A) = \sum_{i=1}^{n} (\mu_A^2(x_i) + \nu_A^2(x_i) + \gamma_A^2(x_i)) \tag{2.12}$$

$$T(B)=\sum_{i=1}^{n}(\mu_B^2(x_i)+\nu_B^2(x_i)+\gamma_B^2(x_i))\cdot \tag{2.13}$$

3. CORRELATION COEFFICIENT OF NEUTROSOPHIC SETS

Let (X, B, P) be a probability space and A be a neutrosophic set in a probability space X, $A=\{\langle x,\mu_A(x),\gamma_A(x),\nu_A(x)\rangle | x\in X\}$, where $\mu_A(x),\gamma_A(x),\nu_A(x): X\to[0,1]$ are, respectively, Borel measurable functions satisfying $^{-}0\le\mu_A(x)+\gamma_A(x)+\nu_A(x)\le 3^{+}$, where $]^{-}0,1^{+}[$ is non-standard unit interval [8].

DEFINITION 3.1. For a neutrosophic sets A, B, we define the correlation of neutrosophic sets A and B as follows:

$$C(A,B)=\int_X(\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B)dP\cdot \tag{3.1}$$

Wher P is the probability measure over X. Furthermore, we define the correlation coefficient of neutrosophic sets A and B as follows:

$$\rho(A,B)=\frac{C(A,B)}{[T(A)\cdot T(B)]^{1/2}}, \tag{3.2}$$

where

$$T(A)=C(A,A)=\int_X(\mu_A^2+\gamma_A^2+\nu_A^2)dP, \text{ and}$$

$$T(B)=C(B,B)=\int_X(\mu_B^2+\gamma_B^2+\nu_B^2)dP\cdot$$

The following proposition is immediate from the definitions.

PROPOSITION 3.1. For neutrosophic sets A and B in X, we have

i. $C(A,B)=C(B,A)$, $\rho(A,B)=\rho(B,A)$.

ii. If A=B, then $\rho(A,B)=1$.

The following theorem generalizes both Theorem 1 [23], Proposition 2.3 [8] and Theorem 1 [[10]] of which the proof is remarkably simple.

THEOREM 3.1. For neutrosophic sets A and B in X, we have

$$0\le\rho(A,B)\le 1. \tag{3.3}$$

PROOF. The inequality $\rho(A,B)\ge 0$ is evident since $C(A,B)\ge 0$ and $T(A),T(B)\ge 0$. Thus, we need only to show that $\rho(A,B)\le 1$, or

$$C(A,B) \le [T(A)\cdot T(B)]^{1/2}$$

For an arbitrary real number k , we have

$$0 \le \int_x (\mu_A - k\mu_B)^2 dP + \int_x (\gamma_A - k\gamma_B)^2 dP + \int_x (\nu_A - k\nu_B)^2 dP$$

$$= \int_X (\mu_A^2 + \gamma_A^2 + \nu_A^2) dP - 2k \int_X (\mu_A\mu_B + \gamma_A\gamma_B + \nu_A\nu_B) dP$$

$$+ k^2 \int_X (\mu_B^2 + \gamma_B^2 + \nu_B^2) dP \cdot$$

Thus by Cauchy Schwarz [22], we can get:

$$\left(\int_X (\mu_A\mu_B + \gamma_A\gamma_B + \nu_A\nu_B) dP \right)^2 \le \int_X (\mu_A^2 + \gamma_A^2 + \nu_A^2) dP \cdot \int_X (\mu_B^2 + \gamma_B^2 + \nu_B^2) dP \cdot$$

$$[C(A,B)]^2 \le T(A)\cdot T(B), C(A,B) \le [T(A)\cdot T(B)]^{1/2}$$

Therefore, we have $\rho(A,B) \le 1$.

THEOREM 3.2. $\rho(A,B) = 1$ iff $A = cB$ for some $c \in IR$.

PROOF. Considering the inequality in the proof of Theorem 3.1, then the equality holds if and only if $P\{\mu_A = c\mu_B\} = P\{\gamma_A = c\gamma_B\} = P\{\nu_A = c\nu_B\} = 1$, for some $c \in IR$. which completes the proof.

THEOREM 3.3. $\rho(A,B) = 0$ iff A and B are non-fuzzy sets and they satisfy the condition:

$$\mu_A + \mu_B = 1 \quad \text{or} \quad \gamma_A + \gamma_B = 1 \quad \text{or} \quad \nu_A + \nu_B = 1 \cdot$$

PROOF. Suppose that $\rho(A,B) = 0$, then $C(A,B) = 0$. Since $\mu_A\mu_B + \gamma_A\gamma_B + \nu_A\nu_B \ge 0$, then $C(A,B) = 0$ implies $P(\mu_A\mu_B + \gamma_A\gamma_B + \nu_A\nu_B = 0) = 1$, which means that $P(\mu_A\mu_B = 0) = 1, P(\gamma_A\gamma_B = 0) = 1$ and $P(\nu_A\nu_B = 0) = 1$.

If $\mu_A(x) = 1$, then we can get $\mu_B(x) = 0$ and $\gamma_A(x) = \nu_A(x) = 0$.

At the same time, if $\mu_B(x) = 1$, then we can get $\mu_A(x) = 0$ and $\gamma_B(x) = \nu_B(x) = 0$, hence, we have $\mu_A + \mu_B = 1$.

Conversely, if A and B are non-fuzzy sets and $\mu_A + \mu_B = 1$. If $\mu_A(x) = 1$, then we can $\mu_B(x) = 0$ and $\gamma_A(x) = \nu_A(x) = 0$. On the other hand, if $\mu_B(x) = 1$, then we can have $\mu_A(x) = 0$ and $\gamma_B(x) = \nu_B(x) = 0$, which implies $C(A,B) = 0$. Similarly we can give the proof when $\gamma_A + \gamma_B = 1$ or $\nu_A + \nu_B = 1$.

THEOREM 3.4. If A is a non-fuzzy set, then $T(A)$ =1.

THE PROOF is obvious.

EXAMPLE. For a continuous universal set X =[1,2], if two neutrosophic sets are written, respectively.

$$A = \{(x, \mu_A(x), \nu_A(x), \gamma_A(x)) | x \in [1,2]\},$$

$$B = \{(x, \mu_B(x), \nu_B(x), \gamma_A(x)) | x \in [1,2]\},$$

where

$$\mu_A(x) = 0.5(x-1) \quad , \quad 1 \le x \le 2 \ , \qquad \mu_B = 0.3(x-1) \quad , \quad 1 \le x \le 2$$

$$\nu_A(x) = 1.9 - 0.9x \quad , \quad 1 \le x \le 2 \ , \qquad \nu_B = 1.4 - 0.4x \quad , \quad 1 \le x \le 2$$

$$\gamma_A(x) = (5-x)/6 \quad , \quad 1 \le x \le 2 \quad , \qquad \gamma_B(x) = 0.5x - 0.3 \quad , \quad 1 \le x \le 2$$

Thus, we have $C(A,B) = 0.79556$, $T(A) = 0.79593)$ and $T(B) = 0.9365.6$, Then we get $\rho(A,B) = 0.936506$. It shows that neutrosophic sets A and B have a good positively correlated.

CONCLUSION

Our main goal of this work is propose a method to calculate the correlation coefficient of neutrosophic sets which lies in [0, 1], give us information for the degree of the relationship between the neutrosophic sets. Further, we discuss some of their properties and give example to illustrate our proposed method reasonable. Thus, we generalize some conclusions in literature

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