Interval neutrosophic multiple attribute decision-making method considering credibility

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Abstract

Considering corresponding credibility on every evaluation value of interval neutrosophic numbers (INNs) in interval neutrosophic decision making, we put forward two interval neutrosophic weighted aggregation operators considering credibility, including a credibility-induced interval neutrosophic weighted arithmetic averaging (CIINWAA) operator and a credibility-induced interval neutrosophic weighted geometric averaging (CIINWGA) operator, and investigate their properties. Based on the Dice similarity measure, we introduce a ranking method for INNs, and then establish a decision-making method based on the CIINWAA and CIINWGA operators and the ranking method to handle multiple attribute decision-making problems with interval neutrosophic information and credibility information. Finally, an illustrative example of investment alternatives and the comparative analysis are provided to demonstrate the application and effectiveness of the developed approach.

Keywords: Interval neutrosophic set; Credibility-induced interval neutrosophic weighted arithmetic

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averaging (CIINWAA) operator; Credibility-induced interval neutrosophic weighted geometric averaging (CIINWGA) operator; Decision making; Dice similarity measure

1. Introduction

Due to more and more complexity of real decision-making problems, the decision information is often incomplete, indeterminate and inconsistent information. Then, the neutrosophic set proposed by Smarandache [1] can be better to express this kind of information and to extend the concepts of fuzzy sets, intuitionistic fuzzy sets, and interval-valued intuitionistic fuzzy sets. To easily apply the neutrosophic set to real scientific and engineering areas, Wang et al. [2, 3] introduced the concepts of an interval neutrosophic set (INS) and a single valued neutrosophic set (SVNS) as the subclasses of a neutrosophic set and provided the set-theoretic operators and various properties of SVNSs and INSs. After that, Ye [4, 5] proposed correlation coefficients between SVNSs and applied them to multiple attribute decision-making problems with single valued neutrosophic information. Ye [6] also proposed a single valued neutrosophic cross-entropy measure and applied it to multiple attribute decision-making in single valued neutrosophic setting. Furthermore, Ye [7] introduced the Hamming and Euclidean distances between INSs and the distances-based similarity measures and applied them to multiple attribute decision-making problems with interval neutrosophic information. Then, Chi and Liu [8] extended a TOPSIS method to interval neutrosophic multiple attribute decision-making problems to rank alternatives. On the other hand, Ye [9] put forward a concept of a simplified neutrosophic set (SNS), which is also a subclass of the neutrosophic set and includes a SVNS and an

INS, and defined basic operational laws of SNSs, and then he developed a simplified neutrosophic weighted arithmetic averaging (SNWAA) operator, a simplified neutrosophic weighted geometric averaging (SNWGA) operator and applied the SNWAA and SNWGA operators to multiple attribute decision-making under simplified neutrosophic environment. Zhang et al. [10] further presented the operations for INSs and defined a comparison approach based on the related research of interval valued intuitionistic fuzzy sets, and then they have introduced two interval neutrosophic number aggregation operators and applied them to multicriteria decision-making problems with interval neutrosophic information. However, current aggregation operators for single valued neutrosophic numbers (SVNNs) and interval neutrosophic numbers (INNs) tend to ignore the knowledge background of the decision maker and his corresponding credibility on every evaluation value of SVNNs/INNs. In order to consider the decision maker's familiarity in professional fields represented by the credibility of the evaluation value, the purposes of this paper, motivated by the credibility-induced hesitant fuzzy aggregation operators [11], are: (1) to proposes a credibility-induced interval neutrosophic weighted arithmetic averaging (CIINWAA) operator and a credibility-induced interval neutrosophic weighted geometric averaging (CIINWGA) operator, taking the importance of attribute weights and the credibility of the evaluation values of attributes into account, (2) to introduce a ranking method based on the Dice similarity measure between INNs for ranking INNs, (3) to apply them to the multiple attribute decision-making problems with interval neutrosophic information and credibility information.

The rest of the paper is organized as follows. Section 2 briefly describes some concepts and operations of INSs. Section 3 proposes two interval neutrosophic weighted aggregation operators considering credibility: a credibility-induced interval neutrosophic weighted arithmetic averaging

(CIINWAA) operator and a credibility-induced interval neutrosophic weighted geometric averaging (CIINWGA) operator, and investigates their properties. In Section 4, we introduce a ranking method based on the Dice similarity measure to rank INNs. Section 5 establishes a multiple attribute decision-making method based on the CIINWAA and CIINWGA operators and the ranking method under interval neutrosophic environment. In Section 6, an illustrative example is presented to demonstrate the application of the proposed method. Section 7 gives related comparative analysis. Finally, Section 8 contains a conclusion and future work.

2. Some concepts and operations of INSs

To express indeterminate information and inconsistent information, Smarandache [1] firstly gave the definition of a neutrosophic set from philosophical point of view, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It is a generalization of fuzzy sets, intuitionistic fuzzy sets, and interval-valued intuitionistic fuzzy sets. However, the neutrosophic theory is difficult to be directly applied in real scientific and engineering areas. To easily use it in science and engineering areas, Wang et al. [2] proposed the concept of INS, which is an instance of a neutrosophic set, and introduced the definition of an INS.

Definition 1 [2]. Let X be a space of points (objects) with generic elements in X denoted by x. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, there are $T_A(x)$ = [inf $T_A(x)$, sup $T_A(x)$] $\subseteq$ [0, 1], $I_A(x)$ = [inf $I_A(x)$, sup $I_A(x)$] $\subseteq$ [0, 1], and $F_A(x)$ = [inf $F_A(x)$, sup $F_A(x)$] $\subseteq$ [0, 1], and the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x)$

≤ 3. Then, an INS A can be expressed as

$$\begin{aligned} A &= \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \\ &= \{\langle x, [\inf T_A(x), \sup T_A(x)], [\inf I_A(x), \sup I_A(x)], [\inf F_A(x), \sup F_A(x)]\rangle \mid x \in X\}. \end{aligned}$$

When the upper and lower ends of the interval values of $T_A(x)$, $I_A(x)$ and $F_A(x)$ in an INS A are equal, the INS A reduces to the SVNS A. However, both SVNSs and INSs are the subclasses of neutrosophic sets.

Whereas, some expressions for INSs A and B are defined as follows [2]:

(1) The complement A^c for an INS A is denoted as $T_A{}^c(x) = F_A(x) = [\inf F_A(x), \sup F_A(x)]$, $I_A{}^c(x) = [1 - \sup I_A(x), 1 - \inf I_A(x)]$, and $F_A{}^c(x) = T_A(x) = [\inf T_A(x), \sup T_A(x)]$ for any x in X.

(2) $A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for any x in X.

(3) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

For convenience, we can use $a = \langle [T^L, T^U], [I^L, I^U], [F^L, F^U]\rangle$ to represent an element in an INS and call it an interval neutrosophic number (INN).

Definition 2 [9, 10]. Let $a_1 = \langle [T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U]\rangle$ and $a_2 = \langle [T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U]\rangle$ be two INNs and $\lambda \geq 0$, then the operational rules of INNs are defined as follows:

(1) $a_1 + a_2 = \langle [T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U]\rangle$,

(2) $a_1 \times a_2 = \langle [T_1^L T_2^L, T_1^U T_2^U], [I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]\rangle$,

(3) $\lambda a_1 = \langle [1 - (1 - T_1^L)^\lambda, 1 - (1 - T_1^U)^\lambda], [(I_i^L)^\lambda, (I_i^U)^\lambda], [(F_i^L)^\lambda, (F_i^U)^\lambda]\rangle$,

(4) $a_1^\lambda = \langle [(T_i^L)^\lambda, (T_i^U)^\lambda], [1 - (1 - I_1^L)^\lambda, 1 - (1 - I_1^U)^\lambda], [1 - (1 - F_1^L)^\lambda, 1 - (1 - F_1^U)^\lambda]\rangle$.

Assume that a_j (j = 1, 2, …, n) is a collection of INNs and w_j is the weight of a_j (j =1, 2, …, n), with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. To aggregate interval neutrosophic information in decision making, Zhang et al. [10] introduced the interval neutrosophic weighted arithmetic averaging

(INWAA) operator and the interval neutrosophic weighted geometric averaging (INWGA) operator, respectively, as follows:

$$INWAA(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j a_j = \left\langle \left[1 - \prod_{j=1}^{n} (1 - T_j^L)^{w_j}, 1 - \prod_{j=1}^{n} (1 - T_j^U)^{w_j} \right], \left[\prod_{j=1}^{n} \left(I_j^L\right)^{w_j}, \prod_{j=1}^{n} \left(I_j^U\right)^{w_j} \right], \left[\prod_{j=1}^{n} \left(F_j^L\right)^{w_j}, \prod_{j=1}^{n} \left(F_j^U\right)^{w_j} \right] \right\rangle \tag{1}$$

$$INWGA(a_1, a_{2,} \cdots, a_n) = \prod_{j=1}^{n} a_j^{w_j} = \left\langle \left[\prod_{j=1}^{n} \left(T_j^L\right)^{w_j}, \prod_{j=1}^{n} \left(T_j^U\right)^{w_j} \right], \left[1 - \prod_{j=1}^{n} (1 - I_j^L)^{w_j}, 1 - \prod_{j=1}^{n} (1 - I_j^U)^{w_j} \right], \left[1 - \prod_{j=1}^{n} (1 - F_j^L)^{w_j}, 1 - \prod_{j=1}^{n} (1 - F_j^U)^{w_j} \right] \right\rangle \tag{2}$$

However, the two weighted aggregation operators for decision making only indicate the weight importance of each INN a_i without considering the credibility of each INN a_i (the evaluation value of an attribute in decision making). In order to consider the decision maker's familiarity in professional fields represented by the credibility of the evaluation value, the following section will propose credibility-induced weighted aggregation operators of INNs for decision making.

3. **Credibility-induced weighted aggregation operators of INNs**

Information aggregation is a necessary tool in multiple attribute decision-making process since the evaluation values of each alternative on attributes need to be aggregated. However, when aggregating the interval neutrosophic information, seldom has a person considered the decision maker's familiarity in the professional field. Existing aggregation operators usually ignore the knowledge background of the decision maker in the aggregation process. In order to consider the decision maker's familiarity in professional fields, motivated by the credibility-induced hesitant fuzzy aggregation operators [11], this section proposes two interval neutrosophic weighted aggregation operators considering credibility, including a CIINWAA operator and a CIINWGA operator.

4.1 Credibility-induced interval neutrosophic weighted arithmetic averaging operator

Considering the synthetic weight of both attribute weights and credibility, we can propose the following CIINWAA operator, which is usually utilized in decision making.

Definition 3. Let a_j (j = 1, 2, …, n) be a collection of INNs and CIINWAA be: $\Omega_n \rightarrow \Omega$. Then, the CIINWAA operator is defined by

$$CIINWAA(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} v_j a_j , \tag{3}$$

where v_j is the synthetic weight $v_j = w_j \times c_j \Big/ \sum_{j=1}^{n} (w_j \times c_j)$, w_j and c_j are the weight and credibility of a_j (j =1, 2, …, n) , with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, $c_j \in [0,1]$, $v_j \in [0,1]$, and $\sum_{j=1}^{n} v_j = 1$.

Theorem 1. Let a_j (j = 1, 2, …, n) be a collection of INNs. Then by Eq. (3) and the operational rules in Definition 2, we have the following result:

$$\begin{aligned} CIINWAA(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} v_j a_j = \Bigg\langle & \left[1 - \prod_{j=1}^{n} (1 - T_j^L)^{v_j}, 1 - \prod_{j=1}^{n} (1 - T_j^U)^{v_j} \right], \\ & \left[\prod_{j=1}^{n} \left(I_j^L\right)^{v_j}, \prod_{j=1}^{n} \left(I_j^U\right)^{v_j} \right], \left[\prod_{j=1}^{n} \left(F_j^L\right)^{v_j}, \prod_{j=1}^{n} \left(F_j^U\right)^{v_j} \right] \Bigg\rangle \end{aligned}, \tag{4}$$

where v_j is the synthetic weight $v_j = w_j \times c_j \Big/ \sum_{j=1}^{n} (w_j \times c_j)$, w_j and c_j are the weight and credibility of a_j (j =1, 2, …, n) , with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, $c_j \in [0,1]$, $v_j \in [0,1]$, and $\sum_{j=1}^{n} v_j = 1$.

Proof. The proof of Eq. (4) can be given by means of mathematical induction.

(1) When n = 2, then,

$$w_1 a_1 = \left\langle \left[1 - \left(1 - T_1^L\right)^{v_1}, 1 - \left(1 - T_1^U\right)^{v_1} \right], \left[\left(I_1^L\right)^{v_1}, \left(I_1^U\right)^{v_1} \right], \left[\left(F_1^L\right)^{v_1}, \left(F_1^U\right)^{v_1} \right] \right\rangle,$$

$$w_2 a_2 = \left\langle \left[1 - \left(1 - T_2^L\right)^{v_2}, 1 - \left(1 - T_2^U\right)^{v_2} \right], \left[\left(I_2^L\right)^{v_2}, \left(I_2^U\right)^{v_2} \right], \left[\left(F_2^L\right)^{v_2}, \left(F_2^U\right)^{v_2} \right] \right\rangle.$$

Thus,

$$\begin{aligned}
CIINWAA(a_1,a_2) &= w_1a_1 + w_2a_2 \\
&= \Big\langle [1-(1-T_1^L)^{v_1} + 1-(1-T_2^L)^{v_2} - (1-(1-T_1^L)^{v_1})(1-(1-T_2^L)^{v_2}), \\
&\quad 1-(1-T_1^U)^{v_1} + 1-(1-T_2^U)^{v_2} - (1-(1-T_1^U)^{v_1})(1-(1-T_2^U)^{v_2})], \\
&\quad [(I_1^L)^{v_1}(I_2^L)^{v_2},(I_1^U)^{v_1}(I_2^U)^{v_2}],[(F_1^L)^{v_1}(F_2^L)^{v_2},(F_1^U)^{v_1}(F_2^U)^{v_2}]\Big\rangle \\
&= \Big\langle [1-(1-T_1^L)^{v_1}(1-T_2^L)^{v_2}, 1-(1-T_1^U)^{v_1}(1-T_2^U)^{v_2}], \\
&\quad [\prod_{j=1}^{2}(I_j^L)^{v_j},\prod_{j=1}^{2}(I_j^U)^{v_j}],[\prod_{j=1}^{2}(F_j^L)^{v_j},\prod_{j=1}^{2}(F_j^U)^{v_j}]\Big\rangle
\end{aligned} \tag{5}$$

(2) When $n = k$, by applying Eq. (4), we get

$$\begin{aligned}
CIINWAA(a_1,a_2,\ldots,a_k) = \Bigg\langle &\left[1-\prod_{j=1}^{k}(1-T_j^L)^{v_j}, 1-\prod_{j=1}^{k}(1-T_j^U)^{v_j}\right], \\
&\left[\prod_{j=1}^{k}\left(I_j^L\right)^{v_j},\prod_{j=1}^{k}\left(I_j^U\right)^{v_j}\right],\left[\prod_{j=1}^{k}\left(F_j^L\right)^{v_j},\prod_{j=1}^{k}\left(F_j^U\right)^{v_j}\right]\Bigg\rangle.
\end{aligned} \tag{6}$$

(3) When $n = k + 1$, by applying Eqs. (5) and (6), we can get

$$\begin{aligned}
CIINWAA(a_1,a_2,\ldots,a_{k+1}) = \Bigg\langle &\Bigg[1-\prod_{j=1}^{k}(1-T_j^L)^{v_j} + 1-(1-T_{k+1}^L)^{v_{k+1}} \\
&-(1-\prod_{j=1}^{k}(1-T_j^L)^{v_j})(1-(1-T_{k+1}^L)^{v_{k+1}}, \\
&1-\prod_{j=1}^{k}(1-T_j^U)^{v_j} + 1-(1-T_{k+1}^U)^{v_{k+1}} \\
&-(1-\prod_{j=1}^{k}(1-T_j^U)^{v_j})(1-(1-T_{k+1}^U)^{v_{k+1}}\Bigg], \\
&\left[\prod_{j=1}^{k+1}\left(I_j^L\right)^{v_j},\prod_{j=1}^{k+1}\left(I_j^U\right)^{v_j}\right],\left[\prod_{j=1}^{k+1}\left(F_j^L\right)^{v_j},\prod_{j=1}^{k+1}\left(F_j^U\right)^{v_j}\right]\Bigg\rangle \\
= \Bigg\langle &\left[1-\prod_{j=1}^{k+1}(1-T_j^L)^{v_j}, 1-\prod_{j=1}^{k+1}(1-T_j^U)^{v_j}\right], \\
&\left[\prod_{j=1}^{k+1}\left(I_j^L\right)^{v_j},\prod_{j=1}^{k+1}\left(I_j^U\right)^{v_j}\right],\left[\prod_{j=1}^{k+1}\left(F_j^L\right)^{v_j},\prod_{j=1}^{k+1}\left(F_j^U\right)^{v_j}\right]\Bigg\rangle .
\end{aligned}$$

Therefore, considering the above results, we have Eq. (4) for any n. This completes the proof. □

Especially, if $c_j = 1$ for $j = 1, 2, \ldots, n$, then the CIINWAA operator degenerates to an interval neutrosophic weighted arithmetic averaging operator, i.e. Eq.(1); if $w_j = 1/n$ and $c_j = 1$ for $j = 1, 2, \ldots, n$ are satisfied at the same time, then the CIINWAA operator degenerates to an interval neutrosophic arithmetic averaging operator.

It is obvious that the CIINWAA operator has the following properties:

(1) Idempotency: Let a_j ($j = 1, 2, \ldots, n$) be a collection of INNs. If a_i ($j = 1, 2, \ldots, n$) is equal, i.e., $a_j = a$ for $j = 1, 2, \ldots, n$, then $CIINWAA(a_1, a_2, \cdots, a_n) = a$.

(2) Boundedness: Let a_j ($j = 1, 2, \ldots, n$) be a collection of INNs, $a_{\min} = \min(a_1, a_2, ..., a_n)$, and $a_{\max} = \max(a_1, a_2, ..., a_n)$ for $j = 1, 2, \ldots, n$, then $a_{\min} \le CIINWAA(a_1, a_2, \cdots, a_n) \le a_{\max}$.

(3) Monotonity: Let a_j ($j = 1, 2, \ldots, n$) be a collection of INNs. If $a_j \le a_j^*$ for $j = 1, 2, \ldots, n$, then $CIINWAA(a_1, a_2, \cdots, a_n) \le CIINWAA(a_1^*, a_2^*, \cdots, a_n^*)$.

Proof. (1) Since $a_j = a$ for $j = 1, 2, \ldots, n$, one obtains

$$\begin{aligned} CIINWAA(a_1, a_2, \cdots, a_n) &= \left\langle \left[1-\prod_{j=1}^{n}(1-T_j^L)^{v_j}, 1-\prod_{j=1}^{n}(1-T_j^U)^{v_j}\right], \right. \\ &\quad \left. \left[\prod_{j=1}^{n}\left(I_j^L\right)^{v_j}, \prod_{j=1}^{n}\left(I_j^U\right)^{v_j}\right], \left[\prod_{j=1}^{n}\left(F_j^L\right)^{v_j}, \prod_{j=1}^{n}\left(F_j^U\right)^{v_j}\right] \right\rangle . \\ &= \left\langle \left[1-(1-T^L)^{\sum_{j=1}^{n}v_j}, 1-(1-T^U)^{\sum_{j=1}^{n}v_j}\right], \left[\left(I^L\right)^{\sum_{j=1}^{n}v_j}, \left(I^U\right)^{\sum_{j=1}^{n}v_j}\right], \left[\left(F^L\right)^{\sum_{j=1}^{n}v_j}, \left(F^U\right)^{\sum_{j=1}^{n}v_j}\right] \right\rangle \\ &= \langle [T^L, T^U], [I^L, I^U], [F^L, F^U] \rangle = a \end{aligned}$$

(2) Since $a_{\min} = \min(a_1, a_2, ..., a_n)$ and $a_{\max} = \max(a_1, a_2, ..., a_n)$ for $j = 1, 2, \ldots, n$, there is $a_{\min} \le a_j \le a_{\max}$. Thus, there exists $\sum_{j=1}^{n} v_j a_{\min} \le \sum_{j=1}^{n} v_j a_j \le \sum_{j=1}^{n} v_j a_{\max}$, and then there is $a_{\min} \le \sum_{j=1}^{n} v_j a_j \le a_{\max}$. Hence $a_{\min} \le CIINWAA(a_1, a_2, \cdots, a_n) \le a_{\max}$.

(3) Since $a_j \le a_j^*$ for $j = 1, 2, \ldots, n$, there is $\sum_{j=1}^{n} v_j a_j \le \sum_{j=1}^{n} v_j a_j^*$, and then $CIINWAA(a_1, a_2, \cdots, a_n) \le CIINWAA(a_1^*, a_2^*, \cdots, a_n^*)$.

Therefore, the proofs of these properties are completed. □

4.2 Credibility-induced interval neutrosophic weighted geometric averaging operator

Considering the synthetic weight of both attribute weights and credibility, we can propose the following CIINWGA operator, which is usually utilized in decision making.

Definition 4. Let a_j (j = 1, 2, …, n) be a collection of INNs and CIINWGA be: $\Omega_n \to \Omega$. Then the CIINWGA operator is defined as

$$CIINWGA(a_1, a_2, \cdots, a_n) = \prod_{j=1}^{n} a_j^{v_j} , \tag{7}$$

where v_j is the synthetic weight $v_j = w_j c_j \Big/ \sum_{j=1}^{n} (w_j c_j)$, w_j and c_j are the weight and credibility of a_j (j =1, 2, …, n) , with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, $c_j \in [0,1]$, $v_j \in [0,1]$, and $\sum_{j=1}^{n} v_j = 1$.

Theorem 2. Let a_j (j = 1, 2, …, n) be a collection of INNs. by Eq. (7) and the operational rules in Definition 2, we have the following result:

$$\begin{aligned} CIINWGA(a_1, a_{2,} \cdots, a_n) = \Bigg\langle & \left[\prod_{j=1}^{n} \left(T_j^L\right)^{v_j}, \prod_{j=1}^{n} \left(T_j^U\right)^{v_j} \right], \\ & \left[1 - \prod_{j=1}^{n} (1 - I_j^L)^{v_j}, 1 - \prod_{j=1}^{n} (1 - I_j^U)^{v_j} \right], \left[1 - \prod_{j=1}^{n} (1 - F_j^L)^{v_j}, 1 - \prod_{j=1}^{n} (1 - F_j^U)^{v_j} \right] \Bigg\rangle \end{aligned} , \tag{8}$$

where v_j is the synthetic weight $v_j = w_j c_j \Big/ \sum_{j=1}^{n} (w_j c_j)$, w_j and c_j are the weight and credibility of a_j (j =1, 2, …, n), with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, $c_j \in [0,1]$, $v_j \in [0,1]$, and $\sum_{j=1}^{n} v_j = 1$.

By the similar proof manner of Theorem 1, we can also give the proof of Theorem 2 (omitted).

Especially, if c_j = 1 for j = 1, 2, …, n, then the CIINWGA operator degenerates to an interval neutrosophic weighted geometric averaging operator, i.e. Eq.(2); while if $w_j = 1/n$ and c_j = 1 for j = 1, 2, …, n are satisfied at the same time, then the CIINWGA operator degenerates to an interval neutrosophic geometric averaging operator.

It is obvious that the CIINWGA operator has the following properties:

(1) Idempotency: Let a_j (j = 1, 2, . . . , n) be a collection of INNs. If a_i (j = 1, 2, . . . , n) is equal, i.e.,

$a_j = a$ for $j = 1, 2, \ldots, n$, then $CIINWGA(a_1, a_2, \cdots, a_n) = a$.

(2) Boundedness: Let a_j ($j = 1, 2, \ldots, n$) be a collection of INNs, $a_{\min} = \min(a_1, a_2, ..., a_n)$, and $a_{\max} = \max(a_1, a_2, ..., a_n)$ for $j = 1, 2, \ldots, n$, then there is $a_{\min} \le CIINWGA(a_1, a_2, \cdots, a_n) \le a_{\max}$.

(3) Monotonity: Let a_j ($j = 1, 2, \ldots, n$) be a collection of INNs. If $a_j \le a_j^*$ for $j = 1, 2, \ldots, n$, then there is $CIINWGA(a_1, a_2, \cdots, a_n) \le CIINWGA(a_1^*, a_2^*, \cdots, a_n^*)$.

Since the process to prove these properties is similar to the above proofs, it does not repeated here.

4. Ranking method of INNs based the Dice similarity measure

The vector similarity measure is one of important tools for the degree of similarity between objects. However, the Dice similarity measure is often used for this purpose. For this, Ye [12] proposed the Dice similarity measure between two INSs.

Definition 5 [12]. Let $a_1 = \langle [T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U] \rangle$ and $a_2 = \langle [T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U] \rangle$ be two INNs, then the Dice similarity measure between the two INNs a_1 and a_2 are defined as follows:

$$D(a_1, a_2) = \frac{2(T_1^L T_2^L + I_1^L I_2^L + F_1^L F_2^L + T_1^U T_2^U + I_1^U I_2^U + F_1^U F_2^U)}{\left(\begin{array}{l} \left[(T_2^L)^2 + (I_2^L)^2 + (F_2^L)^2 + (T_2^U)^2 + (I_2^U)^2 + (F_2^U)^2 \right] \\ + \left[(T_2^L)^2 + (I_2^L)^2 + (F_2^L)^2 + (T_2^U)^2 + (I_2^U)^2 + (F_2^U)^2 \right] \end{array} \right)}. \tag{9}$$

In order to compare two INNs, we propose a method based on the Dice similarity measure between an INN a = <[T^L, T^U], [I^L, I^U], [F^L, F^U]> and an ideal solution a^* = <[1, 1], [0,0], [0, 0]>, which is defined as

$$D(a) = \frac{2(T^L + T^U)}{2 + (T^L)^2 + (I^L)^2 + (F^L)^2 + (T^U)^2 + (I^U)^2 + (F^U)^2}. \tag{10}$$

Thus, the ranking method of INNs based on $D(a)$ can be given by the following definition.

Definition 6. Let $a_1 = \langle [T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U] \rangle$ and $a_2 = \langle [T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U] \rangle$ be two INNs. If $D(a_1) \le D(a_2)$, then $a_1 \le a_2$.

Example 1. Let us compare the following INNs:

(1) Let $a_1 = \langle [0.5, 0.6], [0.3, 0.4], [0.3, 0.4] \rangle$ and $a_2 = \langle [0.4, 0.5], [0.3, 0.4], [0.5, 0.6] \rangle$ be two INNs.

Then, by Eq.(10) we obtain $D(a_1) = 0.7074 > D(a_2) = 0.5505$. Hence $a_1 > a_2$.

(2) Let $a_1 = \langle [0.5, 0.6], [0.5, 0.6], [0.6, 0.7] \rangle$ and $a_2 = \langle [0.5, 0.7], [0.3, 0.4], [0.3, 0.5] \rangle$ be two INNs.

Then, by Eq.(10) we get $D(a_1) = 0.5405 < D(a_2) = 0.7207$. Thus $a_1 < a_2$.

5. Decision making method considering credibility

In a multiple attribute decision-making problem with interval neutrosophic information, there is usually a set of m alternatives $A = \{A_1, A_2, \ldots, A_m\}$, which are to be evaluated based on a set of n attributes $G = \{G_1, G_2, \ldots, G_n\}$. Assume that the weight of the attribute G_j ($j = 1, 2, \ldots, n$), entered by the decision-maker, is w_j, $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. The characteristic of an alternative A_i ($i = 1, 2, \ldots, m$) on an attribute G_j ($j = 1, 2, \ldots, n$) can be represented by the form of an INN $d_{ij} = \langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \rangle$ ($i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$) with its credibility c_j ($j = 1, 2, \ldots, n$), which is usually derived from the decision maker's evaluation considering the credibility. Thus, one can obtain an interval neutrosophic decision matrix $D = (d_{ij})_{m \times n}$ and the credibility given as $\boldsymbol{c} = (c_1, c_2, \ldots, c_n)^{\mathrm{T}}$ in the evaluation of the attributes on the alternatives.

The steps of the decision-making problem based on these conditions are shown as follows:

Step 1: By applying Eq. (4) or Eq. (8), the individual overall INN d_i for A_i ($i = 1, 2, \ldots, m$) is calculated by

$$d_i = \left\langle [T_i^L, T_i^U], [I_i^L, I_i^U], [F_i^L, F_i^U] \right\rangle = CIINWAA(d_{i1}, d_{i2}, \ldots, d_{in})$$
$$= \left\langle \left[1 - \prod_{j=1}^{n}(1 - T_{ij}^L)^{v_j}, 1 - \prod_{j=1}^{n}(1 - T_{ij}^U)^{v_j}\right], \left[\prod_{j=1}^{n}\left(I_{ij}^L\right)^{v_j}, \prod_{j=1}^{n}\left(I_{ij}^U\right)^{v_j}\right], \left[\prod_{j=1}^{n}\left(F_{ij}^L\right)^{v_j}, \prod_{j=1}^{n}\left(F_{ij}^U\right)^{v_j}\right] \right\rangle, \quad (11)$$

or

$$d_i = \left\langle [T_i^L, T_i^U], [I_i^L, I_i^U], [F_i^L, F_i^U] \right\rangle = CIINWGA(d_{i1}, d_{i2}, \ldots, d_{in})$$
$$= \left\langle \left[\prod_{j=1}^{n}\left(T_{ij}^L\right)^{v_j}, \prod_{j=1}^{n}\left(T_{ij}^U\right)^{v_j}\right], \left[1 - \prod_{j=1}^{n}(1 - I_{ij}^L)^{v_j}, 1 - \prod_{j=1}^{n}(1 - I_{ij}^U)^{v_j}\right], \left[1 - \prod_{j=1}^{n}(1 - F_{ij}^L)^{v_j}, 1 - \prod_{j=1}^{n}(1 - F_{ij}^U)^{v_j}\right] \right\rangle. \quad (12)$$

Step 2: Calculate the Dice measure $D(d_i)$ (i = 1, 2, …, m) by using Eq. (10).

Step 3: Rank the alternatives by the measure values of $D(d_i)$ (i = 1, 2, …, m) and obtain the best one(s).

Step 4: End.

6. Illustrative example and comparative analysis

An illustrative example about investment alternatives for a multiple attribute decision-making problem adapted from [7, 8, 10] is used to demonstrate the applications of the proposed decision-making method under interval neutrosophic environment. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the three attributes: (1) G_1 is the risk; (2) G_2 is the growth; (3) G_3 is the environmental impact. The weight vector of the three attributes is given by $\boldsymbol{w}$ = (0.35, 0.25, 0.4)^T [7, 8].

When the four possible alternatives are to be evaluated by the expert under the above three attributes in the form of INNs considering the credibility, one can obtain the following interval neutrosophic decision matrix D [7, 8]:

$$D = \begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.4,0.5],[0.7,0.8],[0.7,0.9]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.8,0.9],[0.5,0.7],[0.3,0.6]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.7,0.9],[0.6,0.8],[0.4,0.5]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.8,0.9],[0.6,0.7],[0.6,0.7]\rangle \end{bmatrix}.$$

At the same time, the credibility is given as $\boldsymbol{c} = (0.8, 0.9, 0.7)^{\mathrm{T}}$ in the evaluation values of the three attributes on the four alternatives.

Then, the developed interval neutrosophic decision-making approach is utilized to obtain the most desirable alternative(s), which is described by the following computational steps:

Step 1: By applying Eq. (11), we can obtain the individual overall INN d_i for A_i (i =1, 2, 3, 4):

$d_1 = \langle[0.4000, 0.5310], [0.2563, 0.4257], [0.3613, 0.5342]\rangle$,

$d_2 = \langle[0.6876, 0.7973], [0.1775, 0.3127], [0.2311, 0.3841]\rangle$,

$d_3 = \langle[0.5301, 0.7560], [0.2959, 0.4257], [0.3324, 0.4331]\rangle$,

$d_4 = \langle[0.7181, 0.8246], [0.0000, 0.2442], [0.1895, 0.3512]\rangle$.

Step 2: By applying Eq. (10), we can obtain $D(d_i)$ (i =1, 2, 3, 4) as follows:

$D(d_1) = 0.5997$, $D(d_2) = 0.8636$, $D(d_3) = 0.7522$, and $D(d_4) = 0.9036$.

Step 3: Since $D(d_4) > D(d_2) > D(d_3) > D(d_1)$, the ranking order of four alternatives is $A_4 > A_2 > A_3 > A_1$. Therefore, we can see that the alternative A_4 is the best choice among all the alternatives.

On the other hand, we can also utilize the CIINWGA operator as the following computational steps:

Step 1': By applying Eq. (12), we compute the individual overall INLN d_i for A_i (i =1, 2, 3, 4):

$d_1 = \langle[0.4000, 0.5268], [0.4168, 0.5523], [0.4624, 0.6833]\rangle$,

$d_2 = \langle[0.6648, 0.7656], [0.2702, 0.4362], [0.2372, 0.4267]\rangle$,

$d_3 = \langle[0.4699, 0.6934], [0.3752, 0.5523], [0.3374, 0.4378]\rangle$,

$d_4 = \langle[0.7024, 0.8030], [0.3002, 0.4120], [0.3261, 0.4573]\rangle$.

Step 2': By using Eq. (10), we can get $D(d_i)$ (i =1, 2, 3, 4) as follows:

$D(d_1) = 0.5153$, $D(d_2) = 0.8105$, $D(d_3) = 0.6738$, and $D(d_4) = 0.8108$.

Step 3': Since $D(d_4) > D(d_2) > D(d_3) > D(d_1)$, the ranking order of four alternatives is $A_4 > A_2 > A_3 > A_1$. Thus, we can see that the alternative A_4 is still the best choice among all the alternatives.

Obviously, we can see that the above two kinds of ranking orders and the best alternative are the same as the results of [7, 8].

7. **Related comparative analysis**

The method proposed in this paper differs from existing approaches for multiple attribute decision-making problems with interval neutrosophic information due to the fact that the proposed method not only uses the CIINWAA and CIINWGA operators considering the credibility represented by decision maker's judgment to an evaluated object and the subjective evaluation value, but also utilizes the ranking method of the Dice similarity measure. Thus, the proposed method makes it have more feasible and practical than the existing neutrosophic decision making methods in real decision-making problems. Therefore, its advantage is easily reflecting the decision maker's familiarity in professional fields represented by the credibility of subjective judgments in the multiple attribute decision-making.

To demonstrate the effectiveness of the proposed method in this paper, we can compare it with the methods proposed by Ye [7], Chi and Liu [8], and Zhang et al [10]. Firstly, the same ranking results are produced by these methods [7, 8]. Secondly, the decision-making method proposed by Ye [7] is based on a similarity measure, and the decision-making method proposed by Chi and Liu [8] is based on an extended TOPSIS method. Therefore, they cannot realize the information aggregation for INNs. Last, the decision-making method proposed in [10] uses the interval neutrosophic weighted aggregation operators without considering the credibility of the evaluation values, while the method proposed in this paper utilizes the credibility-induced weighted aggregation operators of INNs, and it can provide the more general and more credible features as it is assigned the credibility of different evaluation values given by the decision maker in decision making. Therefore, our method is more reasonable than the one in [10].

Furthermore, the literature [11] presented a hesitant fuzzy aggregation operator based on credibility, but it took the absolute credibility into account. This paper puts forward an interval neutrosophic aggregation operator considering the synthetic weight of both attribute weights and credibility. However, the former cannot handle indeterminate information and inconsistent information, while the later can deal with it.

8. Conclusion

Current aggregation operators for SVNNs and INNs tend to ignore the knowledge background of the decision maker and his corresponding credibility on every evaluation value of SVNNs/INNs. In order to consider the decision maker's familiarity in professional fields represented by the credibility of the evaluation value, this paper proposed the CIINWAA and CIINWGA operators and investigated their properties. Then, the ranking method of INNs was introduced based the Dice similarity measure. Furthermore, a decision-making method based on the CIINWAA and CIINWGA operators and the ranking method of the Dice similarity measure was established to handle decision-making problems with interval neutrosophic information and credibility information. Finally, an illustrative example was given to demonstrate the application of the proposed method. Its advantage is easily reflecting the decision maker's familiarity in professional fields represented by the credibility of each evaluation value in the multiple attribute decision-making. The developed method would be more suitable to handle indeterminate information and inconsistent information in complex decision-making problems with interval neutrosophic information and credibility information. In further work, it is necessary and meaningful to develop new aggregation operators of INNs and to investigate their applications such as group decision making, pattern recognition, and medical diagnosis.

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