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Intuitionistic Neutrosophic Set Relations and Some of Its Properties

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Abstract. In this paper, we define intuitionistic neutrosophic set (INSs). In fact, all INSs are neutrosophic set but all neutrosophic sets are not INSs. We have shown by means of example that the definition for neutrosophic sets the complement and union are not true for INSs also give new definition of complement, union and intersection of INSs. We define the relation of INSs and four special type of INSs relations. Finally we have studied some properties of INSs relations.

Keywords: Neutrosophic sets, intuitionistic fuzzy sets, intuitionistic neutrosophic sets, intuitionistic neutrosophic relations

1. Introduction

In 1965 [7], Zadeh first introduced the concept of fuzzy sets. In many real applications to handle uncertainty, fuzzy set is very much useful and in this one real value $\mu_A(x) \in [0,1]$ is used to represent the grade of membership of a fuzzy set A defined on the universe of discorse X . After two decades Turksen [13] proposed the concept of interval-valued fuzzy set. But for some applications it is not enough to satisfy to consider only the membership-function supported by the evident but also have to consider the non-membership-function against by the evident. Atanassov [3] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Intuitionistic fuzzy sets handle incomplete information i.e., the grade of membership function and non-membership function but not the indeterminate information and inconsistent information which exists obviously in belief system.

Wang et.al. [2] introduced another concept of imprecise data called neutrosophic sets. Neutrosophic set is a part of neutrosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrsophic set is a powerful general formal framework that has been recently proposed. In neutrosophic set, indeterminacy by the evident is quantified explicitly and in this concept membership, indeterminacy membership and non-membership functional values are independent. Where membership, indeterminacy membership and non-membership functional values are real standard or non-standard subsets of $]^-0,1^+[$.

In real life problem which is very much useful. For example, when we ask the opinion of an expert about certain statement, he or she may assign that the possibility that the statement true is 0.5 and the statement false is 0.6 and he or she not sure is 0.2 . This idea is very much needful in a various problem in real life situation.

The neutrosophic set generalized the concept of classical set, fuzzy set [7], interval-valued-fuzzy set [13], intuitionistic fuzzy set [3], etc. Recently Bhowmik and Pal et.al. [14] have defined intuitionistic neutrosophic set.

Definition 1 Let X be a fixed set. A FS A of X is an object having the form $A = \{\langle x, \mu_A(x)\rangle / x \in X\}$. where the function $\mu_A : X \to [0,1]$ define the degree of membership of the element $x \in X$ to the set A,

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which is a subset of X .

Definition 2 Let X be a fixed set. An IFS A of X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and degree of nonmembership of the element $x \in X$ to the set A, which is a subset of X and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

An element x of X is called significant with respect to a fuzzy subset A of X if the degree of membership $\mu_A(x) > 0.5$, otherwise, it is insignificant. We see that for a fuzzy subset A both the degrees of membership $\mu_A(x)$ and non- membership $\nu_A(x) = 1 - \mu_A(x)$ can not be significant. Further, for an IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ it is observe that $0 \le \mu_A(x) + \nu_A(x) \le 1$, for all $x \in X$ and hence it is observed that $\min\{\mu_A(x), \nu_A(x)\} \le 0.5$, for all $x \in X$.

Definition 3 [12] Let X be a fixed set. A generalized intuitionistic fuzzy set (GIFS) A of X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and degree of nonmembership of the element $x \in X$ to the set A, which is a subset of X and for every $x \in X$ satisfy the condition

$$\mu_A(x) \wedge \nu_A(x) \le 0.5, \text{ for all } x \in X.$$

This condition is called generalized intuitionistic condition (GIC). In fact, all GIFs are IFSs but all IFSs are not GIFSs.

Having motivated from this definition we propose another concept of neutrosophic set.

In this paper, in Section 2 we recall the non-standard analysis by Abraham Robinson and some definitions of neutrosophic sets of Wang et.al. [2]. In Section 3, we define a new type of neutrosophic sets called intuitionistic neutrosophic sets (INSs)and have shown by means of example, that the definition for neutrosophic sets the complement and union are not true for INSs. Also we define new definition of complement, union and intersection of INS. In section 4, we define the relation of INSs and four special type of INSs relations. Finally we have studied some properties of INSs relations.

2. Preliminaries

In 1960s Abraham Robinson has developed the non-standard analysis, a formalization and a branch of mathematical logic, that rigorously defines the infinitesimals. Informally, an infinitesimal is an infinitely small number. Formally, x is said to be infinitesimal if and only if for all positive integers n one has $|x| < \frac{1}{n}$. Let $\varepsilon > 0$ be a such infinitesimal number. The hyper-real number set is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers $1^+ = 1 + \varepsilon$, where "1" is its standard part and "ε" is non-standard part and $^-0 = 0 - \varepsilon$, where "0" is its standard part and "ε" is non-standard part. Then we call $]^-0,1^+[$ is a non-standard unit interval.

Generally, the left and right boundaries of a non-standard interval $]^-a, b^+[$ are vague and imprecise. Combining the lower and upper infinitesimal non-standard variable of an element we can define as $^-c^+ = \{(c - \varepsilon) \cup (c + \varepsilon)\}$.

Addition of two non-standard finite numbers with themselves or with real numbers defines as:

$$^-a + b = ^-(a+b), a + b^+ = (a+b)^+, ^-a + b^+ = ^-(a+b)^+, \ ^-a + ^-b = ^-(a+b), a^+ + b^+ = (a+b)^+.$$

Similar for subtraction, multiplication, division, root and power of non-standard finite numbers with themselves or real numbers.

Now we recall some definitions of Wang et al.[2].

Let X be a space of points (objects), with a generic elements in X denoted by x. Every element of X is characterized by a truth-membership function T, an indeterminacy function I and a falsity-membership function F . Where $T, I and\ F$ are real standard or non--standard subsets of $]^-0,1^+[$, that is,

$$T : X \to]^-0,1^+[,$$
$$I : X \to]^-0,1^+[,$$
$$F : X \to]^-0,1^+[$$

There is no such restriction on the sum of $T(x), I(x)\ and F(x)$, so ${}^-0 \le T(x) + I(x) + F(x) \le 3^+$.

Definition 4 A neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T(x), I(x), F(x) \rangle$, for all $x \in X$, where $T(x), I(x), F(x) \to]^-0,1^+[$ and ${}^-0 \le T(x) + I(x) + F(x) \le 3^+$.

Definition 5 The complement of a neutrosophic set A is denoted by A' and is defined as $A' = \langle x, T_{A'}(x), I_{A'}(x), F_{A'}(x) \rangle$, where for all x in X

$$T_{A'}(x) = \{1^+\} - T_A(x),$$
$$I_{A'}(x) = \{1^+\} - I_A(x),$$
$$F_{A'}(x) = \{1^+\} - F_A(x).$$

Definition 6 A neutrosophic set A is contained in another neutrosophic set B i.e., $A \subseteq B$, if for all x in X

$$T_A(x) \le T_B(x),$$
$$I_A(x) \le I_B(x),$$
$$F_A(x) \ge F_B(x).$$

Definition 7 The union of two neutrosophic sets A and B is also a neutrosophic set, whose truth-membership, indeterminacy-membership and falsity-membership functions are

$$T_{(A \cup B)}(x) = T_A(x) + T_B(x) - T_A(x)T_B(x),$$
$$I_{(A \cup B)}(x) = I_A(x) + I_B(x) - I_A(x)I_B(x),$$
$$T_{(A \cup B)}(x) = F_A(x) + F_B(x) - F_A(x)F_B(x),\ \textit{for all x in } X.$$

Definition 8 The intersection of two neutrosophic sets A and B is also a neutrosophic set, whose truth-membership, indeterminacy-membership and falsity-membership functions are

$$T_{(A \cap B)}(x) = T_A(x)T_B(x),$$
$$I_{(A \cap B)}(x) = I_A(x)I_B(x),$$
$$T_{(A \cap B)}(x) = F_A(x)F_B(x),\quad \textit{for all x in } X.$$

Definition 9 The difference between two neutrosophic sets A and B is also a neutrosophic set is denoted as $A \setminus B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are

$$T_{(A \setminus B)}(x) = T_A(x) - T_A(x)T_B(x),$$
$$I_{(A \setminus B)}(x) = I_A(x) - I_A(x)I_B(x),$$
$$T_{(A \setminus B)}(x) = F_A(x) - F_A(x)F_B(x),\quad \textit{for all x in } X.$$

Definition 10 The cartesian product of two neutrosophic sets A and B defined on the universes X and Y respectively is also a neutrosophic set which is denoted by $A \times B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined by

$$T_{(A \times B)}(x, y) = T_A(x) + T_B(y) - T_A(x)T_B(y),$$
$$I_{(A \times B)}(x, y) = I_A(x)I_B(y),$$
$$T_{(A \times B)}(x, y) = F_A(x)F_B(y),\quad \textit{for all x in } X \textit{ and } y \textit{ in } Y.$$

3. Intuitionistic neutrosophic sets

Having motivated from the observation, we define an intuitionistic neutrosophic set (INS) as follows:

Definition 11 An element x of X is called significant with respect to neutrsophic set A of X if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $T_A(x)\ or\ F_A(x)\ or\ I_A(x) \geq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant.

We define an intuitionistic neutrosophic set by $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, where

$$\min\{T_A(x), F_A(x)\} \leq 0.5,$$
$$\min\{T_A(x), I_A(x)\} \leq 0.5,$$
$$and\ \min\{F_A(x), I_A(x)\} \leq 0.5,\ for\ all\ x\ in\ X,$$

with the condition $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 2$.

Definition 12 The complement of INS $A = \langle x, T(x), I(x), F(x)\rangle$, for all $x \in X$, is defined as

$$A^{'} = \langle x, F(x), I(x), T(x)\rangle,\ for\ all\ x \in X$$

Definition 13 The intersection of two INSs A^* and B^* is also an INS, whose truth-membership, indeterminacy-membership and falsity-membership functions are

$$T_{(A\cap B)}(x) = \min\{T_A(x), T_B(x)\}$$
$$I_{(A\cap B)}(x) = \min\{I_A(x), I_B(x)\}$$
$$F_{(A\cap B)}(x) = \max\{F_A(x), F_B(x)\},\quad for\ all\ x\ in\ X.$$

Definition 14 The union of two two INSs A and B is also an INS, whose truth-membership, indeterminacy-membership and falsity-membership functions are

$$T_{(A\cup B)}(x) = \max\{T_A(x), T_B(x)\}$$
$$I_{(A\cup B)}(x) = \min\{I_A(x), I_B(x)\}$$
$$F_{(A\cup B)}(x) = \min\{F_A(x), F_B(x)\},\quad for\ all\ x\ in\ X.$$

Here we recall the definitions 5, 7, and we will show by means of examples that they are not valid for INSs.

Example 1 Let $A = \{\langle x_1, 0.8, 0.2, 0.4\rangle, \langle x_2, 0.4, 0.5, 0.8\rangle\}$ and $B = \{\langle x_1, 0.4, 0.3, 0.8\rangle, \langle x_2, 0.4, 0.2, 0.8\rangle\}$ be two INSs of X.

By definition 5, $(A)^{'} = \{\langle x_1, 0.2, 0.8, 0.6\rangle, \langle x_2, 0.6, 0.5, 0.2\rangle\}$.

Here we see that for x_1 both I and F are ≥ 0.5. So $(A)^{'}$ is not a INS.

By definition 7, we get $A \cup B = \{\langle x_1, 0.88, 0.44, 0.88\rangle, \langle x_2, 0.64, 0.6, 0.96\rangle\}$.

Here all T, I, F for x_1 and also for x_2 are ≥ 0.5. So $A \cup B$ is not a INS.

4. Relations on INSs and some of its properties

Here we give the definition relation on of INSs and study some of its properties.

Let $X, Y\ and\ Z$ be three ordinary nonempty sets.

Definition 15 A INS relation (INSR) is defined as a intuitionistic neutrosophic subset of $X \times Y$, having the form

$$R = \{\langle (x, y), T_R(x, y), I_R(x, y), F_R(x, y)\rangle : x \in X, y \in Y\}$$

where,

$$T_R : X \times Y \to [0,1], I_R : X \times Y \to [0,1], F_R : X \times Y \to [0,1]$$

satisfy the conditions

(i) at least one of this $T_R(x, y), I_R(x, y)\ and F_R(x, y)$ is ≥ 0.5 and

(ii) $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 2$.

The collection of of all INSR on $X \times Y$ is denoted as $GR(X \times Y)$.

Definition 16 Let R be a INSR on $X \times Y$, then the inverse relation of R is denoted by R^{-1}, where

$$T_R^{-1}(x,y)=T_R(y,x), I_R^{-1}(x,y)=I_R(y,x), F_R^{-1}(x,y)=F_R(y,x), \forall\,(x,y)\in(X\times Y).$$

Lemma 1 Let $R\in GR(X,Y)$ and $P\in GR(X,Y)$, then

$$\min[\{\max_y\{\min\{T_R(x,y),T_P(y,z)\}\}\},\{\min_y\{\max\{I_R(x,y),I_P(y,z)\}\}\},$$
$$\{\min_y\{\max\{F_R(x,y),F_P(y,z)\}\}\}]\le 0.5$$

Proof. $\min[\{\min\{T_R(x,y),T_P(y,z)\}\},\{\min_{y'}\{\max\{I_R(x,y'),I_P(y',z)\}\}\},$

$$\{\min_{y''}\{\max\{F_R(x,y''),F_P(y'',z)\}\}\}]$$
$$\le \min[\{\min\{T_R(x,y),T_P(y,z)\}\},\{\max\{I_R(x,y),I_P(y,z)\}\},$$
$$\{\max\{F_R(x,y),F_P(y,z)\}\}]$$
$$=\min[\min[\{\min\{T_R(x,y),T_P(y,z)\}\},\{\max\{I_R(x,y),I_P(y,z)\}\}],$$
$$\{\max\{F_R(x,y),F_P(y,z)\}\}]$$
$$=\min[\max[\{\min\{T_R(x,y),T_P(y,z),I_R(x,y)\}\},\{\min\{T_R(x,y),T_P(y,z),I_P(y,z)\}\}],$$
$$\{\max\{F_R(x,y),F_P(y,z)\}\}]$$
$$=\max[\{\min\{F_R(x,y),T_R(x,y),T_P(y,z),I_R(x,y)\}\},$$
$$\{\min\{F_P(y,z),T_R(x,y),T_P(y,z),I_P(y,z)\}\}]$$
$$\le \max[\{\min\{0.5,T_P(y,z)\}\},\{\min\{0.5,T_R(x,y)\}\}]$$
$$[\textit{ since at least one of } F_R(x,y),T_R(x,y) \textit{ and } I_R(x,y)\le 0.5]$$
$$\le 0.5,\ \forall\ y\in Y.$$

Lemma 2 Let $R\in GR(X,Y)$ and $P\in GR(X,Y)$, then

$$\min[\{\min_y\{\max\{T_R(x,y),T_P(y,z)\}\}\},\{\max_y\{\min\{I_R(x,y),I_P(y,z)\}\}\},$$
$$\{\max_y\{\min\{F_R(x,y),F_P(y,z)\}\}\}]\le 0.5$$

Proof. The proof is similar as lemma 1.

Now we define two composit relations of INSs.

Definition 17 Let $P\in GR(X,Y)$ and $R\in GR(Y,Z)$. Then we define two composit relations on $X\times Z$, denoted by $P\circ R$ and $P*R$ and they are defined as

$$P\circ R=\{\langle(x,z),T_{P\circ R}(x,z),I_{P\circ R}(x,z),F_{P\circ R}(x,z)\rangle : x\in X, z\in Z\}, \textit{where}$$
$$T_{P\circ R}(x,z)=\max_y\{\min\{T_P(x,y),T_R(y,z)\}\},$$
$$I_{P\circ R}(x,z)=\min_y\{\max\{I_P(x,y),I_R(y,z)\}\},$$
$$F_{P\circ R}(x,z)=\min_y\{\max\{F_P(x,y),F_R(y,z)\}\}$$
$$\textit{and}\quad P*R=\{\langle(x,z),T_{P*R}(x,z),I_{P*R}(x,z),F_{P*R}(x,z)\rangle : x\in X, z\in Z\}, \textit{where}$$
$$T_{P*R}(x,z)=\min_y\{\max\{T_P(x,y),T_R(y,z)\}\},$$
$$I_{P*R}(x,z)=\max_y\{\min\{I_P(x,y),I_R(y,z)\}\},$$
$$F_{P*R}(x,z)=\max_y\{\min\{F_P(x,y),F_R(y,z)\}\}.$$

By lemma 1 and lemma 2 the sets $P\circ R$ and $P*R$ both satisfy the conditions of INSs.

Theorem 1 If $P \in (X, Y)$ and $R \in (Y, Z)$ be two subsets of INSRs then

(*i*) $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$

(*ii*) $(P * R)^{-1} = R^{-1} * P^{-1}$

Proof. (i) Let $A = P \circ R$ and $B = R^{-1} \circ P^{-1}$, then

$$T_A(x,z) = \max_y \{\min\{T_{(P\circ R)}(x,y), T_{(P\circ R)}(y,z)\}\},$$

$$T_B(x,z) = \max_y \{\min\{T_{(R^{-1}\circ P^{-1})}(z,y), T_{(R^{-1}\circ P^{-1})}(y,x)\}\},$$

$$I_A(x,z) = \max_y \{\min\{I_{(P\circ R)}(x,y), I_{(P\circ R)}(y,z)\}\},$$

$$I_B(x,z) = \max_y \{\min\{I_{(R^{-1}\circ P^{-1})}(z,y), I_{(R^{-1}\circ P^{-1})}(y,x)\}\},$$

$$F_A(x,z) = \max_y \{\min\{F_{(P\circ R)}(x,y), F_{(P\circ R)}(y,z)\}\},$$

$$F_B(x,z) = \max_y \{\min\{F_{(R^{-1}\circ P^{-1})}(z,y), F_{(R^{-1}\circ P^{-1})}(y,x)\}\}.$$

Now, $T_B(x,z) = \max_y \{\min\{T_{(R^{-1}\circ P^{-1})}(x,y), T_{(R^{-1}\circ P^{-1})}(y,z)\}\}$

$$= \max_y [\min[\max_z \{\min\{T_{R^{-1}}(x,z), T_{P^{-1}}(z,y)\}\},$$

$$\max_x \{\min\{T_{R^{-1}}(y,x), T_{P^{-1}}(x,z)\}\}]]$$

$$= \max_y \{\min\{T_R(z,x), T_P(y,z), T_R(x,y), T_P(z,x)\}\}$$

$$= \max_y \{\min\{T_{(R\circ P)}(y,z), T_{(R\circ P)}(x,y)\}\}$$

$$= \max_y \{\min\{T_{(P\circ R)^{-1}}(z,y), T_{(P\circ R)^{-1}}(y,x)\}\}$$

$$= T_{A^{-1}}(z,x)$$

Similarly we can prove $I_B(x,z) = I_{A^{-1}}(x,z)$ and $F_B(x,z) = F_{A^{-1}}(x,z)$.

Hence $(P \circ R)^{-1} = R^{-1} \circ P^{-1}$.

(ii) The proof is similar.

Example 2 Let ,

$$P = \begin{array}{cccc} X \times Y & y_1 & y_2 & y_3 \\ x_1 & \langle (x_1,y_1),0.6,0.2,0.3\rangle & \langle (x_1,y_2),0.3,0.5,0.2\rangle & \langle (x_1,y_3),0.8,0.4,0.1\rangle \\ x_2 & \langle (x_2,y_1),0.2,0.7,0.1\rangle & \langle (x_2,y_2),0.6,0.3,0.4\rangle & \langle (x_2,y_3),0.1,0.5,0.3\rangle \\ x_3 & \langle (x_3,y_1),0.3,0.6,0.5\rangle & \langle (x_3,y_2),0.5,0.7,0.2\rangle & \langle (x_3,y_3),0.2,0.4,0.1\rangle \end{array},$$

and

$$R = \begin{array}{cccc} Y \times Z & z_1 & z_2 & z_3 \\ y_1 & \langle (y_1,z_1),0.4,0.6,0.3\rangle & \langle (y_1,z_2),0.3,0.8,0.2\rangle & \langle (y_1,z_3),0.4,0.2,0.2\rangle \\ y_2 & \langle (y_2,z_1),0.3,0.7,0.2\rangle & \langle (y_2,z_2),0.2,0.4,0.4\rangle & \langle (y_2,z_3),0.8,0.1,0.5\rangle \\ y_3 & \langle (y_3,z_1),0.4,0.2,0.6\rangle & \langle (y_3,z_2),0.3,0.8,0.2\rangle & \langle (y_3,z_3),0.2,0.6,0.4\rangle \end{array}.$$

Then

$$(P\circ R)^{-1}=\begin{array}{cccc} Z\times X & x_1 & x_2 & x_3 \\ z_1 & \langle(z_1,x_1),0.4,0.4,0.2\rangle & \langle(z_1,x_2),0.3,0.5,0.3\rangle & \langle(z_1,x_3),0.3,0.4,0.2\rangle \\ z_2 & \langle(z_2,x_1),0.3,0.5,0.2\rangle & \langle(z_2,x_2),0.2,0.4,0.2\rangle & \langle(z_2,x_3),0.3,0.7,0.2\rangle \\ z_3 & \langle(z_3,x_1),0.4,0.2,0.3\rangle & \langle(z_3,x_2),0.6,0.3,0.2\rangle & \langle(z_3,x_3),0.5,0.6,0.4\rangle \end{array}.$$

Now,

$$P^{-1}=\begin{array}{cccc} Y\times X & x_1 & x_2 & x_3 \\ y_1 & \langle(y_1,x_1),0.6,0.2,0.3\rangle & \langle(y_1,x_2),0.2,0.7,0.1\rangle & \langle(y_1,x_3),0.3,0.6,0.5\rangle \\ y_2 & \langle(y_2,x_1),0.3,0.5,0.2\rangle & \langle(y_2,x_2),0.6,0.3,0.4\rangle & \langle(y_2,x_3),0.5,0.7,0.2\rangle \\ y_3 & \langle(y_3,x_1),0.8,0.4,0.1\rangle & \langle(y_3,x_2),0.1,0.5,0.3\rangle & \langle(y_3,x_3),0.2,0.4,0.1\rangle \end{array},$$

and

$$R^{-1}=\begin{array}{cccc} Z\times Y & y_1 & y_2 & y_3 \\ z_1 & \langle(z_1,y_1),0.4,0.6,0.3\rangle & \langle(z_1,y_2),0.3,0.7,0.2\rangle & \langle(z_1,y_3),0.4,0.2,0.6\rangle \\ z_2 & \langle(z_2,y_1),0.3,0.8,0.2\rangle & \langle(z_2,y_2),0.2,0.4,0.4\rangle & \langle(z_2,y_3),0.3,0.8,0.2\rangle \\ z_3 & \langle(z_3,y_1),0.4,0.2,0.2\rangle & \langle(z_3,y_2),0.8,0.1,0.5\rangle & \langle(z_3,y_3),0.2,0.6,0.4\rangle \end{array}.$$

Then

$$(R^{-1}\circ P^{-1})=\begin{array}{cccc} Z\times X & x_1 & x_2 & x_3 \\ z_1 & \langle(z_1,x_1),0.4,0.4,0.2\rangle & \langle(z_1,x_2),0.3,0.5,0.3\rangle & \langle(z_1,x_3),0.3,0.4,0.2\rangle \\ z_2 & \langle(z_2,x_1),0.3,0.5,0.2\rangle & \langle(z_2,x_2),0.2,0.4,0.2\rangle & \langle(z_2,x_3),0.3,0.7,0.2\rangle \\ z_3 & \langle(z_3,x_1),0.4,0.2,0.3\rangle & \langle(z_3,x_2),0.6,0.3,0.2\rangle & \langle(z_3,x_3),0.5,0.6,0.4\rangle \end{array}.$$

So, $(P\circ R)^{-1}=R^{-1}\circ P^{-1}$.

Definition 18 Let $R\in GR(X\times Y)$, then

(i) R is reflexive of type-1 if $T(x,x)=1, I(x,x)=0\ and F(x,x)=0, \forall\ x\in X$.

(ii) R is reflexive of type-2 if $T(x,x)=1, \forall\ x\in X$, $\max\{I(x,x), I(y,y)\le I(x,y)\}$ $and\ \min\{F(x,x), F(y,y)\le F(x,y)\}, \forall\ x,y\in X$.

(iii) R is reflexive of type-3 if $\min\{T(x,x),T(y,y)\}\ge \max\{0.5, T(x,y)\}$, $\min\{I(x,x),I(y,y)\}\ge\max\{0.5,I(x,y)\}, \forall\ x,y\in X$ and $F(x,x)=0, \forall\ x\in X$.

(iv) R is reflexive of type-4 if $\min\{T(x,x),T(y,y)\}\ge T(x,y), \max\{I(x,x),I(y,y)\le I(x,y)\}$ $and\ \max\{F(x,x),F(y,y)\le F(x,y)\}, \forall\ x,y\in X$.

Definition 19 Let $R\in GR(X\times Y)$, then

(i) R is irreflexive of type-1 if $T(x,x)=0, I(x,x)=0\ and F(x,x)=1, \forall\ x\in X$.

(ii) R is irreflexive of type-2 if $T(x,x)=0, \forall\ x\in X$, $\min\{I(x,x),I(y,y)\}\ge\max\{0.5,I(x,y)\}$ $and\ \min\{F(x,x),F(y,y)\}\le\max\{0.5,F(x,y)\}, \forall\ x,y\in X$.

(iii) R is irreflexive of type-3 if $\max\{T(x,x),T(y,y)\}\le T(x,y)$, $\max\{I(x,x),I(y,y)\}\le I(x,y), \forall\ x,y\in X\ \ and\ F(x,x)=1, \forall\ x\in X$.

(iv) R is irreflexive of type-4 if $\max\{T(x,x),T(y,y)\}\le T(x,y)$,

$$\max\{I(x,x), I(y,y)\} \le I(x,y) \text{ and } \min\{F(x,x), F(y,y)\} \ge F(x,y), \forall\, x, y \in X.$$

Theorem 2

(i) Reflexivity (irreflexivity) of type-1 $\Rightarrow$ Reflexivity (irreflexivity) of type-2, type-3 and type-4.

(ii) Reflexivity (irreflexivity) of type-2 $\Rightarrow$ Reflexivity (irreflexivity) of type-4.

(iii) Reflexivity (irreflexivity) of type-3 $\Rightarrow$ Reflexivity (irreflexivity) of type-4.

The proof follows from the definition. Here we are showing by the numerical example that the above theorems is obvious.

Example 3 Let $R \in GR(X,Y)$ be a reflexive of type-1, where

$$R = \begin{array}{cccc} X \times Y & y_1 & y_2 & y_3 \\ x_1 & \langle (x_1,y_1),1,0,0\rangle & \langle (x_1,y_2),0.6,0.3,0.4\rangle & \langle (x_1,y_3),0.8,0.2,0.1\rangle \\ x_2 & \langle (x_2,y_1),0.3,0.5,0.9\rangle & \langle (x_2,y_2),1,0,0\rangle & \langle (x_2,y_3),0.2,0.4,0.6\rangle \\ x_3 & \langle (x_3,y_1),0.4,0.6,0.3\rangle & \langle (x_3,y_2),0.3,0.7,0.1\rangle & \langle (x_3,y_3),1,0,0\rangle \end{array}.$$

Then R is obviously type-2,type-3 and type-4 reflexive.

Remark 1 It can easily be shown by constructing examples that reflexive (irreflexive) of type-4 $\not\Rightarrow$ reflexive (irreflexive) of type-3 $\not\Rightarrow$ reflexive (irreflexive) of type-2 $\not\Rightarrow$ reflexive (irreflexive) of type-1.

Theorem 3 (i) If $R \in GR(X,X)$ is reflexive of any type then $R \le R \circ R$.

(ii) If $R \in GR(X,X)$ is irreflexive of any type then $R \ge R * R$.

Proof. (i) Let $R = \langle T_R(x_i,x_j), I_R(x_i,x_j), F_R(x_i,x_j)\rangle$ for $i = 1,2,\ldots,n$ *and* $j = 1,2,\ldots,n$ and $A = R \circ R$.

Then $A = \langle T_{R\circ R}(x_i,x_j), I_{R\circ R}(x_i,x_j), F_{R\circ R}(x_i,x_j)\rangle$ for $i = 1,2,\ldots,nn$ *and* $j = 1,2,\ldots,n$. Therefore,

$$T_A(x_i,x_j) = \max_k \{\min\{T_R(x_i,x_k), T_R(x_k,x_j)\}\}$$

$$= \max[\{\min\{T_R(x_i,x_i), T_R(x_i,x_j)\}\}, \max_{k,k\ne i}\{\min\{T_R(x_i,x_k), T_R(x_k,x_j)\}\}]$$

$$= \max[T_R(x_i,x_j), \max_{k,k\ne i}\{\min\{T_R(x_i,x_k), T_R(x_k,x_j)\}\}] \text{ [for any type of reflexivity]}$$

$$\ge T_R(x_i,x_j).$$

$$I_A(x_i,x_j) = \min_k \{\max\{I_R(x_i,x_k), I_R(x_k,x_j)\}\}$$

$$= \min[\{\max\{T_R(x_i,x_i), T_R(x_i,x_j)\}\}, \min_{k,k\ne i}\{\max\{I_R(x_i,x_k), I_R(x_k,x_j)\}\}]$$

$$= \min[I_R(x_i,x_j), \min_{k,k\ne i}\{\max\{I_R(x_i,x_k), I_R(x_k,x_j)\}\}] \text{ [for any type of reflexivity]}$$

$$\le I_R(x_i,x_j).$$

Similarly we can prove $i_A \le I_R(x_i,x_j)$.

Hence $R \le A$ i.e., $R \le R \circ R$.

(ii) Proof is similar to above case.

Note 1 Any special type of reflexivity (irreflexivty) is not a necessary condition for satisfying the theorem 3.

Example 4 Let $R \in GR(X,X)$, where $X = \{x_1, x_2, x_3\}$ and R is not any special type of reflexive , where

$$R = \begin{array}{cccc} X \times X & x_1 & x_2 & x_3 \\ x_1 & \langle (x_1, x_1), 0.4, 0.3, 0.7 \rangle & \langle (x_1, x_2), 0.6, 0.5, 0.3 \rangle & \langle (x_1, x_3), 0.3, 0.6, 0.1 \rangle \\ x_2 & \langle (x_2, x_1), 0.6, 0.3, 0.2 \rangle & \langle (x_2, x_2), 0.7, 0.3, 0.4 \rangle & \langle (x_2, x_3), 0.6, 0.3, 0.4 \rangle \\ x_3 & \langle (x_3, x_1), 0.3, 0.5, 0.1 \rangle & \langle (x_3, x_2), 0.4, 0.5, 0.0 \rangle & \langle (x_3, x_3), 0.3, 0.6, 0.3 \rangle \end{array}.$$

Then

$$R * R = \begin{array}{cccc} X \times X & x_1 & x_2 & x_3 \\ x_1 & \langle (x_1, x_1), 0.3, 0.5, 0.7 \rangle & \langle (x_1, x_2), 0.4, 0.5, 0.3 \rangle & \langle (x_1, x_3), 0.3, 0.6, 0.3 \rangle \\ x_2 & \langle (x_2, x_1), 0.6, 0.3, 0.2 \rangle & \langle (x_2, x_2), 0.6, 0.3, 0.4 \rangle & \langle (x_2, x_3), 0.6, 0.3, 0.4 \rangle \\ x_3 & \langle (x_3, x_1), 0.3, 0.5, 0.1 \rangle & \langle (x_3, x_2), 0.4, 0.5, 0.1 \rangle & \langle (x_3, x_3), 0.3, 0.6, 0.3 \rangle \end{array}.$$

Though R is not any special type of irrflexive then also $R \geq R * R$.

Definition 20 A relation $R \in GR(X, X)$ is called symmetric if $R = R^{-1}$ i.e., if $\forall$ $(x_i, x_j) \in X \times X$,

$$T_R(x_i, x_j) = T_R(x_j, x_i), I_R(x_i, x_j) = I_R(x_j, x_i), F_R(x_i, x_j) = F_R(x_j, x_i).$$

Theorem 4 (i) If $P, R \in GR(X, X)$ are symmetric relations, then $P \circ R = (R \circ P)^{-1}$.

(ii) If $R \in GR(X, X)$ is symmetric relation, then $R \circ R$ is also a symmetric relation.

Proof. (i) Since R and P are both symmetric relation on $X \times X$,

then $P \circ R = P^{-1} \circ R^{-1} == (R \circ P)^{-1}$ [by the theorem 1].

(ii) The proof is obvious.

5. Conclusion

Here we define a new type of neutrosophic setcalled intuitionistic neutrosophic sets (INSs) and have showed by means of examples, that the definition for neutrosophic set the compliment and union are not true for INSs. Also we have define new definition of complement, union and intersection of INS. In section 4, we define the relation of INSs and four special types of INSs relations. Finally we have studied some properties of INSs relations.

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