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# INTUITIONISTIC FUZZY MULTICRITERIA GROUP DECISION-MAKING APPROACH TO QUALITY CLAY-BRICK SELECTION PROBLEMS BASED ON GREY RELATIONAL ANALYSIS

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## ABSTRACT

*This paper presents quality Clay-Brick selection process based on intuitionistic fuzzy multi criteria group decision making through grey relational analysis. Brick plays an important role in construction field. Intuitionistic fuzzy weighted averaging operator is used to aggregate individual opinions of decision makers into a group opinion. Six criteria are considered for selection process and the criteria are obtained from expert opinions. The criteria are namely solidity, color, size and shape, strength of Brick, cost of Bricks and carrying cost. Weights of the criteria are obtained from domain experts by using a questionnaire. The rating of an alternative with respect to certain criterion offered by decision maker is represented by linguistic variable that can be expressed by intuitionistic fuzzy sets. An intuitionistic fuzzy set, which is characterized by membership function (degree of acceptance), non-membership function (degree of rejection) and the degree of indeterminacy or the degree of hesitancy, is a more general and suitable way to deal with imprecise information. Grey relational analysis is used for ranking and selection of alternatives. An illustrative numerical example for quality Brick selection is solved to show the effectiveness of the proposed model.*

**Keywords:** *Bricks, Bricks field, Fuzzy set, Grey relational analysis, Grey relational grade, Grey system theory, Group decision making, Intuitionistic fuzzy set, Linguistic variables, Multi-criteria decision making.*

## INTRODUCTION

Multi criteria decision-making (MCDM) problem generally consists of finding the most satisfactory alternative from all the feasible alternatives. Classical MCDM [20] deals with crisp numbers i.e. the ratings and the weights of criteria are measured by crisp numbers. Fuzzy MCDM [19] deals with fuzzy or intuitionistic fuzzy numbers i.e. the ratings and the weights are expressed by linguistic variables. In 1965, Zadeh [31] published his seminal paper studying with fuzzy sets. In 1986, Atanassov [2] extended the concept of fuzzy sets to intuitionistic fuzzy sets (IFSs).

Brick selection process from various Brick fields is a special case of material selection to construct a building structure. In traditional way, we select Bricks roughly based on its color, size and total cost of Brick, without considering other attributes of Brick. In that case the building construction may have some problems regarding low rigidity, longevity, etc., which be dangerous. So it is necessary to formulate a scientific based selection method. In order to select the most suitable Brick to construct a building, the following criteria of Bricks will have to consider. The criteria are namely, solidity, color, size and shape, strength of Brick, cost of Brick and carrying cost.

Liang and Wang [21] presented a fuzzy MCDM algorithm for personnel selection. Karsak (18) developed a fuzzy MCDM approach based on ideal and anti-ideal solutions for the selection of the most suitable candidate. Gibney and Shang [14] and Günör et al. [15] presented the use of the analytical hierarchy process (AHP) in the personnel selection process, respectively. Dağdeviren (9) proposed a hybrid model, which employs analytical network process (ANP) and modified technique for order preference by similarity to ideal solution (TOPSIS) for supporting the personnel selection process in the manufacturing systems. Dursun and Karsak [12] presented a fuzzy MCDM approach by using TOPSIS with 2-tuples for personnel selection. Robertson and Smith [24] presented good reviews on personnel selection studies. They investigated the role of job analysis, contemporary models of work performance, and set of criteria used in personnel selection process. A comprehensive survey of the state of the art in MCDM can be found in the book authored by Ehrgott and Gandibleux [13].

In this study, we present an intuitionistic fuzzy multi criteria group decision-making model with grey relational analysis for quality Brick selection for constructional field. A good quality Brick should be regular in shape and size, with smooth even sides and no cracks or defects. Normally poor quality Bricks are a result of using poor techniques when making the Bricks but these errors can often be easily corrected. Poor quality Bricks, must have not enough strength to carry the weight of the roof. If Bricks have been well- made and well-fired, you will hear a metallic sound or ring when they are knocked together. If they make a dull sound, it could mean that they are either cracked or under fired [26, 27, 28]. Rest of the paper is constructed in the following manner. Section 2 presents preliminaries of fuzzy sets. Section 3 describes the conversion of linguistic variables into intuitionistic fuzzy numbers. Section 4 presents the definition of operation terms. Section 5 explains grey relational analysis. Section 6 is devoted to present intuitionistic fuzzy multi-criteria group decision making based on grey relational analysis for Brick selection process. Section 7 illustrates example for Brick selection process. Sensitivity analysis of weight structure and ranking of Bricks are presented in Section 8. Section 9 describes the advantage of the proposed approach. Section 10 presents conclusion and future direction of research work.

## 2. PRELIMINARIES OF FUZZY SETS

In 1965, Zadeh [31] first introduced the concept of fuzzy sets as a mathematical form for representing impreciseness.

**Definition 2.1: Fuzzy set:** A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pair $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here, $\mu_{\tilde{A}}(x) : x \to [0,1]$ is a mapping called the membership value of x∈X in a fuzzy set $\tilde{A}$.

**Definition 2.2: Intuitionistic Fuzzy set:** An intuitionistic fuzzy set (IFS) $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}} \rangle : x \in X\}$ , where the functions $\mu_{\tilde{A}}(x)$: $X \to [0,1]$ and $v_{\tilde{A}}(x)$:$X \to [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in X$ to the set $\tilde{A}$ that is a subset of X, and every $x \in X$, $0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}^{i}(x) \le 1$.

**Definition 2.3:** The value of $\pi_{\tilde{A}}^{i}$ (x) = 1-$(\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x))$ is called the degree of non-determinacy (or uncertainty or hesitancy) of the element $x \in X$ to the intuitionistic fuzzy set.

**Definition 2.4:** Hamming distance is defined as $H\left(\tilde{A}, \tilde{B}\right) = \delta$

$$\frac{1}{2}\sum_{x \in E}\left(\left|\mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)\right| + \left|v_{\tilde{A}}(x) - v_{\tilde{B}}\right| + \left|\pi_{\tilde{A}}(x) - \pi_{\tilde{B}}\right|\right)$$

## 3. CONVERSION BETWEEN LINGUISTIC VARIABLES AND INTUITIONISTIC FUZZY NUMBERS (IFNs)

The description of linguistic variable is more realistic when we discuss a problem in intuitionistic fuzzy environment. For example, the ratings of alternative with respect to qualitative criteria could be expressed using linguistic variables such as very bad, bad, good, fair, very good etc. Linguistic variable can be converted into IFNs.

## 4. OPERATIONAL DEFINITIONS OF THE CRITERIA OF BRICK

i) **Solid clay Brick**: Loam soil is used to prepare rigid Brick. An ideal extended solid rigid body whose size and shape are definitely fixed and remain unaltered when forces are applied. The distance between any two given points of a rigid body remains constant in time when external forces applied on it. Basically solid Brick implies more and more rigidity of Brick. If we soap a Brick in water and fall downwards from 3 or 4 feet heights [3, 27], it remains unbroken.

ii) **Color:** Quality Brick has a color reddish or light maroon.

iii) **Size and shape**: All Bricks will be more or less same size and shape. A Brick has a length, width and height. The size or dimensions of a Brick are determined by how it is used in construction. Standard size of a Brick is about $190mm \times 90mm \times 40mm$ .

**Width:**

The width of a Brick should be small enough to allow a Bricklayer to lift the Brick with one hand and place it on a bed of mortar. If the Brick was wider, the Bricklayer would have to put down the trowel while building the wall to pick up the Brick with two hands and as a result, time would be wasted. In addition, a wider Brick would weigh more and therefore tire the mason more quickly. For the average Brick, the width should not be more than 115 mm.

**Length:**

There is a very important relationship between the length of a Brick and its width because of how we use Bricks to build a wall. The length of a Brick should be equal to twice its width plus 10 mm (for the mortar joint). A Brick with this length will be easier to build with because it will provide an even surface on both sides of the wall. For example, if an individual follows the rule of the length and width of the Brick, if breadth is set as 115 mm, then the ideal length would be 240 mm.

**Height:**

The height of a Brick, though of less importance, also has a relationship with the length of the Brick. The height of three Bricks plus two 10 mm joints should be equal to the length of a Brick. This allows a Bricklayer to lay Bricks on end (called a soldier course) and join them into the wall without having to cut the Bricks. To determine the height of a Brick, subtract 20 mm (the thickness of the two 10 mm mortar joints) from the length and divide the result by three (this represents the three Bricks).

**Possible Brick Sizes**:

Therefore, using these rules, the largest size Brick that would still permit a Bricklayer to comfortably pick it up with one hand, would be 240 mm in length, 115 mm in width and 73 mm in height. A Brick of this size would weight about 3.5 kg. Every country in the world seems to have a different size of Brick. The sizes are a result of centuries of tradition or custom but almost all use the same rules and lie within the limits mentioned. No one size is better than the other. In India the standard Brick size is 190 mm x 90 mm x 40 mm while the British standard is 215 mm x 102.5 mm x 65 mm. To choose your Brick size, first contact the local public works department to see if your country has a standard size. If not, you will have to choose according to need and desires and practical utility of Bricks. Possible Brick sizes are shown in the Table 1.

**Table1. Possible sizes of Bricks**

| Length | Width | Height |
|---|---|---|
| 240mm | 115mm | 73mm |
| 230mm | 110mm | 70mm |
| 220mm | 105mm | 67mm |
| 215mm | 102mm | 65mm |
| 210mm | 100mm | 63mm |

iv) **Well dried and burnt (Strength of Brick):** Raw Brick must be well dried in sunshine and then properly burnt. If any two Bricks from a blend are touched with some force then it occur material sound [3]. Bricks must have enough strength to carry the weight of the roof. If Bricks have been well- made and well-fired, you will hear a metallic sound or ring when they are knocked together. If they make a dull sound, it could mean that they are either cracked or under fired. A simple test for strength is to drop a Brick from a height of 1.2 meters (shoulder height). A good Brick will not break. This test should be repeated with a wet Brick (a Brick soaked in water for one week). If the soaked Brick does not break when dropped, the quality is good enough to build structures.

v) **Brick Cost:** Decision makers will try to purchase Brick at minimum cost. The Brick must be in reasonable cost as for as possible. Quality Brick with reasonable price must be more acceptable.
vi) **Carrying Cost**: There must be a reasonable distance between Brick field and construction site for maintaining reasonable carrying cost.

## 5. GREY RELATIONAL ANALYSIS

The calculation process for grey relational analysis (GRA) [10] can be presented as follows:

Suppose G is a factor set of grey relation, $G = \{G_0, G_1, \cdots, G_m\}$ where $G_0 \in G$ represents the referential sequence; $G_i \in G$

Denotes the comparative sequence $G_i$ ,i = 1, 2, ..., m. $G_0$ and $G_i$ consist of n elements and can be expressed as follows:

$G_0 = (G_0(1), G_0(2), \cdots, G_0(k), \cdots, G_0(n))$,

$G_i = (G_i(1), G_i(2), \cdots, G_i(k), \cdots, G_i(n))$, where i =1, 2, ..., m; k = 1, 2, ..., n;

$n \in N$ , and $G_0(k)$ and $G_i(k)$ are the numbers of referential sequences and comparative sequences at point k, respectively. In practical applications, the referential sequence can be an ideal objective and the comparative sequences are alternatives. The best alternative corresponds to the largest degree of grey relation. If the grey relational coefficient (GRC) of the referential sequences and comparative sequences at point k is $\tau(G_0, G_i)$ subject to the four conditions:

**1.** $0 \le \tau(G_0, G_i) \le 1$

$\tau(G_0, G_i) = 1 \Leftrightarrow G_0 = G_i$

$\tau(G_0, G_i) = 0 \Leftrightarrow G_0, G_i \in \Phi$ , where $\Phi$ is empty set.

**2**. $G_0, G_i \in G$ , then

$\tau(G_0, G_i) = \tau(G_i, G_0) \Leftrightarrow G = \{G_0, G\}$

**3.** $\tau(G_0, G_i) \overset{\text{often}}{\neq} \tau(G_i, G_0)$

**4.** If $|G_0(k) - G_i(k)|$ is large, $\tau(G_0(k), G_i(k))$ becomes smaller. The essential condition and quantitative model for grey relation are produced based on the above four prerequisites. The grey relational coefficient of the referential sequences and comparative sequences at point k can be expressed as follows:

$$\tau(G_0(k), G_i(k)) = \frac{\min_i \min_k |G_0(k) - G_i(k)| + \rho \max_i \max_k |G_0(k) - G_i(k)|}{|G_0(k) - G_i(k)| + \rho \max_i \max_k |G_0(k) - G_i(k)|} \qquad (1)$$

The symbol $\rho$ in equation (1) represents "contrast control," "environmental coefficient", or the "distinguishing coefficient". This coefficient is a free parameter. Its value, over a broad appropriate range of values, does not affect the ordering of the grey relational grade values, but a good value of the contrast control is needed for clear identification of key system factors. For the end points 0 and 1, i.e. for $\rho = 1$, the comparison environment is unaltered and for $\rho = 0$, the comparison environment disappears. In cases where data variation is large, $\rho$ usually ranges from 0.1 to 0.5 for reducing the influence of extremely large $\max_i \max_k |G_0(k) - G_i(k)|$. Generally, $\rho = 0.5$ is considered for decision-making situation.

## 6. INTUTIONISTIC FUZZY MULTI-CRITERIA GROUP DECISION-MAKING BASED ON GREY RELATIONAL ANALYSIS

We present here intuitionistic fuzzy multi-criteria group decision-making (MCGDM) using grey relational analysis. For a MCGDM problem, let $A = \{A_1, A_2, \cdots, A_m\}$ $(m \geq 2)$ be a finite set of alternatives, $D = \{D_1, D_2, \cdots, D_t\}$ $(t \geq 2)$ be a set of decision makers, $C = \{C_1, C_2, \cdots, C_n\}$ be a set of criteria. The weight information of the criteria and the decision maker are completely unknown. Let us denote $M = \{1,2,\cdots,m\}$, $T = \{1,2,\cdots,t\}$, $N = \{1,2,\cdots,n\}$. The proposed approach is based on the following steps.

**Step1**. **Formation of Intuitionistic fuzzy decision matrices**:

We assume that the rating of alternative $A_i$ $(i \in M)$ with respect to the criteria $C_j$ $(j \in N)$ offered by the t-th decision maker $(t \in T)$ is linguistic variable $x_{ij}^t$ that can be expressed by IFSs. A MCGDM problem can then be expressed by the following decision matrix:

$$G = \left(G_{ij}^t\right) = \begin{bmatrix} G_{11}^t, G_{12}^t, \cdots, G_{1n}^t \\ G_{21}^t, G_{22}^t, \cdots, G_{2n}^t \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ G_{m1}^t, G_{m2}^t, \cdots, G_{mn}^t \end{bmatrix}, \quad t \in T \tag{2}$$

Here, $\left(G_{ij}^t\right) = \left(\mu_{ij}^t, v_{ij}^t, \pi_{ij}^t\right)$.

**Step2. Determination of the weights of the decision makers**:

Case1. We assume that the decision making group consists of T decision makers. The importance of the decision makers in the selection committee may not be equal. The importance of decision makers are considered as linguistic variables expressed by intuitionistic fuzzy numbers (IFNs). Let $D_t = (\mu_t, v_t, \pi_t)$ be an intuitionistic IFN that represents the rating of the t-th decision maker. Then the weight of the t-th decision maker can be determined as:

$$\gamma_t = \frac{\left(\mu_t + \pi_t\left(\frac{\mu_t}{\mu_t + v_t}\right)\right)}{\sum_{t=1}^{T}\left(\mu_t + \pi_t\left(\frac{\mu_t}{\mu_t + v_t}\right)\right)}, \text{ where } \sum_{t=1}^{T}\gamma_t = 1 \tag{3}$$

The linguistic variables for the importance of the decision makers are provided in the Table 2.

Case 2. If the importance of all the decision makers is same namely extremely importance, the rating of the t-th decision maker can be expressed as (1, 0, 0). Then the weight of each decision maker will be 1/T.

**Step3.**

Formation of the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers: Let G= $\left(G_{ij}^{t}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix of the t-th decision maker. $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_T\right)$, be the weight set of the decision-makers and $\sum_{t=1}^{T}\gamma_t=1$, $\gamma_t\in[0,1]$. In the group decision-making process, all individual decisions needed to be fused into a group opinion to construct an aggregate intuitionistic fuzzy decision matrix. In order to do, we use intuitionistic fuzzy weighted average (IFWA) operator due to Xu [30] as follows:

$$G_{ij}=IFGA_{\gamma}\left(G_{ij}^{(1)},G_{ij}^{(2)},\cdots,G_{ij}^{(t)}\right)=\gamma_1 G_{ij}^{(1)}\oplus\gamma_2 G_{ij}^{(2)}\oplus\cdots\oplus\gamma_t G_{ij}^{(t)}$$

$$=\left(1-\prod_{t=1}^{T}\left(1-\mu_{ij}^{(t)}\right)^{\gamma_t},\prod_{t=1}^{T}\left(v_{ij}^{(t)}\right)^{\gamma_t},\prod_{t=1}^{T}\left(1-\mu_{ij}^{(t)}\right)^{\gamma_t}-\prod_{t=1}^{T}\left(v_{ij}^{(t)}\right)^{\gamma_t}\right) \quad (4)$$

The aggregate intuitionistic fuzzy decision matrix then can be written as:

$$G=\begin{bmatrix}\left(\mu_{11},v_{11},\pi_{11}\right)\left(\mu_{12},v_{12},\pi_{12}\right)\cdots\left(\mu_{1n},v_{1n},\pi_{1n}\right)\\ \left(\mu_{21},v_{21},\pi_{21}\right)\left(\mu_{22},v_{22},\pi_{22}\right)\cdots\left(\mu_{2n},v_{2n},\pi_{2n}\right)\\ \cdots\cdots\cdots\cdots\cdots\cdots\\ \cdots\cdots\cdots\cdots\cdots\cdots\\ \left(\mu_{m1},v_{m1},\pi_{m1}\right)\left(\mu_{m2},v_{m2},\pi_{m2}\right)\cdots\left(\mu_{mn},v_{mn},\pi_{mn}\right)\end{bmatrix}=\begin{bmatrix}G_{11}G_{12}\cdots G_{1n}\\ G_{21}G_{22}\cdots G_{2n}\\ \cdots\cdots\cdots\\ \cdots\cdots\cdots\\ G_{m1}G_{m2}\cdots G_{mn}\end{bmatrix} \quad (5)$$

Here $G_{ij}=\left(\mu_{ij},v_{ij},\pi_{ij}\right)$ $\left(i=1,2,\cdots,m\right)$ and $\left(j=1,2,\cdots,n\right)$ is an element of the aggregate intuitionistic fuzzy decision matrix. $\mu_{ij}=1-\prod_{t=1}^{T}\left(1-\mu_{ij}^{(t)}\right)^{\gamma_t}$, $\nu_{ij}=\prod_{t=1}^{T}\left(\nu_{ij}^{t}\right)^{\gamma^t}$, $\pi_{ij}=\prod_{t=1}^{T}\left(1-\mu_{ij}^{(t)}\right)^{\gamma_t}-\prod_{t=1}^{T}\left(v_{ij}^{(t)}\right)^{\gamma_t}$ $, i\in M, j\in N$

**Step4. Determination weights of the criteria**:

In the decision making situation, decision makers may feel that all criteria of Bricks are not equal importance. Here the importance of the criteria is obtained from expert opinion through questionnaire method i.e. the weights of the criteria are previously determined such that the sum of the weights of the criteria is equal to unity. Incidental sampling is employed to collect data. Data was collected from 10 constructional engineers, 2 earth quake specialist of Nadia district and 10 Brick fields of surrounding areas. After extended interviews and discussions with the experts, the list of criteria of Brick that are identified as playing important role in model formulation of the problem. After structured procedures of filling in specific questionnaire by our domain experts, the criteria for Brick selection are identified as solidity, color, size, strength of Brick, cost of Bricks and carrying cost. We have average weight of each criteria $w_j$ j=1, 2,… 6 as

$w_1=.275, w_2=.175, w_3=.2, w_4=.1, w_5=.05, w_6=.2$ and $\sum_{j=1}^{6} w_j = 1$. Alternately, the entropy weights of the criteria may be used. In order to obtain weight, a set of grades of importance, intuitionistic fuzzy entropy may be used due to Vlachos & Sergiadis [29] as follows:

$$E_j = -\frac{1}{n \ln 2}\sum_{i=1}^{m}\left[\mu_{ij}\ln\mu_{ij} + v_{ij}\ln v_{ij} - (1-\pi_{ij})\ln(1-\pi_{ij}) - \pi_{ij}\ln 2\right] \quad (6)$$

The entropy weight of the j-th criteria is defined as follows:

$$w_j = \frac{1-E_j}{n - \sum_{j=1}^{n} E_{ij}} \quad (7)$$

**Step5**. Determination of the reference sequence based on IFNs:

$$\tilde{G}^+ = \left((\mu_1^+, v_1^+, \pi_1^+), (\mu_2^+, v_2^+, \pi_2^+), \cdots, (\mu_n^+, v_n^+, \pi_n^+)\right) \quad (8)$$

Here $\tilde{G}_{ij}^+ = (\mu_j^+, v_j^+, \pi_j^+) = \left(\max_i \mu_{ij}, \min_i v_{ij}, \min_i \pi_{ij}\right)$, j = 1, 2,...,n $\quad (9)$

Reference sequence should be the optimal sequence of the criteria values. Since the aspired level of the membership value, non-membership value and indeterminacy value are 1, 0, 0 respectively, the point consisting of highest membership value, minimum non-membership value and minimum indeterminacy value would represent the reference value or ideal point or utopia point. In the intuitionistic fuzzy decision matrix, the maximum value $\tilde{G}_j^+ = (1,0,0)$ can be used as the reference value. Then the reference sequence is expressed as $\tilde{G}^+ = \{(1,0,0),(1,0,0)\cdots,(1,0,0)\}$ $\quad (10)$

**Step6**.

Calculation of the grey relational coefficient []of each alternative from positive ideal solution (PIS) using the following equation:

$$\tau_{ij} = \frac{\min_{1\le i\le m}\min_{1\le j\le n}\delta\left(\tilde{G}_{ij}, \tilde{G}_{ij}^+\right) + \rho\max_{1\le i\le m}\max_{1\le j\le n}\delta\left(\tilde{G}_{ij}, \tilde{G}_{ij}^+\right)}{\delta\left(\tilde{G}_{ij}, \tilde{G}_{ij}^+\right) + \rho\max_{1\le i\le m}\max_{1\le j\le n}\delta\left(\tilde{G}_{ij}, \tilde{G}_{ij}^+\right)} \quad (11)$$

$\tau_{ij}$ is the grey relational coefficient between $\tilde{G}_{ij}$ and $\tilde{G}_{ij}^+$. $\rho \in [0,1]$ is the distinguishing coefficient or the identification coefficient. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. $\rho$ is used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients.

**Step7.**

Calculation of the degree of grey relational coefficient of each alternative from PIS using the following equation:

$$\eta_i = \sum_{j=1}^{n} w_{ij}\tau_{ij}, i = 1,2,...,n. \quad (12)$$

**Step8**.

Ranking all the alternatives: We rank all the alternatives $A_i$ (i = 1, 2, ..., m) according to the decreasing order of their grey relational grades $\eta_i$ $(i = 1,2,...,m)$. Greater value of $\eta_i$ reflects the better alternative $A_i$.

**Step9**.

End.

## 7. ILLUSTRATIVE EXAMPLE FOR BRICK SELECTION PROCESS

Suppose that the administration of an authority is going to construct a building. For this purpose it is necessary to collect quality Bricks from various Brick Fields. After initial screening, five types of Bricks (alternatives) $A_1$, $A_2$, $A_3$, $A_4$, $A_5$ remain for further selection. A selection committee is formed with five decision makers or experts $D_1$, $D_2$, $D_3$, $D_4$, $D_5$. Six criteria of Bricks obtained from expert opinions, namely, Solidity $C_1$ color $C_2$, size and Shape $C_3$, strength of Brick $C_4$, cost of Bricks $C_5$ and carrying cost $C_6$ are considered for selection criteria. Decision maker $D_t$ (t = 1, 2, 3, 4, 5) uses linguistic variable to evaluate the ratings of the five types of Bricks $A_t$ (t = 1, 2, 3, 4, 5) with respect to the criterion $C_j$ (j= 1, 2, 3, 4, 5, 6). They construct the decision matrix $G^{(t)} = (G^t_{ij})_{5\times 6}$ (t= 1, 2, 3, 4, 5) shown in the tables 4, 5, 6, 7, & 8.

**Table2. Conversion between linguistic variables and IFNs**

| Linguistic variables(Brick quality) | IFNs |
|---|---|
| Extreme high(EH) | $(0.95,0.05,0.00)$ |
| Very high(VH) | $(0.85,0.10,0.05)$ |
| High(H) | $(0.75,0.15,0.10)$ |
| Medium high(MH) | $(0.65,0.25,0.10)$ |
| Medium(M) | $(0.50,0.40,0.10)$ |
| Medium low(ML) | $(0.35,0.55,0.10)$ |
| Low(L) | $(0.25,0.65,0.10)$ |
| Very low(VL) | $(0.15,0.80,0.05)$ |
| Extreme low(EL) | $(0.05,0.95,0.00)$ |

**Table3. Linguistic variable for the importance of the experts or decision makers**

| Linguistic variables | IFNs |
|---|---|
| Very important | $(1,0,0)$ |
| Important | $(0.75,0.20,0.05)$ |
| Medium | $(0.50,0.40,0.10)$ |
| Unimportant | $(0.25,0.60,0.15)$ |
| Very unimportant | $(0.10,0.80,0.10)$ |

**TABLE 4: Decision matrix $G^{(1)}$**

| $A_i$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | EH | VH | VH | H | H | M |
| $A_2$ | EH | H | VH | VH | M | ML |
| $A_3$ | VH | H | H | H | MH | ML |
| $A_4$ | VH | H | H | VH | M | H |
| $A_5$ | VH | VH | H | H | M | H |

**Table 5: Decision matrix $G^{(2)}$**

| $A_i$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | EH | VH | VH | H | M | MH |
| $A_2$ | VH | H | EH | H | H | M |
| $A_3$ | VH | VH | EH | H | H | ML |
| $A_4$ | EH | EH | VH | VH | M | ML |
| $A_5$ | VH | VH | H | H | H | H |

**Table 6: Decision matrix $G^{(3)}$**

| $A_i$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | VH | H | VH | MH | MH | M |
| $A_2$ | VH | H | H | MH | H | ML |
| $A_3$ | H | VH | H | H | M | ML |
| $A_4$ | H | H | H | H | M | H |
| $A_5$ | VH | VH | H | H | H | H |

**Table 7: Decision matrix $G^{(4)}$**

| $A_i$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | VH | VH | H | H | M | ML |
| $A_2$ | H | VH | H | MH | M | ML |
| $A_3$ | VH | H | VH | MH | MH | H |
| $A_4$ | VH | H | VH | H | MH | M |
| $A_5$ | H | VH | H | MH | M | M |

**Table 8: Decision matrix $G^{(5)}$**

| $A_i$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| $A_1$ | EH | H | VH | H | H | M |
| $A_2$ | VH | H | MH | H | MH | H |
| $A_3$ | VH | VH | MH | MH | M | MH |
| $A_4$ | H | H | H | MH | M | M |
| $A_5$ | VH | H | VH | MH | MH | M |

The selection process is done based on following the steps

**1.** Construct the intuitionistic fuzzy decision matrices of each decision maker. Convert the linguistic evaluation shown in Table 4-8 into IFNs by using Table 2. Then, the intuitionistic fuzzy decision matrix $G^t$ (t = 1, 2, 3, 4, 5) of each decision maker is constructed.

**2.** Determine the weight of the decision makers. The importance of the decision makers in the group decision-making process is shown in Table 3. These intuitionistic fuzzy linguistic variables can be converted into IFNs. Here, importance of decision maker is considered as very important i.e. (1, 0, 0). Using equation (3), we obtain the weights of the decision makers $\gamma_t = 0.2$, (t =1, 2, 3, 4, 5).

**3.** Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers. Utilize the IFWA operator given by the equation (4) to aggregate the intuitionistic fuzzy decision matrices G(t) (t = 1, 2, 3, 4, 5) into a complex intuitionistic fuzzy decision matrix X .

**4.** Consider the weights of the criteria obtained from expert opinions. We have average weight of each criterion $w_j, (j = 1,2,\cdots,6)$ as $w_1 = 0.275, w_2 = 0.175, w_3 = 0.2, w_4 = 0.1, w_5 = 0.05, w_6 = 0.2$ such that $\sum_{j=1}^{6} w_j = 1$.

**5.** Determine the reference sequence based on IFNs. The reference sequence can be presented as $\tilde{G}^+ = [(1,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0)(1,0,0)]$.

**6.** Calculate the grey relational coefficient of each alternative from PIS using the equation (11).

$$(\tau_{ij})_{5\times6} = \begin{bmatrix} 1.0000 & 0.7826 & 0.8182 & 0.7347 & 0.5625 & 0.4675 \\ 0.8571 & 0.7200 & 0.7500 & 0.6923 & 0.6667 & 0.4444 \\ 0.8182 & 0.7826 & 0.7500 & 0.6429 & 0.5455 & 0.4615 \\ 0.8182 & 0.7500 & 0.7500 & 0.7200 & 0.4865 & 0.5143 \\ 0.8182 & 0.8182 & 0.7200 & 0.6429 & 0.6102 & 0.5806 \end{bmatrix}$$

Calculation of $\min \delta_{ij}$ and $\max \delta_{ij}$

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $\min \delta_{ij}$ | $\max \delta_{ij}$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_{1j}$ | .09 | .19 | .17 | .22 | .37 | .50 | .09 | .50 |
| $\delta_{2j}$ | .15 | .23 | .21 | .25 | .27 | .54 | .15 | .54 |
| $\delta_{3j}$ | .17 | .19 | .21 | .29 | .39 | .51 | .17 | .51 |
| $\delta_{4j}$ | .17 | .21 | .21 | .23 | .47 | .43 | .17 | .47 |
| $\delta_{5j}$ | .17 | .17 | .23 | .29 | .32 | .35 | .17 | .35 |
| $\delta_{min}$ | | | | | | | .09 | |
| $\delta_{max}$ | | | | | | | | .54 |

**7.** Calculate the degree of grey relational coefficient of each alternative from PIS using the following equation:

$$\eta_i = \sum_{j=1}^{n} w_j \tau_{ij}\ ,\ j = 1.2,3,4,5,6\ \ i = 1,2,3,4,5$$

$\eta_1 = 0.7707, \eta_2 = 0.7032, \eta_3 = 0.6957, \eta_4 = 0.7187, \eta_5 = 0.7231$

Greater the value of $\eta_i$ implies the better alternative $A_i$.

Here, the relationship between grey relational grades is as follows:

$\eta_1 > \eta_5 > \eta_4 > \eta_2 > \eta_3$

Then, the five Bricks are ranked as:

$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$

Therefore, the most appropriate Brick is $A_1$.

## 8. SENSITIVITY ANALYSIS

With the change of weights of the criteria, it is observed that ranking order is sensitive with weight vectors (see table 9). We consider another weight structure of the criteria as follows:

$w_1 = 0.05 \quad w_2 = 0.22 \quad w_3 = 0.14 \quad w_4 = 0.19 \quad w_5 = 0.17 \quad w_6 = 0.23$

Then using the equation (12), we obtained $\eta_1 = 0.6795 \quad \eta_2 = 0.6533 \quad \eta_3 = 0.6391$ $\eta_4 = 0.6487 \quad \eta_5 = 0.6811$

The relationship between grey relational grades is as follows:

$\eta_5 \succ \eta_1 \succ \eta_2 \succ \eta_4 \succ \eta_3$

Therefore, the most appropriate Brick is $A_5$.

So, it is observed the change in weights of the criteria will produce change in the ranking order.

**Note1:** From grey relational coefficient matrix we have seen that for more priority level of the first four Brick criteria (Solidity, Color, Shape and Size, Strength), $A_1$ will be the best alternative.

**TABLE 9: Sensitivity analysis of weight structure and ranking of Bricks**

| Serial No. | Weight Structure: $w_1, w_2, w_3, w_4, w_5, w_6$ | Grey Relational Grades: $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$ | Ranking of Bricks | Best Alternative |
|---|---|---|---|---|
| 1 | .25, .2, .2, .15, .1, .1 | .7833,.7232,.7082, .7126,.7277 | $A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$ | $A_1$ |
| 2 | .167, .167, .167, .167, .165, .167 | .7276,.6884,.6668, .6732,.6984 | $A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$ | $A_1$ |
| 3 | .2, .2, .3, .2, .05, .05 | .8004,.7344,.7241, .7327,.7314 | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ | $A_1$ |
| 4 | .15, .1, .1, .45, .1, .1 | .7437,.6982,.6660, .6968,.6849 | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ | $A_1$ |

## 9. ADVANTAGES OF THE PROPOSED APPROACH

The proposed approach is very flexible. New criteria could easily be incorporated in the model based on the need, desire and new situations. In this paper, we showed how the proposed approach could provide a well-structured, coherent, and justifiable selection practice.

## 10. CONCLUSION

Grey relational analysis based intuitionistic fuzzy multi-criteria group decision-making approach is a practical, versatile and powerful tool that identifies the criteria and offers a consistent structure and process for selecting Bricks by employing the concept of acceptance, rejection and indeterminacy of Intuitionistic fuzzy sets simultaneously. In this study, we demonstrated how the proposed approach could provide a well-structured, coherent, and scientific selection practice. Therefore, in future, the proposed approach can be used for dealing with multi-criteria decision-making problems such as project evaluation, supplier selection, manufacturing system, and many other areas of management decision problems. After emergence of fuzzy sets, the paradigm shift occurred in decision-making arena. In intuitionistic fuzzy sets, although degree of rejection (non membership) is independent of degree of acceptance (membership) but degree of indeterminacy (hesitancy) is dependent on degree of acceptance and rejection. However, in reality degree of indeterminacy may be independent of degree of acceptance and rejection. Therefore, the researchers feel that the degree of indeterminacy with independent characteristics should be incorporated in the selection process. In this sense, the concept of neutrosophic set due to Smarandache [25] appears to be a promising one to deal with realistic selection process. Some studies due to Ye [16, 17], and Biswas et al. [5, 6] open up new avenue of research in the field of multiple attribute decision making in neutrosophic environment. It is hoped that if grey system theory and neutrosophic logic are used simultaneously, new area of research may be opened. Although this paper has shown the effectiveness of the proposed approach, many areas need to be explored and developed in neutrosophic environment [5, 6, 16, 17] which is very realistic in nature.

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