NSS

An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set

Pingping Chi[1,2] and Peide Liu[2,3,*]

[1]China-Asean International College, Dhurakij Pundit University, Bangkok 10210, Thailand
[2]Institute of Non-governmental Higher Education, Shandong Yingcai University, Jinan 250104, P.R. of China
[3] School of Management Science and Engineering, Shandong University of Finance and Economics, No.7366, Erhuandong Road, Jinan, 250014, P.R. of China.
*E-mail: peide.liu@gmail.com (Corresponding author)

Abstract. The interval neutrosophic set (INS) can be easier to express the incomplete, indeterminate and inconsistent information, and TOPSIS is one of the most commonly used and effective method for multiple attribute decision making, however, in general, it can only process the attribute values with crisp numbers. In this paper, we have extended TOPSIS to INS, and with respect to the multiple attribute decision making problems in which the attribute weights are unknown and the attribute values take the form of INSs, we proposed an expanded TOPSIS method. Firstly, the definition of INS and the operational laws are given, and distance between INSs is defined. Then, the attribute weights are determined based on the Maximizing deviation method and an extended TOPSIS method is developed to rank the alternatives. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: interval neutrosophic set; TOPSIS; multiple attribute decision making; Maximizing deviation method; Hamming distance.

1 Introduction

In real decision making, there exist many multi-criteria decision-making (MCDM) problems. Because of the ambiguity of people's thinking and the complexity of objective things, the attribute values of the MCDM problems cannot always be expressed by crisp numbers, and it maybe is easier to be described by fuzzy information. The fuzzy set (FS) theory, which is proposed by Zadeh [1], is one of the most effective tools for processing fuzzy information; however, its disadvantage is that it only has a membership, and is unable to express non-membership. On the basis of FS, Atanassov [2,3] proposed the intuitionistic fuzzy set (IFS) by adding a non-membership function, i.e., there are membership (or called truth-membership) $T_A(x)$ and non-membership (or called falsity-membership) $F_A(x)$ in intuitionistic fuzzy sets, and they satisfy the conditions $T_A(x), F_A(x) \in [0,1]$ and $0 \le T_A(x) + F_A(x) \le 1$. Further, Atanassov and Gargov [4], Atanassov [5] proposed the interval-valued intuitionistic fuzzy set (IVIFS) by extending the truth-membership function and falsity-membership function to interval numbers. IFSs and IVIFSs can only handle incomplete information not the indeterminate information and inconsistent information. In IFSs, the indeterminacy is $1\text{-}T_A(x)\text{-}F_A(x)$ by default. However, in practice, the decision information is often incomplete, indeterminate and inconsistent information. In order to process this kind of information, Smarandache [6] further proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership on the basis of IFS, which is a generalization of fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and so on. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent.

Recently, NSs have become an interesting research topic and attracted widely attentions. Wang et al. [7] proposed a single valued neutrosophic set (SVNS) from scientific or engineering point of view, which is an instance of the neutrosophic set. Ye [8] proposed the correlation coefficient and weighted correlation coefficient for SVNSs, and he have proved that the cosine similarity degree is a special case of the correlation coefficient in SVNS. Ye [8a] proposed Single valued neutrosophic cross-entropy for multicriteria decision making problems. Similar to IVIFS, Wang et al. [9] proposed interval neutrosophic sets (INSs) in which the truth-membership, indeterminacy-membership, and false-membership were extended to interval numbers, and discussed some properties and comparing method of INSs. Ye [10] proposed the similarity measures between INSs based on the Hamming and Euclidean distances, and developed a multicriteria decision-making method based on the similarity degree. However, so far, there has been no research on extending TOPSIS for INSs.

TOPSIS (The Order Performance technique based on Similarity to Ideal Solution), which was proposed by Hwang and Yoon [11] is one of popular decision making methods. In last 20 years, many researchers have extended this method and proposed different modifications, and it has been applied usefully in the practice to solve many

problems in different fields for decision makers.
Chen [12] extended the TOPSIS for group decision making problems in which the importance weights of various criteria and ratings of alternatives with respect to these criteria take the form of linguistic variables. The key of the proposed method is that these variables are transformed into triangular fuzzy numbers. Jin et al. [13] extended TOPSIS method to MADM problems in which the attribute values are the intuitionistic fuzzy sets, and applied it to the evaluation of human resources. Wei and Liu [14] extended TOPSIS method to the uncertain linguistic variables, and applied it to the risk evaluation of High-technology. Liu [15] proposed an extended TOPSIS method to resolve the multi-attribute decision-making problems in which the attribute weights and attribute values are all interval vague value. Firstly, the ideal and negative ideal solutions are calculated based on the score function. Then the distance between the interval Vague values is defined, and the distances between each alternative and the ideal and negative ideal solutions are calculated. The relative closeness degree is calculated by TOPSIS method, and then the ordering of the alternatives is confirmed according to the relative closeness degree. Liu and Su [16] proposed an extended TOPSIS based on trapezoid fuzzy linguistic variables, and gave the method for determining attribute weights. Liu [17] proposed an extended TOPSIS method for multiple attribute group decision making based on generalized interval-valued trapezoidal fuzzy numbers. Mohammadi et al. [18] used fuzzy group TOPSIS method for selecting adequate security mechanisms in e-business processes. Verma et al. [19] proposed an interval-valued intuitionistic fuzzy TOPSIS method for solving a facility location problem.

Obviously, because TOPSIS is an important decision making method, and the interval neutrosophic set can be easier to express the incomplete, indeterminate and inconsistent information, it is important to establish an extended TOPSIS method based on INS. In this paper, we will establish an extended TOPSIS method for the multiple attribute decision making problems in which the attribute weights are unknown and attribute values take the form of INSs. In order to do so, the remainder of this paper is shown as follows. In section 2, we briefly review some basic concepts and operational rules of INS and propose the Hamming distance and the Euclidian distance between interval neutrosophic values (INVs) or interval neutrosophic sets, and give a proof of Hamming distance and a calcualtion example. In Section 3, we propose a method for determining the attribute weights based on the Maximizing deviation method and extend the TOPSIS method to rank the alternatives, and give the detail decision steps. In Section 4, we give an example to illustrate the application of proposed method, and compare the developed method with the existing method. In Section 5, we conclude the paper.

2 The Interval Neutrosophic Set

2.1 The Definition of the Interval Neutrosophic Set

Definition 1 [6]. Let X be a universe of discourse, with a generic element in X denoted by x. A neutrosophic set (NS) A in X is

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \qquad (1)$$

where, T_A , I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $\left]0^-, 1^+\right[$.

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

The NS was presented from philosophical point of view. Obviously, it was difficult to use in the actual applications. Wang [7] further proposed the single valued neutrosophic set (SVNS) from scientific or engineering point of view, which is a generalization of the existing fuzzy sets, such as classical set, fuzzy set, intuitionistic fuzzy set and paraconsistent sets etc., and it was defined as follows.

Definition 2 [7]. Let X be a universe of discourse, with a generic element in X denoted by x. A single valued neutrosophic set A in X is

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \qquad (2)$$

where, T_A , I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. For each point x in X, we have $T_A(x), I_A(x), F_A(x) \in [0,1]$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

In the actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp values, and they may be easier to be expressed by interval numbers. Wang et al. [9] further defined interval neutrosophic sets (INSs) shown as follows.

Definition 3 [7]. Let X be a universe of discourse, with a generic element in X denoted by x. A interval neutrosophic set A in X is

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \qquad (3)$$

where, T_A , I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. For each point x in X, we have $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$, and $0 \le \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \le 3$.

For convenience, we can use $x=([T^L,T^U],[I^L,I^U],[F^L,F^U])$ to represent a value in INS, and call interval neutrosophic value (INV).

2.2 The Operational Rules of the Interval Neutrosophic Values

Definition 4. Let $x=([T_1^L,T_1^U],[I_1^L,I_1^U],[F_1^L,F_1^U])$ and $y=([T_2^L,T_2^U],[I_2^L,I_2^U],[F_2^L,F_2^U])$ be two INVs, then the operational rules are defined as follows.

(1) The complement of x is

$$\bar{x}=([F_1^L,F_1^U],[1-I_1^U,1-I_1^L],[T_1^L,T_1^U]) \quad (4)$$

(2)
$$x\oplus y=\left(\left[T_1^L+T_2^L-T_1^LT_2^L,T_1^U+T_2^U-T_1^UT_2^U\right],\left[I_1^LI_2^L,I_1^UI_2^U\right],\left[F_1^LF_2^L,F_1^UF_2^U\right]\right) \quad (5)$$

(3)
$$x\otimes y=\left([T_1^LT_2^L,T_1^UT_2^U],[I_1^L+I_2^L-I_1^LI_2^L,I_1^U+I_2^U-I_1^UI_2^U],\left[F_1^L+F_2^L-F_1^LF_2^L,F_1^U+F_2^U-F_1^UF_2^U\right]\right) \quad (6)$$

(4)
$$nx=\left(\left[1-(1-T_1^L)^n,1-(1-T_1^U)^n\right],\left[(I_1^L)^n,(I_1^U)^n\right],\left[(F_1^L)^n,(F_1^U)^n\right]\right)\ n>0 \quad (7)$$

(5)
$$x^n=\left(\left[(T_1^L)^n,(T_1^U)^n\right],\left[1-(1-I_1^L)^n,1-(1-I_1^U)^n\right],\left[1-(1-F_1^L)^n,1-(1-F_1^U)^n\right]\right)\ n>0 \quad (8)$$

2.2 The Distance between two INSs

In the following, we will discuss the distance between two INSs.

Definition 5. Let $x=([T_1^L,T_1^U],[I_1^L,I_1^U],[F_1^L,F_1^U])$, $y=([T_2^L,T_2^U],[I_2^L,I_2^U],[F_2^L,F_2^U])$ and $z=([T_3^L,T_3^U],[I_3^L,I_3^U],[F_3^L,F_3^U])$ be three INVs, S be a collection of all INVs, and f be a mapping with $f:\hat{S}\times\hat{S}\to R$. If $d(x,y)$ meets the following conditions.

(1) $0\le d(x,y)\le 1$, $d(x,x)=0$

(2) $d(x,y)=d(y,x)$

(3) $d(x,y)+d(y,z)\ge d(x,z)$

Then we can call $d(x,y)$ a distance between twoINVs x and y.

Definition 6. Let $x=([T_1^L,T_1^U],[I_1^L,I_1^U],[F_1^L,F_1^U])$, and $y=([T_2^L,T_2^U],[I_2^L,I_2^U],[F_2^L,F_2^U])$ be two INVs, then

(1) The Hamming distance between x and y is defined as follows

$$d_H(x,y)=\frac{1}{6}\left(\left|T_1^L-T_2^L\right|+\left|T_1^U-T_2^U\right|+\left|I_1^L-I_2^L\right|+\left|I_1^U-I_2^U\right|+\left|F_1^L-F_2^L\right|+\left|F_1^U-F_2^U\right|\right) \quad (9)$$

Proof.

Obviously, (9) can meet the above conditions (1) and (2) in Definiation 5.

In the following, we will prove (9) meets condition (3).

For any an INV $z=([T_3^L,T_3^U],[I_3^L,I_3^U],[F_3^L,F_3^U])$, we have

$$\begin{aligned}d_H(x,z)&=\frac{1}{6}\left(\left|T_1^L-T_3^L\right|+\left|T_1^U-T_3^U\right|+\left|I_1^L-I_3^L\right|+\left|I_1^U-I_3^U\right|+\left|F_1^L-F_3^L\right|+\left|F_1^U-F_3^U\right|\right)\\&=\frac{1}{6}\left(\left|T_1^L-T_2^L+T_2^L-T_3^L\right|+\left|T_1^U-T_2^U+T_2^U-T_3^U\right|\right.\\&\quad+\left|I_1^L-I_2^L+I_2^L-I_3^L\right|+\left|I_1^U-I_2^U+I_2^U-I_3^U\right|\\&\quad\left.+\left|F_1^L-F_2^L+F_2^L-F_3^L\right|+\left|F_1^U-F_2^U+F_2^U-F_3^U\right|\right)\\&\le\frac{1}{6}\left(\left|T_1^L-T_2^L\right|+\left|T_2^L-T_3^L\right|+\left|T_1^U-T_2^U\right|+\left|T_2^U-T_3^U\right|\right.\\&\quad+\left|I_1^L-I_2^L\right|+\left|I_2^L-I_3^L\right|+\left|I_1^U-I_2^U\right|+\left|I_2^U-I_3^U\right|\\&\quad\left.+\left|F_1^L-F_2^L\right|+\left|F_2^L-F_3^L\right|+\left|F_1^U-F_2^U\right|+\left|F_2^U-F_3^U\right|\right)\end{aligned}$$

and

$$\begin{aligned}&\frac{1}{6}\left(\left|T_1^L-T_2^L\right|+\left|T_2^L-T_3^L\right|+\left|T_1^U-T_2^U\right|+\left|T_2^U-T_3^U\right|\right.\\&\quad+\left|I_1^L-I_2^L\right|+\left|I_2^L-I_3^L\right|+\left|I_1^U-I_2^U\right|+\left|I_2^U-I_3^U\right|\\&\quad\left.+\left|F_1^L-F_2^L\right|+\left|F_2^L-F_3^L\right|+\left|F_1^U-F_2^U\right|+\left|F_2^U-F_3^U\right|\right)\\&=\frac{1}{6}\left(\left|T_1^L-T_2^L\right|+\left|T_1^U-T_2^U\right|+\left|I_1^L-I_2^L\right|+\left|I_1^U-I_2^U\right|+\left|F_1^L-F_2^L\right|+\left|F_1^U-F_2^U\right|\right)\\&\quad+\frac{1}{6}\left(\left|T_2^L-T_3^L\right|+\left|T_2^U-T_3^U\right|+\left|I_2^L-I_3^L\right|+\left|I_2^U-I_3^U\right|+\left|F_2^L-F_3^L\right|+\left|F_2^U-F_3^U\right|\right)\\&=d_H(x,y)+d_H(y,z)\end{aligned}$$

i.e., $d_H(x,y)+d_H(y,z)\ge d_H(x,z)$.

(2) The Euclidian distance between x and is defined as follows.

$$d_E(x,y)=\sqrt{\frac{1}{6}\left(\left(T_1^L-T_2^L\right)^2+\left(T_1^U-T_2^U\right)^2+\left(I_1^L-I_2^L\right)^2+\left(I_1^U-I_2^U\right)^2+\left(F_1^L-F_2^L\right)^2+\left(F_1^U-F_2^U\right)^2\right)} \quad (10)$$

The proof is similar to that of (9), it is omitted here.

Further, we extend the distance between two INVs x and y to two INSs.

Definition 7 Let $X=([T_i^L,T_i^U],[I_i^L,I_i^U],[F_i^L,F_i^U])$ $(i=1,2,\cdots,n)$ and $Y=([\dot{T}_i^L,\dot{T}_i^U],[\dot{I}_i^L,\dot{I}_i^U],[\dot{F}_i^L,\dot{F}_i^U])$ $(i=1,2,\cdots,n)$ be two INSs, then

(1) The Hamming distance between X and Y is defined as follows

$$d_H(X,Y)=\frac{1}{6n}\sum_{i=1}^{n}\left(\left|T_i^L-\dot{T}_i^L\right|+\left|T_i^U-\dot{T}_i^U\right|+\left|I_i^L-\dot{I}_i^L\right|+\left|I_i^U-\dot{I}_i^U\right|+\left|F_i^L-\dot{F}_i^L\right|+\left|F_i^U-\dot{F}_i^U\right|\right) \tag{11}$$

(2) The Euclidian distance between X and Y is defined as follows

$$d_E(X,Y)=\sqrt{\frac{1}{6n}\sum_{i=1}^{n}\left(\left(T_i^L-\dot{T}_i^L\right)^2+\left(T_i^U-\dot{T}_i^U\right)^2+\left(I_i^L-\dot{I}_i^L\right)^2+\left(I_i^U-\dot{I}_i^U\right)^2+\left(F_i^L-\dot{F}_i^L\right)^2+\left(F_i^U-\dot{F}_i^U\right)^2\right)} \tag{12}$$

For example, if two INSs X and Y are (([0.5,0.6],[0.2,0.3],[0.9,0.9]), ([0.8,0.9],[0.4,0.4],[0.2,0.3]), ([0.3,0.4],[0.8,0.9],[0.7,0.8])) and (([0.7,0.8], [0.4,0.5],[0.2,0.3]),([0.5,0.6],[0.5,0.5],[0.3,0.4]),([0.1,0.2],[0.2,0.4],[0.3,0.4])), then the distances of Hamming and Euclidian between X and Y can be calculated as follows.

$$\begin{aligned}
d_H(X,Y)&=\frac{1}{6\times 3}\big(|0.5-0.7|+|0.6-0.8|+|0.2-0.4|\\
&+|0.3-0.5|+|0.9-0.2|+|0.9-0.3|+|0.8-0.5|+|0.9-0.6|\\
&+|0.4-0.5|+|0.4-0.5|+|0.2-0.3|+|0.3-0.4|+|0.3-0.1|\\
&+|0.4-0.2|+|0.8-0.2|+|0.9-0.4|+|0.7-0.3|+|0.8-0.4|\big)\\
&=0.3
\end{aligned}$$

$$\begin{aligned}
d_E(X,Y)&=SQRT\Bigg(\frac{1}{6\times 3}\big((0.5-0.7)^2+(0.6-0.8)^2\\
&+(0.2-0.4)^2+(0.3-0.5)^2+(0.9-0.2)^2+(0.9-0.3)^2\\
&+(0.8-0.5)^2+(0.9-0.6)^2+(0.4-0.5)^2+(0.4-0.5)^2\\
&+(0.2-0.3)^2+(0.3-0.4)^2+(0.3-0.1)^2+(0.4-0.2)^2\\
&+(0.8-0.2)^2+(0.9-0.4)^2+(0.7-0.3)^2+(0.8-0.4)^2\big)\\
&=0.26
\end{aligned}$$

3 An extended TOPSIS Method for multiple attribute decision making based on INSs

For a multiple attribute decision problem, let $A=(A_1,A_2,\cdots,A_m)$ be a discrete set of alternatives, $C=(C_1,C_2,\cdots,C_n)$ be the set of attributes, $W=(w_1,w_2,\cdots,w_n)^T$ be the weighting vector of the attributes, and meents $\sum_{j=1}^{n}w_j=1, w_j\geq 0$. where w_j is unknown. Suppose that $X=\left[x_{ij}\right]_{m\times n}$ is the decision matrix, where $x_{ij}=([T_{ij}^L,T_{ij}^U],[I_{ij}^L,I_{ij}^U],[F_{ij}^L,F_{ij}^U])$ takes the form of the INVs for alternative A_i with respect to attribute C_j.

The steps of the ranking the alternatives based on these conditions are shown as follows

Step 1. Standardized decision matrix

In general, there are two types in attributes, the more the attibute value is, the better the alternative is, this type is called benifit type; on the contrary, the more the attribute value is, the worse the alternative is, this type is called cost type.

In order to to eliminate the influence of the attribute types, we need convert the cost type to benifit type. Suppose the standardized matrix is expressed by $R=\left[r_{ij}\right]_{m\times n}$, where $r_{ij}=([\dot{T}_{ij}^L,\dot{T}_{ij}^U],[\dot{I}_{ij}^L,\dot{I}_{ij}^U],[\dot{F}_{ij}^L,\dot{F}_{ij}^U])$, then we have

$$\begin{cases} r_{ij}=x_{ij} & \text{if the attrbute } j \text{ is benifit type}\\ r_{ij}=\overline{x}_{ij} & \text{if the attrbute } j \text{ is cost type}\end{cases} \tag{13}$$

Where, $\overline{x}$ is the complement of x.

Step 2. Calculate attribute weights

Because the attribute weights are completely unkown, we need to determine the attribute weights. The maximizing deviation method, which is proposed by Wang [20], is a good tool to calculate the attribute weights for MADM problems with numerical information. The principle of this method is described as follows.

For a MADM problem, if the attribute values for all alternatives have little differences, such an attribute will play a small important role in ranking the alternatives, especially, for an attribute, if the attribute values for all alternatives are equal, the attribute has no effect on the rankng results. Contrariwise, if attribute values for all alternatives under an attribute have obvious differences, such an attribute will play an important role in ranking the alternatives. Based on this view, if the attribute values of all alternatives for a given attribute have a little deviations, we can assign a little weight for thsi attribute; otherwise, the attribute which makes larger deviations should be set a bigger weight. Especially, if the attribute values of all alternatives are all equal with respect to a given attribute, then the weight of such an attribute may be set to 0.

For a MADM problem, the deviation values of alternative A_i to all the other alternatives under the attribute C_j can be defined as $D_{ij}(w_j)=\sum_{l=1}^{m} d(r_{ij},r_{lj})w_j$, then $D_j(w_j)=\sum_{i=1}^{m} D_{ij}(w_j)=\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})w_j$ represents the total deviation values of all alternatives to the other alternatives for the attribute C_j. $D(w_j)=\sum_{j=1}^{n} D_j(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})w_j$ represents the deviation of all attributes for all alternatives to the other alternatives. The optimize model is constructed as follows:

$$\begin{cases} \max D(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})w_j \\ s.t\sum_{j=1}^{n} w_j^2=1, w_j\geq 0, j=1,2\ldots\ldots n \end{cases} \tag{14}$$

Then we can get

$$w_j=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d^2(r_{ij},r_{lj})}} \tag{15}$$

Furthermore, we can get the normalized attribute weight based on this model:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij},r_{lj})} \tag{16}$$

Step 3. Use the extended TOPSIS method to rank the alternatives

The basic principle of TOPSIS is that the best alternative should have the shortest distance to the positive ideal solution and the farthest distance to the negative ideal solution. The positive ideal solution (marked as V^+) is a best solution in which each attribute value is the best one of all alternatives, and the negative ideal solution (marked as V^-) is another worst solution in which each attribute value is the worst value of all alternatives. The steps of ranking the alternatives by the extended TOPSIS are shown as follows.

(1) calculate the weighted matrix

$$Y=(y_{ij})_{m\times n}=\begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \ldots & w_n r_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \tag{17}$$

Where $y_{ij}=([\ddot{T}_{ij}^L,\ddot{T}_{ij}^U],[\ddot{I}_{ij}^L,\ddot{I}_{ij}^U],[\ddot{F}_{ij}^L,\ddot{F}_{ij}^U])$

(2) Determine the positive ideal solution and negative ideal solution:

According to the definition of INV, we can define the absolute positive ideal solution and negative ideal solution shown as follows.

$$\begin{cases} y_j^+=([1,1],[0,0],[0,0]) \\ y_j^-=([0,0],[1,1],[1,1]) \end{cases} \quad j=1,2,\cdots,n \tag{18}$$

or we can select the virtual positive ideal solution and negative ideal solution by selecting the best values for each attribute from all alternatives.

$$\begin{cases} y_j^+=([\max_i \ddot{T}_{ij}^L,\max_i \ddot{T}_{ij}^U],[\min_i \ddot{I}_{ij}^L,\min_i \ddot{I}_{ij}^U], \\ \quad [\min_i \ddot{F}_{ij}^L,\min_i \ddot{F}_{ij}^U]) \\ y_j^-=([\min_i \ddot{T}_{ij}^L,\min_i \ddot{T}_{ij}^U],[\max_i \ddot{I}_{ij}^L,\max_i \ddot{I}_{ij}^U], \\ \quad [\max_i \ddot{F}_{ij}^L,\max_i \ddot{F}_{ij}^U]) \end{cases} \tag{19}$$

$$j=1,2,\cdots,n$$

(3) Calculate the distance between the alternative A_i and positive ideal solution/ Negative ideal solution

The distance between the alternative A_i and positive ideal solution/ negative ideal solution is:

$$\begin{cases} d_i^+=\sum_{j=1}^{n} d(y_{ij},y_j^+) \\ d_i^-=\sum_{j=1}^{n} d(y_{ij},y_j^-) \end{cases} \quad i=1,2,\cdots,m \tag{20}$$

(4) Calculate the relative closeness coefficient

$$RCC_i=\frac{d_i^+}{d_i^+ + d_i^-}.(i=1,2,\cdots,m) \tag{21}$$

(5) Rank the alternatives

Utilize the relative closeness coefficient to rank the alternatives. The smaller RCC_i is, the better alternative A_i is.

4 An application example

In order to demonstrate the application of the proposed method, we will cite an example about the investment selection of a company (adapted from [10]). There is a company, which wants to invest a sum of money to an industry. There are 4 alternatives which can be considered by a panel, including: (1) A1 is a car company; (2) A2 is a food company; (3) A3 is a computer company; (4) A4 is an arms company. The evaluation on the alternatives is based on three criteria: (1) C1 is the risk; (2) C2 is the growth; (3) C3 is the environmental impact. where C1 and C2 are benefit criteria, and C3 is a cost criterion. Suppose the criteria weights are unkown. The final decision information can be obtained by the INVs, and shown in table 1.

Table 1 The evaluation values of four possible alternatives with respect to the three criteria

	C_1	C_2	C_3
A_1	([0.4,0.5],[0.2,0.3],[0.3,0.4])	([0.4,0.6],[0.1,0.3],[0.2,0.4])	([0.7,0.9],[0.2,0.3],[0.4,0.5])
A_2	([0.6,0.7],[0.1,0.2],[0.2,0.3])	([0.6,0.7],[0.1,0.2],[0.2,0.3])	([0.3,0.6],[0.3,0.5],[0.8,0.9])
A_3	([0.3,0.6],[0.2,0.3],[0.3,0.4])	([0.5,0.6],[0.2,0.3],[0.3,0.4])	([0.4,0.5],[0.2,0.4],[0.7,0.9])
A_4	([0.7,0.8],[0.0,0.1],[0.1,0.2])	([0.6,0.7],[0.1,0.2],[0.1,0.3])	([0.6,0.7],[0.3,0.4],[0.8,0.9])

4.1 Ranking the alternatives in this example

We adopt the proposed method to rank the alternatives.

To get the best alternative(s), the following steps are involved:

(1) Convert the cost criterion to benefit criterion. Since C3 is a cost criterion, we can replace $x_{i3}\ (i=1,2,3,4)$ with $\overline{x}_{i3}\ (i=1,2,3,4)$, and get the decision matrix R :

$$R=\begin{bmatrix} ([0.4,0.5],[0.2,0.3],[0.3,0.4]) & ([0.4,0.6],[0.1,0.3],[0.2,0.4]) & ([0.4,0.5],[0.7,0.8],[0.7,0.9]) \\ ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.8,0.9],[0.5,0.7],[0.3,0.6]) \\ ([0.3,0.6],[0.2,0.3],[0.3,0.4]) & ([0.5,0.6],[0.2,0.3],[0.3,0.4]) & ([0.7,0.9],[0.6,0.8],[0.4,0.5]) \\ ([0.7,0.8],[0.0,0.1],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2],[0.1,0.3]) & ([0.8,0.9],[0.6,0.7],[0.6,0.7]) \end{bmatrix}$$

(2) Calculate attribute weights

About the distance in formula (16), we can use the Hamming distance defined in (9), and get $d(r_{ij},r_{lj})\ i,l=1,2,3,4;\ j=1,2,3.$

$$d(r_{11},r_{11})=d(r_{12},r_{12})=d(r_{13},r_{13})=0$$

$$d(r_{21},r_{11})=0.133, d(r_{22},r_{12})=0.083, d(r_{23},r_{13})=0.300$$

$$d(r_{31},r_{11})=0.033, d(r_{32},r_{12})=0.050, d(r_{33},r_{13})=0.250$$

$$d(r_{41},r_{11})=0.233, d(r_{42},r_{12})=0.100, d(r_{43},r_{13})=0.217$$

$$d(r_{11},r_{21})=0.133, d(r_{12},r_{22})=0.083, d(r_{13},r_{23})=0.300$$

$$d(r_{21},r_{21})=d(r_{22},r_{22})=d(r_{23},r_{23})=0$$

$$d(r_{31},r_{21})=0.133, d(r_{32},r_{22})=0.100, d(r_{33},r_{23})=0.083$$

$$d(r_{41},r_{21})=0.100, d(r_{42},r_{22})=0.017, d(r_{43},r_{23})=0.083$$

$$d(r_{11},r_{31})=0.033, d(r_{12},r_{32})=0.050, d(r_{13},r_{33})=0.250$$

$$d(r_{21},r_{31})=0.133, d(r_{22},r_{32})=0.100, d(r_{23},r_{33})=0.083$$

$$d(r_{31},r_{31})=d(r_{32},r_{32})=d(r_{33},r_{33})=0$$

$$d(r_{41},r_{31})=0.233, d(r_{42},r_{32})=0.117, d(r_{43},r_{33})=0.100$$

$$d(r_{11},r_{41})=0.233, d(r_{12},r_{42})=0.100, d(r_{13},r_{43})=0.217$$

$$d(r_{21},r_{41})=0.100, d(r_{22},r_{42})=0.017, d(r_{23},r_{43})=0.083$$

$$d(r_{31},r_{41})=0.233, d(r_{32},r_{42})=0.117, d(r_{33},r_{43})=0.100$$

$$d(r_{41},r_{41})=d(r_{42},r_{42})=d(r_{43},r_{43})=0$$

Then according to (16), we can get the attribute weights shown as follows.

$$w_1=0.366,\ w_2=0.197, w_3=0.437$$

(3) Use the extended TOPSIS method to rank the alternatives

(i) calculate the weighted matrix

In formula (17), we can calculate $w_j r_{ij}\ \ (i=1,2,3,4;\ j=1,2,3)$ by formula (7). For example, we can calculate

$$w_1 r_{11}=([1-(1-0.4)^{0.366},1-(1-0.5)^{0.366}],[0.2^{0.366},0.3^{0.366}],[0.3^{0.366},0.4^{0.366}])$$
$$=([0.171,0.224],[0.555,0.643],[0.643,0.715])$$

Then we can get the weighted matrix Y

$$Y = \begin{bmatrix} ([0.171,0.224],[0.555,0.643],[0.643,0.715]) & ([0.096,0.165],[0.635,0.789],[0.728,0.835]) & ([0.200,0.261],[0.856,0.907],[0.856,0.955]) \\ ([0.285,0.357],[0.430,0.555],[0.555,0.643]) & ([0.165,0.211],[0.635,0.728],[0.728,0.789]) & ([0.505,0.634],[0.739,0.856],[0.591,0.800]) \\ ([0.122,0.285],[0.555,0.643],[0.643,0.715]) & ([0.128,0.165],[0.728,0.789],[0.789,0.835]) & ([0.409,0.634],[0.800,0.907],[0.670,0.739]) \\ ([0.357,0.445],[0.000,0.430],[0.430,0.555]) & ([0.165,0.211],[0.635,0.728],[0.635,0.789]) & ([0.505,0.634],[0.800,0.856],[0.800,0.856]) \end{bmatrix}$$

(ii) Determine the positive ideal solution and negative ideal solution.

According to (19), we can get the virtual positive ideal solution and negative ideal solution shown asa follows.

$$y^{+} = (([0.357,0.445],[0.000,0.430],[0.430,0.555]) \\ ([0.165,0.211],[0.635,0.728],[0.635,0.789]) \\ ([0.505,0.634],[0.739,0.856],[0.591,0.739]))$$

$$y^{-} = (([0.122,0.224],[0.555,0.643],[0.643,0.715]) \\ ([0.096,0.165],[0.728,0.789],[0.789,0.835]) \\ ([0.200,0.261],[0.856,0.907],[0.856,0.955]))$$

(iii) Calculate the distance between the alternative A_i and positive ideal solution/ Negative ideal solution

According to (19), we can get the distance between the alternative A_i and positive ideal solution/ negative ideal solution shown as follows.

$$d_1^{+} = 0.532,\ d_2^{+} = 0.180,\ d_3^{+} = 0.377, d_4^{+} = 0.065$$

$$d_1^{-} = 0.034,\ d_2^{-} = 0.385,\ d_3^{-} = 0.189, d_4^{-} = 0.501$$

(iv) Calculate the relative closeness coefficient

According to (21), we can calculate the the relative closeness coefficient shown as follows.

$$RCC_1 = 0.941,\ RCC_2 = 0.319, RCC_3 = 0.666, RCC_4 = 0.114$$

(v) Rank the alternatives

According to the relative closeness coefficient, we can get the ranking from the best to worst.

$$A_4 \succ A_2 \succ A_3 \succ A_1$$

4.2 Compare with the existing method

In order to further illustrate the effectiveness of the proposed method in this paper, we compare with method proposed by Ye [10]. However, because the attribute weights and positive ideal solution/ Negative ideal solution are different from Ye [10], the ranking result is different; in addition, Ye [10] only consider the similarity measure between each alternative and positive ideal solution. If we adopt the same attribute weights and positive ideal solution ideal solution, and only consider the distance between each alternative and positive ideal solution, we can get the same ranking result from these two methods. Comparing with the method proposed by Ye [10], the method proposed in this paper can solve the multiple attribute problems with unknown weights, and can provide a compromise solution which considers the distances to positive ideal solution and Negative ideal solution. In addition, it is simpler in calculation process than Ye [10].

5 Conclusions

The interval neutrosophic set can be easier to express the incomplete, indeterminate and inconsistent information, and it is a generalization of fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and so on. This paper proposed the operational laws of the interval neutrosophic set, and defined the Hamming distance and the Euclidian distance. Then Maximizing deviation method is used to determine the attribute weights and the TOPSIS method is extended to interval neutrosophic set. Finally, an illustrative example has been given to show the steps of the developed method. It shows that this method is simple and easy to use and it constantly enriches and develops the theory and method of multiple attribute decision making, and proposed a new idea for solving the MADM problems. In the future, we shall continue working in the extension and application of the proposed method.

Acknowledgements

This paper is supported by the National Natural Science Foundation of China (No. 71271124), the Humanities and Social Sciences Research Project of Ministry of Education of China (No. 13YJC630104), the Natural Science Foundation of Shandong Province (No. ZR2013GQ011, No. ZR2011FM036), Social Science Planning Project of Shandong Province (No. 13BGLJ10), and Graduate education innovation projects in Shandong Province (SDYY12065).

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control 8(1965)338- 356.
[2] K.T. A tanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.
[3] K.T. Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems 33(1989)37-46.
[4] K.T. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 3(1989)343-349.
[5] K.T. Atanassov, Operators over interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64(1994)159-174.

[6] F Smarandache, A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic, American Research Press, Rehoboth , 1999.

[7] H.Wang, F. Smarandache, Y. Zhang R. Sunderraman, Single valued neutrosophic sets, Proc Of 10th 476 Int Conf on Fuzzy Theory and Technology, Salt Lake City, 477 Utah, 2005.

[8] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems 42(4) (2013) 386-394.

[8a] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, Applied Mathematical Modelling, (2013) doi: 10.1016/j.apm.2013.07.020.

[9] H. Wang, F. Smarandache, Y.Q. Zhang, et al., Interval neutrosophic sets and logic: Theory and applications in computing, Hexis, Phoenix, AZ, 2005.

[10] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, Journal of Intelligent & Fuzzy Systems, DOI: 10.3233/IFS-120724 (2013).

[11] C.L.Hwang and K. Yoon, Multiple Attribute Decision Making and Applications, Springer, New York, NY,1981.

[12] T.C. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114(2000) 1-9.

[13] F. Jin, P.D., Liu, X. Zhang, Evaluation Study of Human Resources Based on Intuitionistic Fuzzy Set and TOPSIS Method, Journal of Information and Computational Science 4(2007) 1023-1028.

[14] Y.Q. Wei, P.D. Liu, Risk Evaluation Method of High-technology Based on Uncertain Linguistic Variable and TOPSIS Method, Journal of Computers 4 (2009) 276-282.

[15] P.D. Liu, Multi-Attribute Decision-Making Method Research Based on Interval Vague Set and TOPSIS Method, Technological and Economic Development of Economy 15 (2009) 453–463.

[16] P.D. Liu, Y. Su, The extended TOPSIS based on trapezoid fuzzy linguistic variables, Journal of Convergence Information Technology 5 (2010) 38 – 53.

[17] P.D. Liu, An Extended TOPSIS Method for Multiple Attribute Group Decision Making based on Generalized Interval-valued Trapezoidal Fuzzy Numbers, Informatica 35 (2011) 185-196.

[18] S. Mohammadi, S. Golara, and N. Mousavi, Selecting adequate security mechanisms in e-business processes using fuzzy TOPSIS, International Journal of Fuzzy System Application 2(2012) 35-53.

[19] A.K. Verma, R. Verma, and N.C. Mahanti, Facility location selection: an interval valued intuitionistic fuzzy TOPSIS approach, Journal of Modern Mathematics and Statistics 4 (2010) 68-72.

[20] Y.M.Wang, Using the method of maximizing deviations to make decision for multi-indices, System Engineering and Electronics 7(1998) 24–26, 31.

Received: November 30, 2013. Accepted: December 8, 2013.