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Fault Diagnosis Based on the Updating Strategy of Interval-Valued Belief Structures*

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Abstract — This paper presents the dynamic method for fault diagnosis based on the updating of Interval-valued belief structures (IBSs). The classical Jeffrey's updating rule and the linear updating rule are extended to the framework of IBSs. Both of them are recursively used to generate global diagnosis evidence with the form of Interval basic belief assignment (IBBA) by updating the previous evidence with the incoming evidence. The diagnosis decision can be made by global diagnosis evidence. In the process of evidence updating, the similarity factors of evidence are used to determine switching between the extended Jeffrey's and linear updating rules, and to calculate the linear combination weights. The diagnosis examples of machine rotor show that the proposed method can provide more reliable and accurate results than the diagnosis methods based on Dempster-Shafer evidence theory.

Key words — Fault diagnosis, Dempster-Shafer evidence theory, Interval-valued belief structures, Evidence updating.

I. Introduction

Equipment fault diagnosis depends on monitoring information collected by sensors. In generally, the monitoring information or data is inherently incomplete, uncertain, and imprecise because of some unavoidable factors including the random disturbances in environment and the system errors of sensor instrument, *etc*[1−3]. Therefore, it is imperative to devise a fusion mechanism for minimizing such imprecision and uncertainty. Dempster-Shafer evidence theory (DST) can robustly deal with incomplete data and allows the representation of both imprecision and uncertainty[4]. It also provides Dempster's rule of combination to fuse multi-source information so as to reduce the uncertainty and yield more accurate diagnosis results than any single-source information[1−3].

When DST is used to diagnose faults, there are three steps[1−3]. Step one is to construct the frame of discernment including all possible fault modes. Step two is to obtain the Basic belief assignment (BBA) describing the degree of uncertainty that on-line monitoring information supports every fault mode and the subset of fault modes. Such a BBA can be also named as a piece of diagnosis evidence. Step three is to fuse these BBAs coming from different information sources and make diagnosis decision according to the fused results. The researches on this field mainly focus on the second and third steps. The related results can be found in recent publications[1−3,5].

Although these researches have given a distinct impetus to the application of DST in fault diagnosis, there are still two practical and vital questions[1,5]: (1) the BBA is required to have single-valued (crisp) belief degrees and belief structure, which may be too coarse and imperfect to measure uncertainty of fault information. It will miss useful information and even lead to incorrect diagnosis results; (2) The fused result is obtained by fusing the multiple local BBAs collected at same time step. However, in order to ensure the reliability and stability of decision-making, on-line diagnosis should consider dynamical relationship between the current fused result and those results at its adjacent historical time steps.

The first is about measure of uncertainty. Due to the uncertainties in human being's subjective knowledge and sensor observations, maintenance engineers and sensors are not definitely sure about their own judgments. So it is inappropriate for them to assign the precise single-valued belief degrees to every fault mode and the subset of fault modes[5]. In this case, Interval-valued belief structures (IBSs), as the extension of DST, provide the Interval-valued basic belief assignment (IBBA) and the corresponding combination rule to deal with this question. Compared with single-valued BBA, IBBA can contain more fault information and meets human's general understandings and conceptions[5−7]. Xu *et al.*[5] presented a diagnosis method based on IBBA to show IBSs can enhance accuracy of DST-based diagnosis system.

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The second is involved with the dynamic requirement of real-time diagnosis. Actually, the fused result of the above DST/IBSs-based diagnosis methods are all static, as it only synthesizes several pieces of diagnosis evidence collected at the same time step. However, running status of equipment usually varies with time. There are two main variations which should be considered[1]: (1) although equipment always keeps in normal status in operational cycle, some intermittent or abrupt external disturbances are so strong that the DST/IBSs-based system temporarily makes false judgments. Actually, these disturbances never lead to the internal faults of equipment, so right now, a diagnosis system should always make the correct judgments (no fault); (2) equipment may undergo a slow change from the normal status to a certain fault, or may abruptly jump from the normal status to a certain fault. In this case, a diagnosis system should make prompt and stable responses to these changes. A feasible way of solving the second question is to introduce updating strategy of evidence[1], based on which, the updated result at each time step can dynamically integrate the current static fused result with the historical fused results so as to make the global and stable judgment.

Some scholars have devoted to theoretical research on the updating strategies in different ways. Shafer *et al.* [8] and Smets *et al.*[9] respectively presented Jeffrey's rule of conditioning and Transferable belief model (TBM) on the assumption that the current (incoming) evidence is certain. Dubois *et al.*[10] reinterpreted Jeffrey's rule and gave the Jeffrey-like rule to update the Basic belief assignment (BBA) function and the corresponding belief function (Bel). Kulasekere *et al.*[11] gave the linear updating rule of evidence to combine the current BBA with the historical (original) BBA. However, these theoretical methods can not completely fit for dynamic diagnosis. For examples, the updated results given by the Jeffrey-like rule are excessively determined by the current diagnosis evidence. The linear updating rule is available, but the skills for selecting the linear combination weights seem impractical for dynamic diagnosis[1].

In order to solve the above two questions, we present the dynamic fault diagnosis method based on the updating of IBSs. Firstly, the static fusion method in Ref.[5] is used to generate the fused IBBA (the static diagnosis evidence) at each step. Secondly, the Jeffrey's and linear updating rules are extended to the framework of IBSs. Both of them are recursively used to generate the updated IBBA at kth step by updating the updated IBBA at the $(k-1)$th step with the incoming fused IBBA at the kth step, and then, the diagnosis decision can be made by the updated IBBA (the global diagnosis evidence) at every step. In the process of updating, the similarity factors of diagnosis evidence are used to determine switching between the extended Jeffrey's and linear updating rules, and to calculate the linear combination weights. Finally, the diagnosis examples of machine rotor show that the new updating strategy can provide more reliable and accurate results than the static DST/IBSs-based diagnosis methods.

II. Basics of Interval Belief Structures

Let $\Theta = \{\theta_j | j = 1, 2, \ldots, n\}$ be a finite nonempty set of mutually exclusive elements (propositions). It is called the frame of discernment. According to Denoeux's published work in Ref.[7], interval-valued belief structure, *i.e.* IBBA is defined as Definition 1.

Definition 1 Interval-valued belief structure[5−6]

Let $A_1, \ldots, A_N$ be N subsets of Θ and $[a_i, b_i]$ be n intervals with $0 \le a_i \le b_i \le 1$, $i = 1, \ldots, N$, an interval-valued belief structure, *i.e.* An IBBA is defined as a function Im

$$Im(A_i) = [a_i, b_i] \tag{1}$$

such that the following hold:

① $a_i \le m(A_i) \le b_i$, $m(A_i) \in Im(A_i)$, $0 \le a_i \le b_i \le 1$;

② $\sum_{i=1}^{N} a_i \le 1$ and $\sum_{i=1}^{N} b_i \ge 1$;

③ $m(H) = 0, \forall H \notin \{A_1, \ldots, A_N\}$;

According to the above definition, each subset A_i such that $a_i > 0$ is called a focal element of an IBBA. If $a_i = b_i = m(A_i)$, then an IBBA reduced to a BBA. Hence IBSs generalizes the concept of BBA. If $\sum_{i=1}^{N} a_i > 1$ or $\sum_{i=1}^{N} b_i > 1$, then Im is empty and invalid. Invalid IBBA cannot be interpreted as belief structure and thus need to be revised or adjusted[7].

Definition 2 Normalization of IBBA[6]

For a valid IBBA Im, if a_i and b_i satisfy respectively $\sum_{j=1}^{N} b_j - (b_i - a_i) \ge 1$, $\sum_{j=1}^{N} a_j + (b_i - a_i) \le 1$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, then Im is called to be normalized.

In the following, whenever we use the IBBA, we always suppose that it is valid and normalized, unless it is explicitly stated otherwise.

Definition 3 Combination rule of IBBAs[6]

Let Im_1 and Im_2 be two IBBAs with the intervals of belief masses $[a_i, b_i]$ $(a_i \le m_1(A_i) \le b_i,\ i = 1, \ldots, N_1)$ and $[c_j, d_j]$ $(c_j \le m_2(A_j) \le d_j,\ j = 1, \ldots, N_2)$ respectively. Their combination, denoted as $Im_1 \oplus Im_2$, is also an IBBA defined by

$$[Im_1 \oplus Im_2](C) = \begin{cases} 0, & C = \varnothing \\ [(m_1 \oplus m_2)^-(C), (m_1 \oplus m_2)^+(C)], & C \ne \varnothing \end{cases} \tag{2}$$

where $(m_1 \oplus m_2)^-(C)$ and $(m_1 \oplus m_2)^+(C)$ are respectively the minimum and maximum of the following pair of optimization problems:

$$\begin{aligned} &\max / \min \quad [Im_1 \oplus Im_2](C) = \frac{\sum\limits_{A_i \cap A_j = C} m_1(A_i) m_2(A_j)}{1 - \sum\limits_{A_i \cap A_j = \varnothing} m_1(A_i) m_2(A_j)} \\ &\text{s.t.} \quad \sum_{i=1}^{N} m_1(A_i) = 1,\ a_i \le m_1(A_i) \le b_i, i = 1, 2, \ldots, N_1 \\ &\qquad \sum_{j=1}^{N} m_2(A_j) = 1,\ c_j \le m_2(A_j) \le d_j, j = 1, 2, \ldots, N_2 \end{aligned} \tag{3}$$

Referring to Ref.[6], the combination of two IBBAs in Definition 3 can also be extended to the situation of multiple IBBAs.

Actually, for an IBBA Im, if it's any $m(A_i)$ satisfies the constraint $\sum_{i=1}^{N} m(A_i) = 1$, then m is the crisp BBA of this

IBBA. So, the main idea of the combination rule in Eq.(3) can be interpreted as: the crisp BBAs selected respectively from the two IBBAs are combined by using the classical Dempster combination rule. Thus, the fused IBBA can be obtained from maximizing/minimizing the crisp fused BBAs. Each of the above pair of models (max/min) simultaneously considers the combination and normalization of two IBBAs and optimizes them together rather than separately. The reason for doing so is to capture the true belief mass intervals of the combined focal elements[6]. This optimality approach for combining and normalizing IBBAs is more effective and efficient than the existing approaches given in Ref.[7]. Related numerical examples can be found in Ref.[10].

III. The Updating Strategies of IBSs

So far, the Jeffrey-like rule and the linear updating rule of evidence are the most popular updating strategies in DST[10−11]. In this section, both of them are introduced and extended to IBSs for dynamical diagnosis.

1. The extended Jeffrey-like rule in the framework of IBSs

Let m_1 and m_2 be two BBA on Θ with the sets $X_1 = \{B_i|i = 1,\ldots,N_1\}$ and $X_2 = \{A_j|j = 1,\ldots,N_2\}$, B_i and A_j are the focal elements of m_1 and m_2 respectively. Assume that m_1 and m_2 are the original and incoming BBAs respectively.

The Jeffrey-like rule in DST is defined as[10]

$$m(C|m_2) = \sum_{A_j\in X_2} m_2(A_j)m_1(C|A_j) \tag{4}$$

$$m_1(C|A_j) = \sum_{\varnothing\neq C=B_i\cap A_j} m_1(B_i)/Pl_1(A_j) \tag{5}$$

where $C \in X_1\cap X_2$, Pl is Plausibility function corresponding to BBA. By updating m_1 with m_2, Eq.(4) gives the updated belief masses of C, the intersections between B_i and A_j. Actually, this rule re-distributes the belief masses to those propositions simultaneously supported by m_1 and m_2. It can be seen that $m(C|m_2)$ is the weighted sums of the belief masses of those focal elements related to C in X_2. So, the updated BBA is largely determined by the incoming BBA. Specially, when the focal elements of m_2 make a partition of Θ, the updated BBA is equal to m_2, namely, the current updated result has nothing to do with the historical information. Obviously, this rule only applies to the case that current updated result and historical result have the similar opinion about running status of equipment. However, in most cases, the running status of equipment usually varies with time[1−2], so the Jeffrey-like rule cannot be completely suitable for dynamic fault diagnosis.

Following the spirit of optimization in Definition 3, we present the extended Jeffrey-like rule in the framework of IBSs shown in Definition 4.

Definition 4 The extended Jeffrey-like rule of IBBAs

Let Im_1 and Im_2 be two IBBAs with the intervals of belief masses $[a_i, b_i]$ ($a_i \le m_1(A_i) \le b_i$, $i = 1,\ldots,N_1$) and $[c_j, d_j]$ ($c_j \le m_2(A_j) \le d_j$, $j = 1,\ldots,N_2$) respectively. $X_1 = \{B_i|i = 1,\ldots,N_1\}$ and $X_2 = \{A_j|j = 1,\ldots,N_2\}$ are the sets of the focal elements of Im_1 and Im_2 respectively. Assume that Im_1 and Im_2 are the original and incoming IBBAs respectively. The extended Jeffrey-like rule of IBBAs is defined as

$$Im_1\overleftarrow{\otimes}Im_2(C) = \begin{cases} 0, & C\notin X_1\cap X_2 \\ [(m_1\overleftarrow{\otimes}m_2)^-(C), (m_1\overleftarrow{\otimes}m_2)^+(C)], & C\in X_1\cap X_2 \end{cases} \tag{6}$$

where $(m_1\overleftarrow{\otimes}m_2)^-(C)$ and $(m_1\overleftarrow{\otimes}m_2)^+(C)$ are respectively the minimum and maximum of the pair of optimization problems:

$$\begin{aligned} \max/\min\quad [Im_1\overleftarrow{\otimes}Im_2](C) &= \sum_{A_j\in X_2} m_2(A_j)m_1(C|A_j) \\ &= \sum_{A_j\in X_2} m_2(A_j)\Big(\sum_{\varnothing\neq C=B_i\cap A_j} m_1(B_i)/Pl_1(A_j)\Big) \\ \text{s.t.}\quad & \sum_{i=1}^{N_1} m_1(B_i) = 1,\ a_i \le m_1(B_i)\le b_i, i = 1,2,\ldots,N_1 \\ & \sum_{j=1}^{N_2} m_2(A_j) = 1,\ c_j \le m_2(A_j)\le d_j, j = 1,2,\ldots,N_2 \end{aligned} \tag{7}$$

For m_2 and $m_1(\cdot|A)$ are normalized in Eq.(7)[10], the crisp BBA $m(\cdot|m_2)$ calculated by Eq.(4) is also normalized. The updated IBBA can be obtained from maximizing/ minimizing the crisp updated BBAs.

2. The extended linear updating rule in the framework of IBSs

Fagin *et al.*[12] defined the notions of conditional belief and plausibility functions. For any two focal elements $A \in 2^{\Theta}$, $B \in 2^{\Theta}$, the conditional belief and plausibility functions are defined respectively as

$$\begin{aligned} Bel(B|A) &= \frac{Bel(A\cap B)}{Bel(A\cap B) + Pl(A-B)} \\ Pl(B|A) &= \frac{Pl(A\cap B)}{Pl(A\cap B) + Bel(A-B)} \end{aligned} \tag{8}$$

Based on which, Kulasekere *et al.*[11] directly deduced conditional BBA on the assumption $B \subseteq A$

$$m(B|A) = \frac{\sum_{C:C\subseteq B} m(C)}{Pl(A) - \sum_{E:E\in\ell(B)} m(E)} - \sum_{C:C\subsetneq B} m(C|A) \tag{9}$$

where, $\ell(B) = \{E \subseteq \Theta : E = D\cup C \text{ s.t. } \varnothing \neq D \subseteq \bar{A}, \varnothing \neq C \subseteq B \subseteq A\}$ and when $\bar{A}\bigcap B \neq \varnothing$, $m(B|A) = 0$. Especially, for all $B \subseteq A$, s.t. $m(B) = Bel(B)$, then Eq.(10) is reduced to

$$m(B|A) = \frac{m(B)}{Pl(A) - \sum_{E:E\in\ell(B)} m(E)} = \frac{m(B)}{m(B) + Pl(A-B)} \tag{10}$$

For example, assume that the belief mass distribution of the original BBA m is $m(\{\theta_1\}) = 0.1$, $m(\{\theta_2\}) = 0.3$, $m(\{\theta_3\}) = 0.4$, $m(\{\theta_2,\theta_3\}) = 0.2$. There is an incoming piece of evidence with focal element $A = \{\theta_2,\theta_3\}$. When $B \in \{\{\theta_1\},\{\theta_2\},\{\theta_3\},\{\theta_2,\theta_3\}\}$, the corresponding conditional Bel, Pl and m given the conditioning proposition A can be calculated by Eq.(9) or (10), as shown in Table 1. It can be seen that the belief masses of those propositions included in the complement of the conditioning proposition A are be annulled, on the other hand, the belief masses of the remaining propositions related to A are be re-distributed by the conditioning operation. It implied that when one attempts to make decisions by the conditional BBA, the conditioning proposition A derived from the incoming evidence should have the maximal mass, or definitely $m(A) = 1$, *i.e.*, the new evidence completely supports the proposition A, which can be confirmed in the example of a distributed decision-making network illustrated in Ref.[11].

Table 1. The conditional *Bel*, *Pl* and *m*

B	$Bel(B)$	$Pl(B)$	$m(B)$	$Bel(B\|A)$	$Pl(B\|A)$	$m(B\|A)$
θ_1	0.1	0.1	0.1	0	0	0
θ_2	0.3	0.5	0.3	0.3/0.9	0.5/0.9	0.3/0.9
θ_3	0.4	0.6	0.4	0.4/0.9	0.6/0.9	0.4/0.9
$\{\theta_2, \theta_3\}$	0.9	0.9	0.2	1	1	0.2/0.9

Kulasekere *et al.*[11] defined the linear updating rule of evidence, *i.e.* a linear combination of the original BBA and the incoming conditional BBA, as follow:

$$m_A(B) = \alpha_A m(B) + \beta_A m(B|A) \tag{11}$$

$m(B)$ is the original belief mass to $B \in 2^{\Theta}$, $m(B|A)$ quantifies the degree that an incoming BBA with the definite mass "$m(A) = 1$" supports the focal element B. $m_A(B)$ is the updated belief mass of B conditional to A. The linear combination weights $\{\alpha_A, \beta_A\}$ can be interpreted as the measures indicating the flexibility or inertia of the original evidence (BBA) to updating when presented with the incoming conditioning proposition A. Some basic strategies for selecting $\{\alpha_A, \beta_A\}$ were introduced in Ref.[11]:

(1) The choice $\{\alpha_A, \beta_A\} = \{1, 0\}$ is called the infinite inertia based updating strategy. The original evidence has complete inflexibility towards changes. For example, the original evidence is derived from a vast collection of reliable data, but the incoming evidence is completely unreliable, which leads to a high inertia, *etc.*

(2) The choice $\{\alpha_A, \beta_A\} = \{0, 1\}$ is called the zero inertia based updating strategy. The original evidence has the complete flexibility towards changes. For instance, the original evidence is derived from little or no credible knowledge, but the incoming evidence is completely reliable, *etc.*

(3) The choice $\{\alpha_A, \beta_A\} = \{N/(N+1), 1/(N+1)\}$ is called the proportional inertia based updating strategy. N refers to the number of "pieces" of evidence that the original evidence is based upon. In this case, the gathered evidence and the incoming evidence have equal inertia.

In practical application of fault diagnosis, many pieces of diagnosis evidence are commonly gathered at each step. The updated result is recursively calculated by Eq.(11) at each step, which is related to the current incoming evidence and the previous original evidence. Considering the uncertainty of information or knowledge used to generate evidence and the variability of equipment running status, the above three method are static and seem less flexible and changeable.

Following the spirit of optimization in Definition 3, we present the extended linear updating rule in the framework of IBSs shown in Definition 5.

Definition 5 The extended linear updating rule of IBBAs

For Im_1 and Im_2 given in Definition 4, the extended linear updating rule of IBBAs is defined as

$$Im_1 \overleftarrow{\oplus} Im_2(C) = \begin{cases} 0, & C = \varnothing \\ [(m_1 \overleftarrow{\oplus} m_2)^-(C), (m_1 \overleftarrow{\oplus} m_2)^-(C)], & C \neq \varnothing \end{cases} \tag{12}$$

here, $(m_1 \overleftarrow{\oplus} m_2)^-(C)$ and $(m_1 \overleftarrow{\oplus} m_2)^-(C)$ are respectively the minimum and maximum of the following pair of optimization problems:

$$\begin{aligned} &\max / \min \quad [Im_1/Im_2](C) = \alpha_A m_1(C) + \beta_A m(C|A) \\ &= \alpha_A m_1(C) + \beta_A \Bigg(\Bigg(\sum_{B_i \subseteq C} m_1(B_i) \Big/ \Big(Pl_1(A) \\ &\quad - \sum_{B_i \in \ell(C)} m_1(B_i) \Big) \Bigg) - \sum_{B_i \subseteq C} m_1(B_i|A) \Bigg) \\ &\text{s.t.} \ \sum_{i=1}^{N_1} m_1(B_i) = 1 \ (a_i \le m_1(B_i) \le b_i, i = 1, 2, \ldots, N_1) \\ &\quad\ \ \sum_{j=1}^{N_2} m_2(A_j) = 1 \ (c_j \le m_2(A_j) \le d_j, j = 1, 2, \ldots, N_2) \end{aligned} \tag{13}$$

where, $\ell(B) = \{E \subseteq \Theta : E = D \cup C \text{ s.t. } \varnothing \neq D \subseteq \bar{A}, \varnothing \neq C \subseteq B \subseteq A\}$, A is given by Pignistic transformation $(BetP)$[1,9]

$$A = \arg\max_{A_j}(BetP_{m_2}(A_j)) \tag{14}$$

$$BetP_{m_2}(A) = \sum_{\theta \in A} BetP_{m_2}(\theta) \tag{15}$$

$$BetP_{m_2}(\theta) = \sum_{A_j \in X_2, \theta \in A_j} \frac{1}{|A_j|} \frac{m_2(A_j)}{1 - m(\varnothing)}, \quad m(\varnothing) \neq 1 \tag{16}$$

where $|A_j|$ is the cardinality of focal element A_j. $BetP_m(\theta)$ in Eq.(16) is defined as the Pignistic probability function on Θ[9]. The transformation from m to $BetP_m$ is called the pignistic transformation. $BetP_m(A)$ in Eq.(15) is the extended form of Eq.(16) on 2^{Θ}.

Since the above basic strategies for selecting $\{\alpha_A, \beta_A\}$ given in Ref.[11] are not suitable for dynamic diagnosis fault, so, in our following applications, we present the new method for selecting $\{\alpha_A, \beta_A\}$ based on the similarity measure of IBBAs introduced in next section.

IV. The Similarity Measure of IBSs

In our early work[13], we have given the similarity measure of IBSs based on the extended Pignistic probability function ($IBetP$) and the normalized Euclidean distance.

Let Im be an IBBA with the intervals of belief masses $Im(A_i) = [a_i, b_i]$ $(0 \le a_i \le b_i \le 1)$, $A_i \in \{A_i | i = 1, 2, \ldots, N\}$, the extended Pignistic probability function of IBSs is defined as

$$IBetP_{Im}(\theta) = [BetP_m^-(\theta), BetP_m^+(\theta)] \tag{17}$$

$BetP_m^-(\theta)$ and $BetP_m^+(\theta)$ are respectively the minimum and maximum of the following pair of optimization problems:

$$\max/\min \ \ BetP_m(\theta) = \sum_{A_i \subseteq \Theta, \theta \in A_i} \frac{1}{|A_i|} \frac{m(A_i)}{1 - m(\varnothing)}, m(\varnothing) \neq 1$$

$$\text{s.t.} \quad \sum_{i=1}^{N} m(A_i) = 1,\ a_i \le m(A_i) \le b_i, i = 1, 2, \ldots, N \quad (18)$$

Actually, the extended Pignistic transformation projects the mass intervals of subsets of Θ into a new orthogonal space about the singleton elements of Θ. Each dimension of this new space is defined as $IBetP_{Im}(\theta_i)$, $i = 1, 2, \ldots, n$. In this orthogonal space, we use normalized Euclidean distance to measure the degree of similarity between both *IBetP*s so as to indirectly measure the degree of similarity between their corresponding IBBAs.

Let Im_1, Im_2 are two IBBAs on $\Theta = \{\theta_i | i = 1, 2, \ldots, n\}$. Their corresponding *IBetP*s are denoted as $IBetP_{Im_1}$ and $IBetP_{Im_2}$ respectively. The normalized Euclidean distance can be defined as

$$d(IBetP_{Im_1}, IBetP_{Im_2}) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(I_i^-)^2 + (I_i^+)^2} \quad (19)$$

where, $I_i^- = BetP_{m_1}^-(\theta_i) - BetP_{m_2}^-(\theta_i)$, $I_i^+ = BetP_{m_1}^+(\theta_i) - BetP_{m_2}^+(\theta_i)$, and

$$IBetP_{m_1}(\theta_i) = [BetP_{m_1}^-(\theta_i), BetP_{m_1}^+(\theta_i)]$$

$$IBetP_{m_2}(\theta_i) = [BetP_{m_2}^-(\theta_i), BetP_{m_2}^+(\theta_i)]$$

The smaller $d(IBetP_{Im_1}, IBetP_{Im_2})$ is, the more similar Im_1 and Im_2 are, and vice versa. The degree of similarity between Im_1 and Im_2 can be defined as[13]

$$Sim(Im_1, Im_2) = 1 - d(IBetP_{Im_1}, IBetP_{Im_2}) \quad (20)$$

$Sim(Im_1, Im_2) \in [0, 1]$, $Sim(Im_1, Im_2) = 0$ means that Im_1 and Im_2 are totally different; $Sim(Im_1, Im_2) = 1$ means that Im_1 and Im_2 are completely identical.

If there are N IBBAs on Θ, as $Im_1, Im_2, \ldots, Im_N$, then the degree that Im_i is supported by the other $N - 1$ IBBAs can be defined as

$$Sup(Im_i) = \sum_{j=1, j\ne i}^{N} Sim(Im_i, Im_j) \quad (21)$$

The credibility degree of Im_i is defined as

$$Crd(Im_i) = Sup(Im_i) \Big/ \sum_{i=1}^{N} Sup(Im_i) \quad (22)$$

$\sum_{i=1}^{N} Crd(Im_i) = 1$, thus, the credibility degree is a weight showing the relative importance of the collected evidence.

V. Dynamical Fault Diagnosis Using the Updating Strategy of IBSs

In this section, we propose a new updating procedure of IBSs to solve the uncertainty question (the first two steps) and dynamic question (the third step) in fault diagnosis.

Step 1 Acquire multiple pieces of diagnosis evidence at every time step

This step is to deal with the uncertainty question in fault diagnosis. At every time step, we need to obtain the degree of uncertainty that on-line monitoring information supports every fault mode and the subset of fault modes. In the framework of IBSs, this degree is described as an IBBA. Here, the method in Ref.[8] is used to get multiple pieces of diagnosis evidence from multi-source information.

Step 2 Obtain the incoming fused IBBA by combining the multiple IBBAs at kth step

In this step, the multiple pieces of diagnosis evidence (IBBAs) are fused by using the optimal combination rule in Definition 3. The fused IBBA is the incoming evidence at kth step ($k = 1, 2, 3, \ldots$). The function of combination rule is to reduce the uncertainty of local diagnosis evidence such that the fused IBBA is more certain and precise than any local IBBA.

Step 3 Obtain the updated IBBA at the kth step by updating the updated IBBA at the $(k-1)$th step with the incoming fused IBBA

This step presents the new switching updating strategy based on similarity measure of IBSs. Suppose the updated IBBAs $Im_{1:k-2}$ and $Im_{1:k-1}$ at the $(k-2)$th and $(k-1)$th steps have been recursively obtained respectively and the incoming fused IBBA Im_k at the kth step have been calculated from Step 2. The degrees of similarity between Im_k and $Im_{1:k-2}$, $Im_{1:k-1}$ can be got respectively by Eq.(20)

$$Sim(Im_k, Im_{1:k-1}) = 1 - d(IBetP_{Im_k}, IBetP_{Im_{1:k-1}}) = Sim_{k,1:k-1} \quad (23)$$

$$Sim(Im_k, Im_{1:k-2}) = 1 - d(IBetP_{Im_k}, IBetP_{Im_{1:k-2}}) = Sim_{k,1:k-2} \quad (24)$$

Set the switching threshold as δ, $\delta \in [0.5, 1]$. If $\min(Sim_{k,1:k-2}, Sim_{k,1:k-1}) \ge \delta$, then the extend Jeffery-like rule given in Definition 4 is used to calculate the updated IBBA $Im_{1:k}$ by updating $Im_{1:k-1}$ with Im_k. It means that the current fused result and the previous global results have the similar opinion about the running status of equipment. In this case, in large measure, the current updated IBBA can be determined by the incoming fused IBBA, as analyzed in Section III.1. Therefore, generally speaking, δ should be greater than or equal to 0.5 and can be adjusted according to application environments. Specially, when $k = 1$, we set $Im_{1:k} = Im_k$, when $k = 2$, $\min(Sim_{k,1:k-2}, Sim_{k,1:k-1})$ becomes $Sim_{k,1:k-1}$ directly.

On the other hand, if $\min(Sim_{k,1:k-2}, Sim_{k,1:k-1}) < \delta$, then the extend linear updating rule given in Definition 5 is used to get $Im_{1:k}$. It means that the current fused result and the previous global results are conflict with each other. So, the extend Jeffery-like rule is not applicable any longer. On the contrary, the extended linear updating rule can give the reasonable updated result by adjusting the linear combination weights $\{\alpha_k, \beta_k\}$. But the original methods for choosing $\{\alpha_k, \beta_k\}$ are not available for dynamic fault diagnosis, so we give the new method based on the similarity measure of IBSs. At the kth step, we calculate $Sup(Im_{1:k-2})$, $Sup(Im_{1:k-1})$, $Sup(Im_k)$ by Eq.(21), and $Crd(Im_{1:k-2})$, $Crd(Im_{1:k-1})$, $Crd(Im_k)$ by Eq.(22) respectively and set

$$\alpha_k = Crd(Im_{1:k-2}) + Crd(Im_{1:k-1}) \quad (25)$$

$$\beta_k = Crd(Im_k) \quad (26)$$

Especially, when $k=1$, we set $Im_{1:k}=Im_k$, when $k=2$, $\alpha_k=\beta_k=0.5$ directly.

Step 4 Make diagnosis decision according to the updated (global) IBBA at the kth step

At every time step, a diagnosis decision can be made according to the updated results[5]. There are two popular criterions in diagnosis decision:

(1) For the determined fault proposition, the left and right endpoints of its belief mass are respectively greater than those of belief mass of other fault propositions;

(2) The right endpoint of $Im(\Theta)$ must be smaller than a certain threshold, where, we will use 0.3 experientially.

VI. Fault Diagnosis Experiments of Machine Rotor System

The proposed updating strategy is verified by ZHS-2 machine rotor system used in Ref.[5]. A vibration displacement sensor and a vibration acceleration sensor are respectively installed on the bracket of rotor to collect vibration signals in both vertical and horizontal directions. These signals are inputted into HG-8902 data collector, and then processed by signal conditioning circuits. Finally, the processed signals are inputted into laptop. The fault features can be extracted from these signals by HG-8902 data analysis software (under environment of Labview). The typical faults seeded in the system are rotor unbalance, rotor misalignment and motor bracket loosening, thus, the frame of discernment is $\Theta=\{F_0, F_1, F_2, F_3\}$, where, F_0 = normal condition, F_1 = rotor unbalance, F_2 = rotor misalignment, and F_3 = motor bracket loosening. The amplitudes of fundamental, double, triple vibration acceleration frequencies (denoted as $f_{\times 1}$–$f_{\times 3}$ respectively) and average amplitude of vibration displacement (denoted as d_a) are selected as the fault feature parameters[5].

We give four typical fault experiments, in which, the proposed updating strategy are compared with other methods to show its advantages.

Experiment 1 The rotor system always keeps in normal condition (F_0) at the kth step, $k=1,2,\ldots,10$.

At every time step, the method in Ref.[5] is used to get the four local IBBAs respectively from the monitoring data of $f_{\times 1}$, $f_{\times 1}$, $f_{\times 3}$ and d_a, and then, the optimal combination rule in Definition 3 is used to calculate the incoming fused IBBA m_k by fusing the local IBBAs. In Fig.1, the belief masses of m_k are denoted as Incoming belief mass (IBM). $Im_k(\{F_0\})$, $Im_k(\{F_1\})$, $Im_k(\{F_2\})$ and $Im_k(\{F_3\})$ are shown except $Im_k(\Theta)$, because $Im_k(\Theta)$ usually becomes so small by optimal combination that it rarely influences the decision making. For example, the interval value of belief masses of Im_8 illustrated in Fig.1. The updated IBBAs obtained recursively using four updating or combination rules can be respectively shown in Fig.1, including the Combination rule (CR) in Definition 3, the Extended Jeffery-like rule (EJLR), the Extended linear updating rule (ELUR) and the proposed New updating strategy (NUS).

The system is always normal, so at every time step, $Im_k(\{F_0\})$ is always larger than $Im_k(\{F_1\})$, $Im_k(\{F_2\})$, $Im_k(\{F_3\})$ and $Im_k(\Theta)$. In NUS, the switching threshold δ is taken as 0.75. When $k=1,2,3$, $\min(Sim_{k,1:k-2}, Sim_{k,1:k-1})\geq\delta$, then EJLR is chosen as the updating rule, so both NUS and EJLR have same updated results; When $k=4,5,\ldots,10$, $\min(Sim_{k,1:k-2}, Sim_{k,1:k-1})<\delta$, NUS is switched to ELUR with the linear combination weights $\{\alpha_k,\beta_k\}$ given by Eqs.(25) and (26). In ELUR, according to the proportional inertia based updating strategy, when $k=1$, $\{\alpha_k,\beta_k\}=\{0,1\}$, otherwise, $\{\alpha_k,\beta_k\}=\{(k-1)/k, 1/k\}$ (in the following experiments, the strategy is always used). In CR, the updated IBBA at the kth step is obtained by directly fusing all incoming fused IBBAs from the first step to the kth step. From Fig.1, it can be seen, the updated results of NUS, ELUR and CR are better than the incoming fused results. For the incoming fused IBBAs all support F_0, so CR and ELUR make belief masses focus to F_0. At every time step, the updated IBBA of EJLR is similar with the incoming fused IBBA, because the former are overly determined by the latter. Although NUS does not give better results than CR and ELUR, it is available in accordance with the decision criterions.

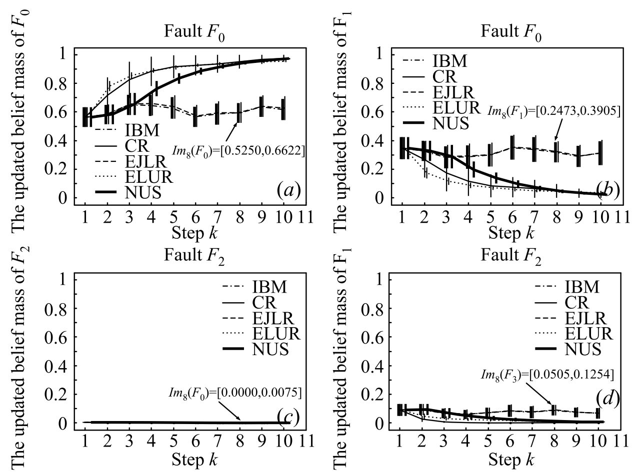

Fig. 1. The updated IBBAs of CR, EJLR, ELUR and NUS in Experiment 1

Experiment 2 The rotor system keeps in normal condition, but encounters the abrupt external disturbances at the certain time steps, and then, returns to normal when the disturbances disappear. There are two detailed cases.

Case 1 The system only encounters the disturbance at the sixth step. It causes the false fault "F_1".

Case 2 The system continuously encounters the disturbances at the sixth and seventh steps. They cause the false faults "F_1" and "F_3" respectively.

The updated results in two cases are shown in Fig.2 and Fig.3 respectively. It can be concluded that, the disturbances are so strong that the incoming fused IBBAs incorrectly support false faults. However, by evidence updating, ELUR and NUS make the correct judgment according to the decision criterions. EJLR makes mistake because its updated results overly depend on the incoming fused IBBA. Because of the conflicts between the incoming fused IBBAs, the interval widths of belief masses given by CR become too large to make decisions. Obviously, NUS still give satisfactory results.

Experiment 3 The rotor system is normal from the first step to the fifth step, but the fault "rotor unbalance (F_1)" suddenly happens at the sixth step and goes on until the tenth step.

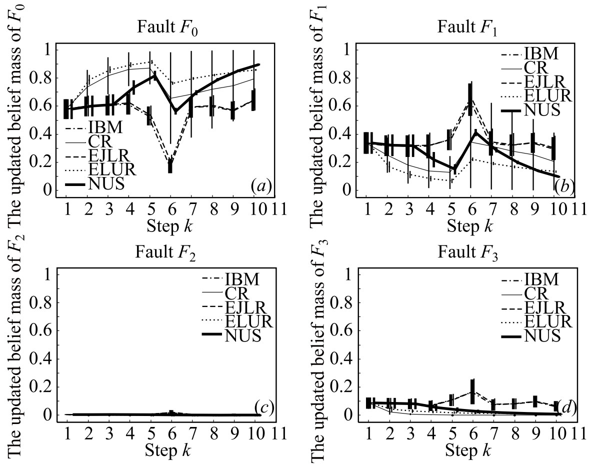

Fig. 2. The updated IBBAs of CR, EJLR, ELUR and NUS in Case 1 of Experiment 2

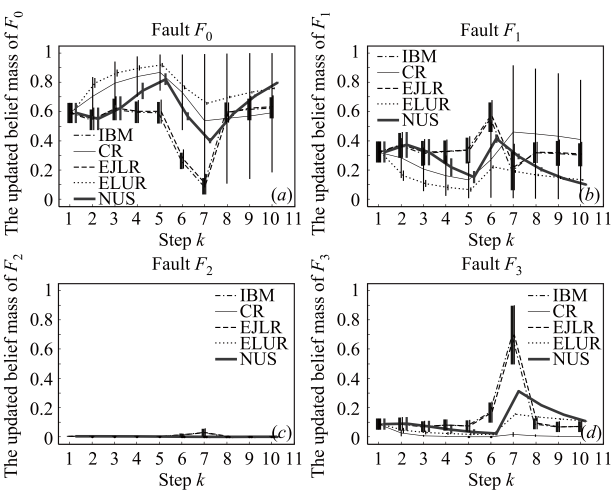

Fig. 3. The updated IBBAs of CR, EJLR, ELUR and NUS in Case 2 of Experiment 2

The updated results are shown in Fig.4. Because adopting the proportional inertia based updating strategy, ELUR becomes insensitive to the sudden changes of running status of the system. Consequently, it always makes incorrect judgments when the fault exists. Although EJLR can make correct judgments, but its updated IBBAs are similar with the incoming fused IBBAs, so the effects of updating are weak. CR is also inapplicable because of the same reason as in Experiment 2. On the contrary, NUS is the tradeoff between EJLR and ELUR. When the system is normal, NUS is consistent with EJLR; when F_1 happens, NUS is switched to ELUR. Although it makes mistake when F_1 just happens at sixth step, the belief masses of NUS can quickly and stably focus to F_1 and the interval widths of belief masses become less than those of EJLR at the following steps, which are advantageous to assure convincing and accurate decision-makings.

Experiment 4 The rotor system goes through the intermediate stage between normal and fault. More specifically, the system is normal from the first step to the third step, from the fourth step to the seventh step, the running status of the system gradually degrades to F_1, and then, F_1 really happens at remaining three steps.

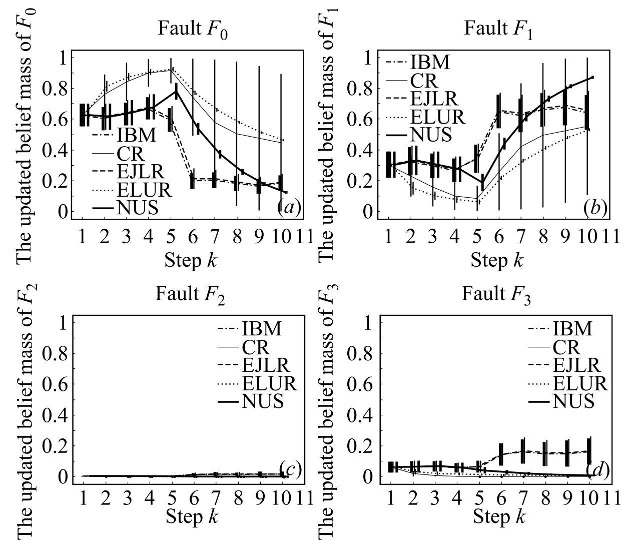

Fig. 4. The updated IBBAs of CR, EJLR, ELUR and NUS in Experiment 3

Fig.5 shows the updated results. Obviously, compared with other methods, NUS has the best performance. Moreover, it can predict fault timely and accurately when the running status of the system just begins to deteriorate.

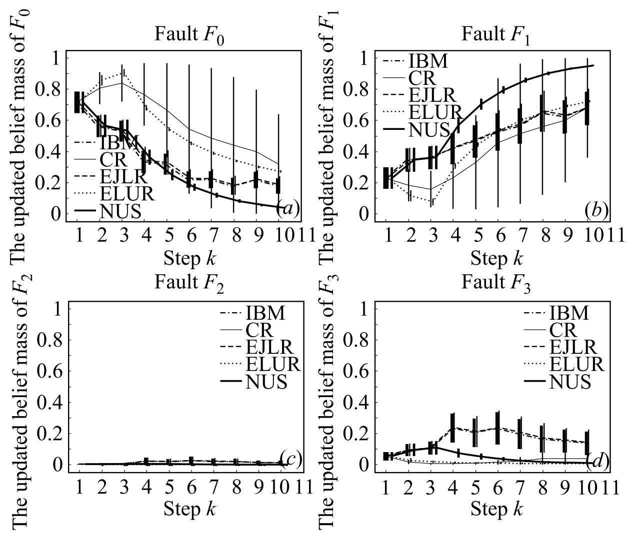

Fig. 5. The updated IBBAs of CR, EJLR, ELUR and NUS in Experiment 4

VII. Conclusion

This paper presents a new updating strategy for dynamic diagnosis based on IBSs. The main contribution includes: (1) The classical Jeffrey's updating rule and the linear updating rule are extended to the framework of IBSs; (2) NUS can adaptively choose appropriate updating rules according to real-time change of incoming fused diagnosis evidence; (3) NUS can adaptively adjust the linear combination weights of ELUR in terms of similarity relationship between the incoming diagnosis evidence and the previous diagnosis evidence. The fault experiments show that NUS has better comprehensive performance than other updating strategies. It is applicable to some typical cases of dynamic diagnosis in the real word.

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