An interesting class of Smarandache generalized Fermat numbers

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I make few observations on a class of Smarandache generalized Fermat numbers, which are the numbers of the form F(k) = a^(b^k) + c, where a, b are integers greater than or equal to 2 and c is integer such that (a, c) = 1. The class that is observed in this paper includes the numbers of the form F(k) = m^(n^k) + n, where k is positive integer and m and n are coprime positive integers, not both of them odd or both of them even.

Observation:

The numbers of the form F(k) = m^(n^k) + n, where k is positive integer and m and n are coprime positive integers, not both of them odd or both of them even are very difficult to be factorized, not just because their obviously large size but because seem to have very few prime factors. This is a known characteristic of Fermat numbers (for instance, the 8-th such number, having 78 digits, is a semiprime) and seems that this class of Smarandache generalized Fermat numbers share this feature also.

Examples:

For F(k) = 4^(3^k) + 3 are obtained the following values:

: F(1) = 67 prime;
: F(2) = 262147 prime;
: F(3) = 18014398509481987*1422061*12667809967 semiprime;
: F(4) = 5846006549323611672814739330865132078623730171907 = 103*56757345139064190998201352726845942510910001669 semiprime;
(the number F(5) has 147 digits)

For F(k) = 2^(7^k) + 7 are obtained the following values:

: F(1) = 16387 = 7*2341 semiprime;
: F(2) = 316912650057057350374175801351 prime;
(the number F(3) has 207 digits)

For F(k) = 2^(9^k) + 9 are obtained the following values:

: F(1) = 521 prime;
: F(2) = 2417851639229258349412361 = 11*219804694475387122673851 semiprime;
(the number F(3) has 220 digits)

For F(k) = 2^(11^k) + 11 are obtained the following values:

: F(1) = 2059 = 29*71 semiprime;
: F(2) = 2658455991569831745807614120560689163 = 13*131*1.561042860581228271172997134797821;
(the number F(3) has 401 digits)

Comment:

Of course, not for every pair of (m, n) that satisfies the conditions described above are obtained relevant results, but I haven't find yet a convincing pattern for a special relation between m and n.

Reference:

Florentin Smarandache, ***Conjecture (General Fermat numbers)***, in Collected Papers, vol. II, Kishinev University Press, Kishinev, 1997.