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Research Paper

A smarandache completely prime ideal with respect to an element of near ring

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Abstract

In this paper we introduce the notion of a smarandache completely prime ideal with respect to an element belated to a near field of a near ring N (b-s-c.p.i) of N. We study some properties of this new concept and link it with some there types of ideals of a near ring.

Keywords: *Smarandache Completely Prime, Near Ring.*

1. Introduction

In 1905, L.E. Drckson began the study of a near ring and later in 1930; Wieland has investigated it [1]. In 1977, G.Pilz, introduced the notion of a prime ideal of near ring [1]. In 1988, N.G. Groenewald introduced of a completely prime ideal of a near ring [5]. In 2002, W.B. Vasanth Kandasamy study samaradache near ring, (samaradache ideal, of a near ring [7]. In 2012 H.H. Abbass and M.A.Mohommed introduced the notion of a completely prime ideal with respect to an element of a near ring [3].
In this work, we introduce a Samaradache completely prime ideal with respect to an element related to a near field of near ring as we mentioned in the abstract.

2. Preliminaries

In this section, we review some basic concepts about a near ring, and some types of fields of a near rind that We need in our work.

Definition 2.1 [1]: *A left near ring is a set N together with two binary operations "+" and"." such that*
1. $(N, +)$ Is a group (not necessarily abelian),
2. $(N, .)$ Is a semi group?
3. $n_1.(n_2 + n_3) = n_1 n_2 + n_1 n_3$, for all $n_1, n_2, n_3, \in N$.

Definition 2.2 [2]: *The left near ring is called a zero symmetric if* $0.x = 0$ *, for allx* $\in N$.

Definition 2.3[7]: *Left* $(N, +, .)$ *be a near-ring. A normal subgroup* I *of* $(N, +)$ *is called a left ideal of* N *if*
1. $N.I \subseteq I$
2. for all $n, n_1 \in N$ and for all $i \in I$,
$(n + i).n - n_1.n \in I$

Remark 2.4: If N is a left near ring, then $x.0 = 0$,for all $x \in N$ (from the left distributire law). Also, we will refer that all near rings and ideals in this work are left.

Definition 2.5 [6]: *Let I be an ideal of a near ring N,then I is called a completely prime ideal of N if for all$x, y \in N$, $x.y \in I$ implies $x \in I$ or $y \in I$,denoted by $c.p.I$ of N.*
The a $\mathrm{b} - \mathrm{c.s.p.I}$ near ring N in example (1.3) is not

Definition 2.6 [3]: *Let N be a near ring, I be an ideal of N and let$b \in N$, then I is called a completely ideal with respect to the element b denoted by $(b - c.p.I)$ of$N$, if for all$x, y \in N$, $b.(x.y) \in I$ implies $x \in I$ or $y \in I$*

Definition 2.7 [7]: *A near ring N is called an integral domain if N has non_zero divisors.*

Definition 2.8 [7]: *Let $(N_1, +, .)$and $(N_2, \dot{+}, \dot{.})$ be two near rings, the mapping $f: N_1 \to N_2$ is called a near ring homomorphism if for all $m, n \in N_1$*
$\mathrm{f(m + n) = f(m)\dot{+}f(n)}$ and $\mathrm{f(m.n) = f(m)\dot{.}f(n)}$

Definition 2.9 [7]: *Anon-empty set N is said to be a near field if N is defined by two binary operations "+" and"." such that*
1. $(\mathrm{N}, +)$ Is a group
2. $(\mathrm{N} \backslash [0], .)$ Is a group
3. $\mathrm{a.(b + c) = a.b + a.c}$, for all $\mathrm{a, b, c,} \in \mathrm{N}$.

Definition 2.10 [7]: *The near ring $(N, +, .)$ is said to be a smarandache near ring denoted by (s-near ring) if it has aproper subset M such that $(M, +, .)$ is a near field.*

Definition 2.11 [7]: *Let N be s-near ring. A normal subgroup I of N is called a smarandache ideal (s-ideal) of N related M if,*
i. For all $\mathrm{x, y} \in \mathrm{M}$ and for all $\mathrm{i} \in \mathrm{I}$,$\mathrm{x(y + i) - xy \in I}$,
Where M is the near field contained inN.
ii. $\mathrm{IM} \subseteq \mathrm{I}$

Remark 2.12 [7]: Let $[\mathrm{I_i}]_{\mathrm{i \in I}}$ be a chain of s-ideals related to a near field M of a near ring N, then
$[\mathrm{I_i}]_{\mathrm{i \in I}}$ Is a s-ideals related to near field M

Remark 2.13 [6]: Let $(N_1, +, .)$and $(N_2, \dot{+}, \dot{.})$ be two s-near rings and let
$f: N_1 \to N_2$ Be an epimomorphism and N_1 has M_1 as near filed. Then $M_2 = f(M_1)$ is a near field ofN_2.

Proposition 2.14 [4]: Let $(N_1, \dot{+}, \dot{.})$and $(N_2, \dot{+}, \dot{.})$ be two s-near rings and
$f: N_1 \to N_2$ Be an epimomorphism and let I be a
S-ideals related to a near field M of a near ring N, and then $f(I)$ is s-ideals related to a near field$f(M)$.

Proposition 2.15 [4]: Let $(N_1, +, .)$ be a s-near ring has a near filed M_1, N_2 be a s-near ring, $f: N_1 \to N_2$ be an epimomorphism and let J be s-ideals related to a near field M_2 of N_2,where $f(M_1) = M_2$ of N_2, then $f^{-1}(J)$ is a s-ideals related to a near field M_1 of N_1.

Definition 2.16 [7]: *Let N is an s-near ring. The s-ideals **I** related to a near field M is called completely prime related to a near field M of N if, for all $x, y \in M$, $x.y \in I$ implies $x \in I$ or $y \in I$.denoted by $(s.c.p.I)$ of N.*

3. The main results

In this section, we define the notion of smarandache completely ideal with respect to an element b
$(b - s.c.p.I)$ And study some properties of this notion, we will discuss the image and pre image of $b - s.c.p.I$ under near rings epimomorphism and explain the relationships between it and $b - s.c.p.I$ of a near ring.

Definition 3.1: *A s-ideals related to a near field M of a s-near ring N is called a samarandache completely ideal with respect to an element b of N $(b - s.c.p.I)$, if $b.(x.y) \in I$ implies $x \in I$ or $y \in I$ for all $x, y \in M$.*

Example 3.2: The left s-near ring with addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

The s-ideal $I = [0, a]$ related to the near field $M = [0, c]$ is $b - s.c.p.I$ of N since $0.(c.c) = 0 \in I, but\ c \notin I$.

Proposition 3.3: Let I be a s-ideal related to a near field M of a s-near ring N,then I is a $\mathrm{s.c.p.I}$ of N if and only if I is $1 - \mathrm{s.c.p.I}$, where 1 is the multiplicative identity element of M.

Proof: Suppose I is a $\mathrm{s.c.p.I}$ ideal of N
And let $\mathrm{x, y} \ \in \mathrm{M}$ such that$1.(\mathrm{x.y}) \in \mathrm{I}$.
Then we have $1.(\mathrm{x.y}) = \mathrm{x.y} \in \mathrm{I}$
$\Longrightarrow \mathrm{x} \in \mathrm{I}$ or $\mathrm{y} \in \mathrm{I}$ [Since I is a $\mathrm{s.c.p.I}$ ofN].
$\Longrightarrow$ I is $1 - \mathrm{s.c.p.I}$ Of N.
Conversely,
Let $\mathrm{x, y} \ \in \mathrm{M}$ such that $\mathrm{x.y} \in \mathrm{I}$
$\Longrightarrow \mathrm{x.y} = 1.(\mathrm{x.y}) \in \mathrm{I} \Longrightarrow \mathrm{x} \in \mathrm{I}$ or $\mathrm{y} \in \mathrm{I}$ [Since I is $1.(\mathrm{x.y})$ of N].

Remark 3.4: In general an S.C.P.I related to a near field M of an s-near ring N may not be b-S.C.P.I related to M of N as in the following example

Example 3.5: Consider the s-near ring of integers mod 6 (z6, t6, .6); the s-ideal I= [0,2,4] is S.C.P.I related to the near field M = [0,3] , but it is not 2-S.C.P.I of N , since 3 $\in$ M and 2.(3.3)=0 $\in$ I but 3 $\notin$ T .

Proposition 3.6: Let I be a b-C.P.I related to a near field M of a s-near ring N. then I is a b-S.C.P.I of N.

Proof: Let x, y $\leftarrow$M, such that b. (x.y) $\in$ I
$\Rightarrow$x,y $\in$ N [since M is a proper subset of N]
$\Rightarrow$x$\in$I or y $\in$I [since I is b-S.C.P.I of N]
$\Rightarrow$I is a b-S.C.P.I of N.

Remark (3.7): The conzerse of proposition (3.6) may not be true as in the following example.

Example 3.8: Consider the s-near ring of integers mod 12 (Z12, t12, i12); s-ideal I = [0,2,4,6,8,10] if z-S.C.P.I related to the near field M= [0,4,8] , but it is not 2-C.P.I , since 3,5$\in$ Z12 and 2.(3.5) = 6$\in$I, but 3 and 5 $\notin$I .

Proposition 3.9: Let N be a s-near ring and let I be a s-ideal related to a near field M of N. then I is a b-S.C.P.I of N if and only if M is a subset of I, for all b$\in$I >

Proof: Suppose I is a b-S.C.P.I, b$\in$I and X$\in$M.
Now,
X^2 = x.x $\in$I , 0$\in$I and 0. x^2 = 0. (x.x) =0$\in$I
x$\in$I [since I is o-S.C.P.I],
$\Rightarrow$M is a subset of I
Conversely,
Let b$\in$I and x ,y $\in$M such that b.(x.y) $\in$I
$\Rightarrow$x or y $\in$ I [since M$\subseteq$I]
$\Rightarrow$I is b-S.C.P.I of N.

Proposition 3.10: Let N be a s-near integral domain . then I =[0] is b-S.C.P.I related to a near field M of N, for all n$\in$ Nl [0] .

Proof: Let b$\in$Nl [0] and x ,y $\in$M , such that b.(x.y) $\in$I
$\Rightarrow$b. (x.y) =0
$\Rightarrow$x.y =0 [since b$\neq$ 0 and N is a near integral domain]
$\Rightarrow$x=0 or y=0 $\Rightarrow$x$\in$I or y$\in$I
$\Rightarrow$x$\in$I or y$\in$I .
$\Rightarrow$I is a b-S.C.P.I of N.

Proposition 3.11: Let N be a zero symmetric s-near ring and let I=[0]. Then I is not o-S.C.P.I of N related to all near fields of N .

Proof: Suppose I is o-S.C.P.I related to a near field M of N .
Since M is a near field $\Rightarrow M \neq [0]$
$\Rightarrow \exists$ X $\in$ M , such that $x \neq 0$.
Now,
$0x^2 = 0.(x.x) = 0 \in I$
$\Rightarrow x \in I \Rightarrow x=0$ and this contradiction [since $x \neq 0$]
$\Rightarrow$ I is not 0- S.C.P.I related to M of N.

Proposition 2.12: Let N be a s-near ring and let $[Ii]_{i \in I}$ be a chain of b-S.C.P.I related to a near field M of N , for all i $\in$ I . then $V_{i \in I} I_i$ is a b-S.C.P.I related to M of N.

Proof : Since $[Ii]_{i \in I}$ is a chain a b-S.C.P.I related to M of N.
$\Rightarrow$ Ii is a s-ideal of N for all i $\in$ I.
$\Rightarrow V_{i \in I}$ Ii is a s-deal of N [By remark (2.12)]
Now,
Let x , y $\in$ M , such that b.(x.y) $\in V_{i \in I} I_i$
$\Rightarrow$ There exists b-S.C.P.I related M $I_k \in [Ii]_{i \in I}$ of N, such that b.(x.y) $\in I_k$
$\Rightarrow x \in I_k$ or $y \in I_k$ [since I_k is a b-S.C.P.I of N]
$\Rightarrow x \in V_{i \in I} I_i$ or $Y \in V_{i \in I} I_i$. $\Rightarrow V_{i \in I} I_i$ is a b-S.C.P.I of N.

Remark 3.13: In general, if $[Ii]_{i \in I}$ is a family of b-S.C.P.I related to a near field M of as near ring N, then $\bigcap_{i \in I} I_i$ and $V_{i \in I} I_i$ may not be b-S.C.P.I
Related to M of N, as in the following example

Example 3.13: Consider the s-near ring of integer's mod12. (Z12,t12, 12) , the s-ideals I=[0,6] and J=[0,4,8] are 3-S.C.P.I. related to the near field M= [0,4,8] of Z12, but the s-ideal $I \cap J = [0]$ is not 3-S.C.P.I related to M of Z12 , since 3.(3.8)=0 $\in$ I, but and 8 $\notin$ I, Also , the subset $I \cup J$= [0,4,6,8] is s-ideal of Z12 and this implies $I \cup J$ is not 3-S.C.P.I related to M of Z12 .

Theorem 3.15: *Let (N1, *, 0) and (N2, t, 0) be two s-near rings , F: N1 $\rightarrow$ N2 be an epimvor phism and let I be a b-S.C.P.I related to near field M of N, then f(I) is f(b) –S.C.P.I related to the near field f(M) of N2.*

Proof : By remark (2.13), we have f (I) is a s-ideal related to a near field f(M)
Now Let f(m1) , f(m2) $\in$ f(m) , such that
f (b) ! (f(m1) ! f(m2) $\in$ f(I)
$\Rightarrow$ f (b (m1 . m2)) $\in$ f (I)
$\Rightarrow$ f (b (m1 . m2)) $\in$ f (I)
$\Rightarrow$ either m1 $\in$ I or m2 $\in$ I or m2 [since I is b- S.C .P. I related to M of N1]
$\Rightarrow$ f (m1) $\in$ f (I) or (m2) $\in$ f(I)
$\Rightarrow$ f (I) is a f (b) - S.C .P. I related to f(M) of N2

Theorem 3.16: *Let (N1 , + , .) be as – near ring has a near field M1 , (N2) be S- near ring , f: N1 $\rightarrow$ N2 be an epimomorphism , and Let J be a b- S.C .P.I related to the near field f(M) of N2 , then f^{1} (I) is a - S.C .P.I related to a near field M of N1 , where b – f (a).*

Prof: By proposition (2.15) , we have f^{-1} (J) is a S – ideal related to M of N1 . Now, Let x,y $\in$ M , such that a. (x.y) $\in$ $^{f-1}$ (J)
$\Rightarrow$ f(x) , f(y) $\in$ f(M) and f(a ! (x y) $\in$ J
$\Rightarrow$ f(x) , f(y) $\in$ f(M) and f(a) ! f(x), f(y)) $\in$ J
$\Rightarrow$ either f(x) $\in$ J or f(y) $\in$ J [since J is b- S.C .P. I related to f(M) of N2]
$\Rightarrow$ either x $\in$ f^{-1} (J) or y f^{-1} (J) or y$\in$ f^{-1} (J)
$\Rightarrow$ f^{-1} (J) is a b- S.C .P. I related to f(M) of N2

Corollary 3.17: Let (N1 ,+,0) be a S- near ring has a near field M, (N2 , +' , .") be a S- near ring , f : N1 , $\rightarrow$ N2 be an e pimomorphism , and if [o^1] be a b- S.C .P. I related to the near field f(M) of N2 The ker(f) is b- S.C .P. I related to a near field M of N1, where

Ker f = [x ∈ N1 : f(x) = 0] and b=f(a)

Proof: Since f^{-1} ([0^{1})] l = ker (f) , then where Rer (f) is a - S.C .P. I related to M of N1
[By theorem (3-16)]

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